ICSE Class 9 Maths Selina Solutions Chapter 10: Here are ICSE Class 9 Maths Selina Solutions Chapter 10. A student's time in class nine is crucial. Understanding the material covered in Class 9 is essential since Class 10 builds on it.

It is recommended that you complete the exercises in every chapter of the Selina publishing book to achieve high scores on the mathematics exam for Class 9. The Selina answers for Maths Class 9 aid students in better comprehending all of the material.

ICSE Class 9 Maths Selina Solutions Chapter 10 Overview

ICSE Class 9 Maths Selina Solutions Chapter 10 introduces isosceles triangles. In this ICSE Class 9 Maths Selina Solutions Chapter 10, students learn that an isosceles triangle has at least two sides of equal length, and the angles opposite these equal sides are also equal.

The ICSE Class 9 Maths Selina Solutions Chapter 10 explains important properties and theorems related to isosceles triangles, such as how the angles opposite the equal sides are equal and how this can be used to find missing angles and sides. By understanding these properties and practicing related problems, students gain the skills to solve various geometric problems involving isosceles triangles.

ICSE Class 9 Maths Selina Solutions Chapter 10

Below we have provided ICSE Class 9 Maths Selina Solutions Chapter 10 -

1. In the triangle

AB = AC

$\angle$A = 48° and

$\angle$ACD = 18°.

Show that BC = CD.

Solution:

In $\triangle$ABC, we have

$\angle BAC + \angle ACB + \angle ABC = 180^0$

$48^0 + \angle ACB + \angle ABC = 180^0$

But, $\angle ACB = \angle ABC$ [Given, AB = AC]

$2\angle ABC = 180^0 - 48^0$

$2\angle ABC = 132^0$

$\angle ABC = 66^0 = \angle ACB$ ……(i)

$\angle ACB = 66^0$

$\angle ACD + \angle DCB = 66^0$

$18^0 + \angle DCB = 66^0$

$\angle DCB = 48^0$ ………(ii)

Now, In ΔDCB,

$\angle DBC = 66^0$ [From (i), Since $\angle ABC = \angle DBC$]

$\angle DCB = 48^0$ [From (ii)]

$\angle BDC = 180^0 - 48^0 - 66^0$

$\angle BDC = 66^0$

Since $\angle BDC = \angle DBC$

Therefore, BC = CD

Equal angles have equal sides opposite to them.

2. Calculate:

(i) $\angle$ADC

(ii) $\angle$ABC

(iii) $\angle$BAC

Solution:

Given: $\angle ACE = 130^0$; AD = BD = CD

Proof:

(i) $\angle ACD + \angle ACE = 180^o$ [DCE is a straight line]

$\angle ACD = 180^o - 130^o$

$\angle ACD = 50^o$

Now,

CD = AD

$\angle ACD = \angle DAC = 50^o$ … (i) [Since angles opposite to equal sides are equal]

In $\triangle ADC$,

$\angle ACD = \angle DAC = 50^o$

$\angle ACD + \angle DAC + \angle ADC = 180^o$

$50^o + 50^o + \angle ADC = 180^o$

$\angle ADC = 180^o – 100^o$

$\angle ADC = 80^o$

(ii) $\angle ADC = \angle ABD + \angle DAB$ [Exterior angle is equal to sum of opposite interior angles]

But, AD = BD

$\therefore \angle DAB = \angle ABD$

$80^o = \angle ABD + \angle ABD$

$2\angle BD = 80^O$

$\angle ABD = 40^O = \angle DAB$ … (ii)

(iii) We have,

$\angle BAC = \angle DAB + \angle DAC$

Substituting the values from (i) and (ii),

$\angle BAC = 40^O + 50^O$

Hence, $\angle BAC = 90^O$

3. In the following figure, AB = AC; BC = CD and DE is parallel to BC. Calculate:

(i) $\angle$CDE

(ii) $\angle$DCE

Solution:

Given, $\angle FAB = 128^O$

$\angle BAC + \angle FAB = 180^O$ [As FAC is a straight line]

$\angle BAC = 180^O – 128^O$

$\angle BAC = 52^{\circ}$

In ΔABC, we have

$\angle A = 52^{\circ}$

$\angle B = \angle C$ [Given AB = AC and angles opposite to equal sides are equal]

Now, by angle sum property

$\angle A + \angle B + \angle C = 180^{\circ}$

$\angle A + \angle B + \angle B = 180^{\circ}$

$52^{\circ} + 2\angle B = 180^{\circ}$

$2\angle B = 128^{\circ}$

$\angle B = 64^{\circ} = \angle C$… (i)

