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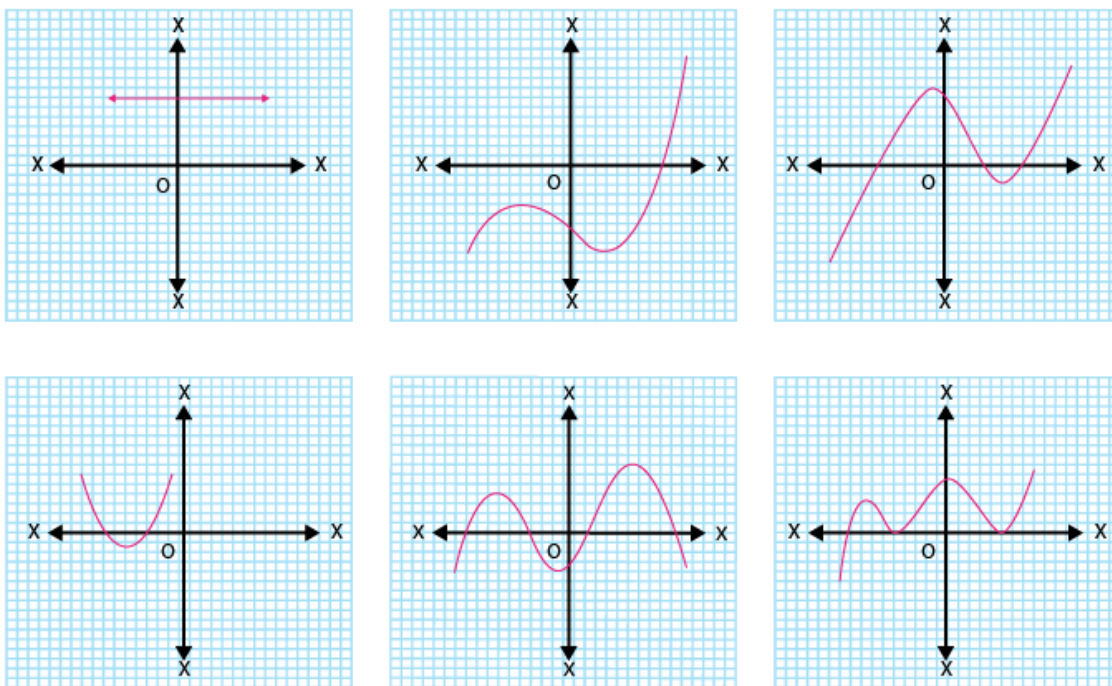
**NCERT Solutions for Class 10 Maths Chapter 2 PDF**

## **NCERT Solutions for Class 10 Maths Chapter 2 Polynomials**

NCERT Solutions for Class 10 Maths Chapter 2 Polynomials provide comprehensive explanations and step-by-step solutions to aid students in mastering polynomial-related concepts and problem-solving techniques.

## **NCERT Solutions for Class 10 Maths Chapter 2 Polynomials Exercise 2.1 Page: 28**

1. The graphs of  $y = p(x)$  are given in Fig. 2.10 below, for some polynomials  $p(x)$ . Find the number of zeroes of  $p(x)$ , in each case.



### Solutions:

#### Graphical method to find zeroes:-

Total number of zeroes in any polynomial equation = total number of times the curve intersects  $x$ -axis.

- (i) In the given graph, the number of zeroes of  $p(x)$  is 0 because the graph is parallel to  $x$ -axis does not cut it at any point.
- (ii) In the given graph, the number of zeroes of  $p(x)$  is 1 because the graph intersects the  $x$ -axis at only one point.
- (iii) In the given graph, the number of zeroes of  $p(x)$  is 3 because the graph intersects the  $x$ -axis at any three points.
- (iv) In the given graph, the number of zeroes of  $p(x)$  is 2 because the graph intersects the  $x$ -axis at two points.
- (v) In the given graph, the number of zeroes of  $p(x)$  is 4 because the graph intersects the  $x$ -axis at four points.
- (vi) In the given graph, the number of zeroes of  $p(x)$  is 3 because the graph intersects the  $x$ -axis at three points.

