Applied Thermodynamics

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(This Module shall only be Used for Educational Purpose.)

Applied Thermodynamics

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1 IC ENGINE CYCLES

1.1 Introduction

1.1.1 Application of TD (IC Engine)

(1) Swept volume $V_S = \frac{\pi}{4} D^2 L_S$

Length of stroke $L_S = 2 \times r_C$ ($r_C \rightarrow$ radius of crank)

(2) Compression ratio $r = \frac{V_T}{V_C} = \frac{V_S + V_C}{V_C} = 1 + \frac{V_S}{V_C} \Rightarrow r > 1$

(3) Clearance ratio $C = \frac{V_C}{V_S} \Rightarrow \frac{1}{C} = \frac{V_S}{V_C} = r - 1 \Rightarrow \boxed{r = 1 + \frac{1}{C}}$

Assumptions while dealing with Engines

1. Mass in cylinder is fixed. (Air is perfect gas)
2. Combustion is replaced by heat transfer process from external source.
3. All the processes involved are internally reversible.
4. Specific heats of the working substance are assumed to be constant.

1. 2 Otto cycle

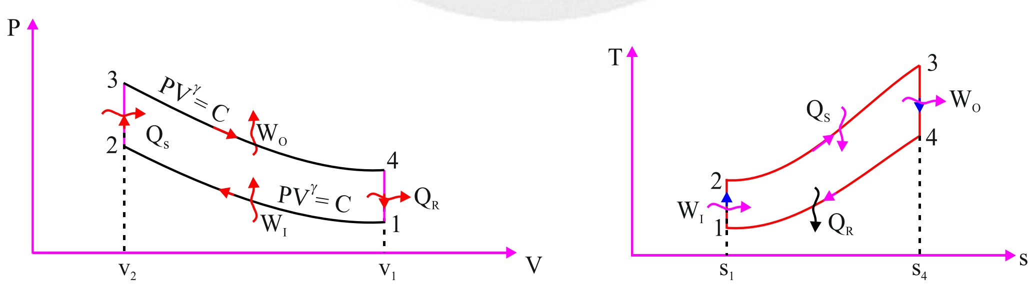


Fig. 1.1. P-V and T-s plots of Otto cycle

Processes involved:
$1 - 2 \rightarrow$ Internally Reversible Isentropic compression
$2 - 3 \rightarrow$ Internally Reversible Constant Volume Heat addition
$3 - 4 \rightarrow$ Internally Reversible Isentropic expansion
$4 - 1 \rightarrow$ Internally Reversible Constant Volume Heat rejection

Clearance Ratio $C = \frac{v_2}{v_1 - v_2}$; $r = 1 + \frac{1}{C} = 1 + \frac{v_1}{v_2} - 1 = \frac{v_1}{v_2}$

Pressure ratio $r_P = \frac{P_3}{P_2} = \frac{P_4}{P_1} = \frac{T_3}{T_2} = \frac{T_4}{T_1}$

$\boxed{T_1T_3 = T_2T_4}$ and $\boxed{P_1P_3 = P_2P_4}$ and $\boxed{v_1v_3 = v_2v_4}$ and $\boxed{s_1s_3 = s_2s_4}$

$w_I = u_2 - u_1$; $w_O = u_3 - u_4$

$q_S = u_3 - u_2$; $q_R = u_4 - u_1$

$$\eta_{th,otto} = \frac{\sum w}{q_S} = \frac{(u_3-u_4)-(u_2-u_1)}{(u_3-u_2)}$$

$$= 1 - \frac{(u_4-u_1)}{(u_3-u_2)} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r^{\gamma-1}}$$

$$\Sigma q = q_S - q_R = (u_3-u_2)-(u_4-u_1) = u_1 - u_2 + u_3 - u_4$$

$$\Sigma w = w_O - w_I = (u_3-u_4)-(u_2-u_1) = u_1 - u_2 + u_3 - u_4$$

$$\Sigma q = \Sigma w$$

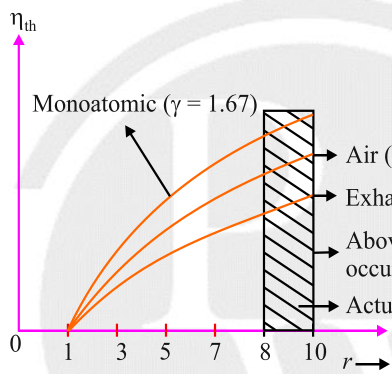


Fig. 1.2. Variation of thermal efficiency with working fluid

In general, η_{th} of spark ignition is from 25% to 30%

- Network output for Otto cycle

$$\Sigma w = w_O - w_I = (u_3-u_4)-(u_2-u_1)$$

$$= u_1 - u_2 + u_3 - u_4$$

$$= c_v(T_1 - T_2 + T_3 - T_4)$$

$$\therefore \quad \Sigma w = c_v.T_1(r^{\gamma-1}-1)(r_P-1)$$

$$= \frac{P_1v_1}{\gamma-1}(r^{\gamma-1}-1)(r_P-1)$$

$$\boxed{\therefore \ \Sigma w = \frac{P_1v_1}{\gamma-1}(r^{\gamma-1}-1)(r_P-1)}$$

For max. network

$$r^{\gamma-1} = \sqrt{\frac{T_{max}}{T_{min}}}$$

and $T_2 = T_4 = \sqrt{T_{max}.T_{min}}$

$$\Sigma w_{max} = c_v\left[\sqrt{T_{max}} - \sqrt{T_{min}}\right]^2$$

$T_1 = T_{min} \rightarrow$ Environmental constraint

$T_3 = T_{max} \rightarrow$ Material constraint

- Mean Effective Pressure: (P_{mean})

$$\Sigma w = w_O - w_I = P_{mean} \times v_s$$

$$\boxed{P_{mean} = \frac{P_1}{(\gamma-1)}\left(\frac{r}{r-1}\right)\left[(r^{\gamma-1}-1)(r_P-1)\right]}$$

- In general, r of S.I is 7 to 10
  Tetra Ethyl Lead (TEL) can take it to 12; but exhaust + TEL is poisonous. So, limited.

1.3 Diesel Cycle

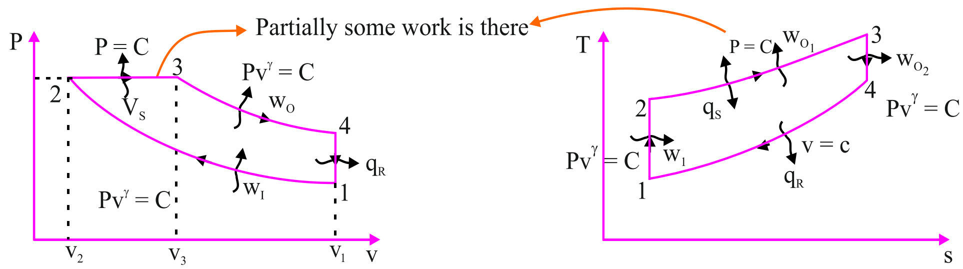


Fig. 1.3. P-v and T-s plots of Diesel cycle

Processes involved: 1 – 2 → Internally Reversible Isentropic compression
2 – 3 → Internally Reversible Constant Pressure Heat addition
3 – 4 → Internally Reversible Isentropic expansion
4 – 1 → Internally Reversible Constant Volume Heat rejection

Clearance ratio $'C' = \frac{v_2}{v_1 - v_2}$; $r = 1 + \frac{1}{C} = \frac{v_1}{v_2}$

Pressure ratio $(r_p) = \frac{P_2}{P_1}$; cut off volume = $v_3 - v_2$

Cut off Ratio $(\rho) = \frac{v_3}{v_2}$; Expansion ratio $\varepsilon = \frac{v_4}{v_3}$

$r = \frac{v_1}{v_2}$; $\varepsilon = \frac{v_4}{v_3} = \frac{v_1}{v_3}$

$\because\ v_3 > v_2$

$\boxed{\Rightarrow r > \varepsilon}$: $\boxed{r = (r_P)^{1/\gamma}}$

$$\boxed{r = \frac{v_1}{v_2} = 1 + \frac{1}{C} = (r_P)^{1/\gamma} = \rho \times \varepsilon}$$

$$\eta_{th,D} = 1 - \frac{(u_4 - u_1)}{(h_3 - h_2)}$$

$$\eta_{th,D} = 1 - \frac{1}{r^{\gamma-1}}[F]$$

Where $F = \frac{\rho^\gamma - 1}{\gamma(\rho - 1)}$;

For same 'r'

→ If F > 1 $\Rightarrow \eta_{otto} > \eta_{Diesel}$

As $\rho\uparrow$; $F\uparrow$

If $\rho = 1 \Rightarrow v_3 = v_2$

$\eta_{otto} = \eta_{Diesel}$

In general:

'r' for Diesel engines is 12 to 24 and η is between 35 to 40%

→ **Net Work**

$\Sigma w = \Sigma q = q_S - q_R$

$= (h_3 - h_2) - (u_4 - u_1)$

$$\boxed{\Sigma w = \frac{P_1 v_1}{\gamma - 1}\left[r^{\gamma-1}\{\gamma \, . \, (\rho - 1)\} - (\rho^{\gamma} - 1)\right]}$$

$$\boxed{p_m = \frac{P_1 r}{(\gamma - 1)(r - 1)}\left[r^{\gamma-1}\{\gamma(\rho - 1)\} - (\rho^{\gamma} - 1)\right]}$$

→ For same initial conditions and same pressure ratio,

on increasing Maximum temperature → **η_{th} will decrease**

→ Σ w increases slightly

But q'_s increases significantly.

