

ICSE Class 9 Maths Selina Solutions Chapter 9: You may find the Selina answers to the questions in the Class 9 Selina textbooks' chapter 9, Triangles, here. Students study triangles in detail in this ICSE Class 9 Maths Selina Solutions Chapter 9, with a particular emphasis on congruency in triangles. Completing every question in the Selina textbook will enable students to receive full credit for the test.

ICSE Class 9 Maths Selina Solutions Chapter 9 is quite simple to comprehend. These answers address every exercise question in the book and follow the syllabus that the ICSE or CISCE has specified. ICSE Class 9 Maths Selina Solutions Chapter 9 is available here in PDF format, which can be accessed online or downloaded. Additionally, students can download these ICSE Class 9 Maths Selina Solutions Chapter 9 for free and use them offline for practice.

ICSE Class 9 Maths Selina Solutions Chapter 9 Overview

ICSE Class 9 Maths Selina Solutions Chapter 9 covers the topic of triangles, focusing on various properties and theorems related to them. The ICSE Class 9 Maths Selina Solutions Chapter 9 delves into different types of triangles (such as equilateral, isosceles, and scalene) and their specific properties. It also explores important concepts like the sum of interior angles in a triangle, the criteria for triangle congruence (such as SSS, SAS, and ASA), and the basic proportionality theorem.

The ICSE Class 9 Maths Selina Solutions Chapter 9 provides a detailed explanation of these properties and theorems, often supported by illustrative examples and exercises to help students understand and apply these concepts effectively in solving problems related to triangles.

ICSE Class 9 Maths Selina Solutions Chapter 9

Below we have provided ICSE Class 9 Maths Selina Solutions Chapter 9 –

1. Which of the following pairs of triangles are congruent? In each case, state the condition of congruency:

(a) In $\triangle ABC$ and $\triangle DEF$, $AB = DE$, $BC = EF$ and $\angle B = \angle E$.

(b) In $\triangle ABC$ and $\triangle DEF$, $\angle B = \angle E = 90^\circ$; $AC = DF$ and $BC = EF$.

(c) In $\triangle ABC$ and $\triangle QRP$, $AB = QR$, $\angle B = \angle R$ and $\angle C = \angle P$.

(d) $\triangle PQR$, $AB = PQ$, $AC = PR$ and $BC = QR$.

(e) In $\triangle ABC$ and $\triangle PQR$, $BC = QR$, $\angle A = 90^\circ$, $\angle C = \angle R = 40^\circ$ and $\angle Q = 50^\circ$.

Solution:

(a) In $\triangle ABC$ and $\triangle DEF$

$$AB = DE \text{ (data)}$$

$$BC = EF \text{ and } \angle B = \angle E \text{ (given)}$$

By SAS criteria of congruency and given data, we can conclude that,

$\triangle ABC$ and $\triangle DEF$ are congruent to each other.

Therefore, $\triangle ABC \cong \triangle DEF$.

(b) Given in $\triangle ABC$ and $\triangle DEF$,

$$\angle B = \angle E = 90^\circ;$$

$$AC = DF$$

That is hypotenuse $AC =$ hypotenuse DF

and $BC = EF$

By right angle hypotenuse side postulate of congruency,

The given triangles $\triangle ABC$ and $\triangle DEF$ are congruent to each other.

Therefore, $\triangle ABC \cong \triangle DEF$.

(c) In $\triangle ABC$ and $\triangle QRP$,

Data: $AB = QR$,

$$\angle B = \angle R \text{ and}$$

$$\angle C = \angle P.$$

By using the SAS postulate and the given data we can conclude that

The given triangles $\triangle ABC$ and $\triangle QRP$ are congruent to each other.

Therefore, $\triangle ABC \cong \triangle QRP$.

(d) In $\triangle ABC$ and $\triangle PQR$,

Data: $AB = PQ$, $AC = PR$ and $BC = QR$.

By using the SSS postulate of congruency and given data we can conclude that

The given triangles $\triangle ABC$ and $\triangle PQR$ are congruent to each other.

Therefore, $\triangle ABC \cong \triangle PQR$.

