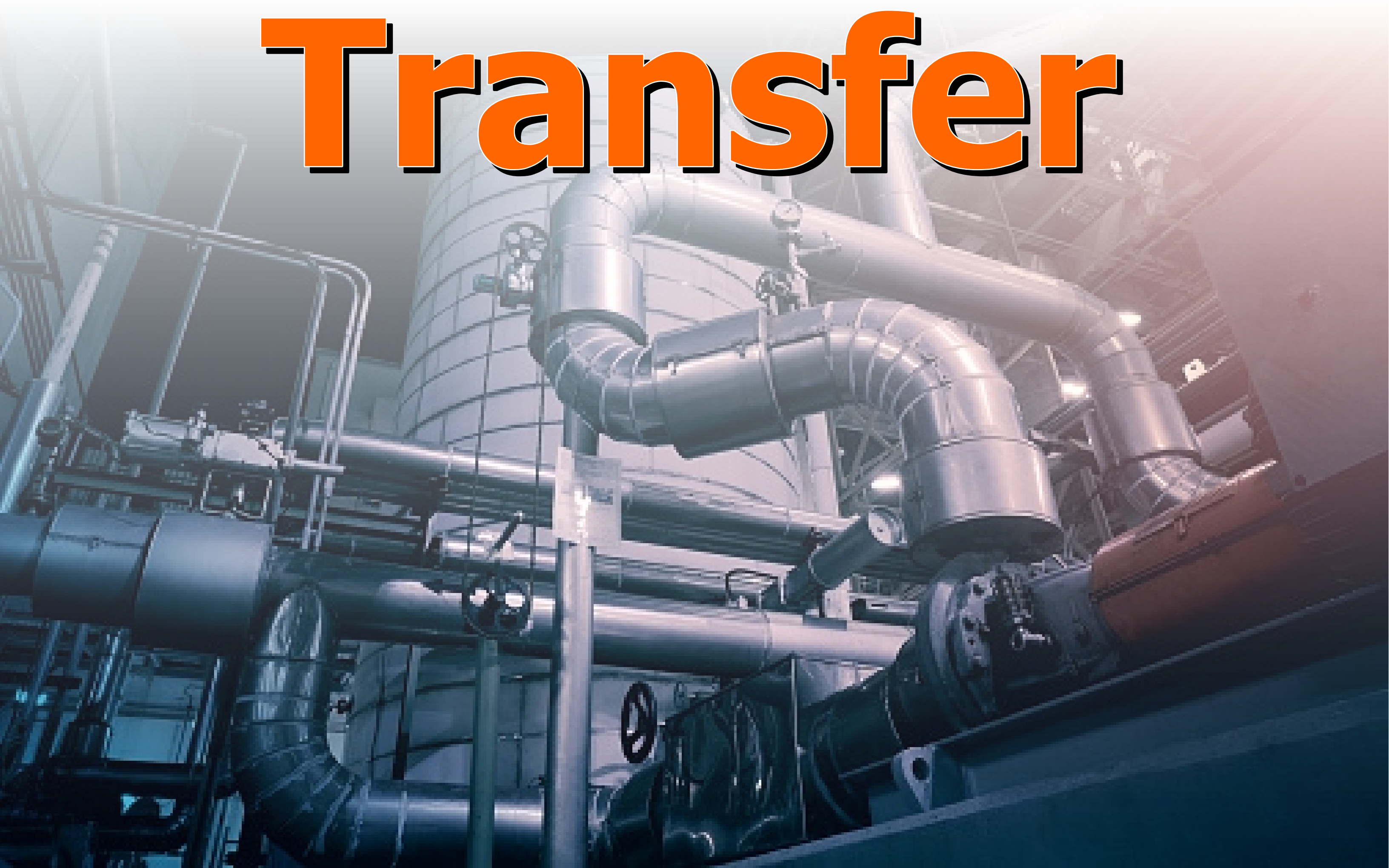
Heat Transfer



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Heat Transfer

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1 BASICS OF HEAT TRANSFER

1.1 Introduction

- Heat transfer is the energy transfer due to temperature difference.
- Heat transfer is governed by the 2^{nd} Law of Thermodynamics.

1.2 Mode of Heat Transfer

- There are three modes of heat transfer
  (i) Conduction heat transfer
  (ii) Convection heat transfer
  (iii) Radiation heat transfer

1.2.1 Conduction:

- Conduction heat transfer is governed by "Fourier's law of heat conduction".
- As per Fourier's law of heat conduction (1 D, Steady state, No Internal heat generation)

$$\dot{Q} \propto A\frac{dT}{dx}$$

$$\dot{Q}_{\text{conduction}} = -kA\frac{dT}{dx}$$

$$\frac{\dot{Q}_{\text{conduction}}}{A} = \dot{q} = -k\frac{dT}{dx}$$

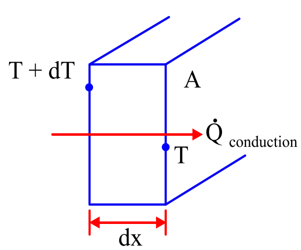

Fig. 1.1. Heat transfer through Plane wall

$\dot{Q}_{\text{conduction}} \Rightarrow$ Rate of conduction heat transfer (W)

k $\Rightarrow$ Thermal conductivity (W/mK) $\left(MLT^{-3}\theta^{-1}\right)$

$A \Rightarrow$ Area normal to direction of conduction heat transfer

$\frac{dT}{dx} \Rightarrow$ Temperature gradient in x-direction

$\dot{q} \Rightarrow$ Conductive heat flux (W/m^2)

Note:
Negative sign shows temperature decreases in the direction of conduction heat transfer.

1.2.2 Convection

- Convection heat transfer is governed by "Newton's law of cooling".
- As per Newton's law of cooling

$$\dot{Q} \propto A(T_s - T_\infty)$$
$$\dot{Q} = hA(T_s - T_\infty)$$
$$\dot{q} = h(T_s - T_\infty)$$

h $\Rightarrow$ Convective heat transfer coefficient (W/m^2 K) ($MT^{-3}\theta^{-1}$)
A $\Rightarrow$ Area responsible for convection heat transfer
T_s $\Rightarrow$ Plate surface temperature
T_∞ $\Rightarrow$ Fluid temperature

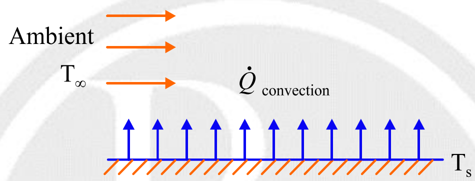

Fig. 1.2. Convection heat transfer from a hot flat plate

- h depends on number of factors
  (a) Free or forced convection
  (b) Laminar or turbulent flow
  (c) Type of fluid (liquid/gas)
  (d) Surface finish of plate (smooth/rough)
  (e) Orientation of plate (horizontal, vertical, inclined)

1.2.3 Radiation

- Radiation heat transfer is governed by "Stefan's Boltzmann Law"
- As per Stefan's Boltzmann Law

$$E_b \propto T^4$$
$$E_b = \sigma_b T^4 \left(\frac{W}{m^2}\right)$$
$$\sigma_b = 5.67 \times 10^{-8} \left(\frac{W}{m^2 K^4}\right)$$

E_b $\Rightarrow$ Total radiation energy emitted by black body per unit time per unit area. (W/m^2)
T $\Rightarrow$ Absolute temperature (in K)

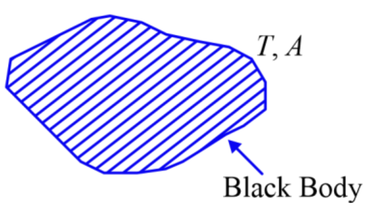

Fig. 1.3. Black body at absolute temperature T

- If body is non-black, then

$$E = \varepsilon E_b$$

$$E = \varepsilon \sigma_b A T^4$$

E ⇒ Radiation energy emitted by a non-black body
ε ⇒ Emissivity

- Radiation heat transfer between two black bodies

$$\dot{Q}_{\text{Radiation}} = A_1 F_{12} \sigma_b \left(T_1^4 - T_2^4\right)$$

F_{12} ⇒ View factor/shape factor/geometric factor

$$A_1F_{12} = A_2F_{21} \qquad \text{(Reciprocity Theorem)}$$

- Radiation heat transfer between two non-black body

$$\dot{Q}_{\text{Radiation}} = A_1 \left(F_g\right)_{12} \sigma_b \left(T_1^4 - T_2^4\right)$$

$(F_g)_{12}$ ⇒ Configuration factor

❑❑❑

2 STEADY-STATE CONDUCTION HEAT TRANSFER

2.1 General heat conduction equation in Cartesian coordinate

$$T(x, y, z, \tau)$$

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial T}{\partial z}\right)+\dot{q}_g=\rho c \frac{\partial T}{\partial \tau}$$

For homogenous and isotropic material $(k_x = k_y = k_z = k)$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}_g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

$$\nabla^2 T+\frac{\dot{q}_g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau} \quad \text{(Fourier-Biot equation)}$$

Where $\dot{q}_g \Rightarrow$ Rate of heat generation per unit volume or volumetric heat generation rate $\left(\frac{W}{m^3}\right)$

$\alpha \Rightarrow$ Thermal diffusivity $\left(\frac{m^2}{s}\right)$

$\alpha = \frac{k}{\rho c}$

$\frac{\partial T}{\partial \tau} \Rightarrow$ Change of temperature with respect to time

Case 1. Steady, No Internal Heat Generation

$$\nabla^2 T = 0 \quad \text{(Laplace Equation)}$$

Case 2. Steady, with Internal Heat Generation

$$\nabla^2 T+\frac{\dot{q}_g}{k}=0 \quad \text{(Poisson's Equation)}$$

Case 3. Unsteady, with No Internal Heat Generation

$$\nabla^2 T=\frac{1}{\alpha}\frac{\partial T}{\partial \tau} \quad \text{(Diffusion Equation)}$$

Case 4. 1-D, Steady, with Internal Heat Generation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}_g}{k} = 0$$

2.2 The Thermal Properties of matter

2.2.1 Thermal Diffusivity (α)

- Ratio of rate of heat energy conducted to heat energy stored. Mathematically,

$$\alpha = \frac{k}{\rho c}$$

c ⇒ Specific heat (J/kg K)

mc ⇒ Heat capacity (J/K)

$\rho c = \frac{mc}{V} \Rightarrow$ Heat capacity per unit volume $\left(J / K\ m^3\right)$

- S.I. Unit of $\alpha = \frac{m^2}{s}\left(L^2 T^{-1}\right)$
- α depends on type of material

$$\alpha_{\text{metal}} > \alpha_{\text{wood}}$$

2. 2. 2 Thermal Conductivity (k)

- Diamond has highest thermal conductivity because it has well-arranged crystal structure.
- In metal, silver (Ag) has highest thermal conductivity
- Descending order of thermal conductivity for some metals

$k_{Ag} > k_{Cu} > k_{Au} > k_{Al} > k_{Iron}$

- Material which is good conductor of heat but bad conductor of electricity is diamond.
- In general, purest form of metal will have higher thermal conductivity than its alloy.

For Example: $k_{\text{copper}} > k_{\text{brass}}$, $k_{\text{iron}} > k_{\text{steel}}$

- In case of metals, as temperature increases thermal conductivity decreases except in case of Al and Ur
- In case of Al, as temperature increases, thermal conductivity increases, then constant and then start to decrease.

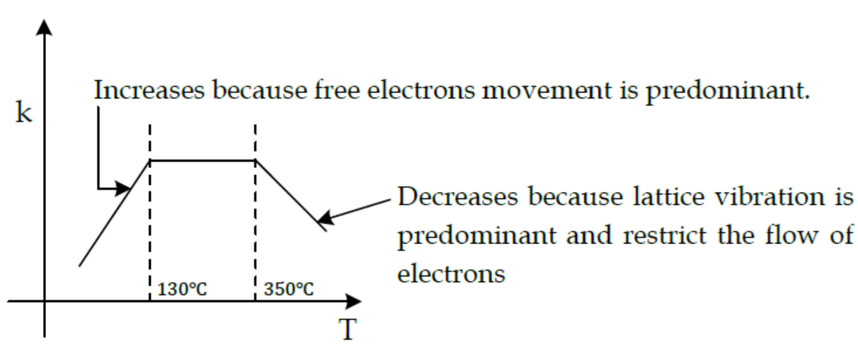

Fig 2.1(a) Temperature variation of thermal conductivity with temperature for Aluminum

- In case of Ur, as temperature increases, thermal conductivity increases.
  (In case of Ur, outer orbit electrons are strongly bonded with nucleus. Hence, lattice vibration is predominant)

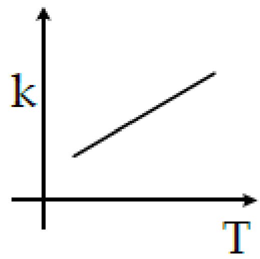

Fig 2.1(b) Temperature variation of thermal conductivity with temperature for Uranium

- In non-metals, as temperature increases, thermal conductivity increases because lattice vibration increases as temperature increases (no free electrons)
- In case of gases:

$$k_{gas} \propto V$$

Where $V \rightarrow$ mean travel velocity, $V = \sqrt{\frac{3\bar{R}T}{M}}$

$\bar{R} \rightarrow$ Universal gas constant, $\left\{\bar{R} = 8.314\frac{kJ}{k\,mol.K}\right\}$

$T \rightarrow$ absolute temperature

$M \rightarrow$ Molecular weight of gas

- For a given gas as temperature increases, the thermal conductivity also increases.
- For a given temperature

$$k_{gas} \propto \frac{1}{M}$$

Therefore, $k_{H_2} > k_{N_2} > k_{O_2} > k_{CO_2}$

- For most liquids as temperature increases, thermal conductivity decreases except water, Hg.
- In general, conduction heat transfer is most predominant in solids, then in liquids and least in gases $\left(k_{solid} > k_{liquid} > k_{gas}\right)$

2.3 Thermal Contact Resistance

- When two surfaces are pressed against each other, the peaks will form good contact but the valleys will form voids filled with air.
- As the air thermal conductivity is low, the interface offers some thermal resistance and this resistance per unit interface area is called thermal contact resistance.
- Mathematically we can write

$$\dot{Q} = h_c\,A\,\Delta T_{interface}$$

Where $h_c \rightarrow$ thermal contact conductance

- For composite wall, we can write

$$\dot{Q} = \frac{T_1 - T_2}{\left(\frac{\delta_1}{k_1 A}\right) + \left(\frac{1}{h_c A}\right) + \left(\frac{\delta_2}{k_2 A}\right)}$$

$$\Rightarrow \dot{q} = \frac{T_1 - T_2}{\left(\frac{\delta_1}{k_1}\right) + R_c + \left(\frac{\delta_2}{k_2}\right)}$$

Where $R_c \rightarrow$ Thermal contact resistance

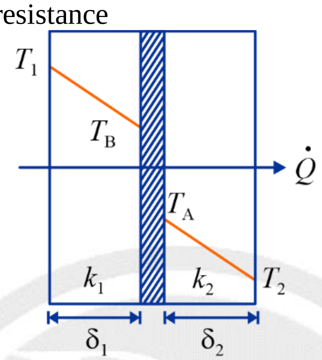

Fig. 2.2 Composite wall with thermal contact resistance

2.4 Conduction heat transfer from plane wall

Assumptions

- Steady
- 1-D (x-direction)
- No internal heat generation
- $k \neq f(T)$

$$\frac{\partial^2 T}{\partial x^2} = 0 \Rightarrow T = C_1 x + C_2$$

$$T = T_1 + (T_2 - T_1)\frac{x}{\delta} \qquad \text{(Linear Variation)}$$

$$\dot{Q}_{\text{conduction}} = -kA\frac{dT}{dx}$$

$$= -kA\left(\frac{T_2 - T_1}{\delta}\right) = \frac{kA(T_1 - T_2)}{\delta}$$

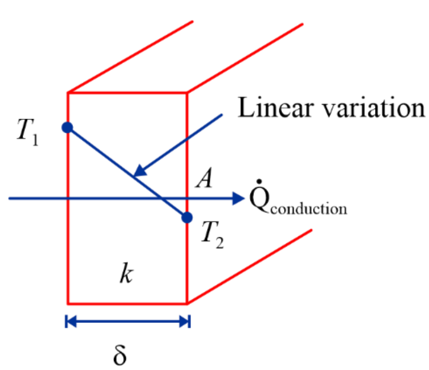

Fig 2.3 Temperature variation in a plane wall

Note:

Consider two identical wall of different material and having same amount of conduction heat transfer. In which there will be higher temperature difference across the wall?