∴ η_{th} **will ↓**

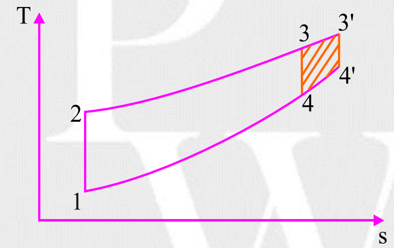
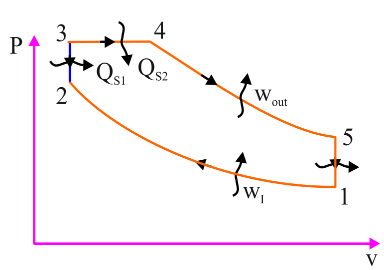


Fig. 1.4. Diesel cycle with increased T_{max}

So, As max. Temp ↑ ; η ↓

Same compression Ratio + any other condition $\Rightarrow \eta_{otto} > \eta_{Diesel}$
Same Maximum Pressure + any other condition $\Rightarrow \eta_{Diesel} > \eta_{otto}$

1.4 Dual cycle (Limited Pressure cycle (or) Mixed cycle)



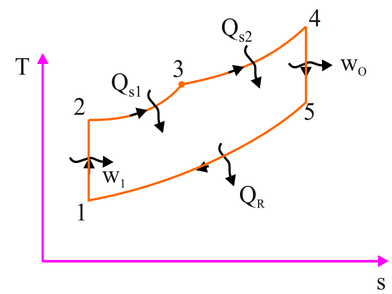


Fig. 1.5. P-v and T-s plots of Dual cycle

Processes involved:
1 – 2 $\rightarrow$ Internally reversible Isentropic compression
2 – 3 $\rightarrow$ Internally Reversible Isochoric Heat addition
3 – 4 $\rightarrow$ Internally Reversible Isobaric Heat Addition
4 – 5 $\rightarrow$ Internally Reversible Isentropic expansion
5 – 1 $\rightarrow$ Internally Reversible Isochoric Heat rejection.

$$\eta_{th} = \frac{\sum W}{Q_s} = \frac{\sum Q}{Q_s} = \frac{Q_{s,1} + Q_{s,2} - Q_R}{Q_{s,1} + Q_{s,2}}$$

$$\eta_{th,\ Cycle} = \frac{mc_v(T_3 - T_2) + mc_p(T_4 - T_3) - mc_v(T_5 - T_1)}{mc_v(T_3 - T_2) + mc_p(T_4 - T_3)}$$

When P_3 tends to P_2, *Dual* $\rightarrow$ Diesel

When V_4 tends to V_3, Dual $\rightarrow$ Otto

For same compression ratio $\rightarrow$ $\eta_{otto} > \eta_{Dual} > \eta_{Diesel}$
For same Peak Pressure $\rightarrow$ $\eta_{Diesel} > \eta_{Dual} > \eta_{otto}$

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2 GAS POWER CYCLES

2.1 Introduction

2.1.1 Gas Power Cycles

Gas Turbine

Rotodynamic machine that converts thermal Energy of gas into Mechanical work.

Closed Cycle Gas Turbine

Joule Proposed it. The main components of a power plant working on Joule cycle are

1. Compressor   2. Heat Exchange   3. Turbine   4. $\boxed{\text{Inter Cooler}}$ ↓

Decrease Specific Volume of Working fluid.

Reason for Intercooling

The lower value of specific volume Decrease the compressor work $(-\int vdp)$

Open Cycle Gas Turbine:

In Brayton cycle, combustion supplies the heat required. (4 –1 Process is heat rejection to atmosphere at constant pressure).

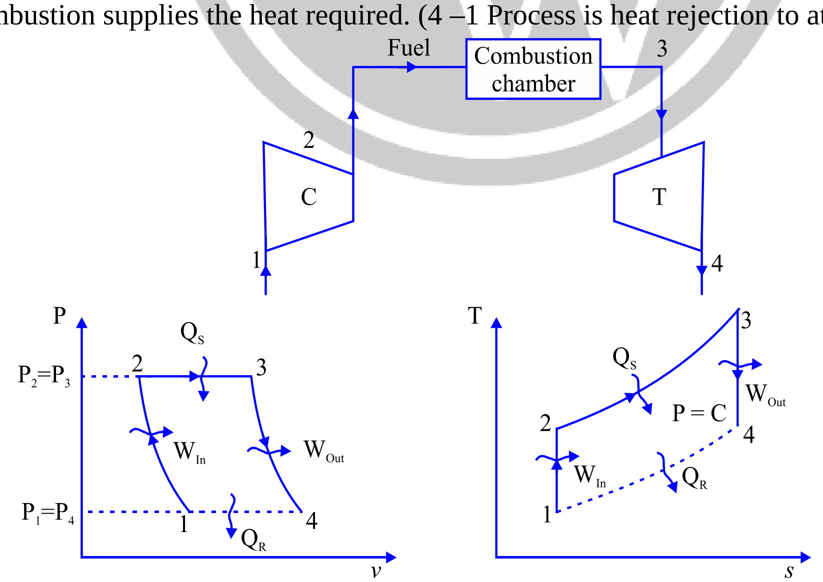

Fig. 2.1. Components of open cycle gas turbine and its graph on P-v and T-s plots

$$W_{In} = C_P(T_2 - T_1);\quad Q_S = C_P(T_3 - T_2);\quad W_{Out} = C_P(T_3 - T_4)$$

$$W_{net} = \Sigma Q = C_P(T_3 - T_4 - T_2 + T_1)$$

$$\boxed{P_1P_3 = P_2P_4}\quad ;\quad \boxed{T_1T_3 = T_2T_4}\quad ;$$

$$\eta_{th} = 1 - \frac{1}{(r_P)^{\left(\frac{\gamma-1}{\gamma}\right)}}$$ where $r_P = \frac{P_2}{P_1}$ = Pressure ratio.

$\therefore$ η_{joule} increases when γ increases and r_P increases

2.1.2 Back Work Ratio: (B.W.R)

$$\text{B.W.R} = \frac{W_C}{W_T} = \frac{C_P(T_2 - T_1)}{C_P(T_3 - T_4)} = \frac{T_2 - T_1}{T_3 - T_4}$$ {Generally around 0.8}

2.1.3 Work Ratio: (W.R)

$$\text{W.R} = \frac{W_{net}}{W_T} = \frac{W_T - W_C}{W_T} = 1 - \frac{W_C}{W_T} = 1 - \text{B.W.R.}$$

$\Rightarrow$ $\boxed{\text{Work Ratio + Back Work Ratio = 1}}$

2.1.4 Condition for maximum work output in Brayton Cycle for given maximum and minimum temperatures.

$$r_P = \frac{P_2}{P_1} \text{ and } W_{net} = C_P\{T_3 - T_4 - T_2 + T_1\}$$

For max. work: $$\frac{dW_{net}}{dr_P} = O \Rightarrow \frac{T_{max}}{T_{min}} = (r_P)^{2\left(\frac{\gamma-1}{\gamma}\right)}$$

So, for Max. work output.

