RD Sharma Solutions Class 10 Maths Chapter 6 Exercise 6.1: Chapter 6 of RD Sharma's Class 10 Maths book covers Trigonometric Identities and provides foundational knowledge on trigonometric functions and their identities.

These identities are essential for simplifying complex trigonometric expressions and solving equations. The exercise includes questions that require students to apply these identities to verify equalities or transform expressions, enhancing problem-solving skills in trigonometry. This section forms the basis for more advanced trigonometric concepts in later chapters.

RD Sharma Solutions Class 10 Maths Chapter 6 Exercise 6.1 Overview

Chapter 6, Exercise 6.1 of RD Sharma's Class 10 Maths book covers fundamental trigonometric identities, which are crucial for simplifying and solving trigonometric equations. This exercise focuses on the primary identities. Understanding these identities is essential as they serve as building blocks for more complex trigonometric problems in higher mathematics.

Mastery of these identities enables students to tackle problems in geometry, physics, and calculus, making them foundational for both academic exams and practical applications in various scientific fields.

RD Sharma Solutions Class 10 Maths Chapter 6 Exercise 6.1 Trigonometric Identities

Below is the RD Sharma Solutions Class 10 Maths Chapter 6 Exercise 6.1 Trigonometric Identities -

Prove the following trigonometric identities:

1. $(1 – \cos^2 A) \text{cosec}^2 A = 1$

Solution:

Taking the L.H.S,

$(1 – \cos^2 A) \text{cosec}^2 A$

$= (\sin^2 A) \text{cosec}^2 A$ [$\because \sin^2 A + \cos^2 A = 1 \Rightarrow 1 – \sin^2 A = \cos^2 A$]

$= 1^2$

$= 1 = $ R.H.S

– Hence Proved

2. $(1 + \cot^2 A) \sin^2 A = 1$

Solution:

By using the identity,

$\text{cosec}^2 A – \cot^2 A = 1 \Rightarrow \text{cosec}^2 A = \cot^2 A + 1$

Taking,

L.H.S $= (1 + \cot^2 A) \sin^2 A$

$= \text{cosec}^2 A \sin^2 A$

$= (\text{cosec } A \sin A)^2$

$= ((1/\sin A) \times \sin A)^2$

$= (1)^2$

$= 1$

= R.H.S

– Hence Proved

3. $\tan^2 \theta \cos^2 \theta = 1 − \cos^2 \theta$

Solution:

We know that,

$\sin^2 \theta + \cos^2 \theta = 1$

Taking,

L.H.S $= \tan^2 \theta \cos^2 \theta$

$= (\tan \theta \times \cos \theta)^2$

$= (\sin \theta)^2$

$= \sin^2 \theta$

$= 1 – \cos^2 \theta$

= R.H.S

– Hence Proved

4. cosec θ √(1 – cos² θ) = 1

Solution:

Using identity,

$\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \sin^2 \theta = 1 – \cos^2 \theta$

Taking L.H.S,

L.H.S = $\text{cosec}\, \theta \sqrt{(1 – \cos^2 \theta)}$

$= \text{cosec}\, \theta \sqrt{(\sin^2 \theta)}$

$= \text{cosec}\, \theta \times \sin \theta$

= 1

= R.H.S

– Hence Proved

5. $(\sec^2 \theta - 1)(\text{cosec}^2 \theta - 1) = 1$

Solution:

Using identities,

$(\sec^2 \theta - \tan^2 \theta) = 1$ and $(\text{cosec}^2 \theta - \cot^2 \theta) = 1$

We have,

L.H.S = $(\sec^2 \theta – 1)(\text{cosec}^2\theta – 1)$

$= \tan^2\theta \times \cot^2\theta$

$= (\tan \theta \times \cot \theta)^2$

$= (\tan \theta \times 1/\tan \theta)^2$

$= 1^2$

= 1

= R.H.S

– Hence Proved

6. tan θ + 1/ tan θ = sec θ cosec θ

Solution:

We have,

L.H.S = tan θ + 1/ tan θ

= $(\tan^2 \theta + 1)/ \tan \theta$

= $\sec^2 \theta / \tan \theta$ [∵ $\sec^2 \theta - \tan^2 \theta = 1$]

= $(1/\cos^2 \theta)$ x 1/ (sin θ/cos θ) [∵ tan θ = sin θ / cos θ]