Let $k_1 > k_2$

- From Fourier law of Heat Conduction

$$\dot{Q} = -kA\frac{dT}{dx}$$

$$\Rightarrow \frac{\dot{Q}}{A} = \dot{q} = -k\frac{dT}{dx} = \text{ constant}$$

- If k is high then $\frac{dT}{dx}$ should be low

Since, $k_1 > k_2$, $(\Delta T)_1 < (\Delta T)_2$, $\theta_1 < \theta_2$

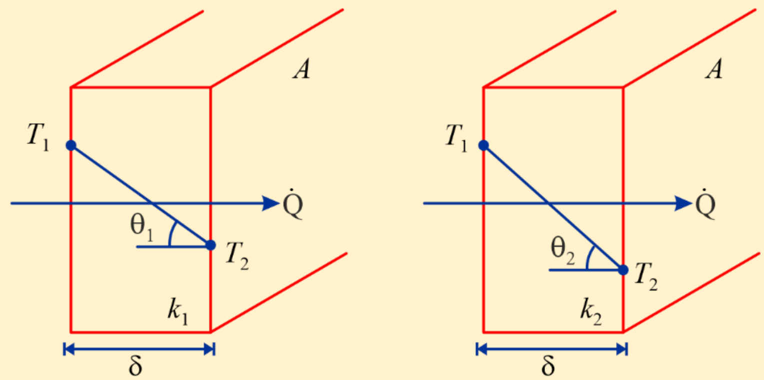

Fig 2.3 Temperature variation in a plane wall with different thermal conductivities

2.4.1 Thermal resistance (R_{th})

- Resistance offered by the material against the heat flow.
- Thermal resistance is analogous to electrical resistance.
- From Ohm's law' $\quad i = \frac{\Delta V}{R}$
- From Fourier's law $\quad \dot{Q} = \frac{\Delta T}{R_{th}}$

$$R_{th} = \frac{\Delta T}{\dot{Q}}\left(\frac{K}{W} / \frac{^\circ C}{W}\right)$$

Conduction Thermal Resistance

- In case of plane wall, heat transfer rate is given by

$$\dot{Q}_{conduction} = kA\frac{\Delta T}{\delta} = \frac{\Delta T}{\delta / kA}$$

$$R_{th} = \delta / kA$$

Convection Thermal Resistance

- Heat transfer rate by convection is given by

$$\dot{Q}_{convection} = hA\Delta T = \frac{\Delta T}{1/hA}$$

$$R_{th} = 1/hA$$

Radiation Thermal Resistance

- Radiation heat transfer in terms of Newton's Law of cooling is given by

$$\dot{Q}_{\text{radiation}} = h_r A(T_1 - T_2) = A_1 F_{12} \sigma_b \left(T_1^{\,4} - T_2^{\,4}\right)$$

$$R_{th} = \frac{1}{h_r A}$$

$h_r \Rightarrow$ Radiative heat transfer coefficient

$$h_r = \sigma_b F_{12}\left(T_1^2 + T_2^2\right)(T_1 + T_2)$$

2.4.2 Composite wall/slab:

- Wall is made of more than one material.

(a) Series Connection

$$\dot{Q}_1 = \dot{Q}_2 = \dot{Q}$$

1. **Equivalent thermal resistance or total thermal resistance**

$$(R_{th})_{eq} = R_{th_1} + R_{th_2} + \cdots$$

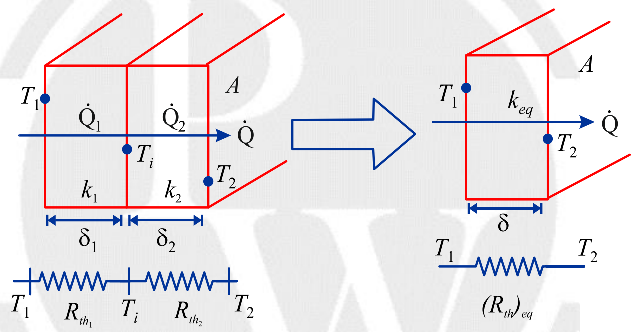

Fig. 2.4 Composite wall in series connection

2. **Equivalent thermal conductivity (k_{eq})**

$$(R_{th})_{eq} = R_{th_1} + R_{th_2}$$

$$\frac{\delta}{k_{eq} A} = \frac{\delta_1}{k_1 A} + \frac{\delta_2}{k_2 A}$$

$$k_{eq} = \frac{\delta}{\frac{\delta_1}{k_1} + \frac{\delta_2}{k_2}}$$

- If N number of slab or wall connected in series then

$$k_{eq} = \frac{\delta_1 + \delta_2 + \ldots + \delta_N}{\frac{\delta_1}{k_1} + \frac{\delta_2}{k_2} + \cdots + \frac{\delta_N}{k_N}}$$

- If thickness of each wall is same then

$$k_{eq} = \frac{\delta_1 N}{\delta_1\left(\frac{1}{k_1} + \frac{1}{k_2} + \cdots + \frac{1}{k_N}\right)}$$

- Two walls of same thickness connected in series then

$$k_{eq} = \frac{2}{1/k_1 + 1/k_2} = \frac{2k_1k_2}{k_1 + k_2}$$

3. Intermediate temperature (T_i)

$$\dot{Q}_1 = \dot{Q}_2 = \dot{Q}$$

$$\frac{T_1 - T_2}{(R_{th})_{eq}} = \frac{T_1 - T_i}{(R_{th})_1} = \frac{T_i - T_2}{(R_{th})_2}$$

4. If $k_2 > k_1$ then $\theta_1 < \theta_2$

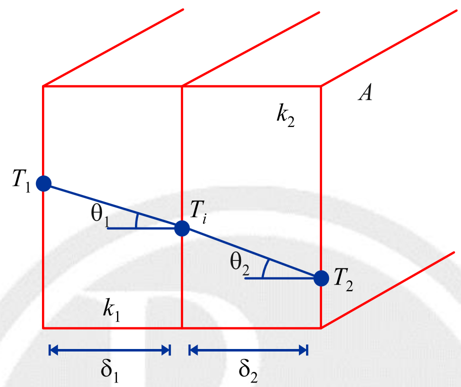

Fig 2.5 Temperature variation for composite wall in series connection

5. Plane wall with convection boundary condition

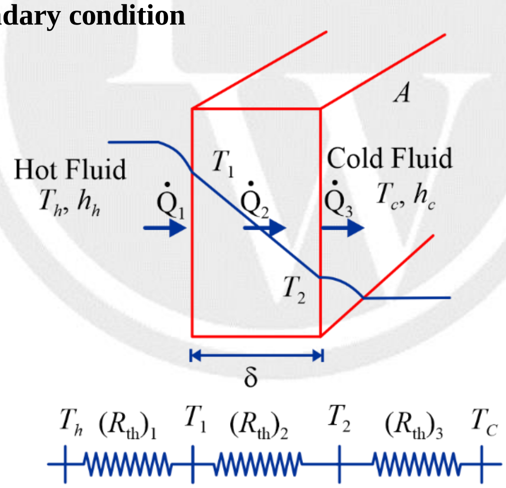

Fig 2.6 Plane wall with convection boundary condition

$$(R_{th})_{eq} = R_{th_1} + R_{th_2} + R_{th_3}$$

$$R_{th_1} = \frac{1}{h_h A}$$

$$R_{th_2} = \frac{\delta}{kA}$$

$$R_{th_3} = \frac{1}{h_c A}$$

$$\dot{Q} = \dot{Q}_1 = \dot{Q}_2 = \dot{Q}_3$$

$$\frac{T_h - T_c}{(R_{th})_{eq}} = \frac{T_h - T_1}{(R_{th})_1} = \frac{T_1 - T_2}{(R_{th})_2} = \frac{T_2 - T_c}{(R_{th})_3}$$

(b) Parallel Connection:

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2$$

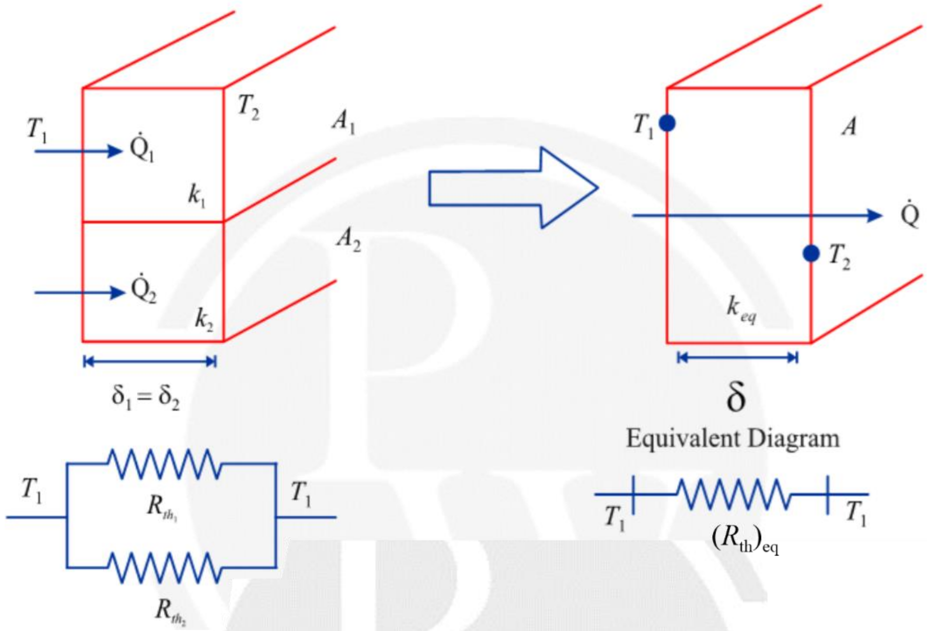

Fig. 2.7 Composite wall in Parallel connection

1. **Equivalent thermal resistance**

$$\frac{1}{(R_{th})_{eq}} = \frac{1}{R_{th_1}} + \frac{1}{R_{th_2}} + \cdots$$

2. **Equivalent thermal conductivity (k_{eq})**

$$\frac{1}{\left(\frac{\delta}{k_{eq}A}\right)} = \frac{1}{\left(\frac{\delta_1}{k_1A_1}\right)} + \frac{1}{\left(\frac{\delta_2}{k_2A_2}\right)}$$

$$\frac{k_{eq}A}{\delta} = \frac{k_1A_1}{\delta_1} + \frac{k_2A_2}{\delta_2}$$

$$k_{eq} = \frac{k_1A_1 + k_2A_2}{A}$$

- If N number of slabs are connected in parallel having same thickness

$$k_{eq} = \frac{k_1A_1 + k_2A_2 + \cdots + k_NA_N}{A_1 + A_2 + \cdots + A_N}$$

Let $A_1 = A_2 = \cdots A_N$

$$k_{eq} = \frac{A_1(k_1 + k_2 + k_3 + \cdots)}{A_1 N}$$

$$k_{eq} = \frac{(k_1 + k_2 + k_3 + \cdots)}{N}$$

- If two slabs of same cross-sectional area are connected in parallel then

$$k_{eq} = \frac{k_1 + k_2}{2}$$

- $\theta_1 = \theta_2$ (Slope of temperature variation in both the wall is same)

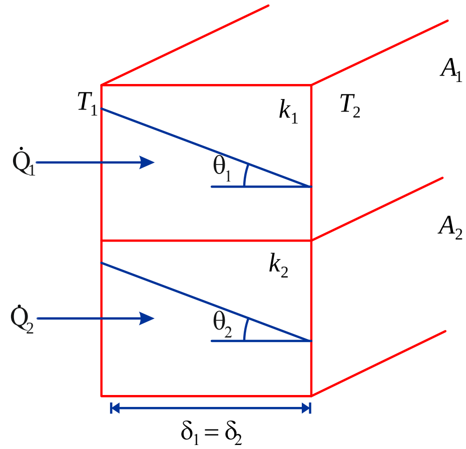

Fig 2.8 Temperature variation for composite wall in Parallel connection

3. Composite wall in parallel connection with convection boundary condition

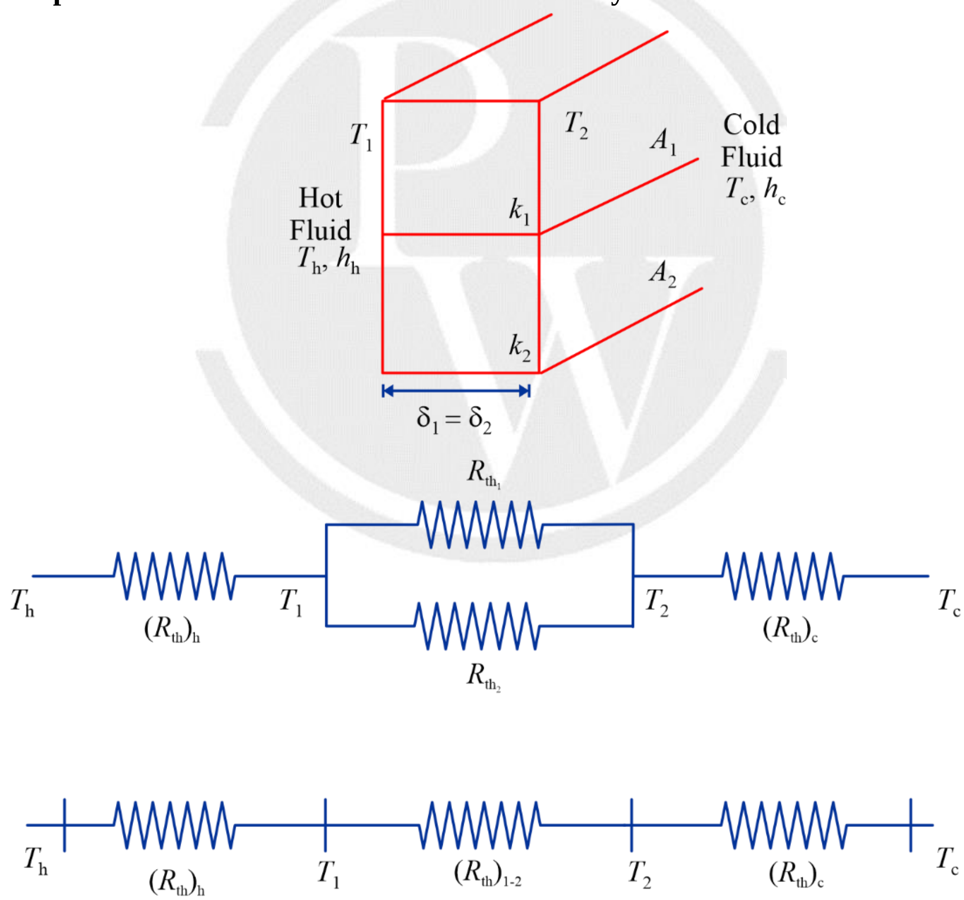

Fig 2.9 Thermal resistance network for composite wall with convection boundary condition

$$\frac{1}{(R_{th})_{1-2}} = \frac{1}{R_{th_1}} + \frac{1}{R_{th_2}}$$

$$(R_{th})_{eq} = (R_{th})_h + (R_{th})_{1-2} + (R_{th})_c$$

$$\dot{Q} = \frac{T_h - T_c}{(R_{th})_{eq}}$$

2. 4. 3 Plane walls with internal heat generation

Assumption

- Steady state
- 1-D
- k ≠ f (T)
- Uniform heat generation rate

Case 1. Wall having different temperature at two faces (Unsymmetric case)