## NCERT Solutions for Class 10 Maths Chapter 2

### Polynomials Exercise 2.2 Page: 33

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

**Solutions:**

**(i)  $x^2-2x-8$**

$$\Rightarrow x^2 - 4x + 2x - 8 = x(x-4) + 2(x-4) = (x-4)(x+2)$$

Therefore, zeroes of polynomial equation  $x^2-2x-8$  are (4, -2)

$$\text{Sum of zeroes} = 4 - 2 = 2 = -(-2)/1 = -(\text{Coefficient of } x)/(\text{Coefficient of } x^2)$$

$$\text{Product of zeroes} = 4 \times (-2) = -8 = -(8)/1 = (\text{Constant term})/(\text{Coefficient of } x^2)$$

**(ii)  $4s^2-4s+1$**

$$\Rightarrow 4s^2 - 2s - 2s + 1 = 2s(2s-1) - 1(2s-1) = (2s-1)(2s-1)$$

Therefore, zeroes of polynomial equation  $4s^2-4s+1$  are (1/2, 1/2)

$$\text{Sum of zeroes} = (1/2) + (1/2) = 1 = -(-4)/4 = -(\text{Coefficient of } s)/(\text{Coefficient of } s^2)$$

$$\text{Product of zeros} = (1/2) \times (1/2) = 1/4 = (\text{Constant term})/(\text{Coefficient of } s^2)$$

**(iii)  $6x^2-3-7x$**

$$\Rightarrow 6x^2 - 7x - 3 = 6x^2 - 9x + 2x - 3 = 3x(2x-3) + 1(2x-3) = (3x+1)(2x-3)$$

Therefore, zeroes of polynomial equation  $6x^2-3-7x$  are (-1/3, 3/2)

$$\text{Sum of zeroes} = -(1/3) + (3/2) = (7/6) = -(\text{Coefficient of } x)/(\text{Coefficient of } x^2)$$

$$\text{Product of zeroes} = -(1/3) \times (3/2) = -(3/6) = (\text{Constant term})/(\text{Coefficient of } x^2)$$

**(iv)  $4u^2+8u$**

$$\Rightarrow 4u(u+2)$$

Therefore, zeroes of polynomial equation  $4u^2 + 8u$  are (0, -2).

$$\text{Sum of zeroes} = 0 + (-2) = -2 = -(8/4) = -(\text{Coefficient of } u)/(\text{Coefficient of } u^2)$$

$$\text{Product of zeroes} = 0 \times -2 = 0 = 0/4 = (\text{Constant term})/(\text{Coefficient of } u^2)$$

**(v)  $t^2 - 15$**

$$\Rightarrow t^2 = 15 \text{ or } t = \pm\sqrt{15}$$

Therefore, zeroes of polynomial equation  $t^2 - 15$  are  $(\sqrt{15}, -\sqrt{15})$

$$\text{Sum of zeroes} = \sqrt{15} + (-\sqrt{15}) = 0 = -(0/1) = -(\text{Coefficient of } t) / (\text{Coefficient of } t^2)$$

$$\text{Product of zeroes} = \sqrt{15} \times (-\sqrt{15}) = -15 = -15/1 = (\text{Constant term}) / (\text{Coefficient of } t^2)$$

**(vi)  $3x^2 - x - 4$**

$$\Rightarrow 3x^2 - 4x + 3x - 4 = x(3x - 4) + 1(3x - 4) = (3x - 4)(x + 1)$$

Therefore, zeroes of polynomial equation  $3x^2 - x - 4$  are  $(4/3, -1)$

$$\text{Sum of zeroes} = (4/3) + (-1) = (1/3) = -(-1/3) = -(\text{Coefficient of } x) / (\text{Coefficient of } x^2)$$

$$\text{Product of zeroes} = (4/3) \times (-1) = (-4/3) = (\text{Constant term}) / (\text{Coefficient of } x^2)$$

**2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes, respectively.**

**(i)  $1/4, -1$**

**Solution:**

From the formulas of sum and product of zeroes, we know,

$$\text{Sum of zeroes} = \alpha + \beta$$

$$\text{Product of zeroes} = \alpha \beta$$

$$\text{Sum of zeroes} = \alpha + \beta = 1/4$$

$$\text{Product of zeroes} = \alpha \beta = -1$$

$\therefore$  If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (1/4)x + (-1) = 0$$

$$4x^2 - x - 4 = 0$$

Thus,  $4x^2 - x - 4$  is the quadratic polynomial.

