CBSE Class 12 Maths Notes Chapter 5: Chapter 5 of CBSE Class 12 Maths Continuity and Differentiability explain the foundational concepts of calculus. This chapter explains the idea of a function being continuous at a point, which means that the function doesn't have any abrupt breaks or jumps at that point.

The chapter also covers differentiability which deals with the ability to find a derivative of a function at any given point. Important concepts include the relationship between continuity and differentiability, derivatives of various types of functions and important theorems like Rolle's Theorem and the Mean Value Theorem. Mastering these topics helps students build a strong understanding of calculus, essential for solving advanced mathematical problems.

CBSE Class 12 Maths Notes Chapter 5 Continuity and Differentiability Overview

These notes for CBSE Class 12 Maths Chapter 5 Continuity and Differentiability have been prepared by subject experts of Physics Wallah.

They provide a clear and simple explanation of the important concepts such as continuity of functions, differentiability and the relationship between the two. These notes will support you in mastering the chapter and performing well in your exams.

CBSE Class 12 Maths Notes Chapter 5 Continuity and Differentiability PDF

The PDF for CBSE Class 12 Maths Notes Chapter 5 Continuity and Differentiability is available below.

The notes are designed to help you grasp the chapter's topics easily and improve your problem-solving skills. Click the link below to access the PDF and strengthen your preparation for the exams.

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Here we have provided CBSE Class 12 Maths Notes Chapter 5 Continuity and Differentiability-

Introduction

Continuity and Differentiability are fundamental concepts in calculus, forming the backbone of many mathematical applications. Continuity refers to a function that is smooth and unbroken over its domain, without any gaps or jumps.

Differentiability, on the other hand, deals with the rate of change of a function and focuses on finding the derivative, or slope, at any given point. Together, these concepts help in understanding the behavior of functions and solving complex problems involving rates of change, motion, and optimization.

Continuity at a Point

A function f(x) is said to be continuous at a point x = a, if Left hand limit of f(x) at(x = a) = Right hand limit of f(x) at (x = a) = Value of f(x) at (x = a)
i.e. if at x = a, LHL = RHL = f(a)
where, LHL = limx→a–f(x) and RHL = limx→a+f(x)
Note: To evaluate LHL of a function f(x) at (x = o), put x = a – h and to find RHL, put x = a + h.

Continuity in an Interval

A function y = f(x) is said to be continuous in an interval (a, b), where a < b if and only if f(x) is continuous at every point in that interval.

- Every identity function is continuous.
- Every constant function is continuous.
- Every polynomial function is continuous.
- Every rational function is continuous.
- All trigonometric functions are continuous in their domain.

Algebra of Continuous Functions
Suppose f and g are two real functions, continuous at real number c. Then,

- f + g is continuous at x = c.
- f – g is continuous at x = c.
- f.g is continuous at x = c.
- cf is continuous, where c is any constant.
- (fg) is continuous at x = c, [provide g(c) ≠ 0]

Suppose f and g are two real valued functions such that (fog) is defined at c. If g is continuous at c and f is continuous at g (c), then (fog) is continuous at c.

Differentiability

A function f(x) is said to be differentiable at a point x = a, if
Left hand derivative at (x = a) = Right hand derivative at (x = a)
i.e. LHD at (x = a) = RHD (at x = a), where Right hand derivative, where

Note: Every differentiable function is continuous but every continuous function is not differentiable.

Differentiation: The process of finding a derivative of a function is called differentiation.

(a) Let us consider a function $\mathrm{y}=f(\mathrm{x})$ defined in a certain interval. It has a definite value for each value of the independent variable x in this interval.