Let $\dot{q}_g \rightarrow$ Uniform rate of heat generation per unit volume $\left(W/m^3\right)$ or volumetric heat generation rate

$$\nabla^2 T + \frac{\dot{q}_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau} \quad \text{(General heat conduction equation)}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}_g}{k} = 0$$

$$T = -\frac{\dot{q}_g}{2k}x^2 + C_1 x + C_2$$

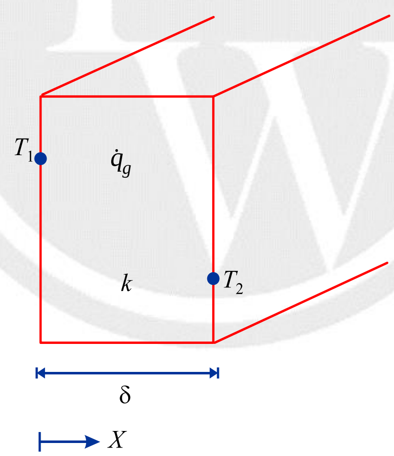

Fig 2.10 Plane wall with uniform heat generation

Where C_1 and C_2 are integration constant and evaluate by using boundary conditions

At $x = 0, T = T_1$

At $x = \delta, T = T_2$

$$\boxed{T = T_1 + \left(T_2 - T_1\right)\frac{x}{\delta} + \frac{\dot{q}_g}{2k}\left(\delta x - x^2\right)}$$

To get location of maximum temperature $\frac{dT}{dx} = 0$

Let

At x $= x', T = T_{\max}$ $\qquad x' = \frac{\delta}{2} - \frac{\left(T_1 - T_2\right)k}{\dot{q}_g \times \delta}$

Subcase 1.

$T_1 > T_2$

$x' < \delta / 2$

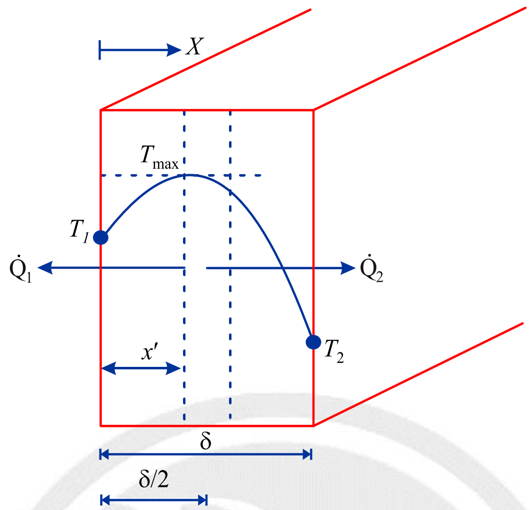

Fig 2.11 Temperature profile for different wall temperatures ($T_1 > T_2$)

Subcase 2. $T_2 > T_1$

$x' > \delta / 2$

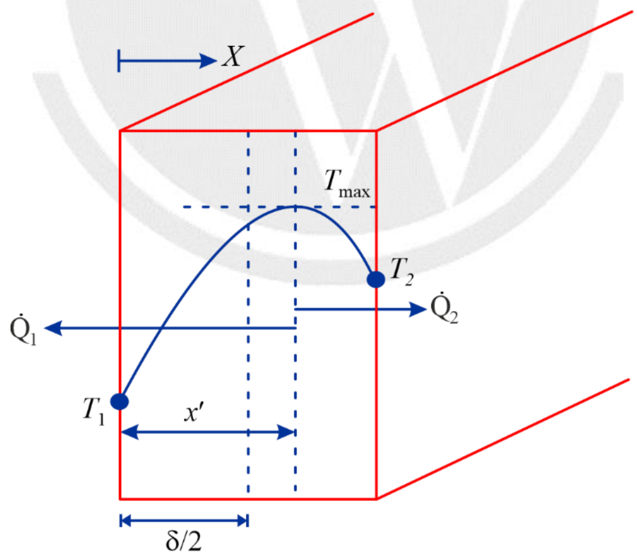

Fig 2.12 Temperature profile for different wall temperatures ($T_2 > T_1$)

Case 2. Wall having same temperature (T_w) at two faces (Symmetric case)

- $T_1 = T_2 = T_w$

$$T = T_w + \frac{\dot{q}_g}{2k}\left(\delta x - x^2\right)$$

Maximum temperature occurs at $x = \delta/2$

$$T_{max} = T_w + \frac{\dot{q}_g \delta^2}{8k}$$

- $\dot{Q}_1 = \dot{Q}_2$ (Due to symmetry)

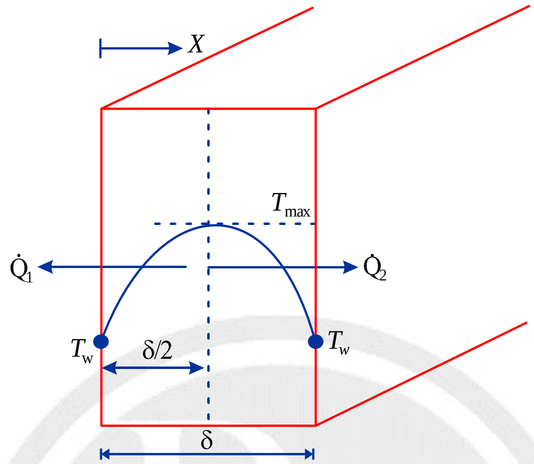

Fig 2.13 Temperature profile for same wall temperatures (T_w)

Case 3. One face is adiabatic and other face has temperature T_w,

Boundary conditions,

- At $x = \delta, T = T_w$
- At $x = 0, \dot{Q}_{conduction} = 0$

$$-kA\frac{dT}{dx} = 0$$

$$\frac{dT}{dx} = 0$$

$$T = T_w + \frac{\dot{q}_g}{2k}\left(\delta^2 - x^2\right)$$

At $x = 0, T = T_{max}$

$$T_{max} = T_w + \frac{\dot{q}_g}{2k}\delta^2$$

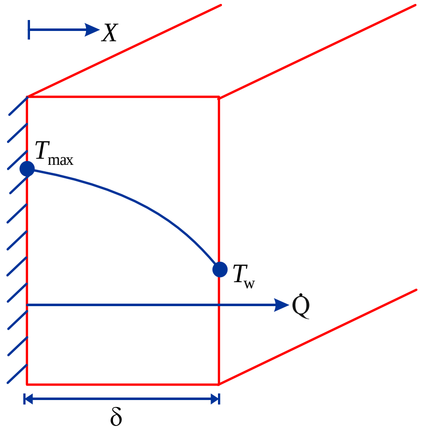

Fig 2.14 Temperature profile for adiabatic wall at one end

2. 4. 4 Effect of Variable Thermal Conductivity

Let

$$k = k_0(1+\beta T)$$

Where

k_0 = Thermal conductivity at 0°C
β = Constant (depends on type of material)

- Value of β may be Positive, may be Negative or zero.

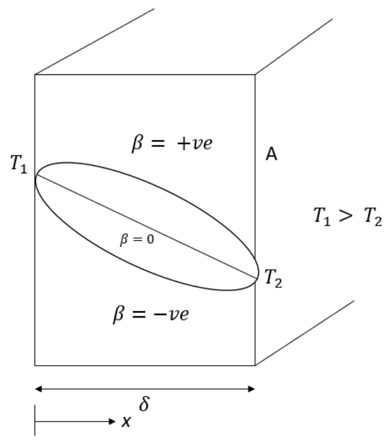

Fig 2.15 Temperature profile for plane wall with variable thermal conductivity

- Rate of conduction Heat Transfer

$$\dot{Q} = k_m A\left[\frac{T_1 - T_2}{\delta}\right] \quad \left(\begin{array}{l}\text{Applicable only when thermal conductivity} \\ \text{varies linearly with temperature}\end{array}\right)$$

k_m = Thermal conductivity at mean temperature

$$k_m = k_0[1+\beta T_m]$$

$$T_m = \frac{T_1 + T_2}{2}$$

2.5 General Heat Conduction Equation in Cylindrical Coordinate System

T (r, θ , z, τ)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rk_r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(k_\theta\frac{\partial T}{\partial\theta}\right)+\frac{\partial}{\partial z}\left(k_z\frac{\partial T}{\partial z}\right)+\dot{q}_g = \frac{\partial}{\partial\tau}(\rho cT)$$

If material properties k, ρ & c are constant then

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}_g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial\tau}$$

Poisson Equation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial\theta^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}_g}{k}=0$$

Laplace Equation $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}=0$

Diffusion Equation $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}=\frac{1}{\alpha}\frac{\partial T}{\partial \tau}$

2.6 Conduction Heat Transfer through Hollow cylinder

- At $r = R_1, T = T_1$
- At $r = R_2, T = T_2$

  Let $T_1 > T_2$

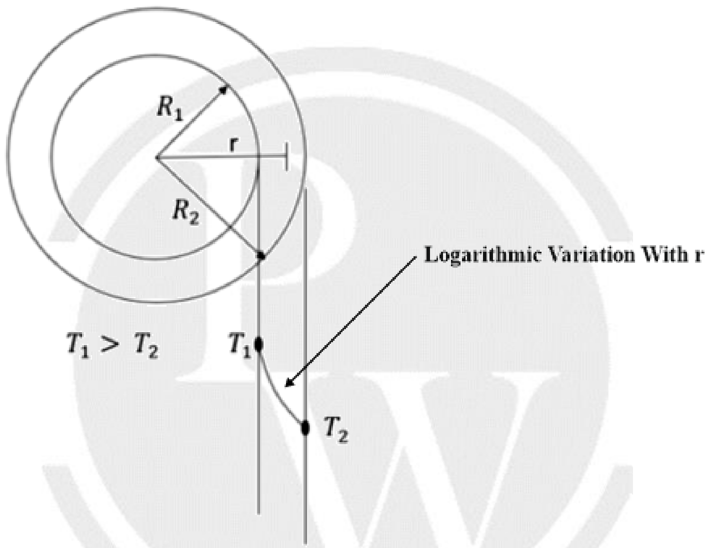

Fig 2.16 Hollow cylinder

- at radial distance r, temperature is T

Assumption –

1. Material is homogenous & isotropic
2. One dimensional (Radial)
3. No internal heat generation
4. Steady State
5. k is Constant. $k \neq f(T)$

- Temperature Distribution

$$\frac{T(r)-T_1}{T_2-T_1} = \frac{\ln(r/R_1)}{\ln(R_2/R_1)} \text{ (Logarithmic temperature variation with } r\text{)}$$

- Thermal resistance in case of a hollow cylinder

$$R_{th} = \frac{\ln(R_2/R_1)}{2\pi kL}$$

- Conduction heat transfer through hollow cylinder

$$\dot{Q} = \frac{T_1 - T_2}{\frac{\ln(R_2/R_1)}{2\pi kL}}$$

2.6.1 Hollow Cylinder with Convection Boundary Condition

- $\dot{Q} = \frac{T_h - T_1}{(R_{th})_1} = \frac{T_1 - T_2}{(R_{th})_2} = \frac{T_2 - T_a}{(R_{th})_3} = \frac{T_h - T_a}{(R_{th})_{eq}}$

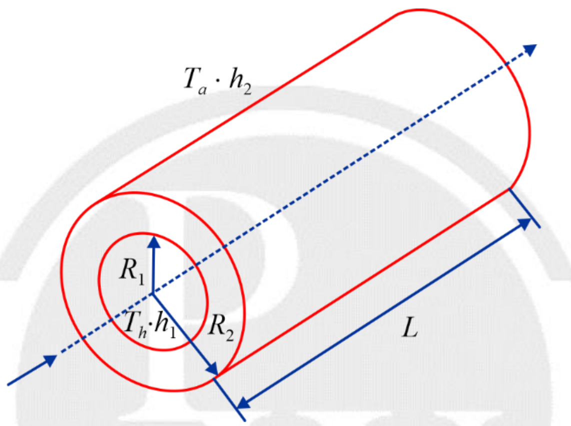

Fig 2.17 Hot fluid carrying hollow cylinder exposed to the ambient

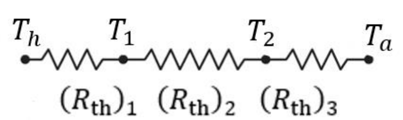

Fig 2.18 Thermal resistance diagram for hollow cylinder

- $(R_{th})_{eq} = (R_{th})_1 + (R_{th})_2 + (R_{th})_3$

Where

$$(R_{th})_1 = \frac{1}{h_1 \cdot A_1} = \frac{1}{h_1 \cdot 2\pi R_1 L}$$

$$(R_{th})_2 = \frac{\ln(R_2/R_1)}{2\pi kL}$$

$$(R_{th})_3 = \frac{1}{h_2 \cdot A_2} = \frac{1}{h_2 \cdot 2\pi R_2 L}$$

2.6.2 Equivalent Area for hollow cylinder

- In case of hollow cylinder equivalent area is equal to logarithmic mean area of A_1 *and* A_2

$$A = \frac{A_2 - A_1}{\ln(A_2/A_1)}$$

2.6.3 Solid Cylinder with Uniform Internal Heat Generation: -

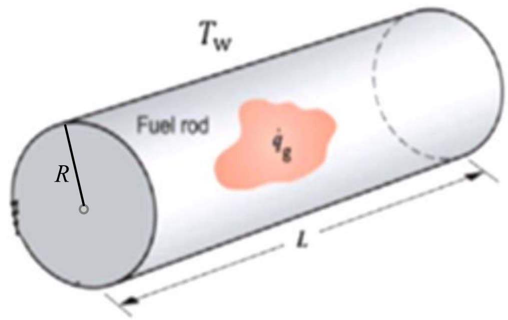

Fig 2.19 Solid cylinder with uniform heat generation

Assumption

- 1-D (Radial Direction)
- Steady State
- Uniform Heat generation
- k is constant

$$T(r) = T_w + \frac{\dot{q}_g}{4k}(R^2 - r^2) \text{ (Parabolic Variation)}$$

At r = 0, T = T_{Max}

$$T_{Max} = T_w + \frac{\dot{q}_g}{4k} R^2$$

2.7 General Heat Conduction Equation in Spherical Coordinate System

T (r, θ , ϕ, τ)

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(k_r r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(k_\phi \frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k_\theta \sin\theta \frac{\partial T}{\partial \theta}\right) + \dot{q}_g = \frac{\partial}{\partial \tau}(\rho c T)$$

If material properties k, ρ & c are constant then

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{\dot{q}_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

Poisson Equation
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{\dot{q}_g}{k} = 0$$