(ii)  $\sqrt{2}, 1/3$

**Solution:**

$$\text{Sum of zeroes} = \alpha + \beta = \sqrt{2}$$

$$\text{Product of zeroes} = \alpha \beta = 1/3$$

$\therefore$  If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (\sqrt{2})x + (1/3) = 0$$

$$3x^2 - 3\sqrt{2}x + 1 = 0$$

Thus,  $3x^2 - 3\sqrt{2}x + 1$  is the quadratic polynomial.

(iii)  $0, \sqrt{5}$

**Solution:**

Given,

$$\text{Sum of zeroes} = \alpha + \beta = 0$$

$$\text{Product of zeroes} = \alpha \beta = \sqrt{5}$$

$\therefore$  If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly

as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (0)x + \sqrt{5} = 0$$

Thus,  $x^2 + \sqrt{5}$  is the quadratic polynomial.

(iv)  $1, 1$

**Solution:**

Given,

$$\text{Sum of zeroes} = \alpha + \beta = 1$$

Product of zeroes =  $\alpha \beta = 1$

∴ If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - x + 1 = 0$$

Thus,  $x^2 - x + 1$  is the quadratic polynomial.

(v)  $-1/4, 1/4$

**Solution:**

Given,

$$\text{Sum of zeroes} = \alpha + \beta = -1/4$$

$$\text{Product of zeroes} = \alpha \beta = 1/4$$

∴ If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - (-1/4)x + (1/4) = 0$$

$$4x^2 + x + 1 = 0$$

Thus,  $4x^2 + x + 1$  is the quadratic polynomial.

(vi) 4, 1

**Solution:**

Given,

$$\text{Sum of zeroes} = \alpha + \beta = 4$$

$$\text{Product of zeroes} = \alpha\beta = 1$$

∴ If  $\alpha$  and  $\beta$  are zeroes of any quadratic polynomial, then the quadratic polynomial equation can be written directly as:-

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$x^2 - 4x + 1 = 0$$

Thus,  $x^2-4x+1$  is the quadratic polynomial.

## NCERT Solutions for Class 10 Maths Chapter 2 Polynomials Exercise 2.3 Page: 36

1. Divide the polynomial  $p(x)$  by the polynomial  $g(x)$  and find the quotient and remainder in each of the following:

(i)  $p(x) = x^3-3x^2+5x-3$  ,  $g(x) = x^2-2$

**Solution:**

Given,

Dividend =  $p(x) = x^3-3x^2+5x-3$

Divisor =  $g(x) = x^2-2$

$$\begin{array}{r} x-3 \\ x^2-2 \overline{) x^3-3x^2+5x-3} \\ \underline{x^3+0x^2-2x} \phantom{-3} \\ -3x^2+7x-3 \\ \underline{-3x^2+0x+6} \\ 7x-9 \end{array}$$

Therefore, upon division we get,

Quotient =  $x-3$

Remainder =  $7x-9$

(ii)  $p(x) = x^4-3x^2+4x+5$  ,  $g(x) = x^2+1-x$

**Solution:**

Given,

Dividend =  $p(x) = x^4-3x^2+4x+5$

Divisor =  $g(x) = x^2 + 1 - x$

$$\begin{array}{r}
 x^2 - x + 1 \overline{) \begin{array}{l} x^4 + 0x^3 - 3x^2 + 4x + 5 \\ \underline{x^4 - x^3 + x^2} \\ x^3 - 4x^2 + 4x + 5 \\ \underline{x^3 - x^2 + x} \\ -3x^2 + 3x + 5 \\ \underline{-3x^2 + 3x - 3} \\ 8 \end{array} }
 \end{array}$$