$$\boxed{\frac{T_{max}}{T_{min}} = (r_P)^{\frac{2(\gamma-1)}{\gamma}}} \quad \text{and} \quad \boxed{(r_P)_{optimal} = \left(\frac{T_{max}}{T_{min}}\right)^{\frac{\gamma}{2(\gamma-1)}}}$$

and $T_2 = T_4 = \sqrt{T_1.T_3}$ and $\boxed{W_{max} = C_P.\left\{\sqrt{T_{max}} - \sqrt{T_{min}}\right\}^2}$

$$\boxed{\eta = 1 - \sqrt{\frac{T_{min}}{T_{max}}}}$$

2.1.4 Actual Gas Turbine Cycle

In Practice, compression & expansion at such high temperature are not Isentropic

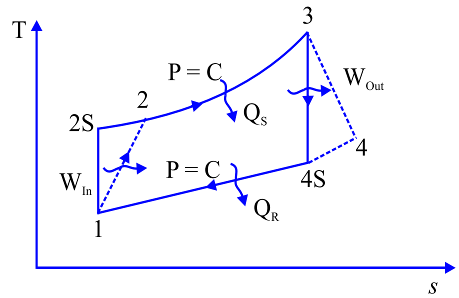

Fig. 2.2 T-s diagram of Actual gas turbine cycle

$$\eta_{compressor} = \frac{T_{2S} - T_1}{T_2 - T_1}; \quad \eta_{turbine} = \frac{T_3 - T_4}{T_3 - T_{4S}}$$

$$W_{net} = C_P \cdot \eta_{turbine}(T_3 - T_{4S}) - \frac{C_P}{\eta_{compressor}}(T_{2S} - T_1)$$

For Max. Work output.

$$\frac{dW_{net}}{dr_P} = O \quad \Rightarrow \quad (r_P)_{optimal} = \left\{\eta_T \cdot \eta_C \left(\frac{T_{max}}{T_{min}}\right)\right\}^{\frac{\gamma}{2(\gamma-1)}}$$

2.2 Methods for improvement in Performance of open cycle Gas Turbine:

2.2.1 Regeneration

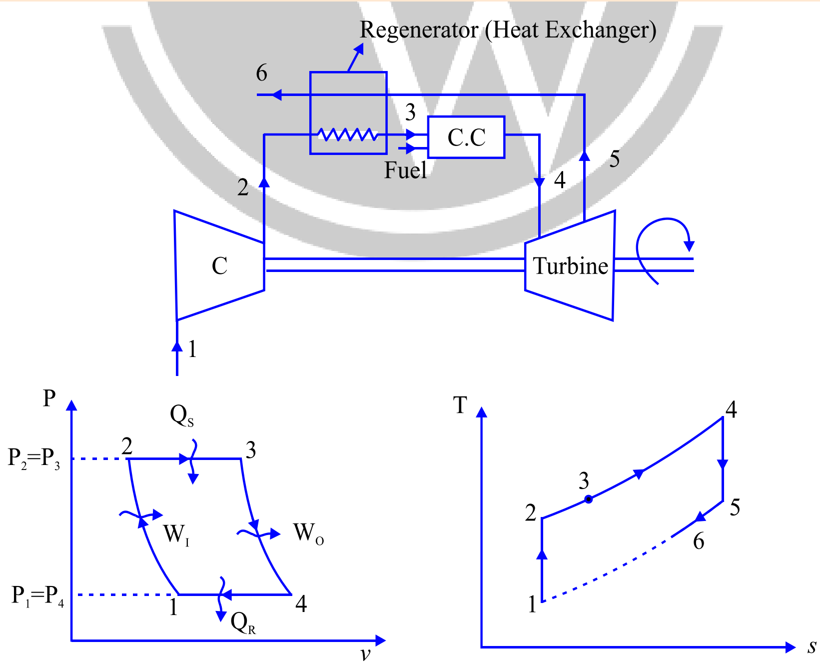

Fig. 2.3. Components of regenerative open cycle gas turbine and its graph on P-v and T-s plot

Temperature of gases leaving the turbine is very high. A counter flow heat exchanger called Regenerator is used to transfer the heat available in exhaust gases to the intake air. This results in decrease in the heat supply in combustion chamber for the same work output from the turbine. So $\eta_{th}\uparrow$.

- Regeneration is possible only when $T_5 > T_2$. {Turbine exit Temp > Compressor exit Temp)

Effectiveness of Regeneration: $\varepsilon = \dfrac{C_P(T_3 - T_2)}{C_P(T_5 - T_2)}$

i.e. $\varepsilon = \dfrac{\text{Actual H.T to air}}{\text{Max. Possible H.T to air}}$

$\therefore$ For regenerative cycle: $\eta_{th} = \dfrac{W_{net}}{Q_{3-4}}$;

For ideal Regeneration case: $\boxed{\eta_{th} = 1 - \dfrac{(r_P)^{\frac{\gamma-1}{\gamma}}}{\left(\dfrac{T_{max}}{T_{min}}\right)}}$

For a given cycle

This is in contradiction to Brayton Cycle.

When $r_P = 1$; $\boxed{\eta_{Regeneration} = \eta_{carnot}}$

In Regeneration, Heat supply to the system decreases for same work output from the cycle. So thermal efficiency increases.

2.2.2 Reheating

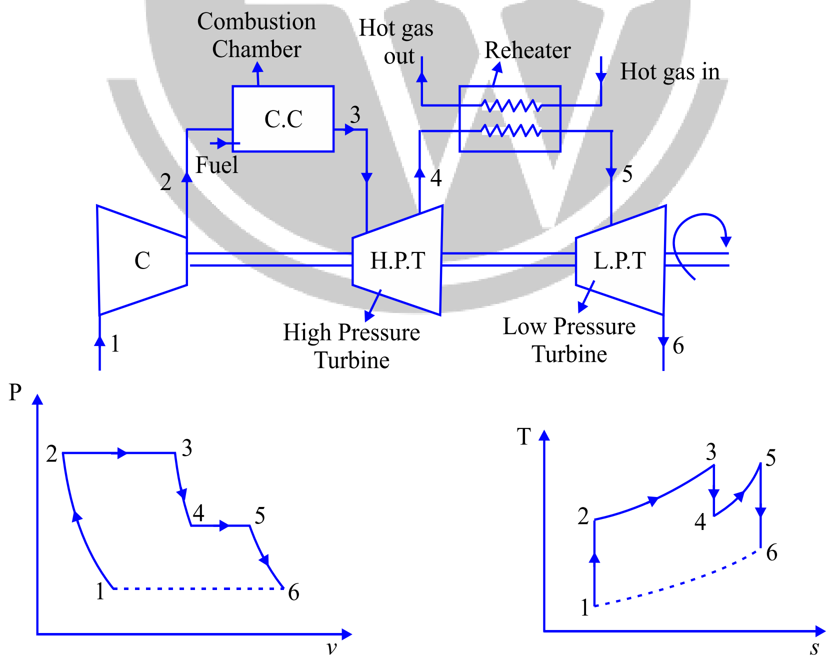

Fig. 2.4. Components of open cycle gas turbine with reheating and its graph on P-v and T-s plot

$$W_{net} = C_P(T_3 - T_4) + C_P(T_5 - T_6) - C_P(T_2 - T_1)$$

$$Q_S = C_P(T_3 - T_2) + C_P(T_5 - T_4).$$

Here $W_{net} \uparrow; Q_s \uparrow; \eta \downarrow$

2.2.3 Intercooling

The cooling of air between two compressors is called Intercooling.