$\angle B = \angle ADE$ [Given DE II BC]

(i) Now, $\angle ADE + \angle CDE + \angle B = 180^{\circ}$ [As ADB is a straight line]

$64^{\circ} + \angle CDE + 64^{\circ} = 180^{\circ}$

$\angle CDE = 180^{\circ} + 128^{\circ}$

$\angle CDE = 52^{\circ}$

(ii) Given DE II BC and DC is the transversal

$\angle CDE = \angle DCB = 52^{\circ}$… (ii)

Also, $\angle ECB = 64^{\circ}$… [From (i)]

But,

$\angle ECB = \angle DCE + \angle DCB$

$64^{\circ} = \angle DCE + 52^{\circ}$

$\angle DCE = 64^{\circ} – 52^{\circ}$

$\angle DCE = 12^{\circ}$

4. Calculate x:

Solution:

(i) Let the triangle be ABC and the altitude be AD.

In ΔABD, we have

$\angle DBA = \angle DAB = 37^o$ [Given BD = AD and angles opposite to equal sides are equal]

Now,

$\angle CDA = \angle DBA + \angle DAB$ [Exterior angle is equal to the sum of opposite interior angles]

$\angle CDA = 37^o + 37^o$

$\therefore \angle CDA = 74^o$

Now, in ΔADC, we have

$\angle CDA = \angle CAD = 74^o$ [Given CD = AC and angels opposite to equal sides are equal]

Now, by angle sum property

$\angle CAD + \angle CDA + \angle ACD = 180^o$

$74^o + 74^o + x = 180^o$

$x = 180^o – 148^o$

$x = 32^o$

(ii) Let triangle be ABC and altitude be AD.

In ΔABD, we have

$\angle DBA = \angle DAB = 50^o$ [Given BD = AD and angles opposite to equal sides are equal]

Now,

$\angle CDA = \angle DBA + \angle DAB$ [Exterior angle is equal to the sum of opposite interior angles]

$\angle CDA = 50^o + 50^o$

$\therefore \angle CDA = 100^o$

In ΔADC, we have

$\angle DAC = \angle DCA = x$ [Given AD = DC and angels opposite to equal sides are equal]

So, by angle sum property

$\angle DAC + \angle DCA + \angle ADC = 180^o$

$x + x + 100^o = 180^o$

$2x = 80^o$

$x = 40^o$

5. In the figure, given below, AB = AC. Prove that: ∠BOC = ∠ACD.

Solution:

Let's assume ∠ABO = ∠OBC = x and ∠ACO = ∠OCB = y

In ABC, we have

∠BAC = 180° – 2x – 2y…(i)

As, ∠B = ∠C [Since, AB = AC]

½ ∠B = ½ ∠C

⇒ x = y

Now,

∠ACD = 2x + ∠BAC [Exterior angle is equal to sum of opposite interior angle]

= 2x + 180° – 2x – 2y [From (i)]

∠ACD = 180° – 2y… (ii)

In ∆OBC, we have

∠BOC = 180° – x – y

∠BOC = 180° – y – y [Since x = y]

∠BOC = 180° – 2y… (iii)

Thus, from (ii) and (iii) we get

∠BOC = ∠ACD

6. In the figure given below, LM = LN; ∠PLN = 110°. Calculate:

(i) ∠LMN

(ii) ∠MLN

Solution:

Given, LM = LN and ∠PLN = 110°

(i) We know that the sum of the measure of all the angles of a quadrilateral is 360°.

In quad. PQNL,

$\angle QPL + \angle PLN + LNQ + \angle NQP = 360^o$

$90^o + 110^o + \angle LNQ + 90^o = 360^o$

$\angle LNQ = 360^o - 290^o$

$\angle LNQ = 70^o$

$\angle LNM = 70^o$… (i)

In ΔLMN, we have

LM = LN [Given]

$\Rightarrow \angle LNM = \angle LMN$ [Angles opposite to equal sides are equal]

$\angle LMN = 70^o$…(ii) [From (i)]

(ii) In ΔLMN, we have

$\angle LMN + \angle LNM + \angle MLN = 180^o$

But, $\angle LNM = \angle LMN = 70^o$ [From (i) and (ii)]

$\Rightarrow 70^o + 70^o + \angle MLN = 180^o$

$\angle MLN = 180^o - 140^o$

$\therefore \angle MLN = 40^o$

7. An isosceles triangle ABC has AC = BC. CD bisects AB at D and $\angle CAB = 55^o$.

Find: (i) $\angle DCB$ (ii) $\angle CBD$.