= cos θ/ $(\sin \theta \times \cos^2 \theta)$

= 1/ cos θ x 1/ sin θ

= sec θ x cosec θ

= sec θ cosec θ

= R.H.S

– Hence Proved

7. cos θ/ (1 – sin θ) = (1 + sin θ)/ cos θ

Solution:

We know that,

$\sin^2 \theta + \cos^2 \theta = 1$

So, by multiplying both the numerator and the denominator by (1+ sin θ), we get

$$\frac{\cos\theta}{1-\sin\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{\cos\theta}{(1+\sin\theta)(1-\sin^2\theta)}$$

$$= \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$$

$$= \frac{(1+\sin\theta)}{\cos\theta}$$

L.H.S =

= R.H.S

– Hence Proved

8. **cos θ/ (1 + sin θ) = (1 – sin θ)/ cos θ**

Solution:

We know that,

$\sin^2\theta + \cos^2\theta = 1$

So, by multiplying both the numerator and the denominator by (1- sin θ), we get

$$\frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$$= \frac{\cos\theta(1-\sin\theta)}{(1-\sin^2\theta)}$$

$$= \frac{\cos\theta(1-\sin\theta)}{(\cos^2\theta)}$$

$$= \frac{(1-\sin\theta)}{\cos\theta}$$

$$= \frac{(1-\sin\theta)}{\cos\theta}$$

L.H.S =

= R.H.S

– Hence Proved

9. $\cos^2\theta + 1/(1 + \cot^2\theta) = 1$

Solution:

We already know that,

$\text{cosec}^2\theta - \cot^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$

Taking L.H.S,

$$\text{L.H.S} = \cos^2 A + \frac{1}{1+\cot^2 A}$$

$$= \cos^2 A + \frac{1}{\text{cosec}^2 A}$$

$$= \cos^2 A + \left(\frac{1}{\text{cosec } A}\right)^2$$

$= \cos^2 A + \sin^2 A$

= 1

= R.H.S

– Hence Proved

10. $\sin^2 A + 1/(1 + \tan^2 A) = 1$

Solution:

We already know that,

$\sec^2 \theta - \tan^2 \theta = 1$ and $\sin^2 \theta + \cos^2 \theta = 1$

Taking L.H.S,

$$\text{L.H.S} = \sin A^2 + \frac{1}{1 + \tan^2 A}$$

$$= \sin A^2 + \frac{1}{\sec^2 A}$$

$$= \sin A^2 + \left(\frac{1}{\sec A}\right)^2$$

$= \sin^2 A + \cos^2 A$

= 1

= R.H.S

– Hence Proved

11.

$$\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}} = \operatorname{cosec}\theta - \cot\theta$$

Solution:

We know that, $\sin^2 \theta + \cos^2 \theta = 1$

Taking the L.H.S,

$$\text{L.H.S} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}}$$

$$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$$

= cosec θ – cot θ

= R.H.S

– Hence Proved

12. 1 – cos θ/ sin θ = sin θ/ 1 + cos θ

Solution:

We know that,

$\sin^2\theta + \cos^2\theta = 1$

So, by multiplying both the numerator and the denominator by (1+ cos θ), we get

$$\text{L.H.S} = \frac{1-\cos^2\theta}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{(\sin^2\theta)}{(1+\cos\theta)(\sin\theta)}$$

$$= \frac{(\sin\theta)}{(1+\cos\theta)}$$

= R.H.S

– Hence Proved

13. sin θ/ (1 – cos θ) = cosec θ + cot θ

Solution:

Taking L.H.S,

$$\text{L.H.S} = \frac{\sin\theta}{1-\cos\theta}$$

On multiplying by its conjugates, we have

$$= \frac{\sin\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta}$$

$$= \frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}$$

Since, $(1 - \cos^2\theta) = \sin^2\theta$

$$= \frac{\sin\theta + (\sin\theta \times \cos\theta)}{\sin^2\theta}$$

$$= \frac{\sin\theta}{\sin^2\theta} + \frac{\sin\theta \times \cos\theta}{\sin^2\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

= cosec θ + cot θ

= R.H.S

– Hence Proved

14. (1 – sin θ) / (1 + sin θ) = (sec θ – tan θ)^2

Solution:

Taking the L.H.S,

$$\text{L. H. S} = \frac{1 - \sin\theta}{1 + \sin\theta}$$

On multiplying by its conjugate, we have

$$= \frac{1 - \sin\theta}{1 + \sin\theta} \times \frac{1 - \sin\theta}{1 - \sin\theta}$$