Now, the ratio of the function's increment to the independent variable's increment,

$$\frac{\Delta y}{\Delta x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Now, as $\Delta \mathrm{x} \rightarrow 0, \Delta \mathrm{y} \rightarrow 0$ and $\frac{\Delta \mathrm{y}}{\Delta \mathrm{x}} \rightarrow$ finite quantity, then derivative $f(x)$ exists and is denoted by $y^{\prime}$ or $f^{\prime}(x)$ or $\frac{dy}{dx}$ Thus, $f^{\prime}(x)=\lim_{x \rightarrow 0}\left(\frac{\Delta y}{\Delta x}\right)=\lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$ (if it exits) for the limit to exist,

$$\lim_{\mathrm{h} \rightarrow 0} \frac{f(\mathrm{x}+\mathrm{h})-f(\mathrm{x})}{\mathrm{h}}=\lim_{\mathrm{h} \rightarrow 0} \frac{f(\mathrm{x}-\mathrm{h})-f(\mathrm{x})}{-\mathrm{h}}$$

(Right Hand derivative) (Left Hand derivative)

(b) The derivative of a given function f at a point $\mathrm{x}=\mathrm{a}$ of its domain is defined as:

$\underset{h \rightarrow 0}{\mathrm{Limit}} \frac{f(\mathrm{a}+\mathrm{h})-f(\mathrm{a})}{\mathrm{h}}$, provided the limit exists is denoted by $f^{\prime}(\mathrm{a})$

Note that alternatively, we can define

$f^{\prime}(\mathrm{a})=\mathrm{Limit}_{\mathrm{x} \rightarrow \mathrm{a}} \frac{f(\mathrm{x})-f(\mathrm{a})}{\mathrm{x}-\mathrm{a}}$, provided the limit exists.

This method is called first principle of finding the derivative of $f(x)$

Theorems On Derivatives

If u and v are derivable functions of x, then,

(i) Term by term differentiation : $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

(ii) Multiplication by a constant $\frac{d}{dx}(Ku) = K\frac{du}{dx}$, where K is any constant

(iii) "Product Rule" $\frac{d}{dx}(u.v) = u\frac{dv}{dx} + v\frac{du}{dx}$ known as In general,

(a) If $u_1, u_2, u_3, u_4, \ldots, u_n$ are the functions of x, then

$$\frac{d}{dx}(u_1 \cdot u_2 \cdot u_3 \cdot u_4 \ldots . u_n)$$
$$= \left(\frac{du_1}{dx}\right)(u_2u_3u_4 \ldots u_n) + \left(\frac{du_2}{dx}\right)(u_1u_3u_4 \ldots u_n)$$
$$+ \left(\frac{du_3}{dx}\right)(u_1u_2u_4 \ldots u_n) + \left(\frac{du_4}{dx}\right)(u_1u_2u_3u_5 \ldots u_n)$$
$$+ \ldots + \left(\frac{du_n}{dx}\right)(u_1u_2u_3 \ldots u_{n-1})$$

(iv) Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\left(\frac{dv}{dx}\right)}{v^2}$$ where $v \neq 0$ known as

(b) Chain Rule : If $y = f(u), u = g(w), w = h(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dw} \cdot \frac{dw}{dx}$

$$\text{or } \frac{dy}{dx} = f'(u) \cdot g'(\quad) \cdot h'(x)$$

Note:

In general if $y = f(u)$ then $\frac{dy}{dx} = f'(u) \cdot \frac{du}{dx}$

Methods of Differentiation

The use of trigonometrical transforms before differentiation greatly reduces the amount of labour required. The following are some of the most significant findings:

(i) $\sin 2x = 2\sin x\cos x = \dfrac{2\tan x}{1+\tan^2 x}$

(ii) $\cos 2\mathrm{x} = 2\cos^2\mathrm{x} - 1 = 1 - 2\sin^2\mathrm{x} = \dfrac{1-\tan^2\mathrm{x}}{1+\tan^2\mathrm{x}}$

(iii) $\tan 2\mathrm{x} = \dfrac{2\tan\mathrm{x}}{1-\tan^2\mathrm{x}}, \tan^2\mathrm{x} = \dfrac{1-\cos 2\mathrm{x}}{1+\cos 2\mathrm{x}}$

(iv) $\sin 3x = 3\sin x - 4\sin^3 x$

(v) $\cos 3x = 4\cos^3 x - 3\cos x$

(vi) $\tan 3x = \dfrac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

(vii) $\tan\left(\dfrac{\pi}{4} + x\right) = \dfrac{1+\tan x}{1-\tan x}$

(viii) $\tan\left(\dfrac{\pi}{4} - \mathrm{x}\right) = \dfrac{1-\tan\mathrm{x}}{1+\tan\mathrm{x}}$