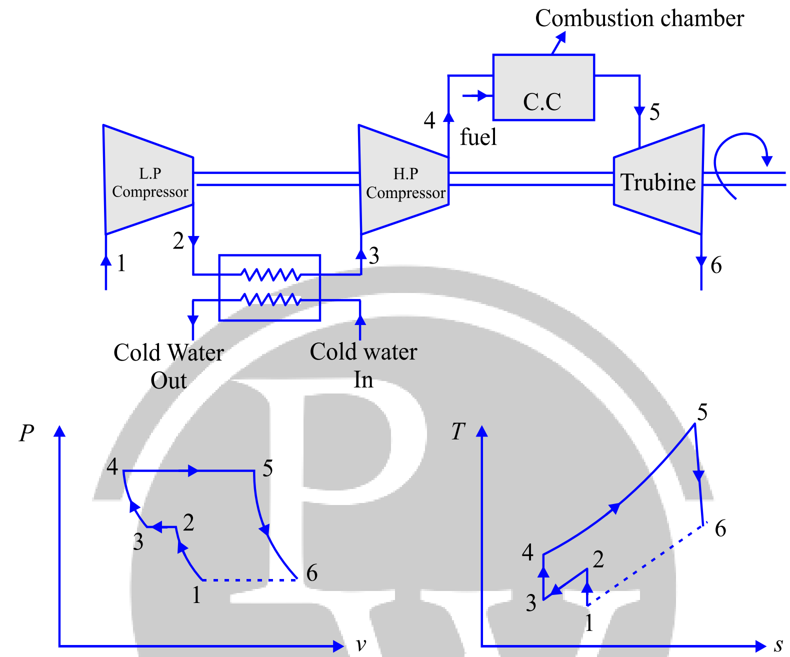

Fig. 2.5 Components of open cycle gas turbine with intercooling and its graph on P-v and T-s plot

$$W_{net} = C_P(T_5 - T_6) - C_P(T_4 - T_5) - C_P(T_2 - T_1)$$

$$Q_S = C_P(T_5 - T_4)$$

Hence, $W_{net} \uparrow$; $Q_s \uparrow$; $\eta \downarrow$ as mean temperature of heat addition decreases.

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3 VAPOUR POWER CYCLES

3.1 Introduction

Cycles are required to serve the need of continuous conversion of Heat to Work.

Phase changes some process

→ Vapor Power Cycle

→ Gas Power Cycle

No Phase change through out

Based on the phase of the working system

3.1.1 Different components of a Thermal Power Plant

Basic Vapour powerplant operates on the principle of Rankine cycle and the basic components of the cycle are

(i) Turbine
(ii) Condenser
(iii) Feed Pump
(iv) Steam Generator (Boiler and Superheater)

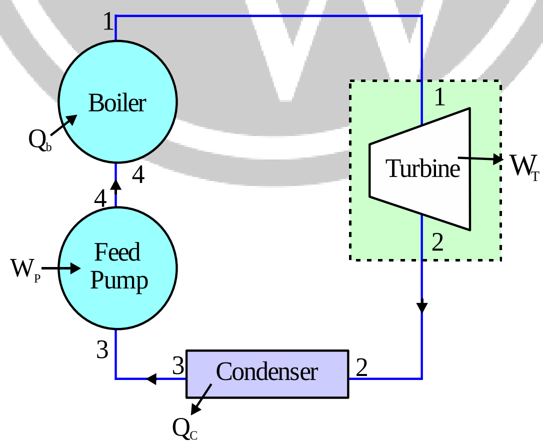


Fig. 3.1. Components of Vapour Power Cycle

(i) Turbine is insulated to avoid unnecessary loss of heat. So (1-2) will be isentropic

(ii) State 1 is at dry saturated state under ideal conditions.

(iii) Isentropic compression is most practically feasible compression process involving minimum work.

So, 3-4 is assumed as isentropic compression.

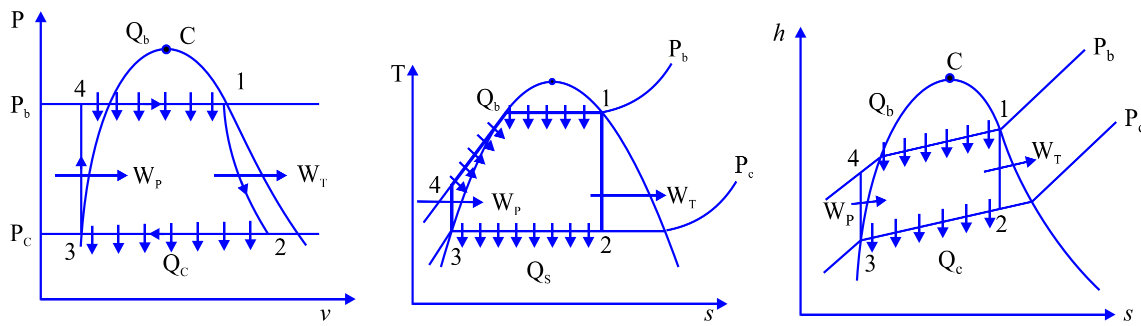


Fig. 3.2 P-v, T-s and h-s diagrams of vapour power cycle with no superheating

The line 3-4 in the P- v is almost straight because pumps handle liquids and liquids are practically incompressible (Constant specific Volume).

Applying S.F.E.E. for different components individually.

We find, $W_T = h_1 - h_2; W_P = h_4 - h_3$ ($W_P \rightarrow$work required per unit mass of working fluid)

$Q_b = h_1 - h_4$ and $Q_C = h_2 - h_3$ (Q_b and Q_c are heat added and rejected respectively per unit mass of working fluid)

$$\text{So,}\quad \eta_{th} = \frac{W_{net}}{Q_b} = \frac{W_T - W_P}{Q_b} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)} = 1 - \frac{(h_2 - h_3)}{(h_1 - h_4)}$$

$$\eta_{th} = \frac{W_{net}}{Q_b} = 1 - \frac{Q_C}{Q_b}.$$ So, h_1, h_2, h_3, h_4 are to be known from the steam tables.

In process 3 – 4, since the specific volume of liquids is very small, we generally neglect pump work. So, in general $h_3 \approx h_4$.

3.1.2 Specific Steam Consumption

The mass of steam required to produce 1 unit of power.

$$\boxed{\text{S.S.C.} = \frac{3600}{W_{net}} \frac{\text{Kg}}{\text{kW} - \text{hr}}}$$

where W_{net} is in kJ/kg.

3. 2 Practicalities of Rankine Cycle

3. 2. 1 Compression and Expansion Process are not adiabatic in general

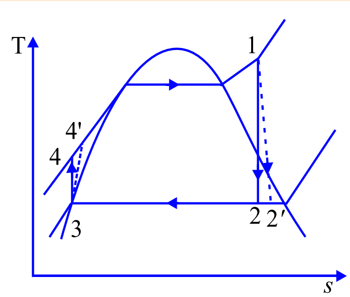


Fig. 3.3 Practical compression and expansion processes in Rankine cycle

Practically 1-2 irreversible and can be approximated to be adiabatic since there is always some possible heat loss from the turbine for a temperature difference with surroundings.

Similarly, there can be irreversibility in feed pump. So, it can end at 4'.

Now considering turbine as control volume.

$$W_T = h_1 - h_2'; \ (W_T)_{ideal} = h_1 - h_2 \text{ and } h_2' > h_1 \text{(always)}.$$

(Because of irreversibility friction increases intermolecular energy and it increases temperature. which increases enthalpy)

$\Rightarrow \quad W_T < (W_T)_{ideal}$

$$\boxed{\therefore \ \eta_{\substack{\text{Turbine,}\\ \text{isertropic}}} = \frac{h_1 - h_2'}{h_1 - h_2}}$$

Actual pump work; $W_P = h_4' - h_3$ (Work needed per unit mass of working fluid)

Ideal Pump work $\Rightarrow (W_P)_{actual} = h_4 - h_3$ and $h_4' > h_4$

$$\boxed{\therefore \ \eta_{\substack{\text{Pump}\\ \text{isertropic}}} = \frac{h_4 - h_3}{h_4' - h_3}}$$

3. 2. 2 Pressure Drops in Boiler

This is due to certain heat loss from the boiler. (But in practice it is very low. So, it can be neglected).

3.3 Methods to increase efficiency of Rankine cycle:

3. 3. 1 Increasing the Mean temperature of Heat Addition

For a heat rejection at temperature T_2 and Mean temperature of Heat addition T_{M_1}, the thermal efficiency of Rankine cycle is given by

$$\eta = 1 - \frac{T_2}{T_{M_1}}$$

So, to increase η we can decrease T_2 (or) increase T_{M1}. But T_2 is fixed by the ambient and condenser operating conditions. ∴ we can increase T_{M1} by super heating.

Increasing Boiler Pressure

On increasing the boiler pressure, the temperature of phase change increases, So the mean temperature of heat addition also increases.

This continuous increase in P_b is restricted because the expansion reduces the dryness fraction of steam.

As P_b increases, dryness fraction of steam after expansion decreases.

But as x decrease, water particles in steam increases, thus increase erosion of blade materials, which can cause drastic damage to turbine. So, *x* has minimum limitation.

3.3.2 Why condenser is used in steam power plant.

We have $\eta = 1 - \dfrac{T_2}{T_M}$ and as T_2 decrease $\Rightarrow \eta$ increases.