(e) In $\triangle ABC$ and $\triangle PQR$,

Data: $BC = QR$, $\angle A = 90^\circ$,

$\angle C = \angle R = 40^\circ$ and $\angle Q = 50^\circ$

But we know that the sum of the angle of the triangle = 180°

Therefore, $\angle P + \angle Q + \angle R = 180^\circ$

$\angle P + 50^\circ + 40^\circ = 180^\circ$

$\angle P + 90^\circ = 180^\circ$

$\angle P = 180^\circ - 90^\circ$

$\angle P = 90^\circ$

In $\triangle ABC$ and $\triangle PQR$,

$\angle A = \angle P$

$\angle C = \angle R$

$BC = QR$

According to the ASA postulate of congruency,

The given triangles $\triangle ABC$ and $\triangle PQR$ are congruent to each other.

Therefore, $\triangle ABC \cong \triangle PQR$.

2. The given figure shows a circle with center O. P is the mid-point of chord AB.

Show that OP is perpendicular to AB.

Solution:

Data: in the given figure O center of the circle.

P is the mid-point of chord AB. AB is a chord

P is a point on AB such that $AP = PB$

Now we have to prove that $OP \perp AB$

Construction: Join OA and OB

Proof: In $\triangle OAP$ and $\triangle OBP$

$OA = OB$ (because radii of the common circle)

$OP = OP$ (common)

$AP = PB$ (data)

To the SSS postulate of congruent triangles

The given triangles $\triangle OAP$ and $\triangle OBP$ are congruent to each other.

Therefore, $\triangle OAP \cong \triangle OBP$.

The corresponding parts of the congruent triangles are congruent.

$\angle OPA = \angle OPB$ (by Corresponding parts of Congruent triangles)

But $\angle OPA + \angle OPB = 180^\circ$ (linear pair)

$\angle OPA = \angle OPB = 90^\circ$

Hence $OP \perp AB$

3. The following figure shows a circle with center O.

If OP is perpendicular to AB, prove that $AP=BP$.

Solution:

Given: In the figure, O is the center of the circle,

And AB is a chord. P is a midpoint on AB such that $AP = PB$

We need to prove that $AP = BP$,

Construction: Join OA and OB

Proof: In right angle triangles $\triangle OAP$ and $\triangle OBP$

Hypotenuse $OA =$ Hypotenuse OB (because radii of the common circle)

Side $OP = OP$ (common)

$AP = PB$ (data)

To SSS postulate of congruent triangles

The given triangles $\triangle OAP$ and $\triangle OBP$ are congruent to each other.

Therefore, $\triangle OAP \cong \triangle OBP$.

The corresponding parts of the congruent triangles are congruent.

$AP = BP$ (by Corresponding parts of Congruent triangles)

Hence the proof.

4. In a triangle ABC, D is the mid-point of BC; AD is produced up to E so that $DE = AD$. Prove that: (i) $\triangle ABD$ and $\triangle ECD$ are congruent.

(ii) $AB = EC$

(iii) AB is parallel to EC.

Solution:

Given $\triangle ABC$ in which D is the mid-point of BC

AD is produced to E so that $DE = AD$

We need to prove that

(i) $\triangle ABD \cong \triangle ECD$

(ii) $AB = EC$

(iii) $AB \parallel EC$

(i) In $\triangle ABD$ and $\triangle ECD$

$BD = DC$ (D is the midpoint of BC)

$\angle ADB = \angle CDE$ (vertically opposite angles)

$AD = DE$ (Given)

By the SAS postulate of congruency of triangles, we have

$\triangle ABD \cong \triangle ECD$

(ii) The corresponding parts of congruent triangles are congruent

Therefore, $AB = EC$ (corresponding parts of congruent triangles)

(iii) Also, we have $\angle DAB = \angle DEC$ (corresponding parts of congruent triangles)

$AB \parallel EC$ [$\angle DAB = \angle DEC$ are alternate angles]

5. A triangle ABC has $\angle B = \angle C$. Prove that:

(i) The perpendiculars from the mid-point of BC to AB and AC are equal.

(ii) The perpendiculars from B and C to the opposite sides are equal.

Solution:

(i) Given $\triangle ABC$ in which $\angle B = \angle C$.

DL is perpendicular from D to AB

DM is the perpendicular from D to AC.

We need to prove that

$$DL = DM$$

Proof:

In $\triangle ABC$ and $\triangle DMC$ (DL perpendicular to AB and DM perpendicular to AC)

$$\angle DLB = \angle DMC = 90^\circ$$

$$\angle B = \angle C \text{ (Given)}$$

$$BD = DC \text{ (D is the midpoint of BC)}$$

By AAS postulate of congruent triangles

$$\triangle DLB \cong \triangle DMC$$

The corresponding parts of the congruent triangles are congruent

$$\text{Therefore } DL = DM$$

(ii) Given $\triangle ABC$ in which $\angle B = \angle C$.