Solution:

In ΔABC, we have

AC = BC [Given]

So, $\angle CAB = \angle CBD$ [Angles opposite to equal sides are equal]

$\Rightarrow \angle CBD = 55^o$

In ΔABC, we have

$\angle CBA + \angle CAB + \angle ACB = 180^o$

But, $\angle CAB = \angle CBA = 55^o$

$55^o + 55^o + \angle ACB = 180^o$

$\angle ACB = 180^o - 110^o$

$\angle ACB = 70^o$

Now,

In ΔACD and ΔBCD, we have

AC = BC [Given]

CD = CD [Common]

AD = BD [Given that CD bisects AB]

$\therefore \Delta ACD \cong \Delta BCD$

So, By CPCT

$\angle DCA = \angle DCB$

$\angle DCB = \angle ACB/2 = 70^o/2$

Thus, $\angle DCB = 35^o$

8. Find x:

Solution:

Let's put markings to the figure as following:

In ΔABC, we have

AD = AC [Given]

$\therefore \angle ADC = \angle ACD$ [Angles opposite to equal sides are equal]

So, $\angle ADC = 42^o$

Now,

$\angle ADC = \angle DAB + DBA$ [Exterior angle is equal to the sum of opposite interior angles]

But,

$\angle DAB = \angle DBA$ [Given: BD = DA]

$\therefore \angle ADC = 2\angle DBA$

$2\angle DBA = 42^o$

$\angle DBA = 21^o$

To find x:

x = ∠CBA + ∠BCA [Exterior angle is equal to the sum of opposite interior angles]

We know that,

∠CBA = 21^{o}

∠BCA = 42^{o}

⇒ x = 21^{o} + 42^{o}

∴ x = 63^{o}

9. In the triangle ABC, BD bisects angle B and is perpendicular to AC. If the lengths of the sides of the triangle are expressed in terms of x and y as shown, find the values of x and y.

Solution:

In ∆ABC and ∆DBC, we have

BD = BD [Common]

∠BDA = ∠BDC [Each equal to 90^{o}]

∠ABD = ∠DBC [BD bisects ∠ABC]

∴ ∆ABD ≅ ∆DBC [ASA criterion]

Therefore, by CPCT

AD = DC

x + 1 = y + 2

x = y + 1… (i)

And, AB = BC

3x + 1 = 5y – 2

Substituting the value of x from (i), we get

3(y+1) + 1 = 5y – 2

3y + 3 + 1 = 5y – 2

3y + 4 = 5y – 2

2y = 6

$y = 3$

Putting $y = 3$ in (i), we get

$x = 3 + 1$

$\therefore x = 4$

10. In the given figure; AE // BD, AC // ED and AB = AC. Find ∠a, ∠b and ∠c.

Solution:

Let's assume points P and Q as shown below:

Given, $\angle PDQ = 58^\circ$

$\angle PDQ = \angle EDC = 58^\circ$ [Vertically opposite angles]

$\angle EDC = \angle ACB = 58^\circ$ [Corresponding angles $\because$ AC II ED]

In $\triangle ABC$, we have

AB = AC [Given]

$\therefore \angle ACB = \angle ABC = 58^\circ$ [Angles opposite to equal sides are equal]

Now,

$\angle ACB + \angle ABC + \angle BAC = 180^\circ$

$58^\circ + 58^\circ + a = 180^\circ$

$\angle a = 180^\circ - 116^\circ$

$\angle a = 64^\circ$

Since, AE II BD and AC is the transversal

$\angle ABC = \angle b$ [Corrosponding angles]

$\therefore \angle b = 58^\circ$

Also, since AE II BD and ED is the transversal

$\angle EDC = \angle c$ [Corrosponding angles]

$\therefore \angle c = 58^\circ$

11. In the following figure; AC = CD, AD = BD and ∠C = 58°.

Find ∠CAB.