Generally, condensers are operated at pressures lower than the atmospheric pressures. Thus, condensation at lower pressures reduces the phase change (or) heat rejection temperature which would have been normal atmospheric temperature in the absence of the condenser. Thus, condensers decrease the T_2 value and increase the efficiency of the plant.

3.4 Methods to improve the performance of Rankine Cycle

3. 4. 1 Concept of Reheat in a Rankine Cycle:

In reheating the work output is increased without sacrificing dryness fraction.

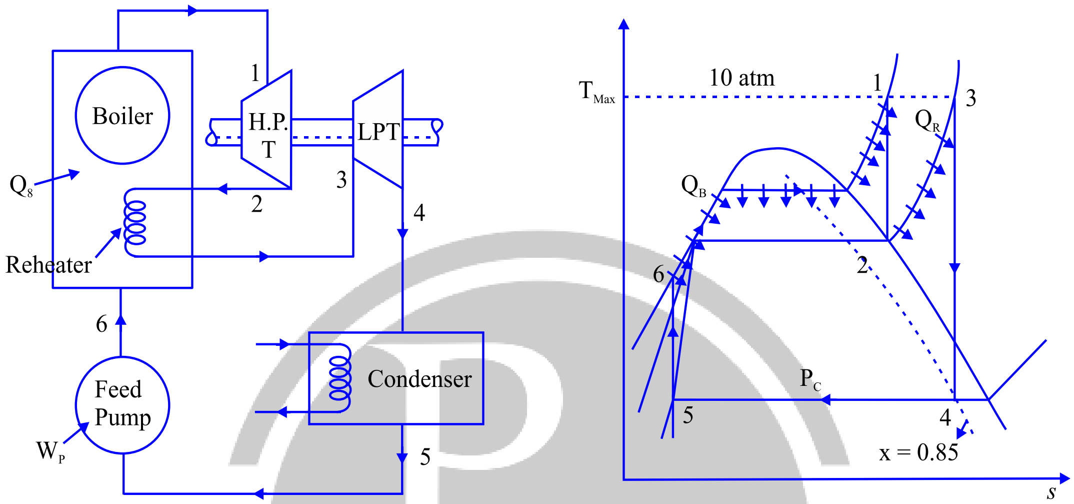


Fig. 3.4. Components of Rankine cycle with reheating and its graph on T-s plot

Heat supply Q increase, Work done also increase

But we can't comment on 'η' (Depends on T_M of 6 to 1) and (2 to 3)

Specific heat added = $(h_1 - h_6) + (h_3 - h_2)$

Specific heat Rejected = $(h_4 - h_5)$

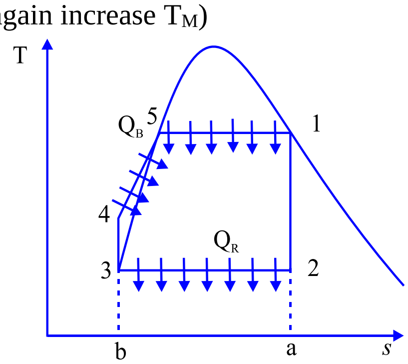
Specific work done by the turbines = $(h_1 - h_2) + (h_3 - h_4)$, specific work input to the pumps can be neglected because of very less specific volume of the liquid.

$$\therefore \eta_{Thermal} = \frac{W_T}{Q_S} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_6) + (h_3 - h_2)}$$

We cannot comment on 'η' until operating conditions are given.

3. 4. 2 Concept of Regeneration:

Need for Regeneration (Principle is to again increase T_M)



The working substance which is in liquid state is heated to state 5 from 4 using the steam that is bled out from the turbine. So, Heat is added from outer source only from point 5 to 1. So, the Heat addition process is obtained to be a near isothermal Heat addition. So, it gets closer to Carnot cycle analysis.

By regeneration, W_{net} decreases because the turbine work decreases and heat supply also decreases. But efficiency of the cycle increases because of increase in the mean temperature of heat addition.

3.5 Practical Circuit for Regeneration:

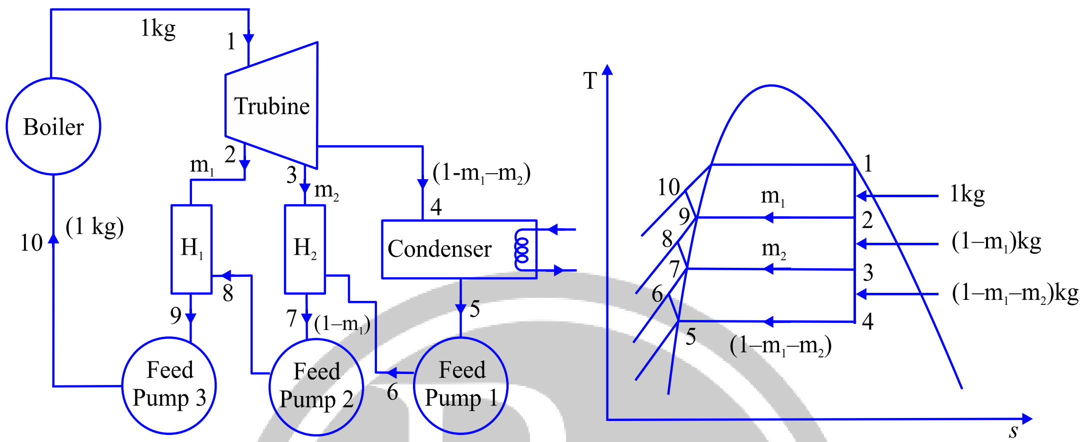


Fig. 3.5 Components of regenerative Rankine cycle and its graph on T-s plot

$$W_T = (h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4)$$

$$Q_{added} = (h_1 - h_{10}); \eta = \frac{W_T - W_P}{Q_{added}}$$

$$W_P = (1 - m_1 - m_2)(h_6 - h_5) + (1 - m_1)(h_8 - h_7) + 1\,.\,(h_{10} - h_9)$$

3.5.1 Calculation of m₁, m₂

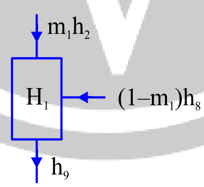


Fig. 3.6 Flow of working fluid through open feed water heater 1

$\Rightarrow \quad m_1 h_2 + (1 - m_1) h_8 = h_9$

$\Rightarrow \quad m_1 (h_2 - h_8) = h_9 - h_8$

$\Rightarrow \quad \boxed{m_1 = \frac{h_9 - h_8}{h_2 - h_8}}$

$$\boxed{h_8 - h_7 = v_7 (P_8 - P_7)}$$

❒❒❒

4 REFRIGERATION AND AIR CONDITIONING

4.1 Introduction

(a) Refrigeration

- The process of producing and maintaining a temperature lower than that of surroundings.

(b) Air-Conditioning

- The process of treating and thus simultaneously controlling the properties of air like temperature, moisture etc.

(c) Cooling by salt solutions

Chemical Reactions
- Exothermic $\rightarrow$ (Liberate Heat)
- $\boxed{\text{Endothermic}}$ $\rightarrow$ (Absorb Heat)

$NaCl + H_2O \longrightarrow$ Capable of Producing – 10°C.

NaCl: Sodium Chloride

$CaCl_2 + H_2O \longrightarrow$ Capable of Producing – 50°C.

$CaCl_2$: Calcium Chloride

(d) Artificial Refrigeration

- Vapour Refrigeration Systems
  - Vapour compression refrigeration system
  - Vapour absorption refrigeration system
- Gas Refrigeration System

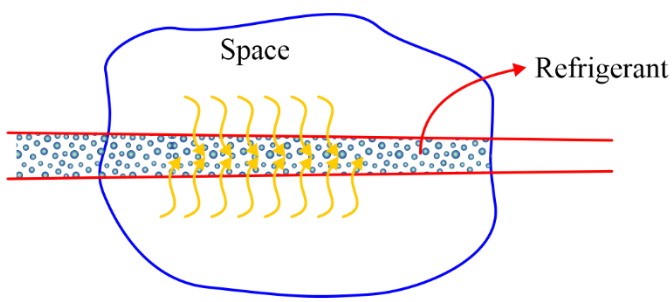


Fig.4.1 Heat absorption by the refrigerant from cooling space

4.2 Refrigeration cycles

The basic components of a refrigeration cycle include-

(a) Evaporator (b) Compressor (c) Condenser (d) Expansion device.