BP is perpendicular from D to AC

CQ is the perpendicular from C to AB.

We need to prove that

$$BP = CQ$$

Proof:

In $\triangle BPC$ and $\triangle CQB$ (BP perpendicular to AC and CQ perpendicular to AB)

$$\angle BPC = \angle CQB = 90^\circ$$

$$\angle B = \angle C \text{ (Given)}$$

$$BC = BC \text{ (common)}$$

By AAS postulate of congruent triangles

$$\triangle BPC \cong \triangle CQB$$

The corresponding parts of the congruent triangles are congruent

Therefore $BP = CQ$

6. The perpendicular bisector of the sides of a triangle AB meet at I. Prove that: $IA = IB = IC$

Solution:

Given triangle ABC in which AD is the perpendicular bisector of BC

BE is the perpendicular bisector of CA

CF is the perpendicular bisector of AB

AD, BE and CF meet at I

We need to prove that

$IA = IB = IC$

Proof:

In $\triangle BID$ and $\triangle CID$

$BD = DC$ (given)

$\angle BDI = \angle CDI = 90^\circ$ (AD is perpendicular bisector of BC)

$BC = BC$ (common)

By SAS postulate of congruent triangles

$\triangle BID \cong \triangle CID$

The corresponding parts of the congruent triangles are congruent

Therefore $IB = IC$

Similarly, In $\triangle CIE$ and $\triangle AIE$

$CE = AE$ (given)

$\angle CEI = \angle AEI = 90^\circ$ (AD is perpendicular bisector of BC)

$IE = IE$ (common)

By SAS postulate of congruent triangles

$\triangle CIE \cong \triangle AIE$

The corresponding parts of the congruent triangles are congruent

Therefore $IC = IA$

Thus, $IA = IB = IC$

7. A line segment AB is bisected at point P and through point P another line segment PQ, which is perpendicular to AB, is drawn. Show that: $QA = QB$.

Solution:

Given triangle ABC in which AB is bisected at P

PQ is the perpendicular to AB

We need to prove that

$QA = QB$

Proof:

In $\triangle APQ$ and $\triangle BPQ$

$AP = PB$ (P is the midpoint of AB)

$\angle APQ = \angle BPQ = 90^\circ$ (PQ is perpendicular to AB)

$PQ = PQ$ (common)

By SAS postulate of congruent triangles

$\triangle APQ \cong \triangle BPQ$

The corresponding parts of the congruent triangles are congruent

Therefore $QA = QB$

8. If AP bisects angle BAC and M is any point on AP, prove that the perpendiculars drawn from M to AB and AC are equal.

Solution:

From M, draw ML such that ML is perpendicular to AB and MN is perpendicular to AC

In $\triangle ALM$ and $\triangle ANM$

$\angle LAM = \angle MAN$ (AP is the bisector of $\angle BAC$)

$\angle ALM = \angle ANM = 90^\circ$ (ML is perpendicular to AB and MN is perpendicular to AC)

$AM = AM$ (common)

By AAS postulate of congruent triangles

$$\triangle ALM \cong \triangle ANM$$

The corresponding parts of the congruent triangles are congruent

$$\text{Therefore } ML = MN$$

Hence the proof.

9. From the given diagram, in which ABCD is a parallelogram, ABL is a line segment and E is mid-point of BC.

Prove that:

$$(i) \triangle DCE \cong \triangle LBE$$

$$(ii) AB = BL.$$

$$(iii) AL = 2DC$$

Solution:

Given ABCD is a parallelogram in which E is the midpoint of BC

We need to prove that

$$(i) \triangle DCE \cong \triangle LBE$$

$$(ii) AB = BL.$$

$$(iii) AL = 2DC$$

$$(i) \text{ In } \triangle DCE \text{ and } \triangle LBE$$

$$\angle DCE = \angle LBE \text{ (DC parallel to AB, alternate angles)}$$

$$CE = EB \text{ (E is the midpoint of BC)}$$

$$\angle DCE = \angle LBE \text{ (vertically opposite angles)}$$

By ASA postulate of congruent triangles

$$\triangle DCE \cong \triangle LBE$$

The corresponding parts of the congruent triangles are congruent

$$\text{Therefore } DC = LB \dots (i)$$

$$(ii) DC = AB \dots (ii)$$

$$\text{From (i) and (ii) } AB = BL \dots (iii)$$

$$(iii) AL = AB + BL \dots (iv)$$

From (iii) and (iv) $AL = AB + AB$

$$AL = 2AB$$

$$AL = 2DC \text{ from (ii)}$$

10. In the given figure, $AB = DB$ and $AC = DC$.

If $\angle ABD = 58^\circ$,

$$\angle DBC = (2x - 4)^\circ,$$

$$\angle ACB = y + 15^\circ \text{ and}$$

$$\angle DCB = 63^\circ; \text{ find the values of } x \text{ and } y.$$

Solution:

Given: In the given figure, $AB = DB$ and $AC = DC$.