Therefore, upon division we get,

Quotient =  $x^2 + x - 3$

Remainder = 8

(iii)  $p(x) = x^4 - 5x + 6$ ,  $g(x) = 2 - x^2$

**Solution:**

Given,

Dividend =  $p(x) = x^4 - 5x + 6 = x^4 + 0x^2 - 5x + 6$

Divisor =  $g(x) = 2 - x^2 = -x^2 + 2$



$$\begin{array}{r}
 -x^2 + 2 \quad \overline{) \quad \begin{array}{cccccc}
 -x^2 & -2 & & & & \\
 x^4 & +0x^3 & +0x^2 & -5x & +6 & \\
 \hline
 x^4 & +0x^3 & -2x^2 & & & \\
 \hline
 & & 2x^2 & -5x & +6 & \\
 & & \hline
 & & 2x^2 & +0x & -4 & \\
 & & \hline
 & & & -5x & +10 & 
 \end{array}
 }
 \end{array}$$

Therefore, upon division we get,

Quotient =  $-x^2 - 2$

Remainder =  $-5x + 10$

**2. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:**

**(i)  $t^2 - 3$ ,  $2t^4 + 3t^3 - 2t^2 - 9t - 12$**

**Solutions:**

Given,

First polynomial =  $t^2 - 3$

Second polynomial =  $2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$\begin{array}{r}
 \begin{array}{c} 2t^2 \quad +3t \quad +4 \\ t^2 - 3 \end{array} \overline{) \begin{array}{r} 2t^4 + 3t^3 - 2t^2 - 9t - 12 \\ - (2t^4 + 0t^3 - 6t^2) \\ \hline 3t^3 + 4t^2 - 9t - 12 \\ - (3t^3 + 0t^2 - 9t) \\ \hline 4t^2 + 0t - 12 \\ - (4t^2 + 0t - 12) \\ \hline 0 \end{array}
 \end{array}$$

As we can see, the remainder is left as 0. Therefore, we say that,  $t^2-3$  is a factor of  $2t^4+3t^3-2t^2-9t-12$ .

(ii)  $x^2+3x+1$  ,  $3x^4+5x^3-7x^2+2x+2$

**Solutions:**

Given,

First polynomial =  $x^2+3x+1$

Second polynomial =  $3x^4+5x^3-7x^2+2x+2$

$$\begin{array}{r}
 x^2 + 3x + 1 \overline{) 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\
 \underline{3x^4 + 9x^3 + 3x^2} \phantom{+ 2x + 2} \\
 -4x^3 - 10x^2 + 2x + 2 \\
 \underline{-4x^3 - 12x^2 - 4x} \phantom{+ 2} \\
 2x^2 + 6x + 2 \\
 \underline{2x^2 + 6x + 2} \\
 0
 \end{array}$$

As we can see, the remainder is left as 0. Therefore, we say that,  $x^2 + 3x + 1$  is a factor of  $3x^4 + 5x^3 - 7x^2 + 2x + 2$ .

(iii)  $x^3 - 3x + 1$ ,  $x^5 - 4x^3 + x^2 + 3x + 1$

**Solutions:**

Given,

First polynomial =  $x^3 - 3x + 1$

Second polynomial =  $x^5 - 4x^3 + x^2 + 3x + 1$

$$\begin{array}{r}
 x^3 - 3x + 1 \overline{) x^5 + 0x^4 - 4x^3 + x^2 + 3x + 1} \\
 \underline{x^5 + 0x^4 - 3x^3 + x^2} \phantom{+ 3x + 1} \\
 -x^3 + 0x^2 + 3x + 1 \\
 \underline{-x^3 + 0x^2 + 3x - 1} \\
 2
 \end{array}$$

As we can see, the remainder is not equal to 0. Therefore, we say that,  $x^3-3x+1$  is not a factor of  $x^5-4x^3+x^2+3x+1$ .