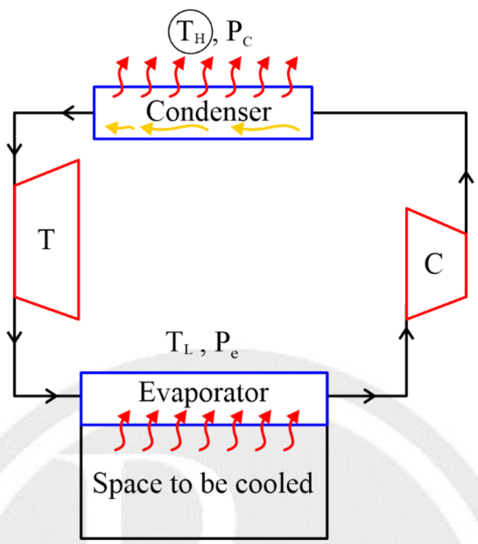


Fig.4.2 Basic components of Refrigeration cycle

4.2.1 Reversed Carnot cycle

Reversed Carnot cycle involves isentropic compression and expansion of the working fluid (refrigerant) and isothermal Heat addition and Heat rejection Processes as shown below.

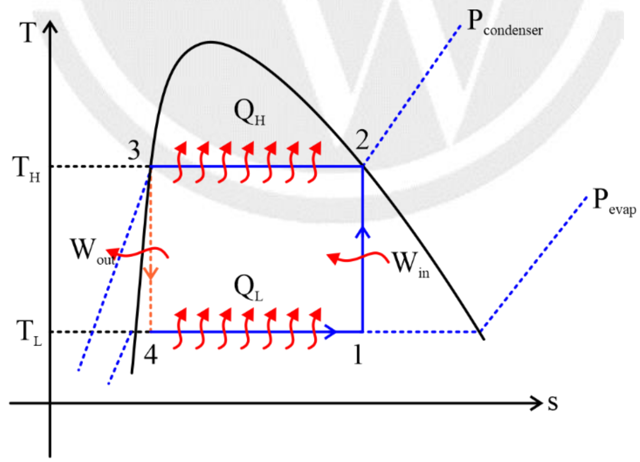


Fig.4.3 T-s diagram of reversed Carnot cycle

4.2.2 Limitations of Reversed Carnot Cycle

Expansion in turbine is practically highly uneconomical because state 3 is saturated liquid for which specific volume is very low and presence of liquid particles will erode turbine blades.

State-1: Wet vapour (Two phase mixture) so compression from 1-2 is difficult.

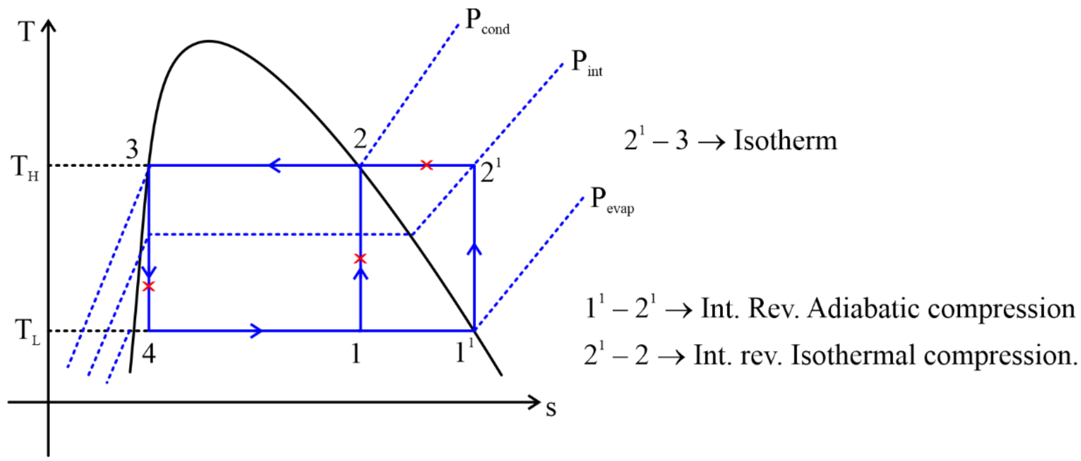
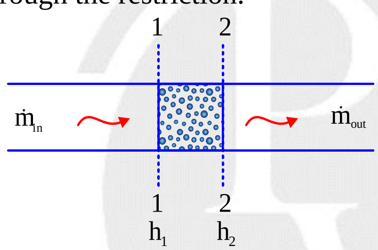


Fig.4.4 T-s diagram of different reversed Carnot cycles

4.3 Throttling Devices

- Throttling devices are used to drop the pressure of the fluid.
  Throttling $\rightarrow$ Passing through the restriction.



Applying SFEE for the porous plug,

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\Rightarrow \cancel{\dot{Q}_{CV}}^{0} - \cancel{\dot{W}_{CV}}^{0} = \cancel{\Delta \dot{KE}}^{0} + \cancel{\Delta \dot{PE}}^{0} + \Delta \dot{H}$$
$$\Rightarrow \Delta \dot{H} = 0$$
$$\Rightarrow \cancel{\dot{m}}(h_2 - h_1) = 0$$

Fig.4.5 Throttling device

Throttling through small devices is generally an isenthalpic process.

4.3.1 Joule Thomson Coefficient

- $\mu_{JT} = \left.\dfrac{\partial T}{\partial p}\right]_h \rightarrow$ Pressure drops

  h = h (T, P) $\rightarrow$ For any substance

$$dh = cpdT + \left[\upsilon - T\left(\frac{\partial \upsilon}{\partial T}\right)_p\right]dp$$

$$\Rightarrow -c_p dT = \left[\upsilon - T\left(\frac{d\upsilon}{\delta T}\right)_p\right]dp$$

$$\Rightarrow \left.\frac{\downarrow dT}{\downarrow dp}\right|_h = \frac{1}{c_p}\left[T\left(\frac{\partial \upsilon}{\partial T}\right)_P - \upsilon\right] = \mu_{JT}$$

In refrigeration cycles, during throttling of refrigerant, the refrigerant temperature should also decrease along with pressure. So (μ_{JT}) of refrigerant (non-ideal fluid) should be positive since both the changes are negative.

4.4 Standard Vapour Compression Refrigeration System (VCRS)

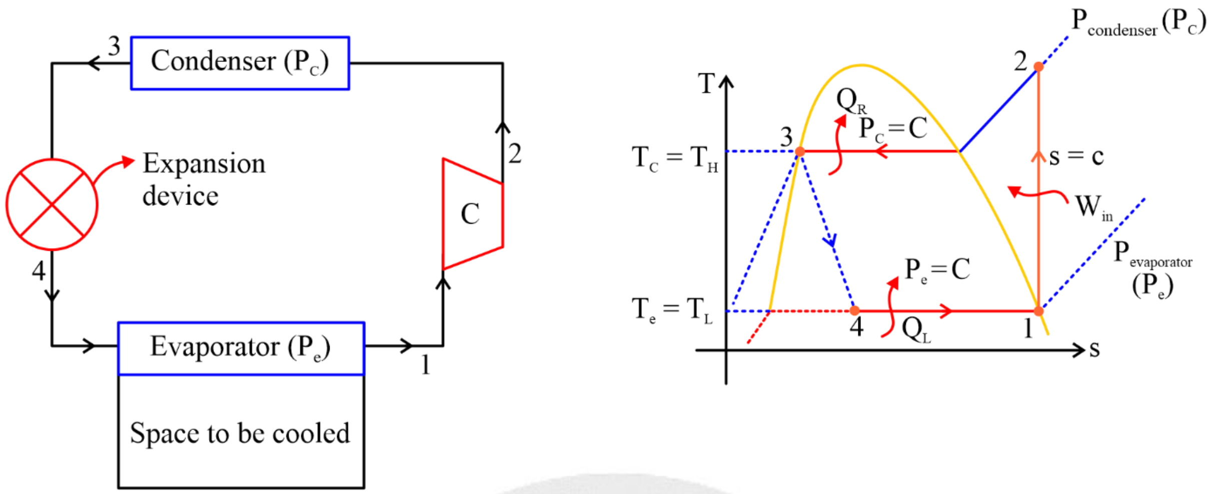


Fig.4.6 Components of standard VCRS cycle and its T-s diagram

- Processes involved in standard VCRS

  1-2: Internally Reversible adiabatic compression

  2-3: Internally Reversible Isobaric heat rejection

  3-4: Irreversible expansion in throttling device

  4-1: Internally reversible isothermal heat addition

Refrigeration effect = $\dot{m}_{\text{ref}}\left(h_1 - h_4\right) = \dot{V}\dfrac{\left(h_1 - h_4\right)}{v}$

$$= \dot{V}\frac{(h_1 - h_4)}{v} = \dot{V}_1 \boxed{\frac{(h_1 - h_4)}{v_1}} = \text{volumetric refrigeration effect}$$