If $\angle ABD = 58^\circ$,

$$\angle DBC = (2x - 4)^\circ,$$

$$\angle ACB = y + 15^\circ \text{ and}$$

$$\angle DCB = 63^\circ;$$

We need to find the values of x and y .

In $\triangle ABC$ and $\triangle DBC$

$$AB = DB \text{ (given)}$$

$$AC = DC \text{ (given)}$$

$$BC = BC \text{ (common)}$$

By SSS postulate of congruent triangles

$$\triangle ABC \cong \triangle DBC$$

The corresponding parts of the congruent triangles are congruent

Therefore

$$\angle ACB = \angle DCB$$

$$y^\circ + 15^\circ = 63^\circ$$

$$y^\circ = 63^\circ - 15^\circ$$

$$y^\circ = 48^\circ$$

$\angle ACB = \angle DCB$ (corresponding parts of the congruent triangles)

But $\angle DCB = (2x - 4)^\circ$

We have $\angle ACB + \angle DCB = \angle ABD$

$$(2x - 4)^\circ + (2x - 4)^\circ = 58^\circ$$

$$4x - 8^\circ = 58^\circ$$

$$4x = 58^\circ + 8^\circ$$

$$4x = 66^\circ$$

$$x = 66^\circ/4$$

$$x = 16.5^\circ$$

Thus, the values of x and y are

$$x = 16.5^\circ \text{ and } y = 48^\circ$$

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1. On the sides AB and AC of triangle ABC, equilateral triangle ABD and ACE are drawn.

Prove that:

(i) $\angle CAD = \angle BAE$

(ii) $CD = BE$.

Solution:

Given triangle ABD is an equilateral triangle

Triangle ACE is an equilateral triangle

Now, we need to prove that

(i) $\angle CAD = \angle BAE$

(ii) $CD = BE$.

Proof:

(i) $\triangle ABD$ is equilateral

Each angle = 60°

$$\angle BAD = 60^\circ \dots\dots(i)$$

Similarly,

$\triangle ACE$ is equilateral

Each angle = 60°

$$\angle CAE = 60^\circ \dots\dots(ii)$$

$$\angle BAD = \angle CAE \text{ from (i) and (ii)} \dots\dots(iii)$$

Adding $\angle BAC$ to both sides, we have

$$\angle BAD + \angle BAC = \angle CAE + \angle BAC$$

$$\angle CAD = \angle BAE \dots\dots(iv)$$

(ii) In $\triangle CAD$ and $\triangle BAE$

$AC = AE$ (triangle ACE is equilateral)

$$\angle CAD = \angle BAE \text{ from (iv)}$$

$AD = AB$ (triangle ABD is equilateral)

By SAS postulate of congruent triangles

$$\triangle CAD \cong \triangle BAE$$

The corresponding parts of the congruent triangles are congruent

Therefore $CD = BE$

Hence the proof.

2. In the following diagrams, $ABCD$ is a square and APB is an equilateral triangle.

In each case,

(i) Prove that: $\triangle APD \cong \triangle BPC$

(ii) Find the angles of $\triangle DPC$.

Solution:

(a)

(i) Proof:

$$AP = PB = AB \text{ [}\triangle APB \text{ is an equilateral triangle]}$$

Also we have,

$$\angle PBA = \angle PAB = \angle APB = 60^\circ \dots\dots\dots (1)$$

Since ABCD is a square, we have

$$\angle A = \angle B = \angle C = \angle D = 90^\circ \dots\dots\dots (2)$$

$$\text{Since } \angle DAP = \angle A - \angle PAB \dots\dots\dots (3)$$

$$\angle DAP = 90^\circ - 60^\circ$$

$$\angle DAP = 30^\circ \text{ [from equation 1 and equation 2] } \dots\dots\dots (4)$$

$$\text{Similarly } \angle CBP = \angle B - \angle PBA$$

$$\angle CBP = 90^\circ - 60^\circ$$

$$\angle CBP = 30^\circ \text{ [from equation 1 and equation 2] } \dots\dots\dots (5)$$

$$\angle DAP = \angle CBP \text{ [from equation 4 and equation 5] } \dots\dots\dots (6)$$