Since, $1 - \sin^2\theta = \cos^2\theta$

$$= \frac{(1 - \sin\theta)^2}{\cos^2\theta}$$

$$= \left(\frac{1 - \sin\theta}{\cos\theta}\right)^2$$

$$= \left(\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}\right)^2$$

$= (\sec\theta – \tan\theta)^2$

= R.H.S

– Hence Proved

15. $$\frac{(1 + \cot^2\theta)\tan\theta}{\sec^2\theta} = \cot\theta$$

Solution:

Taking L.H.S,

$$\text{L. H. S} = \frac{(1 + \cot^2\theta)\tan\theta}{\sec^2\theta}$$

Here, $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$$= \frac{\operatorname{cosec}^2\theta \times \tan\theta}{\sec^2\theta}$$

$$= \frac{1}{\sin^2\theta} \times \frac{\cos^2\theta}{1} \times \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta}$$

$= \cot\theta$

= R.H.S

– Hence Proved

16. $\tan^2\theta - \sin^2\theta = \tan^2\theta \sin^2\theta$

Solution:

Taking L.H.S,

L.H.S = $\tan^2\theta - \sin^2\theta$

$$\text{Since, } \tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$$

$$= \sin^2\theta\left[\frac{1}{\cos^2\theta} - 1\right]$$

$$= \sin^2\theta\left[\frac{1 - \cos^2\theta}{\cos^2\theta}\right]$$

$$= \frac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta$$

$= \tan^2\theta \sin^2\theta$

= R.H.S

– Hence Proved

17. (cosec θ + sin θ)(cosec θ – sin θ) = cot²θ + cos²θ

Solution:

Taking L.H.S = (cosec θ + sin θ)(cosec θ – sin θ)

On multiplying, we get,

$= \operatorname{cosec}^2\theta - \sin^2\theta$

$= (1 + \cot^2\theta) - (1 - \cos^2\theta)$ [Using $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$]

$= 1 + \cot^2\theta - 1 + \cos^2\theta$

$= \cot^2\theta + \cos^2\theta$

= R.H.S

– Hence Proved

18. (sec θ + cos θ) (sec θ – cos θ) = tan²θ + sin²θ

Solution:

Taking L.H.S = (sec θ + cos θ)(sec θ – cos θ)

On multiplying, we get,

$= \sec^2\theta - \sin^2\theta$

$= (1 + \tan^2\theta) - (1 - \sin^2\theta)$ [Using $\sec^2\theta - \tan^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1$]

$= 1 + \tan^2\theta - 1 + \sin^2\theta$

$= \tan^2\theta + \sin^2\theta$

= R.H.S

– Hence Proved

19. sec A(1- sin A) (sec A + tan A) = 1

Solution:

Taking L.H.S = sec A(1 – sin A)(sec A + tan A)

Substituting sec A = 1/cos A and tan A =sin A/cos A in the above, we have,

L.H.S = 1/cos A (1 – sin A)(1/cos A + sin A/cos A)

= 1 – $\sin^2$ A / $\cos^2$ A [After taking L.C.M]

= $\cos^2$ A / $\cos^2$ A [∵ 1 – $\sin^2$ A = $\cos^2$ A]

= 1

= R.H.S

– Hence Proved

20. (cosec A – sin A)(sec A – cos A)(tan A + cot A) = 1

Solution:

Taking L.H.S = (cosec A – sin A)(sec A – cos A)(tan A + cot A)

Putting, $\operatorname{cosec} A = \dfrac{1}{\sin A}, \sec A = \dfrac{1}{\cos A}, \tan A = \dfrac{\sin A}{\cos A}, \cot A = \dfrac{\cos A}{\sin A}$

Subsituting the above in the L.H.S, we get

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$

$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\left(\frac{\sin^2 A + \cos^2 A}{\sin A \ \cos A}\right)$$

= ($\cos^2$ A/ sin A) ($\sin^2$ A/ cos A) (1/ sin A cos A) [∵ $\sin^2\theta + \cos^2\theta = 1$]

= (sin A cos A) (1/ cos A sin A)

= 1

= R.H.S

– Hence Proved

21. $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta) = 1$

Solution:

Taking L.H.S = $(1 + \tan^2\theta)(1 - \sin \theta)(1 + \sin \theta)$

And, we know $\sin^2 \theta + \cos^2 \theta = 1$ and $\sec^2 \theta - \tan^2 \theta = 1$