Solution:

In $\triangle ACD$, we have

AC = CD [Given]

$\therefore$ $\angle CAD = \angle CDA$ [Angles opposite to equal sides are equal]

And,

$\angle ACD = 58^o$ [Given]

By angle sum property, we have

$\angle ACD + \angle CDA + \angle CAD = 180^o$

$58^o + 2\angle CAD = 180^o$

$2\angle CAD = 122^o$

$\angle CAD = \angle CDA = 61^o$… (i)

Now,

$\angle CDA = \angle DAB + \angle DBA$ [Exterior angles is equal to sum of opposite interior angles]

But,

$\angle DAB = \angle DBA$ [Given, AD = DB]

So, $\angle DAB + \angle DAB = \angle CDA$

$2\angle DAB = 61^o$

$\angle DAB = 30.5^o$… (ii)

In $\triangle ABC$, we have

$\angle CAB = \angle CAD + \angle DAB$

$\angle CAB = 61^o + 30.5^o$ [From (i) and (ii)]

$\therefore$ $\angle CAB = 91.5\ ^o$

12. In the figure of Q.11 is given above, if AC = AD = CD = BD; find angle ABC.

Solution:

In $\triangle ACD$, we have

AC = AD = CD [Given]

Hence, ACD is an equilateral triangle

∴ ∠ACD = ∠CDA = ∠CAD = 60°

Now,

∠CDA = ∠DAB + ∠ABD [Exterior angle is equal to sum of opposite interior angles]

But,

∠DAB = ∠ABD [Given, AD = DB]

So, ∠ABD + ∠ABD = ∠CDA

2∠ABD = 60°

∴ ∠ABD = ∠ABC = 30°

13. In △ABC; AB = AC and ∠A: ∠B = 8: 5; find ∠A.

Solution:

Let, ∠A = 8x and ∠B = 5x

Given, In △ABC

AB = AC

So, ∠B = ∠C = 5x [Angles opp. to equal sides are equal]

Now, by angle sum property

∠A + ∠B +C = 180°

8x + 5x + 5x = 180°

18x = 180°

x = 10°

Thus, as ∠A = 8x

∠A = 8 × 10°

∴ ∠A = 80°

14. In triangle ABC; ∠A = 60°, ∠C = 40°, and bisector of angle ABC meets side AC at point P. Show that BP = CP.

Solution:

In △ABC, we have

$\angle A = 60^\circ$

$\angle C = 40^\circ$

$\therefore \angle B = 180^\circ - 60^\circ - 40^\circ$ [By angle sum property]

$\angle B = 80^\circ$

Now, as BP is the bisector of $\angle ABC$

$\therefore \angle PBC = \angle ABC/2$

$\angle PBC = 40^\circ$

In ΔPBC, we have

$\angle PBC = \angle PCB = 40^\circ$

$\therefore$ BP = CP [Sides opposite to equal angles are equal]

15. In triangle ABC; angle ABC = 90° and P is a point on AC such that $\angle PBC = \angle PCB$. Show that: PA = PB.

Solution:

Let's assume $\angle PBC = \angle PCB = x$

In the right-angled triangle ABC,

$\angle ABC = 90^\circ$

$\angle ACB = x$

$\angle BAC = 180^\circ - (90^\circ + x)$ [By angle sum property]

$\angle BAC = (90^\circ - x)$ …(i)

And

$\angle ABP = \angle ABC - \angle PBC$

$\angle ABP = 90^\circ - x$ …(ii)

Thus, in the ΔABP from (i) and (ii), we have

$\angle BAP = \angle ABP$

Therefore, PA = PB [sides opp. to equal angles are equal]

16. ABC is an equilateral triangle. Its side BC is produced upto point E such that C is mid-point of BE. Calculate the measure of angles ACE and AEC.

Solution:

Given, ∆ABC is an equilateral triangle

So, AB = BC = AC

∠ABC = ∠CAB = ∠ACB = 60°

Now, as sum of two non-adjacent interior angles of a triangle is equal to the exterior angle

∠CAB + ∠CBA = ∠ACE

60° + 60° = ∠ACE

∠ACE = 120°

Now,

∆ACE is an isosceles triangle with AC = CF

∠EAC = ∠AEC

By angle sum property, we have

∠EAC + ∠AEC + ∠ACE = 180°

2∠AEC + 120° = 180°

2∠AEC = 180° – 120°

∠AEC = 30°

17. In triangle ABC, D is a point in AB such that AC = CD = DB. If ∠B = 28°, find the angle ACD.

Solution:

From given, we get

∆DBC is an isosceles triangle

⇒ CD = DB

∠DBC = ∠DCB [If two sides of a triangle are equal, then angles opposites to them are equal]

And, ∠B = ∠DBC = ∠DCB = 28°

By angle sum property, we have

∠DCB + ∠DBC + ∠BCD = 180°

$28^\circ + 28^\circ + \angle BCD = 180^\circ$

$\angle BCD = 180^\circ - 56^\circ$

$\angle BCD = 124^\circ$

As sum of two non-adjacent interior angles of a triangle is equal to the exterior angle, we have