Laplace Equation
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) = 0$$

Diffusion Equation
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

2.8 Conduction Heat Transfer through Hollow Sphere

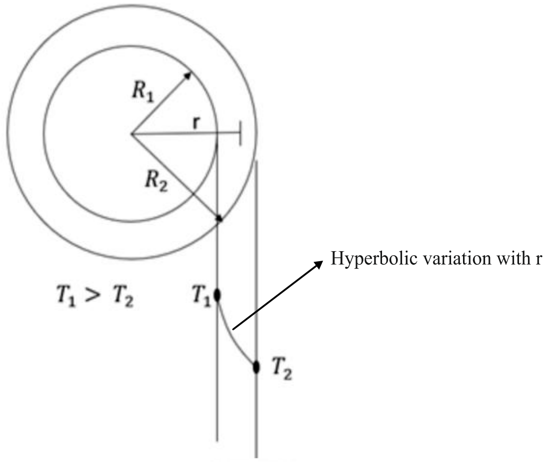

Fig 2.20 Hollow Sphere

Assumption:

- 1D (Radial Direction)
- Steady State
- No internal heat generation
- k is constant

$$\frac{T(r)-T_1}{T_2-T_1} = \frac{\frac{1}{r}-\frac{1}{R_1}}{\frac{1}{R_2}-\frac{1}{R_1}} \quad \text{(Hyperbolic variation with r)}$$

- Thermal Resistance in case of Hollow Sphere

$$(R_{th}) = \frac{(R_2 - R_1)}{4\pi\,k\,R_1 R_2}$$

- Conduction Heat Transfer through Hollow Sphere

$$\dot{Q} = \frac{(T_1 - T_2)}{\frac{R_2 - R_1}{4\pi\,k\,R_1 R_2}}.$$

2.8.1 Hollow Sphere with Convection boundary Condition

- Heat transfer rate $\dot{Q} = \frac{T_h - T_1}{(R_{th})_1} = \frac{T_1 - T_2}{(R_{th})_2} = \frac{T_2 - T_a}{(R_{th})_3} = \frac{T_h - T_a}{(R_{th})_{eq}}$

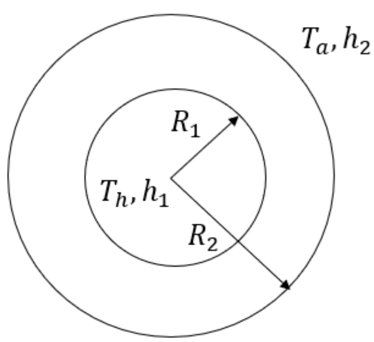

Fig 2.21 Hot fluid carrying hollow sphere exposed to the ambient

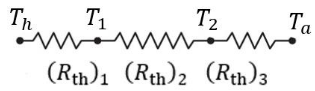

Fig 2.22 Thermal Resistance Diagram for Hollow Sphere

- $(R_{th})_{eq} = (R_{th})_1 + (R_{th})_2 + (R_{th})_3$

  Where
$$(R_{th})_1 = \frac{1}{h_1 \cdot A_1} = \frac{1}{h_1 \cdot 4\pi R_1^2}$$
$$(R_{th})_2 = \frac{(R_2 - R_1)}{4\pi \, k \, R_1 R_2}$$
$$(R_{th})_3 = \frac{1}{h_2 \cdot 4\pi R_2^2}$$

2.8.2 Equivalent Area for Hollow Sphere

- In case of hollow sphere equivalent area is the geometric mean of A_1 *and* A_2 .

$$A = \sqrt{A_1 . A_2}$$

2.8.3 Solid Sphere with uniform internal heat Generation

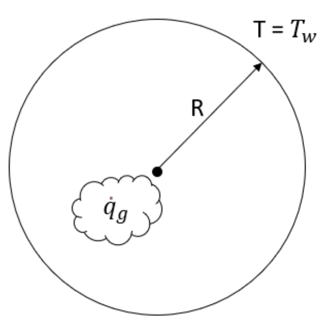

Fig 2.23 Solid sphere with uniform heat generation

Assumption

- 1D (Radial direction)
- Steady State
- Uniform Internal Heat Generation
- k is constant

$$\boxed{T(r) = T_w + \frac{\dot{q}_g}{6k}(R^2 - r^2)}$$ (Parabolic Variation)

At r = 0, T = T_{max}

$$T_{max} = T_w + \frac{\dot{q}_g}{6k} R^2$$

2.9 The Overall Heat Transfer Coefficient

- The heat transfer rate from hot fluid to cold fluid through plane wall is given by

$$\dot{Q} = \frac{T_h - T_c}{\left(\frac{1}{h_h A}\right) + \left(\frac{\delta}{kA}\right) + \left(\frac{1}{h_c A}\right)}$$

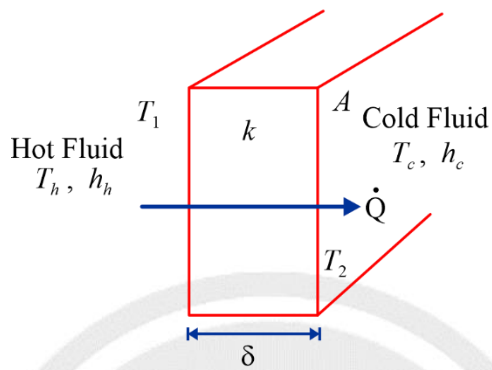

Fig. 2.24 Plane wall

- $\dot{Q}$ can be written in analogous manner to newton's law of cooling as

$$\dot{Q} = UA\,(T_h - T_c)$$

Where $U \rightarrow$ overall heat transfer coefficient $\left(\frac{W}{m^2K}\right)$

$$U = \frac{1}{\frac{1}{h_h} + \frac{\delta}{k} + \frac{1}{h_c}}$$

- Similarly, for hollow cylinder and sphere

$$\dot{Q} = U_i A_i (T_h - T_c) = U_o A_o (T_h - T_c)$$

Where $U_i \rightarrow$ Overall heat transfer coefficient on the basis of inner area.
$U_o \rightarrow$ Overall heat transfer coefficient on the basis of outer area.

2.10 Critical Radius of Insulation

- The critical radius of insulation is the radius at which heat transfer is maximum or thermal resistance minimum.

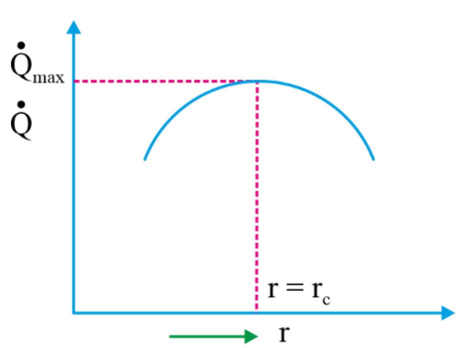

Fig. 2.25 Heat transfer variation with insulation radius

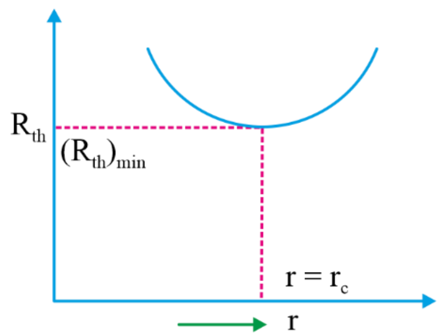

Fig. 2.26 Thermal resistance variation with insulation radius

- In case of cylinder

$$r_c = \frac{k_i}{h}$$

- In case of sphere

$$r_c = 2\frac{k_i}{h}$$

Where k_i = Thermal conductivity of insulating material.

h = Convective heat transfer coefficient

Note:

- Critical thickness: $t_c = r_c - r_o$
  (r_o = outer radius of cylinder/sphere)
- Critical diameter: $d_c = 2r_c$

- With increase in Insulation, total resistance first decreases, attain a minimum value and then increases OR with increase in insulation, rate of heat transfer first increases, attain a maximum value then start to decreases.
- If the outer radius is less than critical radius the heat transfer will be increased by adding more insulation and if outer radius is greater than the critical radius, an increase in insulation thickness will result in a decrease in heat transfer.
- No concept of critical thickness of insulation when insulation is provided inside the pipe or sphere, because providing insulation inside always decreases the heat transfer.
- Similarly, no concept of critical thickness of insulation when insulation is provided in a slab, because providing insulation always decrease the heat transfer.

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3 FINS

3.1 Introduction

- Fins are extended surfaces which are used to enhance the total heat transfer rate between the solid surface and the surrounding fluid

3.2 Fin Analysis

Assumptions

(i) Fin material is homogenous and isotropic

(ii) 1 D (x direction)

(iii) Steady state

(iv) No internal heat generation

(v) k = constant

(vi) Cross sectional area is uniform

(vii) Radiation heat transfer is neglected

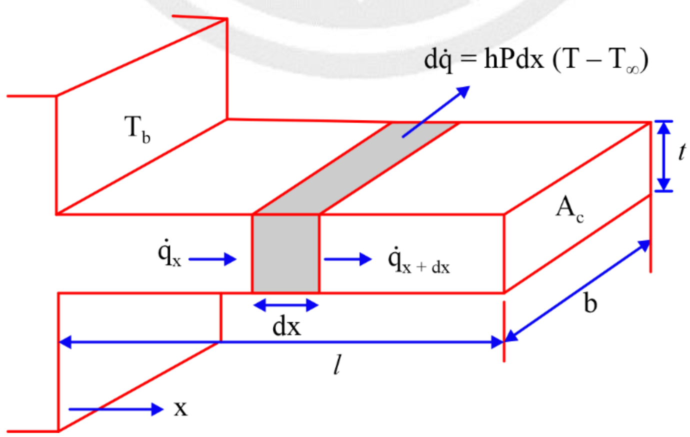

Fig 3.1 Rectangular fin of uniform cross section

Let,

k → Thermal conducting of fin material

h → Convective heat transfer coefficient

$A_c \rightarrow$ Cross sectional area

$A_s \rightarrow$ Surface area

$P \rightarrow$ Perimeter

$T_\infty \rightarrow$ Ambient temperature

$T_b \rightarrow$ Base temperature or root temperature.

$T \rightarrow$ Temperature at a distance x

$\theta = T - T_\infty,\ \theta_b = T_b - T_\infty$

Energy Balance,

$$E_{in} = E_{out}$$

$$\dot{q}_x = \dot{q}_{x+dx} + d\dot{q}_{convection}$$

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta = c_1 e^{mx} + c_2 e^{-mx}$$

Where $m = \sqrt{\frac{hP}{kA_c}}$ and $\theta = T - T_\infty$

3.3 Types of Fins

- Infinitely long fin
- Short Fin with insulated tip
- Short fin without insulated tip

3.3.1 Infinitely long fin (mL > 5)

- Temperature distribution $\frac{\theta}{\theta_b} = \frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}$ (Temperature exponentially decreases with x)
- Heat transfer rate $\dot{Q}_{fin} = -k A_c \left.\frac{dT}{dx}\right|_{x=0}$

$$\dot{Q}_{fin} = \theta_b\sqrt{hPkA_c}$$

3.3.2 Adiabatic or Insulated tip at the fin end

- Temperature distribution

$$\frac{\theta}{\theta_b} = \frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh(mL)}$$ (Temperature exponentially decreases with x)

- Heat transfer rate $\dot{Q}_{fin} = -k A_c \left.\frac{dT}{dx}\right|_{x=0}$

$$= \sqrt{hPkA_c}\,\theta_b \tanh(mL)$$

3.3.3 Convection at the fin end – Using concept of corrected length

- Corrected length $L_c = L + \frac{A_c}{P}$
- Temperature distribution $\frac{\theta}{\theta_b} = \frac{\cos h\, m(L_c - x)}{\cos h(mL_c)}$
- Heat transfer rate $\dot{Q}_{fin} = -k A_c \left.\frac{dT}{dx}\right|_{x=o}$

$$\dot{Q}_{fin} = \sqrt{hPkA_c}\,\theta_b \tan h(mL_c)$$

3.4 Fin Performance parameter

3.4.1 Fin efficiency (η_{fin})

- It is ratio of actual rate of heat transfer through the fin $(\dot{Q}_{act})$ to Maximum rate of heat transfer $(\dot{Q}_{max})$, if entire fin surface is maintained at the base temperature.
- Fin efficiency $\eta_{fin} = \frac{\dot{Q}_{act}}{\dot{Q}_{max}}$

$$= \frac{\dot{Q}_{fin}}{hA_s(T_b - T_\infty)}$$

Where A_S is the surface area of the fin

- $\eta_{fin_\infty} = \frac{1}{mL}$
- $\eta_{fin_{adia}} = \frac{\tanh(mL)}{mL}$
- $\eta_{fin_{non-adiabatic\ tip}} = \frac{\tanh(mLc)}{mLc}$

3.4.2 Fin effectiveness (ε)

- It is ratio of rate heat transfer through the fin $(\dot{Q}_{fin})$ to rate of heat transfer without fin $(\dot{Q}_{without\ fin})$.

$$(\epsilon) = \frac{\dot{Q}_{fin}}{\dot{Q}_{without\ fin}}$$

- $\epsilon_\infty = \sqrt{\frac{kP}{hA_c}}$
- $\epsilon_{adia} = \sqrt{\frac{kP}{hA_c}} \tan h(mL)$

- $\epsilon_{\text{non-adiabatic tip}} = \sqrt{\frac{kP}{hA_c}} \tanh(mL_c)$

- The effectiveness of a fin is increased by

  (i) Using material having high thermal conductivity
  (ii) Low convective heat transfer coefficient
  (iii) Thin and closely packing of fins

3.4.5 Relation Between effectiveness (ε) and efficiency (η)

$$\frac{\text{Fin effectiveness}}{\text{Fin efficiency}} = \frac{A_s}{A_c}$$ (A_s – Surface area, A_c = cross sectional area)

Note:

1. If h = mk, $\dot{Q}_{fin} = \dot{Q}_{w/o\,fin}$ No use of fin
2. If h > mk, $\dot{Q}_{fin} < \dot{Q}_{w/o\,fin}$ Fin acts like insulator i.e., using fin decreases the heat transfer
3. If h < mk, $\dot{Q}_{fin} > \dot{Q}_{w/o\,fin}$ Using fin increases the heat transfer.

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4 TRANSIENT CONDUCTION OR UNSTEADY STATE HEAT CONDUCTION

4.1 Introduction

- To solve the problems of unsteady state conduction heat transfer 3 methods are used.