So,

L.H.S = $(1 + \tan^2 \theta)(1 - \sin \theta)(1 + \sin \theta)$

$= (1 + \tan^2 \theta)\{(1 - \sin \theta)(1 + \sin \theta)\}$

$= (1 + \tan^2 \theta)(1 - \sin^2 \theta)$

$= \sec^2 \theta\ (\cos^2 \theta)$

$= (1/ \cos^2 \theta) \times \cos^2 \theta$

= 1

= R.H.S

– Hence Proved

22. $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A = 1$

Solution:

We know that,

$\cot^2 A = \cos^2 A/ \sin^2 A$ and $\tan^2 A = \sin^2 A/\cos^2 A$

Substituting the above in L.H.S, we get

L.H.S = $\sin^2 A \cot^2 A + \cos^2 A \tan^2 A$

$= \{\sin^2 A (\cos^2 A/ \sin^2 A)\} + \{\cos^2 A (\sin^2 A/\cos^2 A)\}$

$= \cos^2 A + \sin^2 A$

$= 1$ [$\because \sin^2 \theta + \cos^2 \theta = 1$]

= R.H.S

– Hence Proved

(i) $\cot\theta - \tan\theta = \frac{2\cos 2\theta - 1}{\sin\theta * \cos\theta}$

(ii) $\tan\theta - \cot\theta = \left(\frac{2\sin^2\theta - 1}{\sin\theta * \cos\theta}\right)$

23.

Solution:

(i) Taking the L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

L.H.S = cot θ – tan θ

$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta \text{ x} \cos\theta}$$

$$= \frac{\cos^2\theta - (1 - \cos^2\theta)}{\sin\theta \text{ x} \cos\theta}$$

$$= \frac{\cos^2\theta - 1 - \cos^2\theta}{\sin\theta \text{ x} \cos\theta}$$

$$= \left(\frac{2\cos^2\theta - 1}{\sin\theta \text{ x} \cos\theta}\right)$$

= R.H.S

– Hence Proved

(ii) Taking the L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

L.H.S = tan θ – cot θ

$$= \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta - \cos^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta - (1 - \sin^2\theta)}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta - (1 + \sin^2\theta)}{\sin\theta\cos\theta}$$

$$= \left(\frac{2\sin^2\theta - 1}{\sin\theta \quad \cos\theta}\right)$$

= R.H.S

– Hence Proved

24. (cos^2 θ/ sin θ) – cosec θ + sin θ = 0

Solution:

Taking L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$\text{L.H.S} = \frac{\cos^2\theta}{\sin\theta} - \text{cosec}\,\theta + \sin\theta$$

$$= \left(\frac{\cos^2\theta}{\sin\theta} - \text{cosec}\,\theta\right) + \sin\theta$$

$$= \left(\frac{\cos^2\theta}{\sin\theta} - \frac{1}{\sin\theta}\right) + \sin\theta$$

$$= \left(\frac{\cos^2\theta - 1}{\sin\theta}\right) + \sin\theta$$

$$= \left(\frac{-\sin^2\theta}{\sin\theta}\right) + \sin\theta$$

= – sin θ + sin θ

= 0

= R.H.S

- Hence proved

$$\frac{1}{1+\sin A}+\frac{1}{1-\sin A}=2\sec^2 A$$
25.

Solution:

Taking L.H.S,

$$\text{LHS} = \frac{1}{1+\sin A}+\frac{1}{1-\sin A}$$

$$= \frac{(1-\sin A)+(1+\sin A)}{(1+\sin A)(1-\sin A)}$$

$$= \frac{1-\sin A+1+\sin A}{1-\sin^2 A} \quad \because (1+\sin A)(1-\sin A) = 1-\sin^2 A$$

$$= \frac{2}{1-\sin^2 A}$$

$$= \frac{2}{\cos^2 A} \quad [\because 1-\sin^2 A = \cos A]$$

= 2 $\sec^2$ A

= R.H.S

- Hence proved

26. $$\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}=2\sec\theta$$

Solution:

Taking the LHS and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$\text{LHS} = \frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}$$

$$= \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1+\sin^2\theta + 2\sin\theta + \cos^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$

= 2/ cos θ

= 2 sec θ

= R.H.S

- Hence proved

$$\frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta} = \frac{1+\sin^2\theta}{1-\sin^2\theta}$$

27.