$\angle DBC + \angle DCB = \angle DAC$

$28^\circ + 28^\circ = 56^\circ$

$\angle DAC = 56^\circ$

Now,

$\triangle ACD$ is an isosceles triangle with AC = DC

$\Rightarrow \angle ADC = \angle DAC = 56^\circ$

$\angle ADC + \angle DAC + \angle DCA = 180^\circ$ [By angle sum property]

$56^\circ + 56^\circ + \angle DCA = 180^\circ$

$\angle DCA = 180^\circ - 112^\circ$

$\angle DCA = 64^\circ$

Thus, $\angle ACD = 64^\circ$

18. In the given figure, AD = AB = AC, BD is parallel to CA and $\angle ACB = 65°$. Find $\angle DAC$.

Solution:

From figure, it's seen that

$\triangle ABC$ is an isosceles triangle with AB = AC

$\Rightarrow \angle ACB = \angle ABC$

As $\angle ACB = 65^\circ$ [Given]

$\therefore \angle ABC = 65^\circ$

By angle sum property, we have

$\angle ACB + \angle CAB + \angle ABC = 180^\circ$

$65^\circ + 65^\circ + \angle CAB = 180^\circ$

$\angle CAB = 180^\circ - 130^\circ$

$\angle CAB = 50^\circ$

As BD is parallel to CA, we have

$\angle CAB = \angle DBA$ as they are alternate angles

$\Rightarrow \angle CAB = \angle DBA = 50^\circ$

Again, from figure, it's seen that

ΔADB is an isosceles triangle with AD = AB.

$\Rightarrow \angle ADB = \angle DBA = 50^\circ$

By angle sum property, we have

$\angle ADB + \angle DAB + \angle DBA = 180^\circ$

$50^\circ + \angle DAB + 50^\circ = 180^\circ$

$\angle DAB = 180^\circ - 100^\circ$

$\angle DAB = 80^\circ$

Now,

$\angle DAC = \angle CAB + \angle DAB$

$\angle DAC = 50^\circ - 80^\circ$

$\angle DAC = 130^\circ$

19. Prove that a triangle ABC is isosceles, if:

(i) altitude AD bisects angles BAC, or

(ii) bisector of angle BAC is perpendicular to base BC.

Solution:

(i) In
ΔABC, if the altitude AD bisect $\angle BAC$.

Then, to prove: ΔABC is isosceles.

In
ΔADB and ΔADC, we have

$\angle BAD = \angle CAD$ (AD is bisector of $\angle BAC$)

AD = AD (Common)

$\angle ADB = \angle ADC$ (Each equal to 90°)

Therefore, $\Delta ADB \cong \Delta ADC$ by ASA congruence criterion

So, by CPCT

AB = AC

Hence, ΔABC is an isosceles.

(ii) In Δ ABC, the bisector of $\angle BAC$ is perpendicular to the base BC.

Then, to prove: ΔABC is isosceles.

In ΔADB and ΔADC,

$\angle BAD = \angle CAD$ (AD is bisector of $\angle BAC$)

AD = AD (Common)

$\angle ADB = \angle ADC$ (Each equal to 90°)

Therefore, $\Delta ADB \cong \Delta ADC$ by ASA congruence criterion

Thus, by CPCT

AB = AC

Hence, ΔABC is an isosceles.

20. In the given figure; AB = BC and AD = EC.

Prove that: BD = BE.

Solution:

In ΔABC, we have

AB = BC (Given)

So, $\angle BCA = \angle BAC$ (Angles opposite to equal sides are equal)

$\Rightarrow \angle BCD = \angle BAE$ ….(i)

Given, AD = EC

AD + DE = EC + DE (Adding DE on both sides)

⇒ AE = CD ….(ii)

Now, in ΔABE and ΔCBD, we have

AB = BC (Given)

∠BAE = ∠BCD [From (i)]

AE = CD [From (ii)]

Therefore, ΔABE ≅ ΔCBD by SAS congruence criterion

So, by CPCT

BE = BD

ICSE Class 9 Maths Selina Solutions Chapter 10 Exercise 10B

1. If the equal sides of an isosceles triangle are produced, prove that the exterior angles so formed are obtuse and equal.

Solution:

Construction: AB is produced to D and AC is produced to E so that exterior angles ∠DBC and ∠ECB are formed.