  (1) Lumped parameter analysis method

  (2) Heisler Chart Method

  (3) Error function Method

4.2 Biot Number

- It is a dimensionless number which is ratio of internal conduction resistance (ICR) within a solid body to external convection resistance (ECR) between the surface of solid and fluid media.

$$Bi = \frac{\text{Internal Conduction Resistance (ICR)}}{\text{External Convection Resistance (ECR)}} = \frac{\frac{L_c}{k}}{\frac{1}{h}} = \frac{hL_c}{k}$$

- L_c is the characteristic length of body

$$L_c = \frac{\text{Volume}}{\text{Surface area responsible for heat transfer}}$$

4.3 Lumped Parameter analysis method

- Lumped parameter analysis method is used where ICR is very small as compare to ECR hence Bi < 0.1
- Lumped parameter analysis method is applicable where

  (i) h is very less

  (ii) L_c is very less

  (iii) k is very high (infinite thermal conductivity problems/ Negligible temperature gradient Problem)

Note:

- Since negligible temperature gradient exist therefore temperature is only function of time within the solid. T = *f*(time)

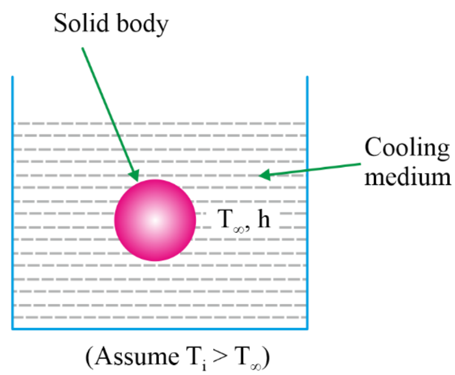

Fig 4.1 Cooling of a hot solid body

Let,

m = mass of the solid body

ρ = density of solid body

c = specific heat of solid body

k = Thermal conductivity of solid

V = Volume of solid body

A = Surface area of solid body

T_∞ = Temperature of fluid

T_i = initial temperature of solid body

T = Temperature of solid body at any time τ

- (Neglect the effect of Radiation)
  From energy balance, at any time τ
  Rate of change of internal energy of solid body = Rate of convection heat transfer

$$-mc\frac{dT}{d\tau} = hA(T - T_\infty)$$

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA}{\rho Vc}\tau} = e^{-B_i F_o} = e^{-\frac{\tau}{\tau^*}}}$$

(Applicable for both heating as well as cooling)

Where

Bi – Biot Number $\left(Bi = \frac{hL_c}{k}\right)$

F_o – Fourier number $\left(F_o = \frac{\alpha\tau}{L_c^2}\right)$,

$\alpha \rightarrow$ Thermal diffusivity $\left(\frac{k}{\rho c}\right)$

τ^* – Thermal time constant $\left(\tau^* = \frac{\rho Vc}{hA}\right)$

- Temperature of solid body exponentially decreases with time or exponentially increases (when body is cool and medium is hot.

$\frac{dT}{d\tau}$ = Rate of heating or cooling

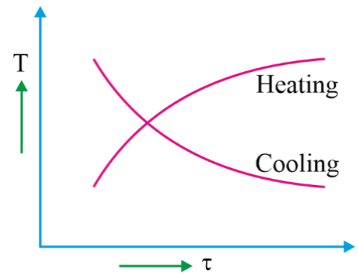

Fig 4.2 Transient temperature response of lumped capacitance solids for heating and cooling

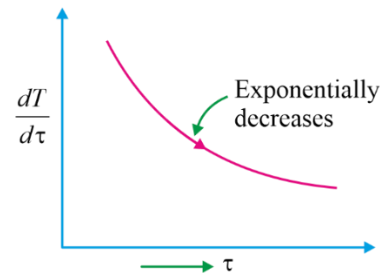

Fig 4.3 Rate of heating or cooling variation with time for lumped capacitance solids

4.3.1 The Characteristic Length (L_c)

$L_c = \frac{\text{Volume (V)}}{\text{Surface area exposed to surrounding (A)}}$	
Characteristic Length for sphere	$L_c = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$
Characteristic Length for solid cylinder	$L_c = \frac{\pi R^2 l}{2\pi R(l+R)}$ if $l >> R, L_c = \frac{R}{2}$
Characteristic Length for cube	$L_c = \frac{L^3}{6L^2} = \frac{L}{6}$
Characteristic Length for rectangular plate	$L_c = \frac{lbt}{2(lb+bt+lt)}$ if $l, b \gg t$ then, $L_c = \frac{t}{2}$
Characteristic Length for hollow cylinder	$L_c = \frac{\pi\left(r_0^2 - r_i^2\right)l}{2\pi r_0 l + 2\pi r_i l + 2\pi\left(r_0^2 - r_i^2\right)}$

4.3.2 Thermal Time Constant or Time Constant

- $\tau^* = \frac{\rho Vc}{hA}$
- Shorter time constant means body respond to temperature change very fast.
- Time constant for thermocouple must be small.

Shorter Time Constant can be achieved by

- Decreasing V/A
- Using low density and low specific heat materials.
- Increasing the heat transfer coefficient

4.4 Heisler Chart Method

- Heisler Chart Method is used to determine the temperature variation and heat flow rate when both conduction and convection resistance are almost of equal importance ($0.1 < Bi < 100$)

4.5 Error Function Method

- Error Function Method is used to determine the temperature variation and heat flow rate when conduction resistance is very high as compare to convection resistance ($Bi > 100$).

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5 CONVECTION HEAT TRANSFER

5.1 Newton's Law of Cooling:

- Convection heat transfer between a surface and fluid is governed by Newton's law of cooling.

$$\dot{Q} = hA(T_s - T_\infty)$$

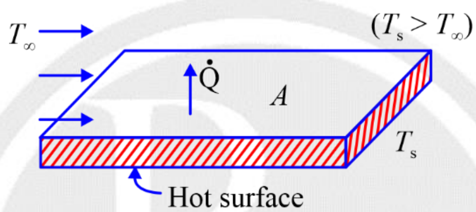

Fig 5.0 Convection heat transfer between hot surface and fluid

Where, $\dot{Q} =$ Rate of convection heat transfer

$A =$ Area exposed to heat transfer

$T_s =$ Surface temperature

$T_\infty =$ Fluid temperature

h = Coefficient of convective heat transfer

5.2 Coefficient of Convective Heat Transfer (h)

- It is defined as amount of heat transfer for a unit temperature difference between the fluid and unit area of surface in unit time.
- The unit of h

$$h = \frac{\dot{Q}}{A(T_s - T_\infty)} \quad W/m^2{}^\circ C \quad \text{or} \quad W/m^2 K$$

- The value of ' $\boldsymbol{h}$ ' depends on the following factors
  (i) Thermodynamic and transport properties (e.g., viscosity. density, specific heat etc.)
  (ii) Nature of convection (Free/forced/mixed)
  (iii) Nature of fluid flow (Laminar/Turbulent)
  (iv) Type of fluid (Liquid/Gas)
  (v) Surface finish of plate (Smooth/Rough)
  (vi) Orientation of surface (Horizontal/Vertical/Inclined)

Type of convection	Range of 'h' (W/m^2-K)
Free convection in gases	3 – 25
Forced convection in gases	25 – 400
Free convection in liquids	50 – 350
Forced convection in liquids	350 – 3000
Condensation	3000 – 25000
Boiling	5000 – 50000

5.3 Criteria for Convection Heat Transfer

- Forced Convection ($Gr/Re^2 << 1$)

  $$Nu = f(Re, Pr)$$

- Free Convection or Natural Convection ($Gr/Re^2 >> 1$)

  $$Nu = f(Gr, Pr)$$

- Mixed Convection ($Gr/Re^2 = 1$)

  $$Nu = f(Re, Pr, Gr)$$

  Where

  Nu = Nusselt number

  Re = Reynolds number

  Pr = Prandtl number

  Gr = Grashof number

5.4 Important Dimensionless numbers used in Convection Heat Transfer

5.4.1 Nusselt Number (Nu)

1. It is ratio of conduction resistance to convection resistance offered by the fluid.

$$Nu = \frac{(R_{th})_{\text{conduction}}}{(R_{th})_{\text{convection}}}$$

2. Nusselt number also defined as ratio of convection heat transfer rate to the conduction heat transfer rate in fluid.

$$Nu = \frac{\dot{Q}_{\text{convection}}}{\dot{Q}_{\text{conduction}}}$$

$$Nu = \frac{hL_c}{k_f}$$

h = convective heat transfer coefficient
Lc = Characteristic length
k_f = thermal conductivity of fluid.

3. Larger Nusselt number means move effective convection heat transfer.

5.4.2 Reynold Number (Re)

- It is ratio of Inertia force to Viscous force.

  Mathematically,

$$Re = \frac{\rho V L_c}{\mu} = \frac{V L_c}{\nu}$$

where ρ = Density of fluid (kg/m^3)

V = Velocity of Fluid (m/s)

L_c = Characteristics length (m)

μ = Viscosity of fluid (Pa-s)

ν = kinematic viscosity (m^2/s)

5.4.3 Prandtl Number (Pr)

- It is ratio of kinematic viscosity (momentum diffusivity) to the thermal diffusivity.

  Mathematically

$$Pr = \frac{\nu}{\alpha} = \frac{\mu c_p}{k_f}$$

Where μ = Viscosity of fluid (Pa-s)

c_p = Specific heat at constant pressure (J/kgK)

k_f = Thermal conductivity of fluid (W/mK)

5.4.4 Grashof Number (Gr)

- It is ratio of Buoyant force to viscous force.

  Mathematically

$$Gr = \frac{\text{Buoyancy force}}{\text{Viscous force}}$$

$$= \frac{g\beta(T_s - T_\infty)L_c^{\,3}}{v^2}$$

Where g = Gravitational acceleration (m/s^2)

β = Coefficient of volume expansion (K^{-1})

T_s = Temperature of surface (K)

T_∞ = Free stream temperature (K)

L_c = Characteristic length of the geometry (m)

v = Kinematic viscosity of the fluid (m^2/s)

5.5 Forced Convection

1. External forced Convection
   (a) Constant Wall Temperature
   (b) Constant heat flux

2. Internal forced Convection
   (a) Constant Wall Temperature
   (b) Constant heat flux

5.5.1 External Forced Convection: Constant Wall Temperature: $T_s \neq f(x)$ = Constant

T_s = Plate temperature/wall temperature

T_∞ = Free stream temperature

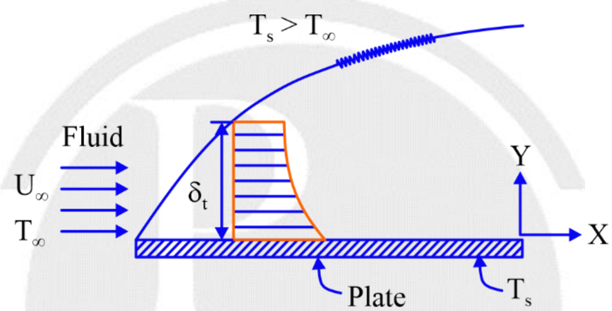

Fig 5.1 Thermal boundary layer for flow over a hot isothermal flat plate

Energy balance

At y = 0

Rate of conduction heat transfer in fluid = Rate of convection heat transfer

$$-k_f A \left.\frac{\partial T}{\partial y}\right|_{y=0} = hA(T_s - T_\infty)$$

$$\Rightarrow \qquad h = \frac{-k_f \left.\dfrac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}$$

Local Nusselt number $\qquad Nu_x = \dfrac{h_x L_c}{k_f}$

$$\Rightarrow \qquad Nu_x = \frac{-\left.\dfrac{\partial T}{\partial y}\right|_{y=0} L_c}{T_s - T_\infty}$$

- Relation between Local convective heat transfer coefficient (h_x) and average convective heat transfer coefficient ($\bar{h}$)

$$\bar{h} = \frac{1}{L}\int_0^L h_x dx$$

5.5.2 Nusselt number in case of constant wall temperature

Laminar Flow	Turbulent Flow
($Re_x \le 5 \times 10^5$)	($Re_x > 5 \times 10^5$)
• $Nu_x = 0.332\,Re_x^{0.5}\,Pr^{0.33}$ $h_x \propto x^{-1/2}$ • $\overline{Nu} = 0.664\,Re_L^{0.5}\,Pr^{0.33}$ $\bar{h} = 2h_x$	• $Nu_x = 0.029\,Re_x^{0.8}\,Pr^{0.33}$ $h_x \propto x^{-1/5}$ • $\overline{Nu} = 0.037\,Re_L^{0.8}\,Pr^{0.33}$ $\bar{h} = \frac{5}{4}h_x$

5.5.3 Nusselt Number in case of constant heat flux

Laminar Flow	Turbulent Flow
• $Nu_x = 0.453\,Re_x^{0.5}\,Pr^{0.33}$	• $Nu_x = 0.0308\,Re_x^{0.8}\,Pr^{0.33}$

Note:
- Nusselt No. & heat transfer coefficient in case of constant heat flux case is approximately 36% more as compare to constant wall temperature case. (In case of laminar flow)
- Nusselt No. & heat transfer coefficient in case of constant heat flux case is approximately 4% more as compare to constant wall temperature case (In case of turbulent flow)

5.5.4 Variation of h_x with nature of flow for a flat plate

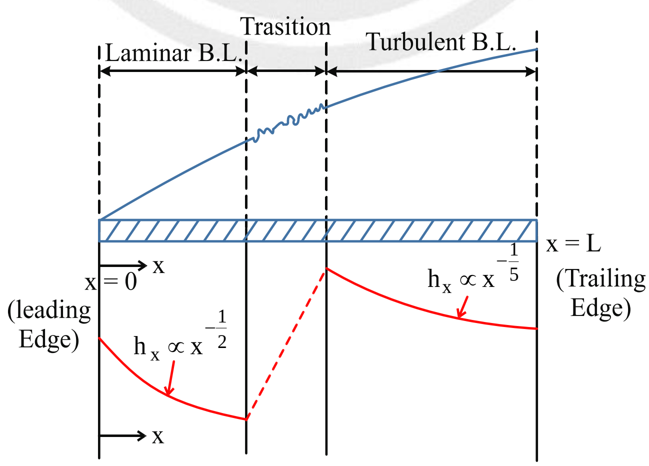

Fig 5.2 Convective heat transfer coefficient variation in flow over flat plate

Note:

Same variation occurs for local friction coefficient (C_{fx}) and for local wall shear stress (τ_{wx}).

5.5.5 Relation between Thermal Boundary Layer thickness and hydrodynamic boundary layer thickness

Laminar Flow	Turbulent Flow
$\frac{\delta_{hx}}{\delta_{tx}} = \text{Pr}^{\frac{1}{3}}$	$\delta_{hx} \approx \delta_{tx}$

Note:

In case of laminar flow 3 cases are possible,

1. $P_r < 1$, $\delta_{hx} < \delta_{tx}$ Ex: Liquid metals.
2. $P_r > 1$, $\delta_{hx} > \delta_{tx}$ Ex: heavy oils
3. $P_r = 1$, $\delta_{hx} = \delta_{tx}$ Ex: gases

5.5.7 Reynold's Analogy

- It relates local convective heat transfer coefficient with local friction coefficient.

$$\frac{Nu_x}{\text{Re}_x \,\text{Pr}} = St_x = \frac{Cf_x}{2} \,(\text{when Pr} = 1)$$

$$\left(St_x = \frac{h_x}{\rho U_\infty c_p} \right)$$

- Reynold's-Colburn Analogy

$$\text{St}_x.\text{Pr}^{2/3} = \frac{Cf_x}{2}$$

Where St_x = Local Stanton number

C_{fx} = Local skin friction coefficient

5.6 Internal Forced Convection

5.6.1 Hydrodynamic Boundary Layer Development

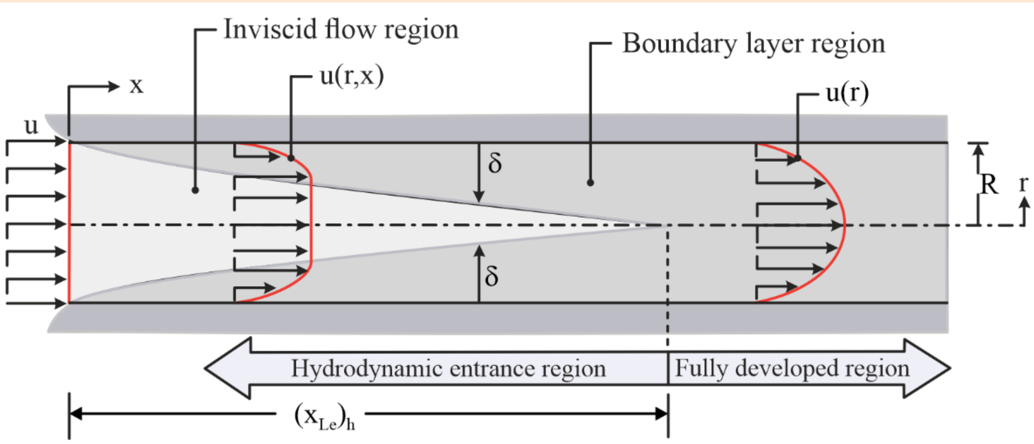

Fig 5.3 Hydrodynamic boundary layer development in internal flow

- In Hydrodynamic entrance region (Developing region) velocity of fluid layer is function of r as well as x, where as in developed region velocity is function of r only.
- In developed region $\left.\frac{du}{dx}\right|_r = 0$.