Solution:

Taking the LHS and using $\sin^2\theta + \cos^2\theta = 1$, we have

$$\text{L.H.S} = \frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{2\cos^2\theta}$$

$$= \frac{(1+2\sin\theta+\sin^2\theta) + (1-2\sin\theta+\sin^2\theta)}{2\cos^2\theta}$$

$$= \frac{1+2\sin\theta+\sin^2\theta+1-2\sin\theta+\sin^2\theta}{2\cos^2\theta}$$

$$= \frac{2+2\sin^2\theta}{2\cos^2\theta}$$

$$= \frac{2(1+\sin^2\theta)}{2(1-\sin^2\theta)}$$

$$= \frac{(1+\sin^2\theta)}{(1-\sin^2\theta)}$$

= R.H.S

- Hence proved

$$\frac{1+\tan^2\theta}{1+\cot^2\theta} = \tan^2\theta$$

28.

Solution:

Taking L.H.S,

$$\frac{1+\tan^2\theta}{1+\cot^2\theta}$$

Using $\sec^2\theta - \tan^2\theta = 1$ and $\mathrm{cosec}^2\theta - \cot^2\theta = 1$

$$= \frac{1 + \tan^2\theta}{1 + \cot^2\theta} = \frac{\sec^2\theta}{\operatorname{cosec}^2\theta}$$

$$= \frac{1}{\cos^2\theta \quad 1}\sin^2\theta = \tan^2\theta$$

= R.H.S

$$\frac{1 + \sec\theta}{\sec\theta} = \frac{\sin^2\theta}{1 - \cos\theta}$$

29.

Solution:

Taking L.H.S and using $\sin^2\theta + \cos^2\theta = 1$, we have

Multiplying by (1 - cos θ) to numerator and denominator

$$\text{LHS} = \frac{1 + \sec\theta}{\sec\theta}$$

$$= \frac{1 + \frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$$

$$= \frac{\cos\theta + 1}{\cos\theta} \cdot \cos\theta$$

$$= 1 + \cos\theta$$

$$= \frac{(1 - \cos\theta)(1 + \cos\theta)}{1 - \cos\theta}$$

$$= \frac{1 - \cos^2\theta}{1 - \cos\theta}$$

$$= \frac{\sin^2\theta}{1 - \cos\theta}$$

= R.H.S

- Hence proved

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\tan\theta+\cot\theta$$

30.

Solution:

Taking LHS, we have

$$\text{LHS}=\frac{\tan\theta}{1-\frac{1}{\tan\theta}}+\frac{\cot\theta}{1-\tan\theta}$$

$$=\frac{\tan^2\theta}{\tan\theta-1}+\frac{\cot\theta}{1-\tan\theta}$$

$$=\frac{1}{1-\tan\theta}\left[\frac{1}{\tan\theta}-\tan^2\theta\right]$$

$$=\frac{1}{1-\tan\theta}\left[\frac{1-\tan^3\theta}{\tan\theta}\right]$$

$$=\frac{1}{1-\tan\theta}\ \frac{(1-\tan\theta)(1+\tan\theta+\tan^2\theta)}{\tan\theta}\ \left[\text{Since, } a^3-b^3=(a-b)(a^2+ab+b^2)\right]$$

$$=\frac{1+\tan\theta+\tan^2\theta}{\tan\theta}$$

$$=\frac{1}{\tan\theta}+\frac{\tan\theta}{\tan\theta}+\frac{\tan^2\theta}{\tan\theta}$$

= 1 + tan θ + cot θ

= R.H.S

- Hence proved

31. $\sec^6\theta = \tan^6\theta + 3\tan^2\theta\sec^2\theta + 1$

Solution:

From trig. Identities we have,

$\sec^2\theta - \tan^2\theta = 1$

On cubing both sides,

$(\sec^2\theta - \tan^2\theta)^3 = 1$

$\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta(\sec^2\theta - \tan^2\theta) = 1$

[Since, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]

$\sec^6\theta - \tan^6\theta - 3\sec^2\theta\tan^2\theta = 1$

$\Rightarrow \sec^6\theta = \tan^6\theta + 3\sec^2\theta\tan^2\theta + 1$

Hence, L.H.S = R.H.S

- Hence proved

32. $\csc^6\theta = \cot^6\theta + 3\cot^2\theta\csc^2\theta + 1$

Solution:

