

CBSE Class 11 Maths Notes Chapter 5: CBSE Class 11 Maths Notes Chapter 5 our knowledgeable instructors prepare complex numbers and quadratic equations.

This CBSE Class 11 Maths Notes Chapter 5 aids students in finding fast, precise, and effective solutions to problems. Additionally, we offer step-by-step solutions for every NCERT problem, guaranteeing that students comprehend them and ace their tests.

CBSE Class 11 Maths Notes Chapter 5 Overview

Several important mathematical theorems and formulas are included in this class 11 revision note for Maths chapter 5 on complex numbers and quadratic equations. To cover all of these topics, the NCERT textbook includes a ton of practice questions that will make it easier for students to understand greater concepts in the future.

We offer answers to all of these issues by outlining each step with appropriate justifications. This specific set of NCERT chapter 5 class 11 maths study notes is intended to assist students who want to pass their maths examinations even at the last minute.

CBSE Class 11 Maths Notes Chapter 5

What is a Complex Number?

A complex number is represented by the notation $a + ib$, where "b" is an imaginary number and "a" is a real number. The complex number that guarantees $i^2 = -1$ is made up of the symbol "i." One-dimensional number lines are added to represent complex numbers. A complex number in a complex plane is typically represented as the point (a, b), and can be written as $a + bi$.

We must understand that a complex number, like i , $-5i$, etc., that has no real part is referred to as being wholly imaginary. Furthermore, a real number is a complex number that has no imaginary component at all.

$\text{Re}(Z) = x$ and $\text{Im}(Z) = y$ are the real and imaginary parts of a complex number, respectively, where x and y are real integers. A complex number is defined as several type $x + iy$.

Properties of Complex Numbers

Below is a list of a complex number's characteristics:

If $a + ib = 0$, then $a = 0$, and $b = 0$ are two real numbers.

If $a + ib = c + id$ holds for the real numbers a , b , and c , then $a = c$ and $b = d$.

z_1, z_2 , and z_3 are a set of three complex numbers that satisfy the distributive, associative, and commutative laws.

Complex numbers are conjugate to each other if the product and sum of any two given complex numbers are both real.

Regarding the two complex numbers, z_1 and z_2 , we have $|z_1 + z_2| \leq |z_1| + |z_2|$.

Any two conjugate complex numbers always add up to a real number.

Any two conjugate complex numbers always have a real product.

When a given number is in the form of $a + ib$, where $a, b \in \mathbb{R}$ and $i = \sqrt{-1}$ it is called a complex number and such number is denoted by ' z '.

$$z = a + ib$$

Where,

a = real part of complex number and,

b = imaginary part of complex number.

1.1 Conjugate of a Complex Number

Consider a complex number $z = a + ib$,

Then its conjugate is written as ' \bar{z} '.

Whose value is defined as $\bar{z} = a - ib$.

Algebra of Complex Numbers

Let $z_1 = a + ib$ and $z_2 = c + id$ be two complex numbers where $a, b, c, d \in \mathbb{R}$ and $i = \sqrt{-1}$.

1. Addition :

$$\begin{aligned} z_1 + z_2 &= (a + bi) + (c + di) \\ &= (a + c) + (b + d)i \end{aligned}$$

2. Subtraction :

$$\begin{aligned} z_1 - z_2 &= (a + bi) - (c + di) \\ &= (a - c) + (b - d)i \end{aligned}$$

3. Multiplication :

$$\begin{aligned} z_1 \cdot z_2 &= (a + bi)(c + di) \\ &= a(c + di) + bi(c + di) \\ &= ac + adi + bci + bdi^2 \\ &= ac - bd + (ad + bc)i \quad (\because i^2 = -1) \end{aligned}$$

Note...

$$\begin{aligned} 1. \quad a + ib &= c + id \\ \Leftrightarrow a &= c \& b = d \end{aligned}$$

$$2. \quad i^{4k+r} = \begin{cases} 1; & r = 0 \\ i; & r = 1 \\ -1; & r = 2 \\ -i; & r = 3 \end{cases}$$

$$3. \quad \sqrt{b}\sqrt{a} = \sqrt{ba} \text{ is only possible if atleast one of either } a \text{ or } b \text{ is non-negative.}$$

Argand Plane

Any complex number $z = a + ib$ can be represented by a unique point $P(a, b)$ in the argand plane.

Modulus and Argument of Complex Number

(i) Distance of z from the origin is referred to as the modulus of complex number z .

It is represented by $r = |z| = \sqrt{a^2 + b^2}$

(ii) Here, θ i.e. The angle made by ray OP with positive direction of the real axis is called the argument of z .