$\triangle APD$ and $\triangle BPC$

$$AD = BC \text{ [sides of square ABCD]}$$

$$\angle DAP = \angle CBP \text{ [from 6]}$$

$$AP = BP \text{ [sides of equilateral triangle APB]}$$

Therefore by SAS criteria of congruency, we have

$$\triangle APD \cong \triangle BPC$$

$$(ii) AP = PB = AB \text{ [}\triangle APB \text{ is an equilateral triangle] } \dots\dots\dots (7)$$

$$AB = BC = CD = DA \text{ [sides of square ABCD] } \dots\dots\dots (8)$$

From equation 7 and 8, we have

$$AP = DA \text{ and } PB = BC \dots\dots\dots (9)$$

In $\triangle APD$,

$$AP = DA \text{ [from 9]}$$

$$\angle ADP = \angle APD \text{ [angles opposite to equal sides are equal]}$$

$$\angle ADP + \angle APD + \angle DAP = 180^\circ \text{ [sum of angles of a triangle = } 180^\circ]$$

$$\angle ADP + \angle APD + 30^\circ = 180^\circ$$

$$\angle ADP + \angle ADP = 180^\circ - 30^\circ \text{ [from 2 and from 10]}$$

$$2 \angle ADP = 150^\circ$$

$$\angle ADP = 75^\circ$$

$$\text{We have } \angle PCD = \angle C - \angle PCB$$

$$\angle PCD = 90^\circ - 75^\circ$$

$$\angle PCD = 15^\circ \dots\dots\dots (13)$$

In triangle DPC

$$\angle PDC = 15^\circ$$

$$\angle PCD = 15^\circ$$

$$\angle PCD + \angle PDC + \angle DPC = 180^\circ$$

$$\angle DPC = 180^\circ - 30^\circ$$

$$\angle DPC = 150^\circ$$

Therefore angles are 15° , 150° and 15°

(b)

(i) Proof: In triangle APB

$$AP = PB = AB$$

Also,

We have,

$$\angle PBA = \angle PAB = \angle APB = 60^\circ \dots\dots\dots (1)$$

Since ABCD is a square, we have

$$\angle A = \angle B = \angle C = \angle D = 90^\circ \dots\dots\dots (2)$$

$$\angle DAP = \angle A + \angle PAB \dots\dots\dots (3)$$

$$\angle DAP = 90^\circ + 60^\circ$$

$$\angle DAP = 150^\circ \text{ [from 1 and 2] } \dots\dots\dots (4)$$

$$\angle CBP = \angle B + \angle PBA \dots\dots\dots (3)$$

$$\angle CBP = 90^\circ + 60^\circ$$

$$\angle CBP = 150^\circ \text{ [from 1 and 2] (5)}$$

$$\angle DAP = \angle CBP \text{ [from 4 and 5] (6)}$$

In triangle APD and triangle BPC

$$AD = BC \text{ [sides of square ABCD]}$$

$$\angle DAP = \angle CBP \text{ [from 6]}$$

$$AP = BP \text{ [sides of equilateral triangle APB]}$$

By SAS criteria we have

$$\triangle APD \cong \triangle BPC$$

$$(ii) AP = PB = AB \text{ [triangle APB is an equilateral triangle] (7)}$$

$$AB = BC = CD = DA \text{ [sides of square ABCD] (8)}$$

From equation 7 and 8, we have

$$AP = DA \text{ and } PB = BC \text{ (9)}$$

In $\triangle APD$,

$$AP = DA \text{ [from 9]}$$

$$\angle ADP = \angle APD \text{ [angles opposite to equal sides are equal] (10)}$$

$$\angle ADP + \angle APD + \angle DAP = 180^\circ \text{ [sum of angles of a triangle = } 180^\circ]$$