In △ABC, we have

AB = AC [Given]

∴ ∠C = ∠B …(i) [Angles opposite to equal sides are equal]

Since, ∠B and ∠C are acute they cannot be right angles or obtuse angles

Now,

∠ABC + ∠DBC = 180^0 [ABD is a straight line]

∠DBC = 180^0 – ∠ABC

∠DBC = 180^0 – ∠B …(ii)

Similarly,

∠ACB + ECB = 180^0 [ABD is a straight line]

∠ECB = 180^0 – ∠ACB

∠ECB = 180^0 – ∠C …(iii)

$\angle$ECB = $180^0 - \angle$B …(iv) [from (i) and (iii)]

$\angle$DBC = $\angle$ECB [from (ii) and (iv)]

Now,

$\angle$DBC = $180^0 - \angle$B

But, $\angle$B is an acute angle

$\Rightarrow$ $\angle$DBC = 180^0 – (acute angle) = obtuse angle

Similarly,

$\angle$ECB = $180^0 - \angle$C

But, $\angle$C is an acute angle

$\Rightarrow$ $\angle$ECB = 180^0 – (acute angle) = obtuse angle

Therefore, exterior angles formed are obtuse and equal.

2. In the given figure, AB = AC. Prove that:

(i) DP = DQ

(ii) AP = AQ

(iii) AD bisects $\angle$A

Solution:

Construction: Join AD.

In $\triangle$ABC, we have

AB = AC [Given]

$\therefore$ $\angle$C = $\angle$B …(i) [Angles opposite to equal sides are equal]

(i) In $\triangle$BPD and $\triangle$CQD, we have

$\angle$BPD = $\angle$CQD [Each = 90^o]

$\angle$B = $\angle$C [Proved]

BD = DC [Given]

Thus, $\triangle$BPD $\cong$ $\triangle$CQD by AAS congruence criterion

$\therefore$ DP = DQ by CPCT

(ii) Since, $\Delta BPD \cong \Delta CQD$

Therefore, BP = CQ [CPCT]

Now,

AB = AC [Given]

AB – BP = AC – CQ

AP = AQ

(iii) In ΔAPD and ΔAQD, we have

DP = DQ [Proved]

AD = AD [Common]

AP = AQ [Proved]

Thus, $\Delta APD \cong \Delta AQD$ by SSS congruence criterion

$\angle PAD = \angle QAD$ by CPCT

Hence, AD bisects angle A.

3. In triangle ABC, AB = AC; BE $\perp$ AC and CF $\perp$ AB. Prove that:

(i) BE = CF

(ii) AF = AE

Solution:

(i) In ΔAEB and ΔAFC, we have

$\angle A = \angle A$ [Common]

$\angle AEB = \angle AFC = 90^{\circ}$ [Given : BE $\perp$ AC and CE $\perp$ AB]

AB = AC [Given]

Thus, $\Delta AEB \cong \Delta AFC$ by AAS congruence criterion

$\therefore$ BE = CF by CPCT

(ii) Since, $\Delta AEB \cong \Delta AFC$

$\angle ABE = \angle AFC$

$\therefore$ AF= AE by CPCT

4. In isosceles triangle ABC, AB = AC. The side BA is produced to D such that BA = AD. Prove that: ∠BCD = 90°

Solution:

Construction: Join CD.

In ∆ABC, we have

AB = AC [Given]

∴ ∠C = ∠B … (i) [Angles opposite to equal sides are equal]

In ∆ACD, we have

AC = AD [Given]

∴ ∠ADC = ∠ACD … (ii)

Adding (i) and (ii), we get

∠B + ∠ADC = ∠C + ∠ACD

∠B + ∠ADC = ∠BCD … (iii)

In ∆BCD, we have

∠B + ∠ADC +∠BCD = 180°

∠BCD + ∠BCD = 180° [From (iii)]

2∠BCD = 180°

∴ ∠BCD = 90°

5. (i) In ∆ABC, AB = AC and ∠A= 36°. If the internal bisector of ∠C meets AB at point D, prove that AD = BC.

(ii) If the bisector of an angle of a triangle bisects the opposite side, prove that the triangle is isosceles.

Solution:

Given, AB = AC and ∠A = 36°

So, ∆ABC is an isosceles triangle.

∠B = ∠C = $(180^0 – 36^o)/2$ = 72°

∠ACD = ∠BCD = 36° [∵ CD is the angle bisector of ∠C]

Now, ∆ADC is an isosceles triangle as ∠DAC = ∠DCA = 36°

∴ AD = CD …(i)

In ∆DCB, by angle sum property we have

∠CDB = 180° – (∠DCB + ∠DBC)

= 180° – (36° + 72°)

= 180° – 108°

= 72°

Now, ∆DCB is an isosceles triangle as ∠CDB = ∠CBD = 72°

∴ DC = BC …(ii)

From (i) and (ii), we get

AD = BC

– Hence Proved.