From trig. Identities we have,

$\csc^2\theta - \cot^2\theta = 1$

On cubing both sides,

$(\csc^2\theta - \cot^2\theta)^3 = 1$

$\csc^6\theta - \cot^6\theta - 3\csc^2\theta\cot^2\theta(\csc^2\theta - \cot^2\theta) = 1$

[Since, $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$]

$\csc^6\theta - \cot^6\theta - 3\csc^2\theta\cot^2\theta = 1$

$\Rightarrow \csc^6\theta = \cot^6\theta + 3\csc^2\theta\cot^2\theta + 1$

Hence, L.H.S = R.H.S

- Hence proved

$$\frac{(1 + \tan^2\theta)\cot\theta}{\csc^2\theta} = \tan\theta$$ **33.**

Solution:

Taking L.H.S and using $\sec^2\theta - \tan^2\theta = 1 \Rightarrow 1 + \tan^2\theta = \sec^2\theta$

$$\text{LHS} = \frac{\sec^2\theta \cdot \cot\theta}{\text{cosec}^2\theta}$$
$$= \frac{1 \cdot \sin^2\theta}{\cos^2\theta} \cdot \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin\theta}{\cos\theta} = \tan\theta$$

= R.H.S

- Hence proved

$$\frac{1 + \cos A}{\sin^2 A} = \frac{1}{1 - \cos A}$$ **34.**

Solution:

Taking L.H.S and using the identity $\sin^2A + \cos^2A = 1$, we get

$\sin^2A = 1 - \cos^2A$

$\Rightarrow \sin^2A = (1 - \cos A)(1 + \cos A)$

$$\text{LHS} = \frac{1 + \cos A}{(1 - \cos A)(1 + \cos A)}$$
$$= \frac{1}{(1 - \cos A)}$$

- Hence proved

$$\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1 + \sin A)^2}$$ **35.**

Solution:

We have,

$$\text{LHS} = \frac{\sec A - \tan A}{\sec A + \tan A}$$

Rationalizing the denominator and numerator with (sec A + tan A) and using $\sec^2\theta - \tan^2\theta = 1$ we get,

$$= \frac{\sec^2 A - \tan^2 A}{(\sec A + \tan A)^2}$$

$$= \frac{1}{(\sec A + \tan A)^2}$$

$$= \frac{1}{(\sec^2 A + \tan^2 A + 2\sec A \tan A)}$$

$$= \frac{1}{\left(\frac{1}{\cos^2 A} + \frac{\sin^2 A}{\cos^2 A} + \frac{2\sin A}{\cos A}\right)}$$

$$\Rightarrow \frac{\cos^2 A}{1 + \sin^2 A + 2\sin A}$$

$$= \frac{\cos^2 A}{(1 + \sin A)^2}$$

= R.H.S

- Hence proved

$$1 + \frac{\cos A}{\sin A} = \frac{\sin A}{1 - \cos A}$$

36.

Solution:

We have,

$$\text{LHS} = \frac{1 + \cos A}{\sin A}$$

On multiplying the numerator and denominator by (1 – cos A), we get

$$= \frac{(1+\cos A)(1-\cos A)}{\sin A(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{\sin A(1-\cos A)}$$

$$= \frac{\sin^2 A}{\sin A(1-\cos A)}$$

$$= \frac{\sin A}{1-\cos A}$$

= R.H.S

- Hence proved

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$$

37. (i)

Solution:

Taking L.H.S and rationalizing the numerator and denominator with √(1 + sin A), we get

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A}} = \sqrt{\frac{(1+\sin A)^2}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)^2}{\cos^2 A}} = \sqrt{\frac{(1+\sin A)}{\cos A}}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A} = \sec A + \tan A$$

= R.H.S

- Hence proved

$$\sqrt{\frac{(1-\cos A)}{(1+\cos A)}}+\sqrt{\frac{(1+\cos A)}{(1-\cos A)}}=2\operatorname{cosec} A$$

(ii)