5.6.2 Thermal Boundary Layer Development

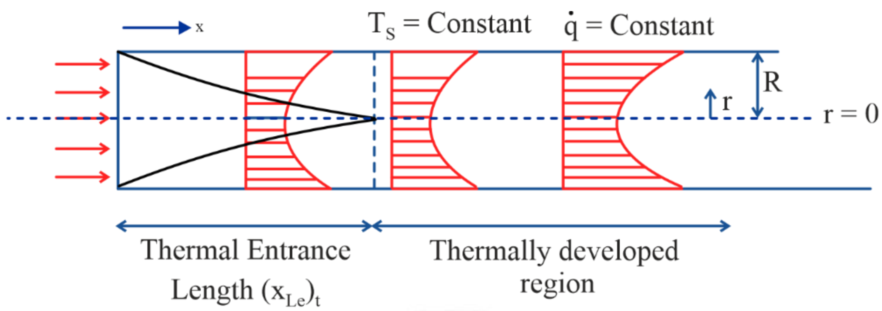

Fig 5.4 Thermal boundary layer development in internal flow

5.6.3 Thermal entrance length $(x_{Le})_t$

Laminar Flow	Turbulent flow
$(x_{Le})_t = 0.05\ \text{Re Pr D}$ $= (x_{Le})_h \text{Pr}$	$(x_{Le})_t \approx 10\ D$

- For Thermally fully developed flow, note the following important points
  - (i) h/k is a constant, h will be a constant, if k = constant
  - (ii) Nu is a constant
  - (iii) θ is not a function of x, where $\theta = \frac{T(x,r)-T_s(x)}{T_m(x)-T_s(x)}$ i.e $\frac{d\theta}{dx} = 0$
- For thermal entry region, h is inversely proportional to x

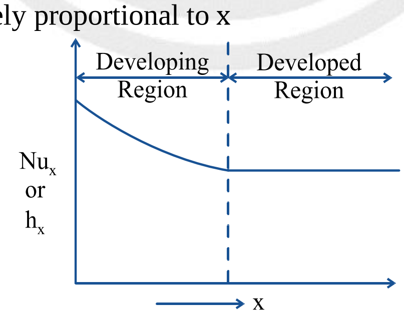

Fig 5.5 Convective heat transfer coefficient variation in internal flow

5.6.4 Average Velocity or Bulk mean velocity or Mean velocity (U_{avg})

- It is the hypothetical uniform velocity at a particular section that gives the same mass flow rate as actual velocity profile gives.

$$U_{avg} = \frac{2}{R^2}\int_0^R u(r).rdr$$

5.6.5 Bulk Mean Temperature /Average Temperature /Mixing cup temperature (T_m)

- It is hypothetical uniform temperature at a particular section that gives the same enthalpy rate as actual temperature profile gives.

$$T_m = \frac{\int_0^R u(r)T(r).rdr}{\int_0^R u(r).rdr}$$

5.6.6 Boundary Conditions for Internal flow Forced Convection

- In internal flow (L_c = D)

$$N_u = \frac{hL_c}{k_f} = \frac{h \times D}{k_f}$$

Laminar		Turbulent
Constant wall temperature $N_u = \frac{hD}{k_f} = 3.66$	Constant heat flux $N_u = \frac{hD}{k_f} = 4.364$	Ditties-Boelter Equation $Nu = \frac{hD}{k_f} = 0.023\ Re^{0.8}\ Pr^n$ $n = 0.4 \Rightarrow$ Heating $n = 0.3 \Rightarrow$ cooling

5.6.7 Mean Temperature Variation in Constant Wall Temperature Case

$T_s > T_{mi}$ (assume)

- T_s = constant, $T_s \neq f(x)$, $T_m = f(x)$
- Temperature distribution

$$\frac{T_m(x) - T_s}{T_{mi} - T_s} = e^{-\frac{hPx}{\dot{m}c_p}}$$

- Rate of heat transfer $(\dot{Q})$

$$\dot{Q} = \dot{m}c_p\left[T_{mi}(x) - T_{me}\right] = hA\left[\frac{\theta_1 - \theta_2}{\ln\frac{\theta_1}{\theta_2}}\right]$$

A = Area responsible for convection

$\theta_1 = T_s - T_{mi}$, $\theta_2 = T_s - T_{me}$

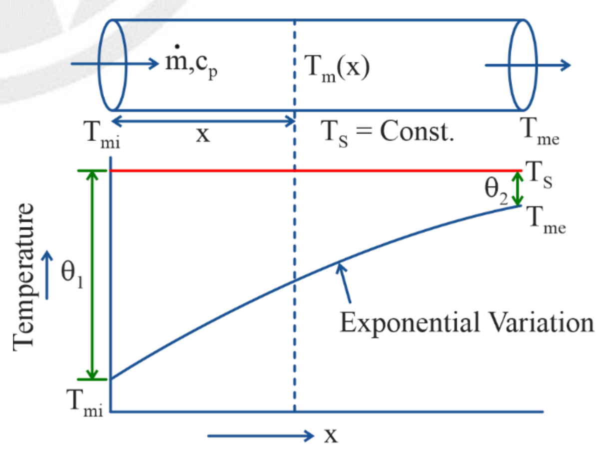

Fig 5.6 Mean temperature variation in internal flow with constant wall temperature

5.6.8 Mean Temperature Variation in Constant Heat Flux Case

$\dot{q}$ = **constant,** $\dot{q} \neq$ **f(x)**

- $\dfrac{dT_m}{dx} = 0$ (T_m varies Linearly)
- In developing region

  $h\downarrow$, $(T_s - T_m)\uparrow$
- In developed region

  h = constant, $(T_s - T_m)$ = constant

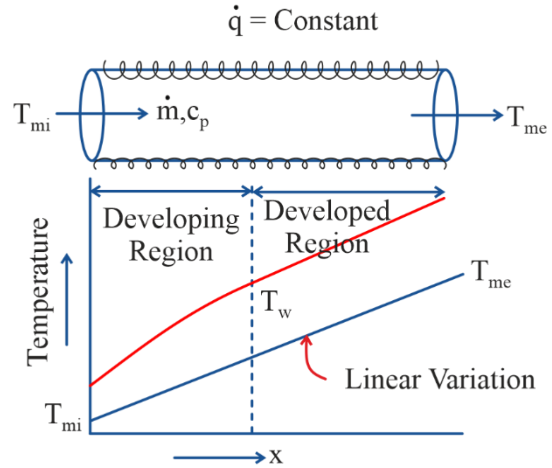

Fig 5.7 Mean temperature variation in internal flow with constant heat flux

5.7 Free or Natural Convection

- In free convection or natural convection motion of fluid particles occurs due to buoyancy which induced due to variation of density in the presence of temperature gradient.

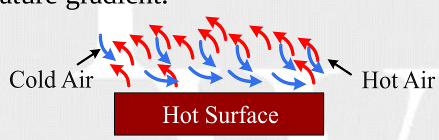

Fig 5.8 Natural convection

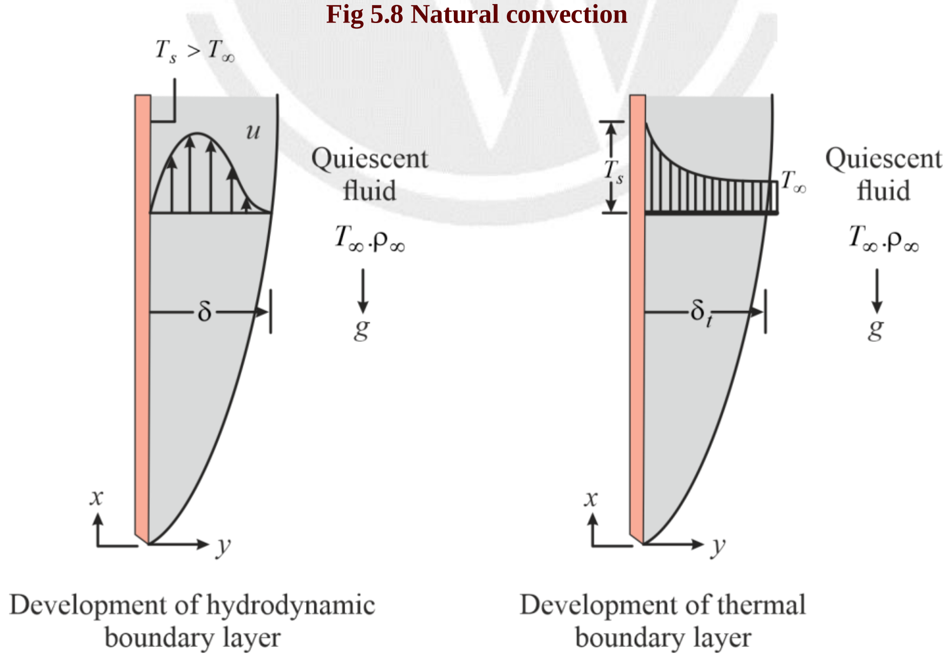

Fig 5.9 Hydrodynamic and thermal boundary layer development on a heated vertical plate.

Note:

If $\dfrac{Gr}{Re_L^2} \gg 1$, Free convection is predominant over forced convection.

5.7.1 Grashof Number (Gr)

- $Gr = \frac{\text{buoyancy force}}{\text{viscous force}} = \frac{g\beta\Delta T L_c^3}{\nu^2} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$

Where

g = Gravitational acceleration (m/s^2)

T_s = Temperature of surface (K)

T_∞ = Temperature of fluid far from the surface (K)

L_c = Characteristic length of the geometry (m)

ν = Kinematic viscosity of the fluid (m^2/s)

β = Coefficient of volume expansion or isobaric expansivity (1/K)

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

$$\beta = \frac{1}{T_f}(\text{For ideal gas})$$

$$T_f = \frac{T_s + T_\infty}{2}$$

T_f = film temperature

5.7.2 Rayleigh Number (Ra)

- Rayleigh number is product of Grashof number (Gr) and Prandtl number (Pr)

$$Ra = Gr \times Pr$$

- $10^4 < Ra < 10^9$ – Laminar Boundry layer

  $Ra > 10^9$ – Turbulent boundary layer

5.7.3 Nusselt Number Expression for Free Convection Case

- Nu = f(Gr, Pr) or Nu = f(Ra)

  $Nu = C\,Ra^{1/4}$ (Laminar)

  $Nu = C\,Ra^{1/3}$ (Turbulent)

  Where C is a Constant.

- For Vertical Plate or vertical Cylinder:

  $Nu = 0.59\,Ra^{1/4}$ (Laminar)

  $Nu = 0.13\,Ra^{1/3}$ (Turbulent)

- For Horizontal Plate/Horizontal Cylinder:

  $Nu = 0.54\,Ra^{1/4}$ (Laminar)

  $Nu = 0.14\,Ra^{1/3}$ (Turbulent)

Note:

In case of vertical cylinder characteristic length is length of cylinder where as in case of horizontal cylinder characteristic length is diameter of cylinder.

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6 HEAT EXCHANGER

6.1 Introduction

- A heat exchanger is a device which is used to transfer the heat energy from one fluid (hot fluid) to another fluid (cold fluid). The cold fluid is heated where as hot fluid gets cooled down.

6.2 Type of Heat Exchanger

6.2.1 On the basis of nature of heat exchange process:

(i) Direct contact type heat exchanger: In Direct contact type heat exchanger hot fluid and cold fluid are directly in contact with each other.

Ex. Cooling Tower, Open feed water heater, Jet Condenser

(ii) Storage type or Regenerator type Heat exchanger: Regenerator is the type of heat exchanger where heat energy from the hot fluid is intermittently stored in a thermal storage medium before it is transferred to the cold fluid. To accomplish this the hot fluid is brought into contact with the heat storage medium, then the hot fluid is replaced with the cold fluid, which absorbs the heat.

Ex. Heat exchanger used in gas turbines, Air Preheaters in steam power plant

(iii) Transfer type or Recuperator Type: Recuperator is a type of heat exchanger which has separate flow paths for each fluid and heat is transferred through the separating walls.

Ex. Concentric Tube type heat exchanger, Shell and tube type heat exchanger etc.

6.2.2 On the basis of relative direction of fluid motion:

1. Parallel flow/Co-current flow Heat Exchanger

- In parallel flow heat exchanger two fluids are moving in the same direction.

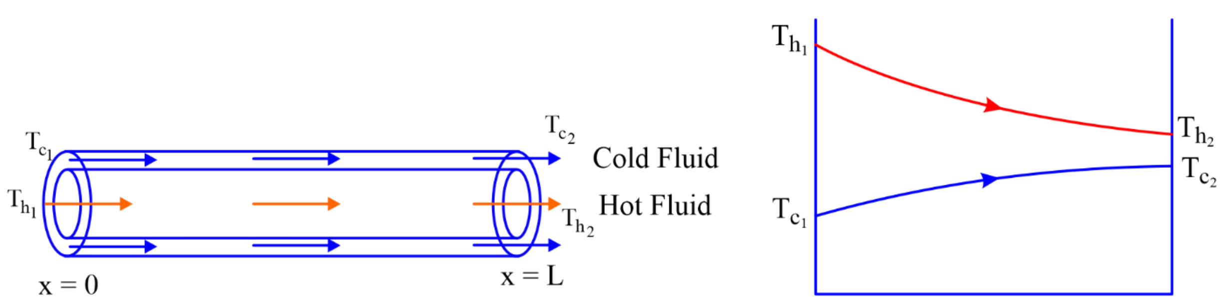

Fig. 6.1 Parallel flow heat exchanger

2. Counter-Flow Heat Exchanger

- In counter-flow heat exchanger two fluids are moving in the opposite direction.

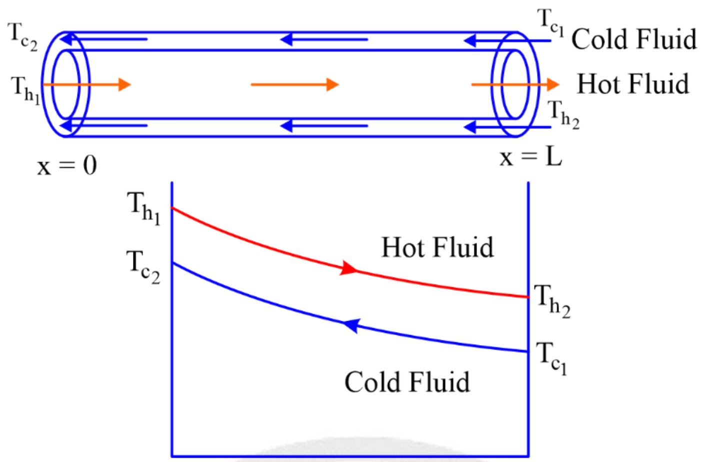

Fig. 6.2. Counter flow heat exchanger

3. Cross-Flow Heat Exchanger

- In cross flow heat exchanger two fluids are crossing at 90°

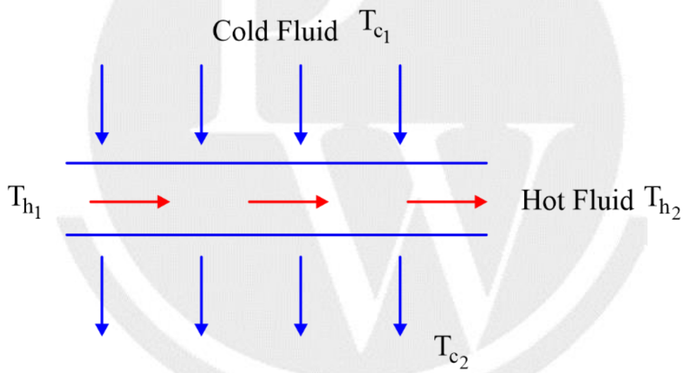

Fig. 6.3 Cross flow heat exchanger

6.2.3 On the basis of Phase change of the fluid

1. Evaporator type:

- Cold fluid gets evaporated by taking the heat energy from hot fluid.