Note.

$z_1 > z_2$ or $z_1 < z_2$ has no meaning but $|z_1| > |z_2|$ or $|z_1| < |z_2|$ holds meaning.

Principal Argument

The argument ' θ ' of complex numbers $z = a + ib$ is called the principal argument of z if $-\pi < \theta \leq \pi$.

Consider $\tan \alpha = \left| \frac{b}{a} \right|$, and θ be the $\arg(z)$.

Polar Form

$$a = r \cos \theta \quad \& b = r \sin \theta$$

where $r = |z|$ and $\theta = \arg(z)$

$$\begin{aligned} \therefore z &= a + ib \\ &= r(\cos \theta + i \sin \theta) \end{aligned}$$

Note...

A complex number z can also be represented as $z = re^{i\theta}$, it is known as Euler's form.

Where,

$$r = |Z| \& \theta = \arg(Z)$$

Some Important Properties

1. $\overline{(\bar{z})} = z$
2. $z + \bar{z} = 2\operatorname{Re}(z)$
3. $z - \bar{z} = 2i\operatorname{Im}(z)$
4. $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$
5. $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$
6. $|z| = 0 \Rightarrow z = 0$
7. $z \cdot \bar{z} = |z|^2$
8. $|z_1 z_2| = |z_1| |z_2|; \left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$
9. $|\bar{z}| = |z| = |-z|$
10. $|z_1 \pm z_2|^2 = |z_1|^2 + |z_2|^2 \pm 2\operatorname{Re}(z_1 \bar{z}_2)$
11. $|z_1 + z_2| \leq |z_1| + |z_2|$ (Triangle Inequality)
12. $|z_1 - z_2| \geq ||z_1| - |z_2||$
13. $|az_1 - bz_2|^2 + |bz_1 + az_2|^2 = (a^2 + b^2) (|z_1|^2 + |z_2|^2)$
14. $\operatorname{amp}(z_1 \cdot z_2) = \operatorname{amp} z_1 + \operatorname{amp} z_2 + 2k\pi; \quad k \in \mathbb{I}$
15. $\operatorname{amp}\left(\frac{y_0}{y_1}\right) = \operatorname{amp} z_1 - \operatorname{amp} z_2 + 2k\pi; \quad k \in \mathbb{I}$
16. $\operatorname{amp}(z^n) = n \operatorname{amp}(z) + 2k\pi; \quad k \in \mathbb{I}$

De-Moivre's Theorem

Statement: $\cos n\theta + i \sin n\theta$ is the value or one of the values of $(\cos \theta + i \sin \theta)^n$ according as if 'n' is integer or a rational number. The theorem is very useful in determining the roots of any complex quantity.

Cube Root of Unity

Roots of the equation $x^3 = 1$ are called cube roots of unity.

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$$x^3 - 1 = 0$$

$$(x - 1)(x^2 + x + 1) = 0$$

$$x = 1 \quad \text{or} \quad x^2 + x + 1 = 0$$

$$\text{i.e } x = \underbrace{\frac{-1 + \sqrt{3}i}{2}}_w \quad \text{or } x = \underbrace{\frac{-1 - \sqrt{3}i}{2}}_{w^2}$$

(i) The cube roots of unity are $1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}$.

(ii) $\omega^3 = 1$

(iii) If ω is one of the imaginary cube roots of unity then $1 + \omega + \omega^2 = 0$.

(iv) In general $1 + \omega^r + \omega^{2r} = 0$; where $r \in \mathbb{I}$ but is not the multiple of 3.

(v) In polar form the cube roots of unity are:

$$\cos 0 + i \sin 0; \cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}, \cos \frac{4\pi}{3} + i \sin \frac{4\pi}{3}$$

LOCI in Complex Plane

- (i) $|z - z_0| = a$ represents the circumference of a circle, centred at z_0 , radius a .
- (ii) $|z - z_0| < a$ represents the interior of the circle.
- (iii) $|z - z_0| > a$ represents the exterior of this circle.
- (iv) $|z - z_1| = |z - z_2|$ represents \perp bisector of segment with endpoints z_1 and z_2 .
- (v) $\left| \frac{-z_1}{-z_2} \right| = k$ represents: $\left\{ \begin{array}{l} \text{circle, } k \neq 1 \\ \perp \text{ bisector, } k = 1 \end{array} \right\}$
- (vi) $\arg(z) = \theta$ is a ray starting from the origin (excluded) inclined at an $\angle \theta$ with a real axis.
- (vii) Circle described on line segment joining z_1 and z_2 as diameter is:
- $$(z - z_1)(\bar{z} - \bar{z}_2) + (z - z_2)(\bar{z} - \bar{z}_1) = 0$$
- (viii) If z_1, z_2, z_3 are the vertices of an equilateral triangle where z_0 is its circumcentre then
- (a) $\frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} + \frac{1}{z_1 - z_2} = 0$
- (b) $z_0^1 + z_1^1 + z_2^1 - z_1 z_2 - z_2 z_3 - z_3 z_1 = 0$
- (c) $z_0^1 + z_1^1 + z_2^1 = 3z_0^1$