**3. Obtain all other zeroes of  $3x^4+6x^3-2x^2-10x-5$ , if two of its zeroes are  $\sqrt{5/3}$  and  $-\sqrt{5/3}$ .**

**Solutions:**

Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

$\sqrt{5/3}$  and  $-\sqrt{5/3}$  are zeroes of polynomial  $f(x)$ .

$$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) = x^2 - (5/3) = 0$$

$(3x^2-5)=0$ , is a factor of given polynomial  $f(x)$ .

Now, when we will divide  $f(x)$  by  $(3x^2-5)$  the quotient obtained will also be a factor of  $f(x)$  and the remainder will be 0.

$$\begin{array}{r}
 \phantom{3x^4+} x^2 + 2x + 1 \\
 3x^2-5 \overline{) 3x^4 + 6x^3 - 2x^2 - 10x - 5} \\
 \underline{3x^4 \phantom{- 2x^2} - 5x^2} \phantom{- 10x - 5} \\
 (-) \phantom{3x^4} (+) \phantom{- 10x - 5} \\
 \hline
 \phantom{3x^4+} + 6x^3 + 3x^2 - 10x - 5 \\
 \underline{- 6x^3 \phantom{+ 3x^2} - 10x} \phantom{- 5} \\
 (+) \phantom{3x^4+} (-) \phantom{- 5} \\
 \hline
 \phantom{3x^4+} 3x^2 \phantom{- 10x} - 5 \\
 \phantom{3x^4+} \underline{3x^2 \phantom{- 10x} - 5} \\
 \phantom{3x^4+} (-) \phantom{3x^2} (+) \\
 \hline
 \phantom{3x^4+} 0
 \end{array}$$

$$\text{Therefore, } 3x^4 + 6x^3 - 2x^2 - 10x - 5 = (3x^2 - 5)(x^2 + 2x + 1)$$

Now, on further factorizing  $(x^2+2x+1)$  we get,

$$x^2+2x+1 = x^2+x+x+1 = 0$$

$$x(x+1)+1(x+1) = 0$$

$$(x+1)(x+1) = 0$$

So, its zeroes are given by:  $x = -1$  and  $x = -1$ .

Therefore, all four zeroes of given polynomial equation are:

$$\sqrt[3]{5/3}, -\sqrt[3]{5/3}, -1 \text{ and } -1.$$

Hence, is the answer.

**4. On dividing  $x^3-3x^2+x+2$  by a polynomial  $g(x)$ , the quotient and remainder were  $x-2$  and  $-2x+4$ , respectively. Find  $g(x)$ .**

**Solution:**

Given,

$$\text{Dividend, } p(x) = x^3-3x^2+x+2$$

$$\text{Quotient} = x-2$$

$$\text{Remainder} = -2x+4$$

We have to find the value of Divisor,  $g(x) = ?$

As we know,

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

$$\therefore x^3-3x^2+x+2 = g(x) \times (x-2) + (-2x+4)$$

$$x^3-3x^2+x+2-(-2x+4) = g(x) \times (x-2)$$

$$\text{Therefore, } g(x) \times (x-2) = x^3-3x^2+3x-2$$

Now, for finding  $g(x)$  we will divide  $x^3-3x^2+3x-2$  with  $(x-2)$

$$\begin{array}{r}
 x^2 - x + 1 \\
 x - 2 \overline{) x^3 - 3x^2 + 3x - 2} \\
 \underline{x^3 - 2x^2} \phantom{+ 3x - 2} \\
 (-) (+) \\
 \phantom{x^3 - } - x^2 + 3x - 2 \\
 \phantom{x^3 - } \underline{- x^2 + 2x} \phantom{- 2} \\
 (+) (-) \\
 \phantom{x^3 - } \phantom{- x^2 + } x - 2 \\
 \phantom{x^3 - } \phantom{- x^2 + } \underline{x - 2} \\
 \phantom{x^3 - } \phantom{- x^2 + } (-) (+) \\
 \phantom{x^3 - } \phantom{- x^2 + } \phantom{x - } 0
 \end{array}$$

