ISC SEMESTER 1 EXAMINATION
SPECIMEN QUESTION PAPER
MATHEMATICS

Maximum Marks: 80

Time allowed: One and a half hours

(*Candidates are allowed additional **15 minutes** for **only** reading the paper.*)

The Question paper consists of three sections A, B and C.

Candidates are required to attempt **all** questions from Section A and all questions **either** from Section B **OR** Section C.

The marks intended for questions or parts of questions are given in brackets [ ].

Select the correct option for each of the following questions.

SECTION A (64 Marks)

(Answer all Questions)

Question 1 **[2]**

The function f: R → R defined by $f(x) = sin\ (3x + 2),\ \forall\ x \in R$ is:

(a) One-One
(b) Onto
(c) Neither one-one nor onto
(d) one-one but not onto.

Question 2 **[2]**

What will be the Principal value of $Cosec^{-1}(-\sqrt{2})$?

(a) $\frac{3\pi}{4}$
(b) $-\frac{\pi}{6}$
(c) $\frac{\pi}{4}$
(d) $-\frac{\pi}{4}$

Question 3 **[2]**

If set A contains 5 elements and set B contains 6 elements, then the number of one-one onto mappings from A to B is:

(a) 720
(b) 120
(c) 0
(d) none of the above.

Question 4 **[2]**

If $\alpha \leq 2Sin^{-1}x + Cos^{-1}x \leq \beta$, then (α, β) is:
(a) $(0, \pi)$
(b) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
(c) $\left(-\frac{3\pi}{2}, \frac{\pi}{2}\right)$
(d) None of the above.

Question 5 **[2]**

Let A be the set of all students of a boy's school. Then the relation R in A is defined by:
R = {(a,b) : a is sister of b} is
(a) an equivalence relation
(b) symmetric relation
(c) an empty relation
(d) a universal relation

Question 6 **[2]**

$\forall\, x \in R, Cot^{-1}(-x) =$
(a) $\pi - cot^{-1}x$
(b) $-tan^{-1}x$
(c) $-cot^{-1}x$
(d) $\pi + cot^{-1}x$

Question 7 **[2]**

The value of $\begin{vmatrix} 1 & \log_a b \\ \log_b a & 1 \end{vmatrix}$ is:

(a) $1 - \log ab$
(b) $1 - \frac{\log b}{\log a}$
(c) 0
(d) $\log ab - 1$

Question 8 **[2]**

From the matrix equation $AB = AC$, it can be concluded that $B = C$ provided:
(a) A is singular matrix
(b) A is non-singular matrix
(c) A is a symmetric matrix
(d) A is a skew symmetric matrix

Question 9 **[2]**

What is the transpose of a column matrix?
(a) Zero matrix
(b) Diagonal matrix
(c) Column matrix
(d) Row matrix

Question 10 **[2]**

What is the multiplicative inverse of matrix A is?

(a) A
(b) A^2
(c) $|A|$
(d) $\frac{adjA}{|A|}$

Question 11 **[2]**

If A and B are two non singular matrices, and AB exists, then $(AB)^{-1}$ is:

(a) $A^{-1}B^{-1}$
(b) $B^{-1}A^{-1}$
(c) AB
(d) None of the above

Question 12 **[2]**

If $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$, then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix}$ is:

(a) Δ
(b) $k\Delta$
(c) $3k\Delta$
(d) $k^3\Delta$

Question 13 **[2]**

If $y = t^2$ and t = x + 3 then $\frac{dy}{dx}$ is equal to:

(a) $(x+3)^2$
(b) $2(x+3)$
(c) $2t$
(d) $2(x+3)^2$

Question 14 **[2]**

The set of points, where the function $f(x) = x\,|x|$ is differentiable in:

(a) $(-\infty, \infty)$
(b) $(-\infty, 0) \cup (0, \infty)$
(c) $(0, \infty)$
(d) $[0, \infty)$

Question 15 **[2]**

If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then $\frac{dy}{dx}$ is equal to:

(a) $\frac{x}{y}$
(b) $-\frac{x}{y}$
(c) $\frac{y}{x}$
(d) $-\frac{y}{x}$

Question 16 **[2]**

The value of $\lim_{x\to 0} \frac{\log(1+x)}{x}$ is equal to:

(a) e
(b) 0
(c) 1
(d) -1

Question 17 **[2]**

What will be the value of *x* for the determinant equation $\begin{vmatrix} 3-x & 6 & 3 \\ -6 & 3-x & 3 \\ 3 & 3 & 3-x \end{vmatrix} = 0$?

(a) 6
(b) 3
(c) 0
(d) -6

Question 18 **[2]**

Any tangent to the curve $y = 3x^7 + 5x + 3$:

(a) is parallel to x – axis
(b) is parallel to y – axis
(c) makes an acute angle with x – axis
(d) makes on obtuse angle with y – axis

Question 19 **[2]**

The second derivative of $y = x^3 - 5x^2 + x$ is:

(a) $10x - 5$
(b) $6x - 10$
(c) $3x^2 - 10x$
(d) $3x^2 - 10x + 1$

Question 20 **[2]**

What will be the derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$?