4.4.1 Coefficient of performance

$$(\text{COP}) = \frac{\text{Desired output}}{\text{Required Input}} = \frac{\text{Refrigeration effect}}{\text{Work input to the compressor}} = \frac{\dot{m}_{ref}(h_1 - h_4)}{\dot{m}_{ref}(h_2 - h_1)}$$

$$since\ h_4 = h_3, COP = \frac{h_1 - h_4}{h_2 - h_1} = \frac{h_1 - h_3}{h_2 - h_1}$$

$$\therefore (COP)_{std.VCRS} = \frac{h_1 - h_3}{h_2 - h_1}$$

4.4.2 P-h diagram of standard VCRS

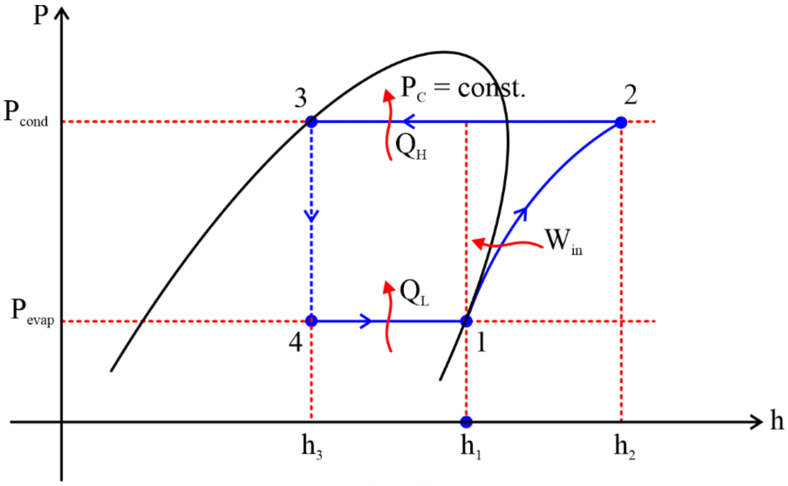


Fig.4.7 P-h diagram of standard VCRS

From Tds equation, $Tds = dh - vdP$

For isentropic process (ds = 0)

$\Rightarrow dh = vdP$

$\Rightarrow \frac{dP}{dh} = \frac{1}{v}$

4.4.3 Reversed Carnot Vs standard VCRS

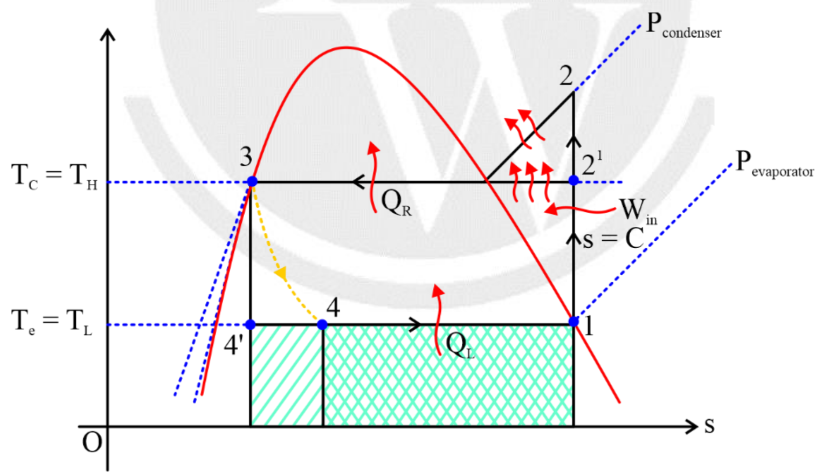


Fig.4.8 Comparison of reversed Carnot and standard VCRS on T-s graph

$(\text{Ref.effect})\big|_{\substack{\text{std.}\\ \text{VCRS}}} < \text{Ref.effect}\big|_{\substack{\text{Rev}\\ \text{Carnot}}}$

Ref. effect in Revised Carnot = $h_1 - h_4, kJ/kg = T_L(s_1 - s_4)$

Ref. effect in Standard VCRS = $h_1 - h_4, kJ/kg = T_L(s_1 - s_4)$

$$(COP)_{\substack{Rev.\\Carnot}} = \frac{T_L}{T_H - T_L} = \frac{1}{\left(\frac{T_H}{T_L}\right) - 1}$$

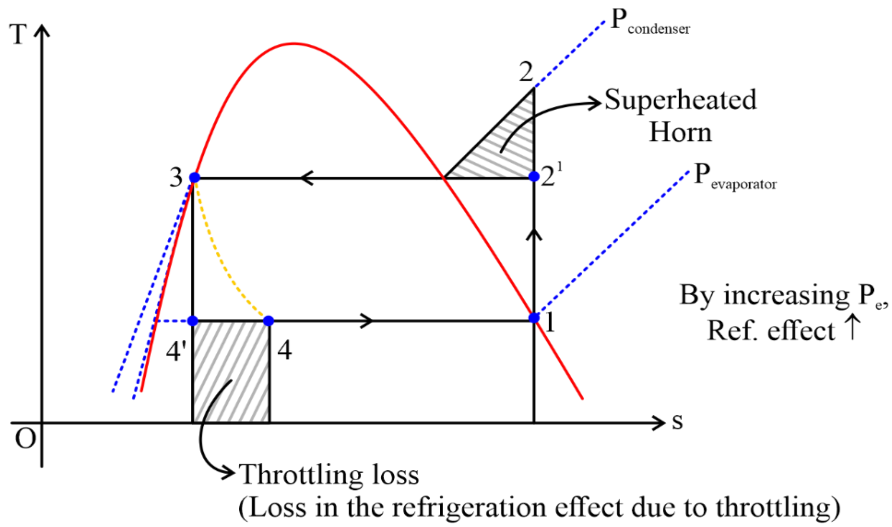


Fig.4.9 Throttling loss and superheated horn in VCRS on T-s graph

4.5 Gas Refrigeration Systems (Rev. Carnot cycle)

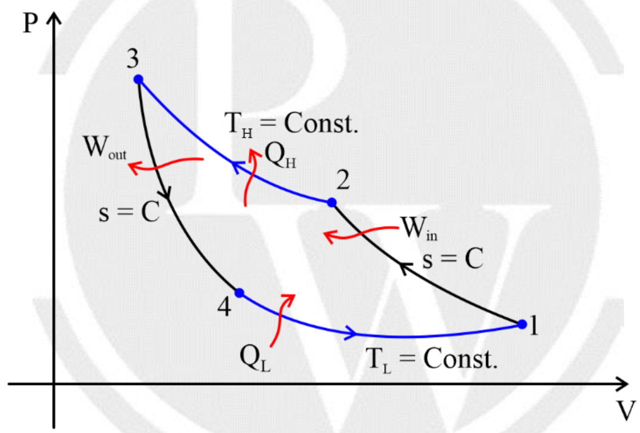


Figure 4.10 P-V diagram of reversed Carnot cycle

- Process:

  1-2: Internally reversible adiabatic compression
  2-3: Internally reversible isothermal heat rejection
  3-4: Internally reversible adiabatic expansion
  4-1: Internally reversible isothermal heat addition.

4.5.1 Reversed Brayton Cycle (or) Joule Cycle (or) Bell-Coleman Cycle

Since isothermal heat additions and heat rejections are practically difficult to achieve at high speeds of compressor, heat interactions are replaced by Isobaric processes with air as working fluid.