Therefore,  $g(x) = (x^2 - x + 1)$

**5. Give examples of polynomials  $p(x)$ ,  $g(x)$ ,  $q(x)$  and  $r(x)$ , which satisfy the division algorithm and**

**(i)  $\deg p(x) = \deg q(x)$**

**(ii)  $\deg q(x) = \deg r(x)$**

**(iii)  $\deg r(x) = 0$**

**Solutions:**

According to the division algorithm, dividend  $p(x)$  and divisor  $g(x)$  are two polynomials, where  $g(x) \neq 0$ . Then we can find the value of quotient  $q(x)$  and remainder  $r(x)$ , with the help of below given formula;

Dividend = Divisor  $\times$  Quotient + Remainder

$\therefore p(x) = g(x) \times q(x) + r(x)$

Where  $r(x) = 0$  or degree of  $r(x) <$  degree of  $g(x)$ .

Now let us proof the three given cases as per division algorithm by taking examples for each.

**(i)  $\deg p(x) = \deg q(x)$**

Degree of dividend is equal to degree of quotient, only when the divisor is a constant term.

Let us take an example,  $p(x) = 3x^2 + 3x + 3$  is a polynomial to be divided by  $g(x) = 3$ .

$$\text{So, } (3x^2+3x+3)/3 = x^2+x+1 = q(x)$$

Thus, you can see, the degree of quotient  $q(x) = 2$ , which also equal to the degree of dividend  $p(x)$ .

Hence, division algorithm is satisfied here.

**(ii)  $\deg q(x) = \deg r(x)$**

Let us take an example,  $p(x) = x^2 + 3$  is a polynomial to be divided by  $g(x) = x - 1$ .

$$\text{So, } x^2 + 3 = (x - 1) \times (x) + (x + 3)$$

Hence, quotient  $q(x) = x$

Also, remainder  $r(x) = x + 3$

Thus, you can see, the degree of quotient  $q(x) = 1$ , which is also equal to the degree of remainder  $r(x)$ .

Hence, division algorithm is satisfied here.

**(iii)  $\deg r(x) = 0$**

The degree of remainder is 0 only when the remainder left after division algorithm is constant.

Let us take an example,  $p(x) = x^2 + 1$  is a polynomial to be divided by  $g(x) = x$ .

$$\text{So, } x^2 + 1 = (x) \times (x) + 1$$

Hence, quotient  $q(x) = x$

And, remainder  $r(x) = 1$

Clearly, the degree of remainder here is 0.

Hence, division algorithm is satisfied here.

## **NCERT Solutions for Class 10 Maths Chapter 2 Polynomials Exercise 2.4 Page: 36**

**1. Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:**

**(i)  $2x^3+x^2-5x+2$ ;  $-1/2$ ,  $1$ ,  $-2$**

**Solution:**

Given,  $p(x) = 2x^3 + x^2 - 5x + 2$

And zeroes for  $p(x)$  are  $= 1/2, 1, -2$

$$\therefore p(1/2) = 2(1/2)^3 + (1/2)^2 - 5(1/2) + 2 = (1/4) + (1/4) - (5/2) + 2 = 0$$

$$p(1) = 2(1)^3 + (1)^2 - 5(1) + 2 = 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2 = 0$$

Hence, proved  $1/2, 1, -2$  are the zeroes of  $2x^3 + x^2 - 5x + 2$ .

Now, comparing the given polynomial with general expression, we get;

$$\therefore ax^3 + bx^2 + cx + d = 2x^3 + x^2 - 5x + 2$$

$$a=2, b=1, c=-5 \text{ and } d=2$$

As we know, if  $\alpha, \beta, \gamma$  are the zeroes of the cubic polynomial  $ax^3 + bx^2 + cx + d$ , then;

$$\alpha + \beta + \gamma = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$\alpha\beta\gamma = -d/a.$$

Therefore, putting the values of zeroes of the polynomial,

$$\alpha + \beta + \gamma = 1/2 + 1 + (-2) = -1/2 = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = (1/2 \times 1) + (1 \times -2) + (-2 \times 1/2) = -5/2 = c/a$$

$$\alpha\beta\gamma = 1/2 \times 1 \times (-2) = -2/2 = -d/a$$

Hence, the relationship between the zeroes and the coefficients are satisfied.