(a) -1
(b) 1
(c) 2
(d) 4

Question 21 **[4x2]**

Ramu purchased 5 pens, 3 bags and 1 instrument box and paid ₹ 16. From the same shop Venkat purchased 2 pens , 1 bag and 3 instrument boxes and paid ₹ 19 while Gopi purchased 1 pen, 2 bags and 4 instrument boxes and paid ₹ 25.

Using the concept of Matrices and Determinants to answer the following questions by choosing the correct option:

(i) If $x, y \,\&\, z$ respectively denotes the cost of pen, bag and instrument box then which of the following is true?
   (a) $5x + 3y + z = 16$
   (b) $2x + y + 3z = 19$

(c) $x + 2y + 4z = 25$
(d) All of the above

(ii) If $A = \begin{pmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{pmatrix}$, $|A|$is:
(a) -22
(b) 22
(c) 0
(d) 20

(iii) If $A = \begin{pmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{pmatrix}$ and $adj\ A = \begin{pmatrix} -2 & x & 8 \\ -5 & 19 & -13 \\ 3 & -7 & y \end{pmatrix}$ then missing value of x and y are:
(a) $x = -10$ & $y = -1$
(b) $x = 10$ & $y = -1$
(c) $x = -10$ & $y = 1$
(d) $x = 10$ & $y = 1$

(iv)The cost of one pen is
(a) ₹ 2
(b) ₹ 5
(c) ₹ 1
(d) ₹ 3

Question 22 **[4x2]**

A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window as shown in the figure given below. The total perimeter of the window is 10 m.

Based on the above information answer the following by choosing the correct option:

(i) If the length and breadth of the rectangle portion of the window is y and x respectively (as shown in the figure above) then the relation between the variable is

(a) y = $\frac{20 + (\pi - 2)x}{4}$
(b) y = $\frac{20 - (\pi + 2)x}{2}$
(c) y = $\frac{20 - (\pi + 4)x}{4}$
(d) y = $\frac{20 - (\pi + 2)x}{4}$

(ii) Let A be the area of the Norman window which admits the sunlight. Then A expressed in terms of x is

(a) A = 5x + $\frac{\pi}{4}x^2 - 2x^2$

(b) A = 5x + $\frac{\pi}{8}x^2 - \frac{1}{2}x^2$

(c) A = 5x - $\frac{\pi}{8}x^2 - \frac{1}{2}x^2$

(d) A = 5x - $\frac{\pi}{2}x^2 - \frac{1}{4}x^2$

(iii) For the maximum value of A what will be the radius of the semicircle?

(a) $\frac{10}{2+\pi}$

(b) $\frac{10}{\pi-2}$

(c) $\frac{10}{4+\pi}$

(d) $\frac{20}{4-\pi}$

(iv) For maximum value of A , the length of the rectangle represented by y will be equal to:

(a) $\frac{10}{4+\pi}$

(b) $\frac{10}{\pi-2}$

(c) $\frac{20}{4+\pi}$

(d) $\frac{20}{4-\pi}$

Question 23 **[4x2]**

Consider the mapping $f: A \to B$ is defined by $f(x) = \frac{x-1}{x-2}$ such that $f(x)$ is one-one onto. Based on the above information, answer the following questions by choosing the correct option.

(i) Domain of $f(x)$ is:

(a) $R - \{2\}$

(b) R

(c) $R - \{1, 2\}$

(d) $R - \{0\}$

(ii) Range of $f(x)$ is:

(a) $R - \{2\}$

(b) R

(c) $R - \{1\}$

(d) $R - \{0\}$

(iii) If $g(x) = 2f(x) - 1$, then $g(x)$ in terms of x is:

(a) $\frac{x+2}{x}$

(b) $\frac{x+1}{x-2}$

(c) $\frac{x-2}{x}$

(d) $\frac{x}{x-2}$

(iv) A function $f(x)$is said to be one-one if:

(a) $f(x_1) = f(x_2) \Longrightarrow x_1 = x_2$

(b) $f(-x_1) = f(-x_2) \Longrightarrow -x_1 = x_2$

(c) $f(x_1) = f(x_2) \Longrightarrow -x_1 = x_2$

(d) $-f(x_1) = f(x_2) \Longrightarrow x_1 = x_2$

SECTION B (16 Marks)

(Answer all Questions)

Question 24 **[2]**

What will be the value of m if the vector $2\hat{\imath} + m\hat{\jmath} + \hat{k}$ is perpendicular to $2\hat{\imath} - \hat{\jmath} + 3\hat{k}$?

(a) 7
(b) 0
(c) 1
(d) -1

Question 25 **[2]**

What will be the angle between the two lines $\frac{-x+2}{-2} = \frac{y-1}{7} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{2y-8}{4} = \frac{z-5}{4}$?

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) 0
(d) π

Question 26 **[2]**

What are the direction ratios of the line passing through two points (-2, 4, 5) and (1, 2, 3)?