$$\angle ADP + \angle APD + 150^\circ = 180^\circ$$

$$\angle ADP + \angle ADP = 180^\circ - 150^\circ \text{ [from 2 and from 10]}$$

$$2 \angle ADP = 30^\circ$$

$$\angle ADP = 15^\circ$$

$$\text{We have } \angle PCD = \angle D - \angle ADP$$

$$\angle PCD = 90^\circ - 15^\circ$$

$$\angle PCD = 75^\circ \text{ (11)}$$

In triangle BPC

$$PB = BC \text{ [from 9]}$$

$$\angle PCB = \angle BPC \text{ (12)}$$

$$\angle PCB + \angle BPC + \angle CPB = 180^\circ$$

$$\angle PCB + \angle PCB = 180^\circ - 150^\circ \text{ [from 2 and from 10]}$$

$$2 \angle PCB = 30^\circ$$

$$\angle PCB = 15^\circ$$

$$\text{We have } \angle PCD = \angle C - \angle PCB$$

$$\angle PCD = 90^\circ - 15^\circ$$

$$\angle PCD = 75^\circ \dots\dots\dots (11)$$

In triangle DPC

$$\angle PDC = 75^\circ$$

$$\angle PCD = 75^\circ$$

$$\angle PCD + \angle PDC + \angle DPC = 180^\circ$$

$$75^\circ + 75^\circ + \angle DPC = 180^\circ$$

$$\angle DPC = 180^\circ - 150^\circ$$

$$\angle DPC = 30^\circ$$

Angles of triangle are 75° , 30° and 75°

3. In the figure, given below, triangle ABC is right-angled at B. ABPQ and ACRS are squares. Prove that:

(i) $\triangle ACQ$ and $\triangle ASB$ are congruent.

(ii) $CQ = BS$.

Solution:

Triangle ABC is right-angled at B.

ABPQ and ACRS are squares.

We need to prove that:

(i) $\triangle ACQ$ and $\triangle ASB$ are congruent.

(ii) $CQ = BS$.

Proof:

(i) $\angle QAB = 90^\circ$ (ABPQ is a square) (1)

$$\angle SAC = 90^\circ \text{ (ACRS is a square) } \dots\dots\dots (2)$$

From (1) and (2) we have

$$\angle QAB = \angle SAC \dots\dots\dots (3)$$

Adding $\angle BAC$ both sides of (3) we get

$$\angle QAB + \angle BAC = \angle SAC + \angle BAC$$

$$\angle QAC = \angle SAB \dots\dots\dots (4)$$

In $\triangle ACQ$ and $\triangle ASB$

$$QA = QB \text{ (sides of a square ABPQ)}$$

$$\angle CAD = \angle BAE \text{ from (iv)}$$

$$AC = AS \text{ (side of a square ACRS)}$$

By AAS postulate of congruent triangles

$$\text{Therefore } \triangle ACQ \cong \triangle ASB$$

(ii) The corresponding parts of the congruent triangles are congruent

$$\text{Therefore } CQ = BS$$

**4. In a $\triangle ABC$, BD is the median to the side AC , BD is produced to E such that $BD = DE$.
Prove that: AE is parallel to BC .**

Solution:

Given in a $\triangle ABC$, BD is the median to the side AC ,

BD is produced to E such that $BD = DE$.

Now we have to prove that: AE is parallel to BC .

Construction: Join AE

Proof:

$$AD = DC \text{ (BD is median to AC)}$$

In $\triangle BDC$ and $\triangle ADE$

$$BD = DE \text{ (Given)}$$

$$\angle BDC = \angle ADE = 90^\circ \text{ (vertically opposite angles)}$$

$$AD = DC \text{ (from 1)}$$

By SAS postulate of congruent triangles

Therefore $\triangle BDC \cong \triangle ADE$

The corresponding parts of the congruent triangles are congruent

$$\angle BDC = \angle ADE$$

But these are alternate angles

And AC is the transversal

Thus, AE parallel to BC

5. In the adjoining figure, OX and RX are the bisectors of the angles Q and R respectively of the triangle PQR.

If $XS \perp QR$ and $XT \perp PQ$; prove that:

(i) $\triangle XTQ \cong \triangle XSQ$

(ii) PX bisects angle $\angle P$.

Solution:

In the adjoining figure,

OX and RX are the bisectors of the angles Q and R respectively of the triangle PQR.

If $XS \perp QR$ and $XT \perp PQ$;

We have to prove that:

(i) $\triangle XTQ \cong \triangle XSQ$

(ii) PX bisects angle $\angle P$.