**(ii)  $x^3 - 4x^2 + 5x - 2$ ; 2, 1, 1**

**Solution:**

Given,  $p(x) = x^3 - 4x^2 + 5x - 2$

And zeroes for  $p(x)$  are 2, 1, 1.



$$\therefore p(2) = 2^3 - 4(2)^2 + 5(2) - 2 = 0$$

$$p(1) = 1^3 - (4 \times 1^2) + (5 \times 1) - 2 = 0$$

Hence proved, 2, 1, 1 are the zeroes of  $x^3 - 4x^2 + 5x - 2$

Now, comparing the given polynomial with general expression, we get;

$$\therefore ax^3 + bx^2 + cx + d = x^3 - 4x^2 + 5x - 2$$

$$a = 1, b = -4, c = 5 \text{ and } d = -2$$

As we know, if  $\alpha, \beta, \gamma$  are the zeroes of the cubic polynomial  $ax^3 + bx^2 + cx + d$ , then;

$$\alpha + \beta + \gamma = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a$$

$$\alpha\beta\gamma = -d/a.$$

Therefore, putting the values of zeroes of the polynomial,

$$\alpha + \beta + \gamma = 2 + 1 + 1 = 4 = -(-4)/1 = -b/a$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = 2 \times 1 + 1 \times 1 + 1 \times 2 = 5 = 5/1 = c/a$$

$$\alpha\beta\gamma = 2 \times 1 \times 1 = 2 = -(-2)/1 = -d/a$$

Hence, the relationship between the zeroes and the coefficients are satisfied.

**2. Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -14 respectively.**

**Solution:**

Let us consider the cubic polynomial is  $ax^3 + bx^2 + cx + d$  and the values of the zeroes of the polynomials be  $\alpha, \beta, \gamma$ .

As per the given question,

$$\alpha + \beta + \gamma = -b/a = 2/1$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = c/a = -7/1$$

$$\alpha\beta\gamma = -d/a = -14/1$$

Thus, from above three expressions we get the values of coefficient of polynomial.

$$a = 1, b = -2, c = -7, d = 14$$

Hence, the cubic polynomial is  $x^3 - 2x^2 - 7x + 14$

**3. If the zeroes of the polynomial  $x^3 - 3x^2 + x + 1$  are  $a - b, a, a + b$ , find  $a$  and  $b$ .**

**Solution:**

We are given with the polynomial here,

$$p(x) = x^3 - 3x^2 + x + 1$$

And zeroes are given as  $a - b, a, a + b$

Now, comparing the given polynomial with general expression, we get;

$$\therefore px^3 + qx^2 + rx + s = x^3 - 3x^2 + x + 1$$

$$p = 1, q = -3, r = 1 \text{ and } s = 1$$

$$\text{Sum of zeroes} = a - b + a + a + b$$

$$-q/p = 3a$$

Putting the values  $q$  and  $p$ .

$$-(-3)/1 = 3a$$

$$a = 1$$

Thus, the zeroes are  $1 - b, 1, 1 + b$ .

$$\text{Now, product of zeroes} = 1(1 - b)(1 + b)$$

$$-s/p = 1 - b^2$$

$$-1/1 = 1 - b^2$$

$$b^2 = 1 + 1 = 2$$

$$b = \pm\sqrt{2}$$

Hence,  $1 - \sqrt{2}, 1, 1 + \sqrt{2}$  are the zeroes of  $x^3 - 3x^2 + x + 1$ .