(a) $< 1, 2, 3 >$
(b) $< -3, 2, 2 >$
(c) $< 2, 4, 5 >$
(d) $< 0, -1, 4 >$

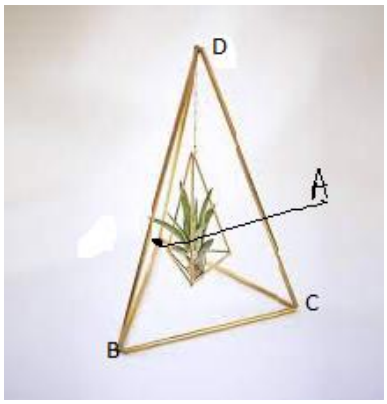
Question 27 **[2]**

The equation of the line passing $(1, -1, 0)$ and parallel to the line $\frac{x-1}{1} = \frac{y+2}{-2} = \frac{z+1}{-1}$ is:

(a) $\frac{x-1}{1} = \frac{y+1}{-2} = \frac{z}{-1}$
(b) $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+1}{-3}$
(c) $\frac{x-6}{1} = \frac{y-2}{-2} = \frac{z+1}{3}$
(d) $\frac{x-2}{-2} = \frac{y+2}{-2} = \frac{z+3}{-1}$

Question 28 **[4x2]**

The given figure shows an air plant holder which is in the shape of a tetrahedron. Let A(1, 1, 1), B(2, 1, 3), C(3, 2, 2) & D(3, 3, 4) are the vertices of air plant holder. Based on the above information answer the following questions.



(i) The vector of $\overrightarrow{AB}$ is:
  (a) $-\hat{\imath} - 2\hat{k}$
  (b) $2\hat{\imath} + \hat{k}$
  (c) $\hat{\imath} + 2\hat{k}$
  (d) $-2\hat{\imath} - \hat{k}$

(ii) The vector of $\overrightarrow{AC}$ is:
  (a) $2\hat{\imath} - \hat{\jmath} - \hat{k}$
  (b) $2\hat{\imath} + \hat{\jmath} + \hat{k}$
  (c) $-2\hat{\imath} - \hat{\jmath} + \hat{k}$
  (d) $\hat{\imath} + 2\hat{\jmath} + \hat{k}$

(iii) Area of ΔABC is:
  (a) $\frac{\sqrt{11}}{2}$ Sq. units
  (b) $\frac{\sqrt{14}}{2}$ Sq. units
  (c) $\frac{\sqrt{13}}{2}$ Sq. units
  (d) $\frac{\sqrt{17}}{2}$ Sq. units

(iv) The unit vector along the $\overrightarrow{AB}$ is:
  (a) $\frac{-2\hat{\imath}-\hat{k}}{\sqrt{5}}$
  (b) $\frac{-\hat{\imath}-2\hat{k}}{\sqrt{5}}$
  (c) $\frac{2\hat{\imath}+\hat{k}}{\sqrt{5}}$
  (d) $\frac{\hat{\imath}+2\hat{k}}{\sqrt{5}}$

SECTION C (16 Marks)

(Answer all Questions)

Question 29 **[2]**

A company sells its product for ₹ 20 per unit. Fixed costs for the company is ₹ 45,000 and variable costs is estimated to run 25% of total revenue. If x denotes number of units produced, then what will be the total cost function?

(a) $45000 + 5x$
(b) $15000 + 4x$
(c) $45000 + 2x$
(d) $4500 + 20x$

Question 30 **[2]**

The demand function for a certain commodity is given by $p = 4000 - 100x$. What will be the total revenue from the sale of 3 units?

(a) 11,100
(b) 1000
(c) 4500
(d) 2000

Question 31 **[2]**

A company sells x packets of biscuits each day at ₹ 10 a packet. The cost of manufacturing these packets is ₹ 5 per packet plus a fixed daily overhead cost of ₹ 700. What will be the profit function?

(a) $6x - 400$
(b) $5x - 700$
(c) $10x - 500$
(d) $5x - 10$

Question 32 **[2]**

The cost function of a firm is given by $(x) = 3x^2 - 2x + 6$. The average cost of the firm at $x = 3$ is:

(a) 11
(b) 17
(c) 9
(d) 27

Question 33 **[4x2]**

The demand function for a certain product is represented by the equation: $p = ax^2 + bx + c$ where x is the number of units demanded and p is the price per unit.

(i) The revenue function $R(x)$ is:
   (a) $ax^3 + bx^2 + cx$
   (b) $ax + b + \frac{c}{x}$
   (c) $ax^3 + bx^2 + cx + d$
   (d) $2ax + b$

(ii) The marginal revenue $MR(x)$ is:

(a) $a - \frac{c}{x^2}$

(b) $3ax^2 + 2bx + c$

(c) $3ax^3 + 2bx^2 + c$

(d) 2a

(iii) The slope of the marginal revenue is:

(a) 0

(b) $6ax + 2b$

(c) $\frac{2c}{x^3}$

(d) $9ax^2 + 4bx$

(iv) Values of x, for which marginal revenue increases is:

(a) $x > \frac{-b}{3a}$

(b) $x < \frac{-b}{3a}$

(c) $x = \frac{-b}{3a}$

(d) $x \leq \frac{-b}{3a}$