6. Prove that the bisectors of the base angles of an isosceles triangle are equal.

Solution:

In ∆ABC, we have

AB = AC [Given]

∴ ∠C = ∠B …(i) [Angles opposite to equal sides are equal]

½∠C = ½∠B

⇒ ∠BCF = ∠CBE …(ii)

Now, in ∆BCE and ∆CBF, we have

∠C = ∠B [From (i)]

∠BCF = ∠CBE [From (ii)]

BC = BC [Common]

∴ ∆BCE ≅ ∆CBF by AAS congruence criterion

Thus, BE = CF by CPCT

7. In the given figure, AB = AC and ∠DBC = ∠ECB = 90°

Prove that:

(i) BD = CE

(ii) AD = AE

Solution:

In △ABC, we have

AB = AC [Given]

∴ ∠ACB = ∠ABC [Angles opposite to equal sides are equal]

⇒ ∠ABC = ∠ACB … (i)

∠DBC = ∠ECB = 90° [Given]

⇒ ∠DBC = ∠ECB …(ii)

Subtracting (i) from (ii), we get

∠DCB – ∠ABC = ∠ECB – ∠ACB

∠DBA = ∠ECA … (iii)

Now,

In ΔDBA and ΔECA, we have

∠DBA = ∠ECA [From (iii)]
∠DAB = ∠EAC [Vertically opposite angles]
AB = AC [Given]
∴ ΔDBA ≅ ΔECA by ASA congruence criterion

Thus, by CPCT

BD = CE

And, also

AD = AE

8. ABC and DBC are two isosceles triangles on the same side of BC. Prove that:

(i) DA (or AD) produced bisects BC at right angle.

(ii) ∠BDA = ∠CDA.

Solution:

DA is produced to meet BC in L

In △ABC, we have

AB = AC [Given]

∴ ∠ACB = ∠ABC … (i) [Angles opposite to equal sides are equal]

In ∆DBC, we have

DB = DC [Given]

∴ ∠DCB = ∠DBC … (ii) [Angles opposite to equal sides are equal]

Subtracting (i) from (ii), we get

∠DCB – ∠ACB = ∠DBC – ∠ABC

∠DCA = ∠DBA …(iii)

Now,

In ∆DBA and ∆DCA, we have

DB = DC [Given]

∠DBA = ∠DCA [From (iii)]

AB = AC [Given]

∴ ∆DBA ≅ ∆DCA by SAS congruence criterion

∠BDA = ∠CDA …(iv) [By CPCT]

In ∆DBA, we have

∠BAL = ∠DBA + ∠BDA …(v) [Exterior angle = sum of opposite interior angles]

From (iii), (iv) and (v), we get

∠BAL = ∠DCA + ∠CDA …(vi) [Exterior angle = sum of opposite interior angles]

In ∆DCA, we have

∠CAL = ∠DCA + ∠CDA …(vi)

From (vi) and (vii)

∠BAL = ∠CAL …(viii)

In ∆BAL and ∆CAL,

∠BAL = ∠CAL [From (viii)]

∠ABL = ∠ACL [From (i)

AB = AC [Given]

∴ ∆BAL ≅ ∆CAL by ASA congruence criterion

So, by CPCT

∠ALB = ∠ALC

And, BL = LC …(ix)

Now,

∠ALB + ∠ALC = 180°

∠ALB + ∠ALB = 180° [Using (ix)]

2∠ALB = 180°

∠ALB = 90°

∴ AL ⊥ BC

Or DL ⊥ BC and BL ⊥ LC

Therefore, DA produced bisects BC at right angle.

9. The bisectors of the equal angles B and C of an isosceles triangle ABC meet at O. Prove that AO bisects angle A.

Solution:

In ∆ABC, we have AB = AC

∠B = ∠C [Angles opposite to equal sides are equal]

½∠B = ½∠C

∠OBC = ∠OCB …(i)

⇒ OB = OC …(ii) [Sides opposite to equal angles are equal]

Now,

In ∆ABO and ∆ACO, we have

AB = AC [Given]

∠OBC = ∠OCB [From (i)]

OB = OC [From (ii)]

Thus, ∆ABO ≅ ∆ACO by SAS congruence criterion

So, by CPCT

∠BAO = ∠CAO

Therefore, AO bisects ∠BAC.