**4. If two zeroes of the polynomial  $x^4 - 6x^3 - 26x^2 + 138x - 35$  are  $2 \pm \sqrt{3}$ , find other zeroes.**

**Solution:**

Since this is a polynomial equation of degree 4, hence there will be total 4 roots.

$$\text{Let } f(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$$

Since  $2 + \sqrt{3}$  and  $2 - \sqrt{3}$  are zeroes of given polynomial  $f(x)$ .

$$\therefore [x - (2 + \sqrt{3})][x - (2 - \sqrt{3})] = 0$$

$$(x - 2 - \sqrt{3})(x - 2 + \sqrt{3}) = 0$$

On multiplying the above equation we get,

$x^2 - 4x + 1$ , this is a factor of a given polynomial  $f(x)$ .

Now, if we will divide  $f(x)$  by  $g(x)$ , the quotient will also be a factor of  $f(x)$  and the remainder will be 0.

$$\begin{array}{r}
 \phantom{x^4 - 6x^3 - 26x^2 + 138x - 35} x^2 - 2x - 35 \\
 x^2 - 4x + 1 \overline{) x^4 - 6x^3 - 26x^2 + 138x - 35} \\
 \underline{x^4 - 4x^3 + \phantom{-26}x^2} \phantom{+ 138x - 35} \\
 (-) \quad (+) \quad (-) \\
 \phantom{x^4 - 6x^3 - 26x^2 + 138x - 35} - 2x^3 - 27x^2 + 138x - 35 \\
 \underline{- 2x^3 + 8x^2 - 2x} \phantom{- 35} \\
 (+) \quad (-) \quad (+) \\
 \phantom{x^4 - 6x^3 - 26x^2 + 138x - 35} - 35x^2 + 140x - 35 \\
 \underline{- 35x^2 + 140x - 35} \\
 (+) \quad (-) \quad (+) \\
 \phantom{x^4 - 6x^3 - 26x^2 + 138x - 35} 0
 \end{array}$$

$$\text{So, } x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$$

Now, on further factorizing  $(x^2 - 2x - 35)$  we get,

$$x^2 - (7-5)x - 35 = x^2 - 7x + 5x + 35 = 0$$

$$x(x - 7) + 5(x - 7) = 0$$

$$(x+5)(x-7) = 0$$

So, its zeroes are given by:

$x = -5$  and  $x = 7$ .

Therefore, all four zeroes of given polynomial equation are:  $2+\sqrt{3}$ ,  $2-\sqrt{3}$ ,  $-5$  and  $7$ .

**Q.5:** If the polynomial  $x^4 - 6x^3 + 16x^2 - 25x + 10$  is divided by another polynomial  $x^2 - 2x + k$ , the remainder comes out to be  $x + a$ , find  $k$  and  $a$ .

**Solution:**

Let's divide  $x^4 - 6x^3 + 16x^2 - 25x + 10$  by  $x^2 - 2x + k$ .

$$\begin{array}{r} x^2 - 2x + k \overline{) x^4 - 6x^3 + 16x^2 - 25x + 10} \quad (x^2 - 4x + (8 - k)) \\ \underline{x^4 - 2x^3 + kx^2} \phantom{- 25x + 10} \\ -4x^3 + (16 - k)x^2 - 25x \phantom{+ 10} \\ \underline{-4x^3 + 8x^2 - 4kx} \phantom{+ 10} \\ (8 - k)x^2 + (4k - 25)x + 10 \\ \underline{(8 - k)x^2 - 2(8 - k)x + k(8 - k)} \\ (4k - 25 + 16 - 2k)x + [10 - k(8 - k)] \end{array}$$

Given that the remainder of the polynomial division is  $x + a$ .

$$(4k - 25 + 16 - 2k)x + [10 - k(8 - k)] = x + a$$

$$(2k - 9)x + (10 - 8k + k^2) = x + a$$

Comparing the coefficients of the above equation, we get;

$$2k - 9 = 1$$

$$2k = 9 + 1 = 10$$

$$k = 10/2 = 5$$

And

$$10 - 8k + k^2 = a$$

$$10 - 8(5) + (5)^2 = a \text{ [since } k = 5]$$

$$10 - 40 + 25 = a$$

$$a = -5$$

Therefore,  $k = 5$  and  $a = -5$ .