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$=\sqrt{\frac{(1-\cos A)(1-\cos A)}{(1+\cos A)(1-\cos A)}}+\sqrt{\frac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$$

$$=\sqrt{\frac{(1-\cos A)^2}{(1-\cos^2 A)}}+\sqrt{\frac{(1+\cos A)^2}{(1-\cos^2 A)}}$$

$$=\sqrt{\frac{(1-\cos A)^2}{(\sin^2 A)}}+\sqrt{\frac{(1+\cos A)^2}{(\sin^2 A)}}$$

$$=\frac{(1-\cos A)}{(\sin A)}+\frac{(1+\cos A)}{(\sin A)}$$

$$=\frac{(1-\cos A+1+\cos A)}{(\sin A)}$$

$$=\frac{(2)}{(\sin A)}$$

= 2 cosec A

= R.H.S

- Hence proved

38. Prove that:

(i) $\sqrt{\frac{(\sec\theta-1)}{(\sec\theta+1)}}+\sqrt{\frac{(\sec\theta+1)}{(\sec\theta-1)}}=2\,\text{cosec}\,\theta$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$=\sqrt{\frac{(\sec\theta-1)(\sec\theta-1)}{(\sec\theta+1)(\sec\theta-1)}}+\sqrt{\frac{(\sec\theta+1)(\sec\theta+1)}{(\sec\theta-1)(\sec\theta+1)}}$$

$$=\sqrt{\frac{(\sec\theta-1)^2}{(\sec^2\theta-1)}}+\sqrt{\frac{(\sec\theta+1)^2}{(\sec^2\theta-1)}}$$

$$=\sqrt{\frac{(\sec\theta-1)^2}{\tan^2\theta}}+\sqrt{\frac{(\sec\theta+1)^2}{\tan^2\theta}}$$

$$=\frac{(\sec\theta-1)}{\tan\theta}+\frac{(\sec\theta+1)}{\tan\theta}$$

$$=\frac{(\sec\theta-1+\sec\theta+1)}{\tan\theta}$$

$$=\frac{(2\cos\theta)}{\cos\theta\sin\theta}$$

$$=\frac{2}{\sin\theta}$$

= 2 cosec θ

= R.H.S

- Hence proved

(ii) $\sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)}}+\sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)}}=2\sec\theta$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$=\sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}}+\sqrt{\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{(1-\sin^2\theta)}}+\sqrt{\frac{(1-\sin\theta)^2}{(1-\sin^2\theta)}}$$

$$=\sqrt{\frac{(1+\sin\theta)^2}{(\cos^2\theta)}}+\sqrt{\frac{(1-\sin\theta)^2}{(\cos^2\theta)}}$$

$$=\frac{(1+\sin\theta)}{(\cos\theta)}+\frac{(1-\sin\theta)}{(\cos\theta)}$$

$$=\sqrt{\frac{(1+\sin\theta+1-\sin\theta)}{(\cos\theta)}}$$

$$=\frac{(2)}{(\cos\theta)}=2\sec\theta$$

= R.H.S

- Hence proved

(iii) $\sqrt{\frac{(1+\cos\theta)}{(1-\cos\theta)}}+\sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)}}=2\ \text{cosec}\ \theta$

Solution:

Taking L.H.S and rationalizing the numerator and denominator with its respective conjugates, we get

$$= \sqrt{\frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}} + \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{(1-\cos^2\theta)}} + \sqrt{\frac{(1+\cos\theta)^2}{(1-\cos^2\theta)}}$$

$$= \sqrt{\frac{(1-\cos\theta)^2}{(\sin^2\theta)}} + \sqrt{\frac{(1+\cos\theta)^2}{(\sin^2\theta)}}$$

$$= \frac{(1-\cos\theta)}{(\sin\theta)} + \frac{(1+\cos\theta)}{(\sin\theta)}$$

$$= \frac{(1-\cos\theta+1+\cos\theta)}{(\sin\theta)}$$

$$= \frac{(2)}{(\sin\theta)}$$

= 2 cosec θ

= R.H.S

- Hence proved

(iv) $\frac{\sec\theta-1}{\sec\theta+1} = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2$

Solution:

Taking L.H.S, we have

$$= \frac{\sec\theta - 1}{\sec\theta + 1} = \frac{1 - \cos\theta}{1 + \cos\theta}$$

On multiplying numerator and denominator by $1 + \cos\theta$, we get

$$= \frac{(1 - \cos\theta)(1 + \cos\theta)}{(1 + \cos\theta)(1 + \cos\theta)}$$

$$= \frac{(1 - \cos^2\theta)}{(1 + \cos\theta)^2}$$

$$= \frac{\sin^2\theta}{(1 + \cos\theta)^2}$$

$$= \left(\frac{\sin\theta}{1 + \cos\theta}\right)^2$$

= R.H.S

- Hence proved

$$(\sec A - \tan A)^2 = \frac{1 - \sin A}{1 + \sin A}$$

39.