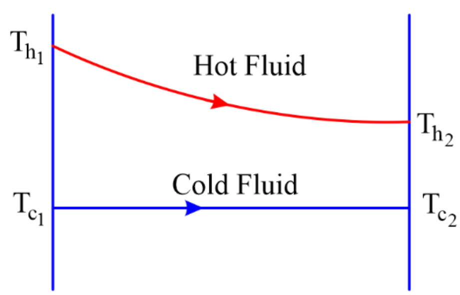

Fig. 6.4 Evaporator type Parallel Flow Heat Exchange

2. Condenser type:

- Hot fluid is condensed and releases latent heat of condensation which is absorb by the cold fluid and gets heated.

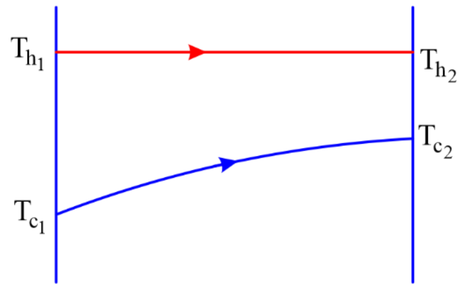

Fig. 6.5 Condenser type Parallel Flow Heat Exchange

6.3 Heat Exchanger Analysis

Let, $\dot{m}$ = Mass flow rate $\left(\frac{kg}{s}\right)$

c_p = Specific heat of fluid at constant pressure $\left(\frac{J}{kg\,°C}\right)$

T = Temperature of fluid

- Subscripts h and c refer to the hot and cold fluids respectively and 1 and 2 corresponds to the inlet and outlet conditions respectively.

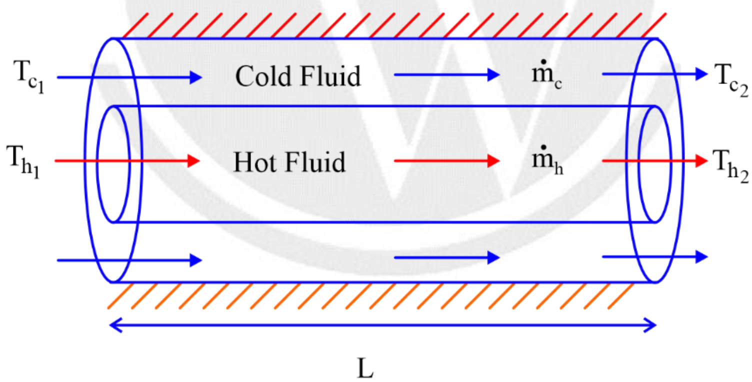

Fig. 6.6. Analysis of heat exchanger

Assumptions

- Flow is steady
- Heat Exchanger is adiabatic
- Change in Kinetic energy and potential energy are negligible.

Rate of heat energy lost by the hot fluid

$$\dot{Q} = \dot{m}_h c_{ph} (T_{h_1} - T_{h_2})$$

Rate of heat energy gained by the cold fluid

$$\dot{Q} = \dot{m}_c c_{pc} (T_{c_2} - T_{c_1})$$

Total heat transfer rate in the heat exchanger

$$\dot{Q} = UA\theta_m$$

By Energy Balance,

$$\dot{m}_c c_{pc}(T_{c_2} - T_{c_1}) = \dot{m}_h c_{ph}(T_{h_1} - T_{h_2}) = UA\theta_m$$

Where U = Overall heat transfer coefficient (W/m^2k)

A = Effective heat transfer area, and (m^2)

θ_m = Logarithmic Mean Temperature Difference (LMTD)

6.3.1 Logarithmic Mean Temperature Difference (LMTD)

- LMTD = $\dfrac{\theta_1 - \theta_2}{\ln \dfrac{\theta_1}{\theta_2}}$

Where θ_1 = Temperature difference between hot and cold fluid at x = 0

θ_2 = Temperature difference between hot and cold fluid at x = L

- For parallel flow heat exchanger

$$\theta_1 = T_{h_1} - T_{c_1}$$

$$\theta_2 = T_{h_2} - T_{c_2}$$

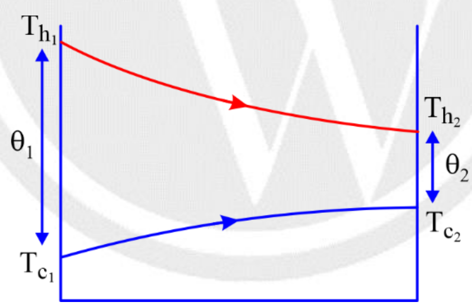

Fig. 6.7 Temperature profile for parallel flow heat exchanger

- LMTD for Counter flow heat exchanger

$$\theta_1 = T_{h_1} - T_{c_2}$$

$$\theta_2 = T_{h_2} - T_{c_1}$$

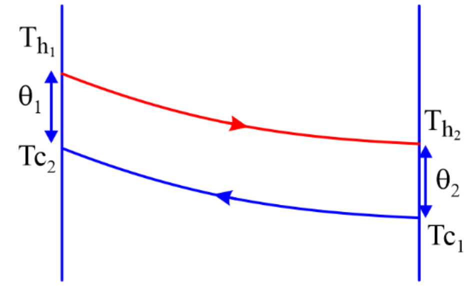

Fig. 6.8 Temperature profile for counter flow heat exchanger

- For Evaporator and condenser type heat exchanger, LMTD will be same either flow is parallel flow or counter flow.

Note:

Balance type heat exchanger

- Balanced type of heat exchanger is a counter flow heat exchanger with equal heat capacity rate.
- Temperature profile for hot fluid and cold fluid are straight line and parallel to each other.

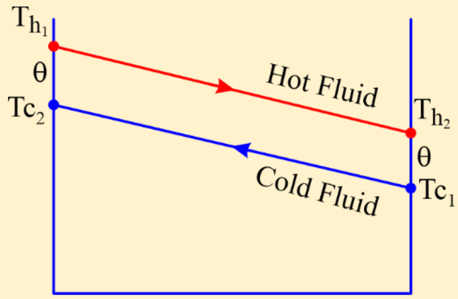

Fig. 6.8. (a) Temperature profile for balanced type heat exchanger

- For Balanced type heat exchanger.

$$\text{LMTD} = \theta_1 = \theta_2$$

6.4 Fouling Factor

- Fouling in relation to heat exchangers is the deposition and accumulation of unwanted material such as scale, suspended solids, insoluble salts and even algae on the internal surfaces of the heat exchanger.
- The fouling factor represents the theoretical resistance to heat flow due to the build-up of a fouling layer on the tube surfaces of the heat exchanger.
- Mathematically $R_f = \frac{1}{U_{foul}} - \frac{1}{U_{clean}}$

Where U_{foul}: Overall heat transfer coefficient for foul surface

U_{clean}: Overall heat transfer coefficient for clean surface

6.5 Effectiveness of Heat Exchanger

- The heat exchanger effectiveness (ε) is defined as the ratio of actual heat transfer to the maximum possible heat transfer. Thus

$$\varepsilon = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}}$$

$$= \frac{\dot{Q}}{\dot{Q}_{\max}}$$

$$\varepsilon = \frac{\dot{m}_h c_{ph}\left(T_{h_1} - T_{h_2}\right)}{\left(\dot{m}c_p\right)_{\min}\left(T_{h_1} - T_{c_1}\right)} = \frac{\dot{m}_c c_{pc}\left(T_{c_2} - T_{c_1}\right)}{\left(\dot{m}c_p\right)_{\min}\left(T_{h_1} - T_{c_1}\right)}$$

6.6 Effectiveness-NTU method

6.6.1 Number of Transfer Unit (NTU)

$$\text{NTU} = \frac{\text{UA}}{(\dot{m}c_p)_{\min}}$$

Where U = Overall heat transfer coefficient (W/m^{2}K)

$\dot{m}$ = Mass flow rate (kg/s)

$\left(\dot{m}c_p\right)_{\min}$ = Minimum heat capacity rate (W/K)

6.6.2 Effectiveness (ε) of parallel/counter flow heat exchanger

$$\epsilon_{\text{parallel}} = \frac{1 - e^{-NTU(1+C)}}{1+C}$$

$$\epsilon_{\text{counter}} = \frac{1 - e^{-NTU(1-C)}}{1 - C \cdot e^{-NTU(1-C)}}$$

Where C = Heat capacity ratio

$$= \frac{\left(\dot{m}c_p\right)_{\min}}{\left(\dot{m}c_p\right)_{\max}}$$

6.6.3 Special cases

- Heat exchanger of equal heat capacity rate (C = 1)

$$\epsilon_{\text{parallel}} = \frac{1 - \exp^{(-2NTU)}}{2}$$

$$\epsilon_{\text{counter}} = \frac{\text{NTU}}{1 + \text{NTU}}$$

- For evaporator and condenser type heat exchanger (C = 0)

$$\epsilon_{\text{parallel}} = \epsilon_{\text{counter}} = 1 - \exp^{(-NTU)}$$

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7 Radiation Heat Transfer

7.1 Introduction

- Radiation is the energy emitted by matter by virtue of their own temperature due to thermal excitation of the molecules.
- Radiation is assumed to propagate in the form of electromagnetic waves. In heat transfer study, we are interested in thermal radiation.
- Thermal Radiation wavelength ranges from 0.1 μm to 100 μm. Thermal Radiation consists

  (a) Some part of UV rays – (0.1 μm – 0.4 μm)

  (b) Complete Visible lights – (0.4 μm – 0.7 μm)

  (c) Infrared rays – (0.7 μm – 100 μm)
- Thermal radiation exhibit characteristics similar to visible light.
- Thermal radiation can be reflected, refracted and are subject to scattering and absorption when they passes through medium.

7.2 Absorptivity (α), Reflectivity (ρ), Transmittivity (τ)

- Absorptivity (α) : Fraction of incident radiation absorbed.
- Reflectivity (ρ) : Fraction of incident radiation reflected.
- Transmittivity (τ) : Fraction of incident radiation transmitted.

Energy Balance

$$\dot{Q}_a + \dot{Q}_t + \dot{Q}_r = \dot{Q}_i$$

$$\frac{\dot{Q}_a}{\dot{Q}_i} + \frac{\dot{Q}_t}{\dot{Q}_i} + \frac{\dot{Q}_r}{\dot{Q}_i} = 1$$

$$\boxed{\alpha + \tau + \rho = 1}$$

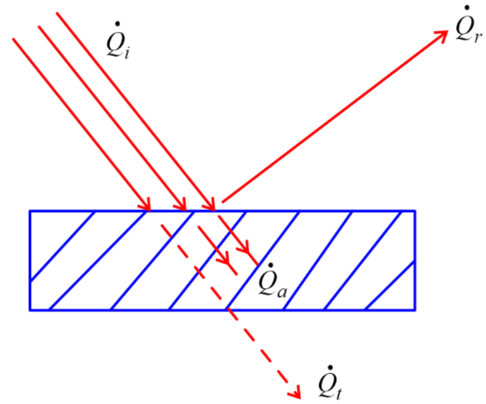

Fig 7.1 Absorption, reflection, and transmission processes associated with a semi-transparent medium.

- For black body

$$\alpha = 1, \rho = 0, \tau = 0$$

- For opaque body

$$\tau = 0, \alpha + \rho = 1$$

- For white body or perfect reflector

$$\rho = 1, \alpha = 0, \tau = 0$$

7.3 Black Body

$$\alpha = 1, \rho = 0, \tau = 0$$

- A body is said to be black if it absorbs all incident radiation.
- The term black is used, since most black coloured surfaces normally shows high value of absorptivity.
- There are some surfaces which absorb nearly all incident radiation but do not appear black.
- Therefore, we can say, black surfaces are black body but black body need not be black in colour.

  Ex. ice, snow etc.

- Black body is a perfect emitter as well as a perfect absorber.
- At a given temperature and wavelength, no surface can emit more radiation energy than a black body.
- A black body is a diffuse emitter which means it emits radiation uniformly in all direction. Also, a black body absorbs all incident radiation regardless of wavelength and direction.
- A large cavity with a small opening closely resembles to a black body.

7.4 Emissivity

7. 4. 1 Total Emissivity or Emissivity

- Emissivity of a surface is defined as the ratio of radiation emitted by the surface to the radiation emitted by a black body at the same temperature.

  Total Emissivity $(\varepsilon) = \dfrac{E(T)}{E_b(T)}$

7. 4. 2 Spectral Emissivity or Monochromatic Emissivity

- Spectral Emissivity of a surface is defined as the ratio of spectral emissive power of the surface to the spectral emissive power a black body at the same temperature.

Spectral Emissivity (ε_λ) = $\dfrac{E_\lambda(T)}{E_{b\lambda}(T)}$

7.5 Radiation Intensity (I_e)

- It is defined as the amount of energy emitted per unit solid angle by per unit area of the radiating surface.

$$I_e = \frac{E}{\pi}$$

7.6 Irradiation (G) & Radiosity(J)

7.6.1 Irradiation (G)

- Total incident radiation on a surface from all directions per unit time and per unit area of surface.

7.6.2 Radiosity (J)

- It refers to all of the radiant energy leaving a surface per unit time, per unit area of surface.

$$J = E + \rho G$$

Where

E = Total emissive power (W/m^2)

ρ = Reflectivity

G = Irradiation (W/m^2)

J = Radiosity (W/m^2)

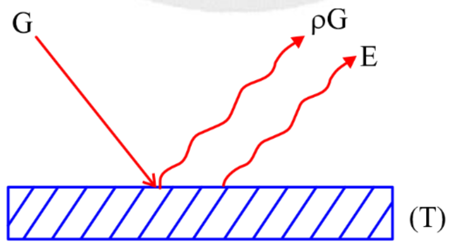

Fig 7.2 Irradiation and radiosity associated with an opaque, non-black surface

7.7 Gray Body

- A body for which emissivity is constant, is known as Gray body

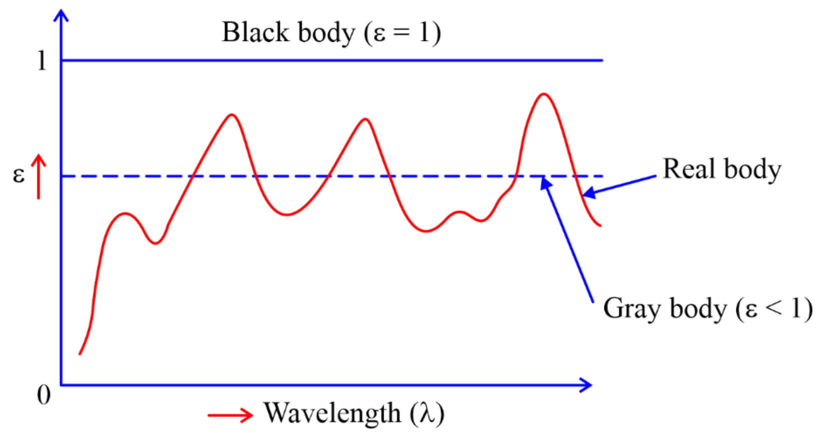

Fig 7.3 Emissivity Distribution for Black, Real and Grey body

7. 8 Various Laws used in Radiation

7. 8. 1 Planck's Law

- Planck's law is the basic law of radiation, other laws are derived from Planck's law.
- As per Planck's law for a given black body, at a given temperature monochromatic emissive power strongly depends on wavelength.