Construction:

Draw If $XZ \perp PR$ and join PX

Proof:

(i) In $\triangle XTQ$ and $\triangle XSQ$

$\angle QTX = \angle QSX = 90^\circ$ (XS perpendicular to QR and XT perpendicular to PQ)

$\angle QTX = \angle QSX$ (QX is bisector of angle Q)

QX = QX (common)

By AAS postulate of congruent triangles

Therefore $\triangle XTQ \cong \triangle XSQ$ (1)

(ii) The corresponding parts of the congruent triangles are congruent

Therefore $XT = XS$ (by c.p.c.t)

In $\triangle XSR$ and $\triangle XZR$

$\angle XSR = \angle XZR = 90^\circ$ (XS perpendicular to SR and angle XSR = 90°)

$\angle SRX = \angle ZRX$ (RX is a bisector of angle R)

$RX = RX$ (common)

By AAS postulate of congruent triangles

Therefore $\triangle XSR \cong \triangle XZR$ (1)

The corresponding parts of the congruent triangles are congruent

Therefore $XS = XZ$ (by c.p.c.t) (2)

From (1) and (2)

$XT = XZ$ (3)

In $\triangle XTP$ and $\triangle XZP$

$\angle XTP = \angle XZP = 90^\circ$ (Given)

$XP = XP$ (common)

$XT = XZ$ (from 3)

By right angle hypotenuse side postulate of congruent triangles

Therefore $\triangle XTP \cong \triangle XZP$

The corresponding parts of the congruent triangles are congruent

$\angle XPT = \angle XPZ$

PX bisects $\angle SRX = \angle ZRX$

6. In the parallelogram ABCD, the angles A and C are obtuse. Points X and Y are taken on the diagonal BD such that the angles XAD and YCB are right angles.

Prove that: $XA = YC$.

Solution:

ABCD is a parallelogram in which $\angle A$ and $\angle C$ are obtuse.

Points X and Y are on the diagonal BD

Such that $\angle XAD = \angle YCB = 90^\circ$

We need to prove that $XA = YC$

Proof:

In $\triangle XAD$ and $\triangle YCB$

$\angle XAD = \angle YCB = 90^\circ$ (Given)

$AD = BC$ (opposite sides of a parallelogram)

$\angle ADX = \angle CBY$ (alternate angles)

By ASA postulate of congruent triangles

Therefore $\triangle XAD \cong \triangle YCB$

The corresponding parts of the congruent triangles are congruent

Therefore $XA = YC$

Hence the proof.

7. ABCD is a parallelogram. The sides AB and AD are produced to E and F respectively, such produced to E and F respectively, such that $AB = BE$ and $AD = DF$.

Prove that: $\triangle BEC \cong \triangle DCF$

Solution:

ABCD is a parallelogram.

The sides AB and AD are produced to E and F respectively,

Such that $AB = BE$ and $AD = DF$.

We need to prove that $\triangle BEC \cong \triangle DCF$

Proof:

$AB = DC$ (opposite sides of a parallelogram) (1)

$AB = BE$ (given) (2)

From (1) and (2) we have

$BE = DC$ (opposite sides of a parallelogram) (3)

$AD = BC$ (opposite sides of a parallelogram) (4)

$$AD = DF \text{ (given) } \dots\dots\dots (5)$$

From (4) and (5) we have

$$BC = DF \dots\dots\dots (6)$$

Since AD parallel to BC the corresponding angles are equal

$$\angle DAB = \angle CBE \dots\dots\dots (7)$$

Since AD parallel to DC the corresponding angles are equal

$$\angle DAB = \angle FDC \dots\dots\dots (8)$$

From (7) and (8)

$$\angle CBE = \angle FDC \dots\dots\dots (9)$$

In $\triangle BEC$ and $\triangle DCF$

$$BE = DC \text{ (from (3))}$$

$$\angle CBE = \angle FDC \text{ (from (9))}$$

$$BC = DF \text{ (from (6))}$$

By SAS postulate of congruent triangles

$$\text{Therefore } \triangle BEC \cong \triangle DCF$$

Hence the proof.

8. In the following figures, the sides AB and BC and the median AD of triangle ABC are equal to the sides PQ and QR and median PS of the triangle PQR. Prove that $\triangle ABC$ and $\triangle PQR$ are congruent.