Solution:

Taking LHS = $(\sec A – \tan A)^2$, we have

$$= \left[\frac{1}{\cos A} - \frac{\sin A}{\cos A}\right]^2$$

$$= \frac{(1 - \sin A)^2}{\cos^2 A}$$

$$= \frac{(1 - \sin A)^2}{1 - \sin^2 A}$$

$$= \frac{(1 - \sin A)^2}{(1 + \sin A)(1 - \sin A)}$$

$$= \frac{(1 - \sin A)}{(1 + \sin A)}$$

= R.H.S

- Hence proved

$$\frac{1 - \cos A}{1 + \cos A} = (\cot A - \operatorname{cosec} A)^2$$

40.

Solution:

Taking L.H.S and rationalizing the numerator and denominator with (1 – cos A), we get

$$= \frac{(1 - \cos A)(1 - \cos A)}{(1 + \cos A)(1 - \cos A)}$$

$$= \frac{(1 - \cos A)^2}{(1 - \cos^2 A)}$$

$$= \frac{(1 - \cos A)^2}{(\sin^2 A)}$$

$$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)^2$$

= (cosec A – cot A)^2

= (cot A – cosec A)^2

= R.H.S

- Hence proved

$$\frac{1}{\sec A - 1} + \frac{1}{\sec A + 1} = 2 \operatorname{cosec} A \cot A$$

41.

Solution:

Considering L.H.S and taking L.C.M and simplifying, we have,

$$= \frac{\sec A + 1 + \sec A - 1}{(\sec A + 1)(\sec A - 1)}$$

$$= \frac{2 \sec A}{(\sec^2 A - 1)}$$

$$= \frac{2 \sec A}{(\tan^2 A)}$$

$$= \frac{2 \cos^2 A}{(\cos A \sin^2 A)}$$

$$= \frac{2 \cos A}{(\sin^2 A)}$$

$$= \frac{2 \cos A}{(\sin A)(\sin A)}$$

= 2 cosec A cot A = RHS

- Hence proved

$$\frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A} = \sin A + \cos A$$

42.

Solution:

Taking LHS, we have

$$= \frac{\cos A}{1 - \tan A} + \frac{\sin A}{1 - \cot A}$$

$$= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} + \frac{\sin A}{1 - \frac{\cos A}{\sin A}}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A}$$

= cos A + sin A

= RHS

- Hence proved

$$\frac{(\text{cosec } A)}{(\text{cosec } A - 1)} + \frac{(\text{cosec } A)}{(\text{cosec } A + 1)} = 2 \sec^2 A$$ **43.**

Solution:

Considering L.H.S and taking L.C.M and simplifying, we have,

$$= \frac{(\text{cosec A})(\text{cosec A} + 1 + \text{cosec A} - 1)}{(\text{cosec}^2\text{A} - 1)}$$

$$= \frac{(2\ \text{cosec}^2\text{A})}{\cot^2\text{A}}$$

$$= \frac{2\ \sin^2\text{A}}{\sin^2\text{A}.\ \cos^2\text{A}}$$

$$= \frac{2}{\cos^2\text{A}}$$

= 2 $\sec^2$ A

= RHS

- Hence proved

Benefits of Solving RD Sharma Solutions Class 10 Maths Chapter 6 Exercise 6.1

Solving RD Sharma Solutions for Class 10 Maths Chapter 6 Exercise 6.1 on Trigonometric Identities offers several benefits for students:

Strengthens Conceptual Understanding: The exercise provides problems based on fundamental trigonometric identities. Practicing these problems helps reinforce the foundational concepts, enabling students to better understand how identities work and how they can be applied.

Improves Problem-Solving Skills: Working through various questions on trigonometric identities hones problem-solving abilities. Students learn techniques to simplify complex trigonometric expressions, which is beneficial not only in trigonometry but also in calculus and other advanced math topics.

Prepares for Competitive Exams: Trigonometry is a significant part of competitive exams and various state-level engineering entrance exams. RD Sharma's problems provide the rigor required to build confidence for these exams.

Boosts Calculation Speed and Accuracy: Practicing RD Sharma exercises improves calculation speed and accuracy, helping students become more efficient at handling trigonometric expressions and equations under time constraints.

Enhances Analytical Thinking: Trigonometric identities require analytical thinking to recognize patterns and apply the right identity. Regular practice with these exercises enhances students' ability to think analytically, which is useful across math and other subjects.