Important Identities

$$(i) x^2 + x + 1 = (x - \omega)(x - \omega^2)$$

$$(ii) x^2 - x + 1 = (x + \omega)(x + \omega^2)$$

$$(iii) x^2 + xy + y^2 = (x - y\omega)(x - y\omega^2)$$

$$(iv) x^2 - xy + y^2 = (x + y\omega)(x + y\omega^2)$$

$$(v) x^2 + y^2 = (x + iy)(x - iy)$$

$$(vi) x^3 + y^3 = (x + y)(x + y\omega)(x + y\omega^2)$$

$$(vii) x^3 - y^3 = (x - y)(x - y\omega)(x - y\omega^2)$$

$$(viii) x^2 + y^2 + z^2 - xy - yz - zx = (x + y\omega + z\omega^2)(x + y\omega^2 + z\omega)$$

$$\text{or } (x\omega + y\omega^2 + z)(x\omega^2 + y\omega + z)$$

$$\text{or } (x\omega + y + z\omega^2)(x\omega^2 + y + z\omega)$$

$$(ix) x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + y\omega + z\omega^2)(x + y\omega^2 + z\omega)$$

Roots of Quadratic Equation

(a) The solution of the quadratic equation,

$$ax^2 + bx + c = 0 \text{ is given by } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The expression $D = b^2 - 4ac$ is called the discriminant of the quadratic equation.

(b) If α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$, then

$$(i) \alpha + \beta = \frac{-b}{a}$$

$$(ii) \alpha\beta = \frac{c}{a}$$

$$(iii) |\alpha - \beta| = \frac{\sqrt{D}}{|a|}$$

(c) A quadratic equation whose roots are α and β is $(x - \alpha)(x - \beta) = 0$ i.e.,

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0 \quad \text{i.e.,}$$

$$x^2 - (\text{sum of roots})x + \text{product of roots} = 0$$

Particular Cases

(a) Quadratic Equation: If α, β be the roots the quadratic equation, then the equation is :

$$x^2 - S_1x + S_2 = 0 \quad \text{i.e.} \quad x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

(b) Cubic Equation: If α, β, γ be the roots the cubic equation, then the equation is :

$$x^3 - S_1x^2 + S_2x - S_3 = 0 \quad \text{i.e.}$$

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma = 0$$

(i) If α is a root of equation $f(x) = 0$, the polynomial $f(x)$ is exactly divisible by $(x - \alpha)$. In other words, $(x - \alpha)$ is a factor of $f(x)$ and conversely.

(ii) Every equation of n th degree ($n \geq 1$) has exactly n roots & if the equation has more than n roots, it is an identity.

Only One Common Root

Let α be the common root of $ax^2 + bx + c = 0$ and $a'x^2 + b'x + c' = 0$, such that $a, a' \neq 0$ and $b' \neq a'b$. Then, the condition for one common root is:

$$(ca' - c'a)^2 = (ab' - a'b)(bc' - b'c)$$

(b) Two common roots

Let α, β be the two common roots of

$$ax^2 + bx + c = 0 \text{ and } a'x^2 + b'x + c' = 0 \text{ such that } a, a' \neq 0.$$

Then, the condition for two common roots is: $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$

Transformation of Equations

- (i) To obtain an equation whose roots are reciprocals of the roots of a given equation, it is obtained by replacing x by $1/x$ in the given equation.
- (ii) Transformation of an equation to another equation whose roots are negative of the roots of a given equation-replace x by $-x$.
- (iii) Transformation of an equation to another equation whose roots are square of the roots of a given equation-replace x by \sqrt{x} .
- (iv) Transformation of an equation to another equation whose roots are cubes of the roots of a given equation-replace x by $x^{1/3}$.

Benefits of CBSE Class 11 Maths Notes Chapter 5

We have produced all of the review notes for class 11 Maths chapter 5 in compliance with the most recent CBSE syllabus to ensure that any changes made by the CBSE board are understood. Teachers with extensive expertise and a thorough understanding of the material have created the Chapter 5 class 11 mathematics revision notes for complex numbers and quadratic equations.

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