10. Prove that the medians corresponding to equal sides of an isosceles triangle are equal.

Solution:

In ∆ABC, we have

AB = AC [Given]

∠C = ∠B … (i) [Angles opposite to equal sides are equal]

Now,

½ AB = ½ AC

BF = CE … (ii)

In ∆BCE and ∆CBF, we have

∠C = ∠B [From (i)]

BF = CE [From (ii)]

BC = BC [Common]

∴ ∆BCE ≅ ∆CBF by SAS congruence criterion

So, CPCT

BE = CF

11. Use the given figure to prove that, AB = AC.

Solution:

In ∆APQ, we have

AP = AQ [Given]

∴ ∠APQ = ∠AQP …(i) [Angles opposite to equal sides are equal]

In ∆ABP, we have

∠APQ = ∠BAP + ∠ABP …(ii) [Exterior angle is equal to sum of opposite interior angles]

In ∆AQC, we have

∠AQP = ∠CAQ + ∠ACQ …(iii) [Exterior angle is equal to sum of opposite interior angles]

From (i), (ii) and (iii), we get

∠BAP + ∠ABP = ∠CAQ + ∠ACQ

But, ∠BAP = ∠CAQ [Given]

∠CAQ + ABP = ∠CAQ + ∠ACQ

∠ABP = ∠CAQ + ∠ACQ – ∠CAQ

∠ABP = ∠ACQ

∠B = ∠C

So, in ∆ABC, we have

∠B = ∠C

⇒ AB = AC [Sides opposite to equal angles are equal]

12. In the given figure; AE bisects exterior angle CAD and AE is parallel to BC.

Prove that: AB = AC.

Solution:

Since, AE || BC and DAB is the transversal

∴ ∠DAE = ∠ABC = ∠B [Corresponding angles]

Since, AE || BC and AC is the transversal

∠CAE = ∠ACB = ∠C [Alternate angles]

But, AE bisects ∠CAD

∴ ∠DAE = ∠CAE

∠B = ∠C

⇒ AB = AC [Sides opposite to equal angles are equal]

13. In an equilateral triangle ABC; points P, Q and R are taken on the sides AB, BC and CA respectively such that AP = BQ = CR. Prove that triangle PQR is equilateral.

Solution:'

Given, AB = BC = CA (Since, ABC is an equilateral triangle) …(i)

and AP = BQ = CR …(ii)

Subtracting (ii) from (i), we get

AB – AP = BC – BQ = CA – CR

BP = CQ = AR …(iii)

∴ ∠A = ∠B = ∠C …(iv) [Angles opposite to equal sides are equal]

In ∆BPQ and ∆CQR, we have

BP = CQ [From (iii)]

∠B = ∠C [From (iv)]

BQ = CR [Given]

∴ ∆BPQ ≅ ∆CQR by SAS congruence criterion

So, PQ = QR [by CPCT] … (v)

In ∆CQR and ∆APR, we have

CQ = AR [From (iii)]

∠C = ∠A [From (iv)]

CR = AP [Given]

∴ ∆CQR ≅ ∆APR by SAS congruence criterion

So, QR = PR [By CPCT] … (vi)

From (v) and (vi), we get

PQ = QR = PR

Therefore, PQR is an equilateral triangle.

Benefits of ICSE Class 9 Maths Selina Solutions Chapter 10

Studying Chapter 10 on Isosceles Triangles from the ICSE Class 9 Maths Selina Solutions offers several benefits:

Understanding Basic Properties: Students gain a clear understanding of the fundamental properties of isosceles triangles, including the fact that the angles opposite the equal sides are equal. This foundational knowledge is crucial for solving more complex geometric problems.

Enhanced Problem-Solving Skills: By applying the properties and theorems related to isosceles triangles, students develop strong problem-solving skills. They learn how to use these properties to find missing angles and sides in various geometric configurations.

Foundation for Advanced Topics: Mastery of isosceles triangles provides a solid foundation for studying more advanced geometric concepts. Understanding these basic principles helps in grasping more complex theorems and proofs in later chapters.

Preparation for Exams: The ICSE Class 9 Maths Selina Solutions Chapter 10 equips students with the knowledge and practice needed for exams. It covers essential theorems and their applications, which are often tested in ICSE exams.

Improved Analytical Thinking: Working through problems involving isosceles triangles enhances analytical and logical thinking skills. Students learn to approach problems systematically and apply theoretical knowledge in practical scenarios.