(ix) $\sqrt{(1 \pm \sin x)} = \left|\cos\dfrac{x}{2} \pm \sin\dfrac{x}{2}\right|$

(x) $\tan^{-1}x \pm \tan^{-1}y = \tan^{-1}\left(\dfrac{x \pm y}{1 \mp xy}\right)$

(xi) $\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right\}$

(xii) $\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\left\{xy \mp \sqrt{1-x^2}\sqrt{1-y^2}\right\}$

(xiii) $\sin^{-1}x + \cos^{-1}x = \tan^{-1}x + \cot^{-1}x = \sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2$

(xiv) $\sin^{-1}x = \operatorname{cosec}^{-1}(1/x); \cos^{-1}x = \sec^{-1}(1/x); \tan^{-1}x = \cot^{-1}(1/x)$

Some standard substitutions:

Expressions **Substitutions**

(i) $\sqrt{(a^2 - x^2)}$ $x = a\sin\theta$ or $a\cos\theta$

(ii) $\sqrt{(a^2 + x^2)} x = a\tan\theta$ or $a\cot\theta$

(iii) $\sqrt{(x^2 - a^2)} x = a\sec\theta$ or $a\,\mathrm{cosec}\,\theta$

(iv) $\sqrt{\left(\frac{a+x}{a-x}\right)}$ or $\sqrt{\left(\frac{a-x}{a+x}\right)} x = a\cos\theta$ or $a\cos 2\theta$

(v) $\sqrt{(a-x)(x-b)}$ or $x = a\cos^2\theta + b\sin^2\theta$

(vi) $\sqrt{\left(\frac{a-x}{x-b}\right)}$ or $\sqrt{\left(\frac{x-}{a-x}\right)}$

(vii) $\sqrt{(x-a)(x-b)}$ or $x = a\sec^2\theta - b\tan^2\theta$

(viii) $\sqrt{\left(\frac{x-a}{x-b}\right)}$ or $\sqrt{\left(\frac{x-}{x-a}\right)}$

(ix) $\sqrt{(2ax - x^2)} x = a(1 - \cos\theta)$

Derivative of Order Two & Three

Let us assume a function $\mathrm{y}=f(\mathrm{x})$ be defined on an open interval (a, b). It's derivative, if it exists on (a, b), is a certain function $f'(\mathrm{x})$ [or $(\mathrm{dy}/\mathrm{dx})$ or y'] is called the first derivative of y w.r.t. $\mathbf{x}$. If it occurs that the first derivative has a derivative on (a, b) then this derivative is called the second derivative of y w.r.t. x is denoted by $f''(x)$ or $\left(d^2y/dx^2\right)$ or y''.

Similarly, the 3^{rd} order derivative of y w.r.t. x, if it exists, is defined by $\frac{\mathrm{d}^3\mathrm{y}}{\mathrm{dx}}=\frac{\mathrm{d}}{\mathrm{dx}}\left(\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2}\right)$ it is also denoted by $f''(\mathrm{x})$ or y''' Some Standard Results :

(i) $\frac{\mathrm{d}^{\mathrm{n}}}{\mathrm{dx}^{\mathrm{n}}}(\mathrm{ax}+\mathrm{b})^{\mathrm{m}}=\frac{\mathrm{m}!}{(\mathrm{m}-\mathrm{n})!}\cdot\mathrm{a}^{\mathrm{n}}\cdot(\mathrm{ax}+\mathrm{b})^{\mathrm{m}-\mathrm{n}},\mathrm{m}\geqslant\mathrm{n}$

(ii) $\frac{\mathrm{d}^{\mathrm{n}}}{\mathrm{dx}^{\mathrm{n}}}\mathrm{x}^{\mathrm{n}}=\mathrm{n}!$

(iii) $\frac{\mathrm{d}^{\mathrm{n}}}{\mathrm{dx}^{\mathrm{n}}}(\mathrm{e}^{\mathrm{mx}})=\mathrm{m}^{\mathrm{n}}\cdot\mathrm{e}^{\mathrm{mx}},\mathrm{m}\in\mathrm{R}$