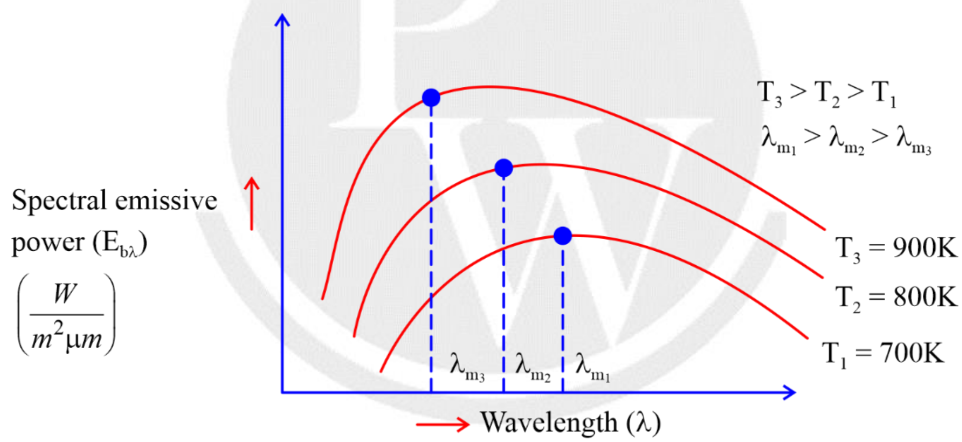

Fig 7.4 Spectral Emissive Power Distributing for a Black Body

According to this,

$$E_{b\lambda} = \frac{2\pi c^2 h\lambda^{-5}}{\exp\left(\frac{ch}{\lambda kT}\right)-1} = \frac{C_1\lambda^{-5}}{\exp\left(\frac{C_2}{\lambda T}\right)-1}$$

where,
$E_{b\lambda}$ = Monochromatic (single wavelength) emissive power of a black body,
c = Velocity of light in vacuum
h = Planck's constant
λ = Wavelength, (in μm)
k = Boltzmann constant
C_1 = 3.74×10^{-16} W-m^2, $C_2 = 1.438 \times 10^{-2}$ mK
T = Absolute temperature, K

Observation:

(i) Corresponding to particular temperature, monochromatic emissive power of a black body first increases, reaches maximum and then start to decrease.

(ii) As temperature increases, wavelength at which monochromatic emissive power is maximum shifted towards the shorter wavelength side.

(iii) At a given temperature, area under the graph ($E_{b\lambda}$ vs λ) gives the total emissive power of a black body at that temperature.

7. 8. 2 Stefan-Boltzmann law

- According to Stefan-Boltzmann law "At a particular temperature, total radiation energy emitted by a black body per unit time per unit area of all possible wavelength in all possible direction is directly proportional to fourth power of absolute temperature.

$$E_{b\lambda} = \int_0^{\infty} E_{b\lambda} \cdot d\lambda \text{ W/m}^2$$

$$E_b = \sigma_b T^4 \text{ (W/m}^2)$$

$$E_b = \sigma_b A T^4 \ (W)$$

where, σ_b = Stefan – Boltzmann constant = 5.67×10^{-8} W/m^2 K^4

7. 8. 3 Wien's Displacement Law

- It gives a relationship between the temperature of a black body and the wavelength at which the maximum value of monochromatic emissive power occurs.
- A peak monochromatic emissive power occurs at a particular wavelength.
- As temperature increases wavelength at which monochromatic emissive power is maximum, shifted towards the shorter wavelength side.

Mathematically, $\lambda_{max} T$ = Constant

$$\lambda_{max} T = 2900 \mu\text{m-K}$$

$$(E_{b\lambda})_{max} = 1.285 \times 10^{-5}\, T^5 \frac{W}{m^3}$$

$$(E_{b\lambda})_{max} \propto T^5$$

- Maximum monochromatic emissive power of a black body is proportional to the 5th power of absolute temperature.

7.8.4 Kirchhoff's Law

- As per Kirchhoff's law, the emissivity of a body is equal to its absorptivity when the body remains in thermal equilibrium with its surroundings.

$$\varepsilon = \alpha$$

7.8.5 Lambert's Cosine Law

- The law states that the intensity of radiation I_θ from a radiating plane surface in any direction is directly proportional to the cosine of the angle between the radiation emission and the normal surface vector.

$$I_\theta = I_n \cos\theta$$

Where I_n = Intensity of radiation in the direction of its normal

θ = Angle subtended by normal to the radiating surface and direction vectors of emission of the receiving surface.

- Equation is only true for a radiation surface whose radiation intensity is constant.

7. 9 Shape Factor

- The shape factor may be defined as "The fraction of radiative energy that is diffused from one surface and strikes the other surface directly with no intervening reflections.

$$F_{1-2} = \frac{\dot{Q}_{1-2}}{\dot{Q}_1}$$

where, $\dot{Q}_1$ = Rate of total energy emitted by surface 1

$\dot{Q}_{1-2}$ = Fraction of rate of energy leaving surface ' 1 ' and reaching to surface '2'.

7. 9. 1 Characteristics of shape factor

- Shape factor only depends only on geometry and orientation of the surface. Shape factor is also called view factor or geometry factor.
- Reciprocity theorem

$$A_1F_{1-2} = A_2F_{2-1}$$

- Shape factor algebra:

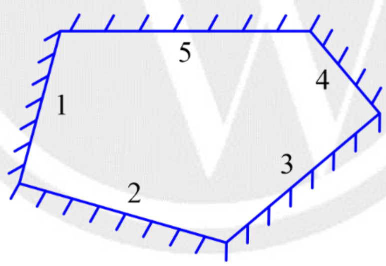

Fig 7.5 Radiation exchange in an enclosure.

$$F_{11} + F_{12} + F_{13} + \ldots\ldots\ldots\ldots\ldots\ldots = 1$$

$$F_{21} + F_{22} + F_{23} + \ldots\ldots\ldots\ldots\ldots\ldots = 1$$

$$F_{31} + F_{32} + F_{33} + \ldots\ldots\ldots\ldots\ldots\ldots = 1$$

& so on.

- Shape factor for some important surface

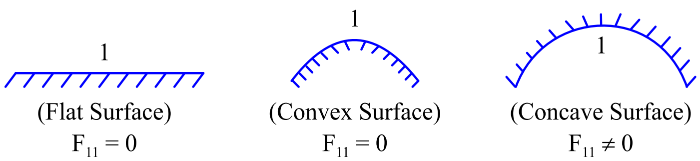

Fig 7.6 Shape factor for various surfaces

- If Emitting surface or radiating surface (1) is divided into two parts 3 and 4,

$$A_1F_{12} = A_3F_{32} + A_4F_{42}$$

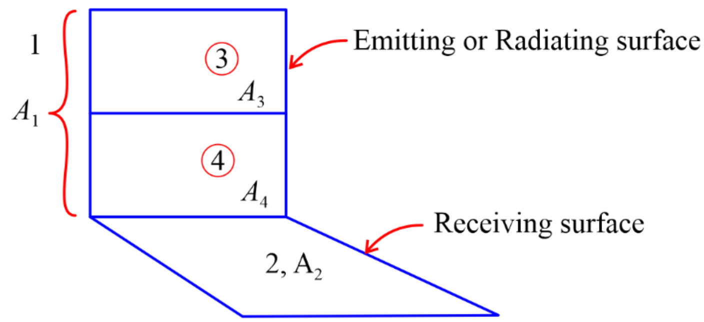

Fig 7.7 View factors for Perpendicular Rectangles with a Common Edge

- If Receiving surface (2) is divided into two parts 3 and 4,

$$F_{12} = F_{13} + F_{14}$$

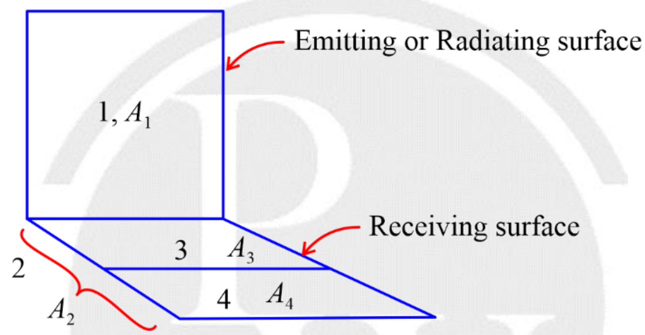

Fig 7.8 View factors for Perpendicular Rectangles with a Common Edge

7.10 Net Radiation Heat Transfer between bodies

7.10.1 Radiation Heat Exchange between two black bodies

$$\dot{Q}_{net} = A_1F_{12}\,\sigma_b\left(T_1^4 - T_2^4\right)$$
$$= A_2F_{21}\,\sigma_b\left(T_1^4 - T_2^4\right)$$

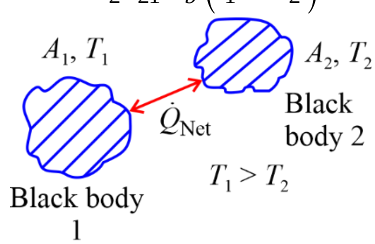

Fig 7.9 Radiation heat transfer between two black body

7.10.2 Radiation Heat exchange between two Non-black body

$$\dot{Q}_{net} = \frac{\sigma_b(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_1}{\varepsilon_2 A_2}}$$

7.10.3 Radiation Heat exchange between Two Infinite Parallel Surfaces

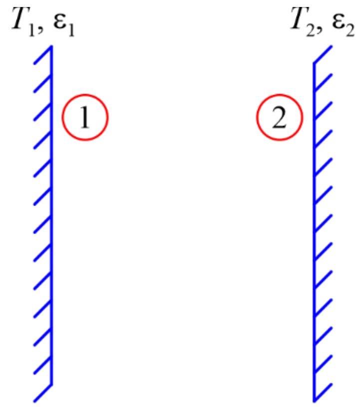

Fig 7.10 Radiation between two infinite parallel surfaces

$$\dot{Q} = A_1 (F_g)_{12} \sigma_b (T_1^4 - T_2^4)$$

$$(F_g)_{12} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot \frac{A_1}{A_2}}$$

$$F_{12} = 1, \frac{A_1}{A_2} = 1$$

$$(F_g)_{12} = \frac{1}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}$$

7.10.4 Radiation Heat exchange between two Concentric Cylinders

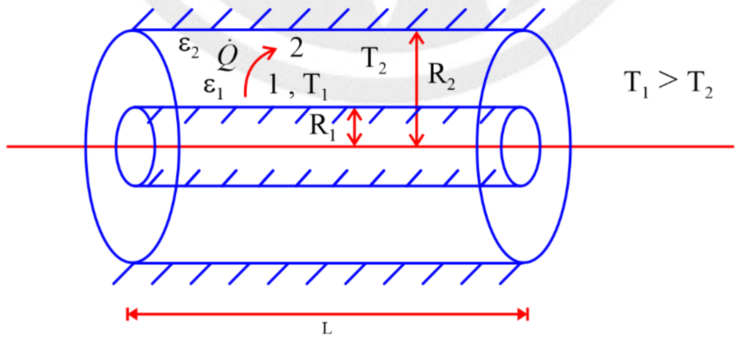

Fig 7.11 Radiation between two concentric cylinders

$$\dot{Q} = A_1 (F_g)_{12} \sigma_b (T_1^4 - T_2^4)$$

$$(F_g)_{12} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{12}} + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right) \frac{A_1}{A_2}}$$

$$F_{12} = 1, \frac{A_1}{A_2} = \frac{R_1}{R_2}$$

7.10.5 Radiation Heat exchange between two Concentric Spheres

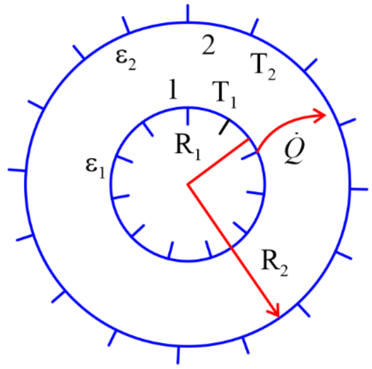

Fig 7.12 Radiation between two concentric sphere

$$\dot{Q} = A_1 (F_g)_{12} \sigma_b (T_1^4 - T_2^4)$$

$$(F_g)_{12} = \frac{1}{\frac{1-\varepsilon_1}{\varepsilon_1} + \frac{1}{F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2} \cdot \frac{A_1}{A_2}}$$

And $F_{12} = 1, \quad \frac{A_1}{A_2} = \frac{R_1^2}{R_2^2}$

7.10.6 Radiation Heat exchange between A Small Convex Object Within A Large Enclosure

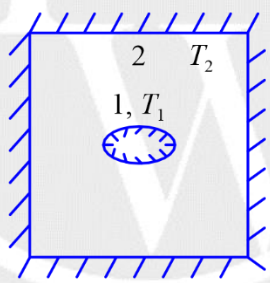

Fig 7.13 Radiation between a small body enclosed by a large body

F_{12} = 1 and $A_1/A_2 = 0$ ($A_2 >> A_1$)

And $\dot{Q} = A_1 \varepsilon_1 \sigma_b (T_1^4 - T_2^4)$

7.11 Radiation Shield Concept

- Radiation shield is used to reduce the heat transfer by increasing the resistance. Plate which is used for radiation shield should have high reflectivity and lower emissivity.

$$\left(\dot{Q}_{net}\right) \text{ with 1R.S.} = \frac{\sigma_b (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2}{\varepsilon_3} - 2} \left(\frac{\text{w}}{\text{m}^2}\right)$$

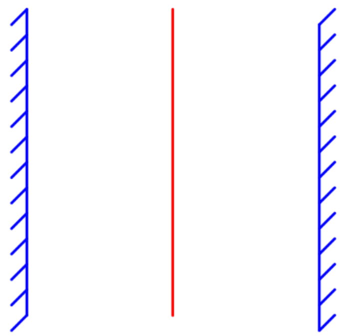

Fig 7.14 Radiation between parallel plate with radiation shield

- If N, radiation shield of different emissivity $\varepsilon_{S_1}, \varepsilon_{S_2} \cdots \varepsilon_{SN}$ are used

$$\left(\dot{Q}_{\text{Net}}\right)_{N\ \text{R.S.}} = \frac{\sigma_b (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + 2\sum_{i=1}^{N} \frac{1}{\varepsilon_{S_i}} - (N+1)} \left(\frac{\text{w}}{\text{m}^2}\right)$$

- If N radiation shield of same emissivity are used

$$\varepsilon_{S_1} = \varepsilon_{S_2} = \varepsilon_{S_N} = \varepsilon$$

$$\dot{Q}_{\text{Net}} = \frac{\sigma_b (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} + \frac{2}{\varepsilon} N - (N+1)}$$

- If shields and plate have same emissivity then

$$\left(\dot{Q}_{\text{Net}}\right)_{\text{with } N\ \text{R.S.}} = \frac{1}{N+1} \left(\dot{Q}_{\text{Net}}\right)_{\text{without R.S.}}$$

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