Solution:

$$\text{Since } BC = QR$$

We have $BD = QS$ and $DC = SR$ (D is the midpoint of BC and S is the midpoint of QR)

In $\triangle ABD$ and $\triangle PQS$

$$AB = PQ \dots\dots\dots (1)$$

$$AD = PS \dots\dots\dots (2)$$

$$BD = QS \dots\dots\dots (3)$$

By SSS postulate of congruent triangles

Therefore $\triangle ABD \cong \triangle PQS$

Similarly

In $\triangle ADC$ and $\triangle PSR$

$$AD = PS \dots\dots\dots (4)$$

$$AC = PR \dots\dots\dots (5)$$

$$DC = SR \dots\dots\dots (6)$$

By SSS postulate of congruent triangles

Therefore $\triangle ADC \cong \triangle PSR$

We have

$$BC = BD + DC \text{ (D is the midpoint of BC)}$$

$$= QS + SR \text{ (from (3) and (6))}$$

$$= QR \text{ (S is the midpoint of QR) } \dots\dots\dots (7)$$

Now again consider the triangles $\triangle ABC$ and $\triangle PQR$

$$AB = PQ \text{ (from 1)}$$

$$BC = QR \text{ (from 7)}$$

$$AC = PR \text{ (from 7)}$$

By SSS postulate of congruent triangles

Therefore $\triangle ABC \cong \triangle PQR$

Hence the proof.

9. In the following diagram, AP and BQ are equal and parallel to each other.

Prove that

(i) $\triangle AOP \cong \triangle BOQ$

(ii) AB and PQ bisect each other

Solution:

In the figure AP and BQ are equal and parallel to each other

Therefore $AP = BQ$ and AP parallel to BQ

We need to prove that

(i) $\triangle AOP \cong \triangle BOQ$

(ii) AB and PQ bisect each other

(i) since AP parallel to BQ

$$\angle APO = \angle BQO \text{ (alternate angles) } \dots\dots\dots (1)$$

$$\text{And } \angle PAO = \angle QBO \text{ (alternate angles) } \dots\dots\dots (2)$$

Now in $\triangle AOP$ and $\triangle BOQ$

$$\angle APO = \angle BQO \text{ (from 1)}$$

$$AP = BQ \text{ (given)}$$

$$\angle PAO = \angle QBO \text{ (from 2)}$$

By ASA postulate of congruent triangles,

$$\triangle AOP \cong \triangle BOQ$$

(ii) the corresponding parts of the congruent triangles are congruent

$$\text{Therefore } OP = OQ \text{ (by c.p.c.t)}$$

$$OA = OB \text{ (by c.p.c.t)}$$

Hence AB and PQ bisect each other

10. In the following figure, $OA = OC$ and $AB = BC$.

(i) $\angle P = 90^\circ$

(ii) $\triangle AOD \cong \triangle COD$

Solution:

Given $OA = OC$ and $AB = BC$.

Now we have to prove that,

(i) $\angle P = 90^\circ$

(ii) $\triangle AOD \cong \triangle COD$

(iii) $AD = CD$

(i) In $\triangle ABO$ and $\triangle CBO$

$AB = BC$ (given)

$AO = CO$ (given)

$OB = OB$ (common)

By SSS postulate of congruent triangles

Therefore $\triangle ABO \cong \triangle CBO$

The corresponding parts of the congruent triangles are congruent

$\angle ABO = \angle CBO$ (by c.p.c.t) Hence $\angle ABD = \angle CBD$

$\angle AOB = \angle CBO$ (by c.p.c.t)

We have $\angle ABO + \angle CBO = 180^\circ$ (linear pair)

$\angle ABO = \angle CBO = 90^\circ$

And AC perpendicular to BD

(ii) In $\triangle AOD$ and $\triangle COD$

$OD = OD$ (common)

$\angle AOD = \angle COD$ (each = 90°)

$AO = CO$ (given)

By SAS postulate of congruent triangles

Therefore $\triangle AOD \cong \triangle COD$

(iii) The corresponding parts of the congruent triangles are congruent

Therefore $AD = CD$ (by c.p.c.t)

Hence the proof.

Benefits of ICSE Class 9 Maths Selina Solutions Chapter 9

Using the ICSE Class 9 Maths Selina Solutions for Chapter 9 on Triangles offers several benefits for students:

Clear Understanding of Concepts: The ICSE Class 9 Maths Selina Solutions Chapter 9 provide step-by-step explanations of problems, helping students grasp the underlying concepts of triangle properties, congruence criteria, and theorems.

Enhanced Problem-Solving Skills: By working through a variety of problems and seeing the ICSE Class 9 Maths Selina Solutions Chapter 9, students can improve their problem-solving skills and learn different approaches to tackle similar questions.

Preparation for Exams: The ICSE Class 9 Maths Selina Solutions Chapter 9 help students practice and understand the types of questions that may appear in exams, thus boosting their confidence and readiness for tests.

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