(iv) $\frac{\mathrm{d}^{\mathrm{n}}}{\mathrm{dx}^{\mathrm{n}}}(\sin(\mathrm{ax}+\mathrm{b}))=\mathrm{a}^{\mathrm{n}}\sin\left(\mathrm{ax}+\mathrm{b}+\frac{\mathrm{n}\pi}{2}\right),\mathrm{n}\in\mathrm{N}$

(v) $\frac{\mathrm{d}^{\mathrm{n}}}{\mathrm{dx}^{\mathrm{n}}}(\cos(\mathrm{ax}+\mathrm{b}))=\mathrm{a}^{\mathrm{n}}\cos\left(\mathrm{ax}+\mathrm{b}+\frac{\mathrm{n}\pi}{2}\right),\mathrm{n}\in\mathrm{N}$

(vi) $\frac{\mathrm{d}^{\mathrm{n}}}{\mathrm{dx}^{\mathrm{n}}}\{\mathrm{e}^{\mathrm{ax}}\sin(\mathrm{bx}+\mathrm{c})\}=\mathrm{r}^{\mathrm{n}}\cdot\mathrm{e}^{\mathrm{ax}}\cdot\sin(\mathrm{bx}+\mathrm{c}+\mathrm{n}\phi),\mathrm{n}\in\mathrm{N}$

where $r=\sqrt{(a^2+b^2)},\phi=\tan^{-1}(b/a)$

(vii) $\frac{\mathrm{d}^{\mathrm{n}}}{\mathrm{dx}^{\mathrm{n}}}\{\mathrm{e}^{\mathrm{ax}}\cdot\cos(\mathrm{bx}+\mathrm{c})\}=\mathrm{r}^{\mathrm{n}}\cdot\mathrm{e}^{\mathrm{ax}}\cdot\cos(\mathrm{bx}+\mathrm{c}+\mathrm{n}\phi),\mathrm{n}\in\mathrm{N}$

where $r=\sqrt{(a^2+b^2)},\phi=\tan^{-1}(b/a)$

The **derivative of order two** and **derivative of order three** refer to taking the derivative of a function multiple times. These are also known as **second-order** and **third-order derivatives**.

Second-Order Derivative:

- The second-order derivative is the derivative of the first derivative. If the first derivative represents the rate of change of a function, the second-order derivative gives the rate of change of that rate. It's often used to analyze the concavity of a function.

Third-Order Derivative:

- The third-order derivative is the derivative of the second derivative. It gives the rate of change of the second derivative, which can help in more advanced analysis, such as in physics to understand acceleration changes.

A List of Continuous Functions

	Function f(x)f(x)	Interval in which f(x) is continuous
1	Constant (c)	(−∞,∞)(−∞,∞)
2	xn,n is an integer xn,n is an integer	(−∞,∞)(−∞,∞)
3	x−n,n is a positive integer x−n,n is a positive integer	(−∞,∞)−{0}(−∞,∞)−{0}
4	|x−a||x−a|	(−∞,∞)(−∞,∞)
5	P(x)=a0xn+a1xn−1+…..+anP(x)=a0xn+a1xn−1+…..+an	(−∞,∞)(−∞,∞)
6	sinxsinx	(−∞,∞)(−∞,∞)
7	cosxcosx	(−∞,∞)(−∞,∞)
8	tanxtanx	(−∞,∞)−{(2n+1)π2:n∈I}(−∞,∞)−{(2n+1)π2:n∈I}
9	cotxcotx	(−∞,∞)−{nπ:n∈I(−∞,∞)−{nπ:n∈I
10	secxsecx	(−∞,∞)−{(2n+1)(−∞,∞)−{(2n+1)

11	cosecxcosecx	π/2:n∈Iπ/2:n∈I
12	exex	(−∞,∞)−{nπ:n∈I}(−∞,∞)−{nπ:n∈I }
13	logcxlogcx	(−∞,∞)(−∞,∞) &(0,∞)(0,∞)

Class 12 Chapter 5 Continuity and Differentiability Important Questions with Solutions

Question 1:

Explain the continuity of the function f(x) = sin x . cos x

Solution:

We know that sin x and cos x are continuous functions. It is known that the product of two continuous functions is also a continuous function.

Hence, the function f(x) = sin x . cos x is a continuous function.

Question 2:

Determine the points of discontinuity of the composite function y = f[f(x)], given that, f(x) = 1/x-1.

Solution:

Given that, f(x) = 1/x-1

We know that the function f(x) = 1/x-1 is discontinuous at x = 1

Now, for x ≠1,

f[f(x)]= f(1/x-1)

= 1/[(1/x-1)-1]

= x-1/ 2-x, which is discontinuous at the point x = 2.

Therefore, the points of discontinuity are x = 1 and x=2.

Question 3:

If f (x) = |cos x|, find f’(3π/4)

Solution:

Given that, f(x) = |cos x|

When $\pi/2 < x < \pi$, cos x < 0,

Thus, |cos x| = -cos x

It means that, f(x) = -cos x

Hence, f’(x) = sin x

Therefore, f’(3π/4) = sin (3π/4) = 1/√2

f’(3π/4) = 1/√2

Question 4:

Verify the mean value theorem for the following function f (x) = (x – 3) (x – 6) (x – 9) in [3, 5]

Solution:

f(x)=(x−3)(x−6)(x−9)

$=(x-3)(x^2-15x+54)$

$=x^3-18x^2+99x-162$

fc∈(3,5)

f′(c)=f(5)−f(3)/5−3

f(5)=(5−3)(5−6)(5−9)

=2(−1)(−4)=−8

f(3)=(3−3)(3−6)(3−9)=0

f′(c)=8−0/2=4

$\therefore f'(c)=3c^2-36c+99$

$3c^2-36c+99=4$

$3c^2-36c+95=0$

$ax^2+bx+c=0$

$a=3$

$b=-36$

$c=95$

$c=36\pm\sqrt{(36)^2-4(3)(95)}/2(3)$

$=36\pm\sqrt{1296-1140}/6$

$=36\pm12.496$

$c=8.8 \& c=4.8$

$c\in(3,5)$

$f(x)=(x-3)(x-6)(x-9)$ on $[3,5]$

Question 5:

Explain the continuity of the function f = |x| at x = 0.

Solution:

From the given function, we define that,

f(x) = {-x, if x<0 and x, if x≥0

It is clearly mentioned that the function is defined at 0 and f(0) = 0. Then the left-hand limit of f at 0 is

$\text{Lim}_{x\to0^-} f(x)= \lim_{x\to0^-} (-x) = 0$

Similarly for the right hand side,

$\text{Lim}_{x\to0^+} f(x)= \lim_{x\to0^+} (x) = 0$

Therefore, for the both left hand and the right hand limit, the value of the function coincide at the point x = 0.

Therefore, the function f is continuous at the point x =0.

Question 6:

If y= tan x + sec x , then show that $d^2y / dx^2 = \cos x / (1-\sin x)^2$

Solution:

Given that, y= tan x + sec x

Now, the differentiate wih respect to x, we get

$dy/dx = \sec^2 x + \sec x \tan x$

$= (1/ \cos^2 x) + (\sin x/ \cos^2 x)$

= (1+sinx)/ (1+sinx)(1-sin x)

Thus, we get.

dy/dx = 1/(1-sin x)

Now, again differentiate with respect to x, we will get

$d^2y / dx^2 = -(-\cos x)/(1- \sin x)^2$

$d^2y / dx^2 = \cos x / (1-\sin x)^2$.

Benefits of CBSE Class 12 Maths Notes Chapter 5 Continuity and Differentiability

Focused and Concise Content: The notes provide a summary of important concepts and formulas, helping students focus on the most relevant material for exams without unnecessary details.

Quick Revision: These notes are ideal for last-minute revision allowing students to quickly go over key points and refresh their understanding of the chapter before the exam.

Time-Saving: With organized content students can save time by studying only the important topics, ensuring they use their revision time efficiently.

Step-by-Step Solutions for Problems: The notes include solved examples that guide students through typical exam-style problems, helping them understand the correct approach and boosting their confidence for solving similar questions.

Enhances Problem-Solving Speed: Regular practice with these notes helps students become more familiar with the types of problems they will face in the exam, allowing them to solve questions more quickly and effectively.