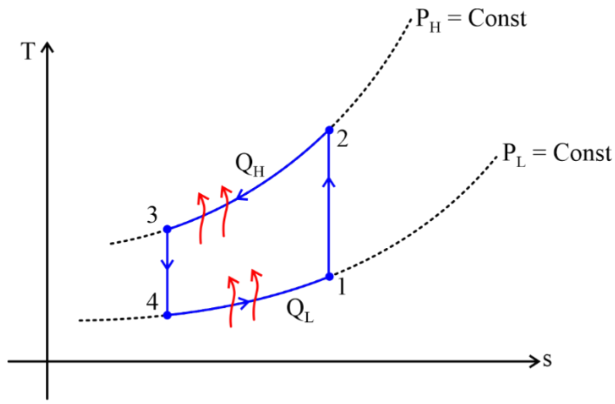


Fig.4.11 Bell- Coleman cycle

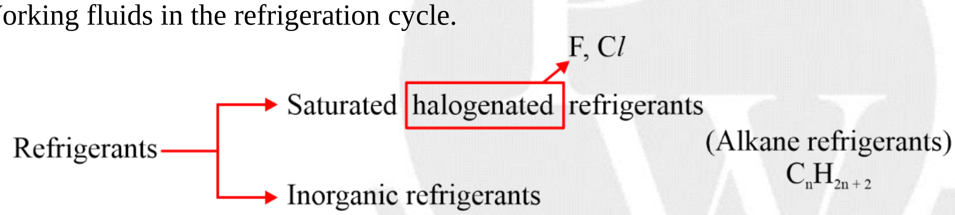
Refrigeration effect = $\dot{m}_{ref}(h_1 - h_4)$

$= \dot{V}_1 \boxed{\left(\frac{h_1 - h_4}{v_1}\right)}$ $\rightarrow$ volumetric refrigeration capacity

4.6 Refrigerants

Refrigerants

Working fluids in the refrigeration cycle.



4.6.1 Designation of a refrigerant

Any organic refrigerant is generally designated as R-XYZ

Number of carbon atoms = X+ 1

Number of hydrogen atoms = Y – 1

Number of Fluorine atoms = Z

Remaining atoms are chlorine atoms = (2x + 4) – (y – 1) – z

Ex.

(i) R-1 3 4

Number of C atoms = x + 1 = 1 + 1 = 2

Number of H atoms = y – 1 = 3 – 1 = 2

Number of F atoms = 4

R – 134 $\rightarrow$ $C_2H_2F_4$ (Tetra fluoro ethane)

(ii) R- 0XY

R 012$\Rightarrow$ Number of C atoms = 0 + 1 = 1

Number of H atoms = 1 – 1 = 0

Number of F atoms = 2

Number of Cl atoms = 2

Inorganic Refrigerants:

R-7XY

XY $\rightarrow$ Molecular weight of the refrigerant.

$NH_3 \rightarrow$ R717

$H_2O \rightarrow$ R718

4.7 Unit of Refrigeration

1 Tonn of refrigeration = 211 kJ/min of heat removal from a space.

$$= 211\frac{kJ}{60\sec}$$

$$= \frac{211}{60}kW$$

1 Tonn of refrigeration = 3.516 Kw

4.8 Air Conditioning

- **Air-Conditioning:** The process of treating and thus simultaneously controlling the properties of air like temperature, moisture content etc.

Air $\rightarrow$ Dry air $\rightarrow$ Mixture of gases ($N_2 + O_2$) — 78% by vol (N_2), 21% by vol (O_2)

Air $\rightarrow$ Atmospheric air $\rightarrow$ Mixture of gases ($N_2 + O_2$ + other gases) + $\boxed{\text{Water vapour.}}$ $\rightarrow$ Generally available in superheated vapor state

- **Daltons law of partial pressure:**

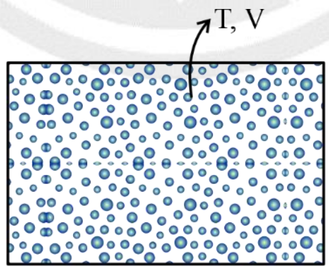


Fig.4.12 A volume containing different gases at temperature T

$$P_{tot}(T,V) = P_1(T,V) + P_2(T,V) + P_3(T,V)$$

$$\boxed{PV = nR_uT}$$

$$P_i = \bar{x}_i . P_{tot}$$

$$n_{tot} = n_1 + n_2 + n_3$$

$$\frac{P_{tot}.V}{P_1.V} = \frac{n_{tot}.R_u.T}{n_1.R_u.T}$$

$$\Rightarrow \frac{P_1}{P} = \frac{n_1}{n_{tot}} \rightarrow \text{Mole fraction of gas- 1}$$

$$\Rightarrow \frac{\mathrm{P}_1}{\mathrm{P}_{\mathrm{tot}}} = \bar{x}_1 \Rightarrow P_i = \bar{x}_1 P_{\mathrm{tot}}$$

Atmospheric Air $\rightarrow \underset{\text{Ideal gas}}{\text{Dry air}} + \underset{\text{Ideal gas}}{\text{Water Vapour}}$

$h_a(T°C) = 1.005T(°C)$

$h_a(0°C) \approx 0$ (Reference)

Mol. Wt. of air $= \frac{100}{\left(\frac{78.5}{28}\right)+\left(\frac{21.5}{32}\right)} = 28.9$ kg/kmol

$dh_v = c_P dT$

$\Rightarrow h_v(T°C) - h_v(0°C) = 1.82(T-0)$

$\Rightarrow h_v(T°C) = 2500.9 + 1.82T(°C)$

Mol. Weight of vapour = 18 kg/kmol.

4.8.1 Enthalpy of moist air

$H = H_a + H_v$

$\Rightarrow H = m_a.h_a + m_v.h_v$

For 1 kg of dry air

$\Rightarrow \frac{H}{m_a} = \frac{m_a}{m_a}.h_a + \frac{m_v}{m_a}.h_v$

$\Rightarrow h = h_a + \left(\frac{m_v}{m_a}\right).h_v$

$h(T°C) = 1.005.T(°C) + \left(\frac{m_v}{m_a}\right)[2500.9 + 1.82.T(°C)]$

$\Rightarrow \boxed{h(T°C) = 1.005.T(°C) + \omega[2500.9 + 1.82.T(°C)]}$

4.8.2 Specific Humidity (or) Absolute Humidity

- The mass of vapor present per kg of dry air is called specific humidity and it is denoted by 'ω.'
  It is given by

$$\omega = \frac{m_v}{m_a} = \frac{\left(\frac{P_v.V}{R_v.T}\right)}{\left(\frac{P_a.V}{R_a.T}\right)}$$

$$\Rightarrow \omega = \frac{P_v}{P_a}.\frac{R_a}{(R_v)} \Rightarrow \omega = \frac{\left(\frac{R_u}{MW_a}\right)}{\left(\frac{R_u}{MW_v}\right)} \times \frac{P_v}{P_a}$$

$$\Rightarrow \omega = \left(\frac{MW_v}{MW_a}\right) \times \frac{P_v}{P_a} = \left(\frac{18}{28.9}\right) \times \frac{P_v}{P_a} = 0.622\frac{P_v}{P_a}$$

$$\therefore \omega = 0.622\frac{P_v}{P_a}$$

$$\Rightarrow \boxed{\omega = 0.622\frac{p_v}{p_{tot} - p_v}}$$

4.8.3 Relative Humidity (ϕ)

- The ratio of mass of water vapour present in the air to the maximum amount of vapor that the air can with stand at the same condition.

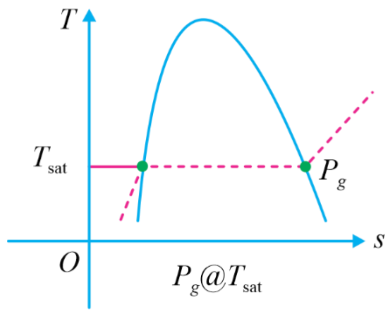


Fig.4.13 Pressure of saturated vapour at T_{sat}

It is denoted by ϕ and is given by

$$\phi = \frac{m_v}{(m_v)_{max}} = \frac{\left(\frac{P_v.V}{R_vT}\right)}{\left(\frac{P_g.V}{R_v.T}\right)}$$

$\Rightarrow$ $\boxed{\phi = \frac{p_v}{p_g}}$ where $P_g = P_{sat}$ at given T°C

As air is heated, ϕ decreases

$$h = 1.005.T(°C) + \frac{m_v}{m_a}[2500.9 + 1.82.T(°C)]$$

$$h = 1.005.T(°C) + \omega[2501 + 1.82.T(°C)]$$

$$\omega = \frac{m_v}{m_a}; \phi = \frac{P_v}{P_g}$$

$0 < \phi \le 1$

$\boxed{0\% < \phi \le 100\%}$

4.8.4 DBT, WBT AND DPT

Dry Bulb Temperature: (DBT)

The actual/normal temperature of the air measured with a thermometer.

Wet Bulb Temperature: (T_{WBT})

The temperature measured by the thermometer when the bulb is covered with a wet cotton wick.

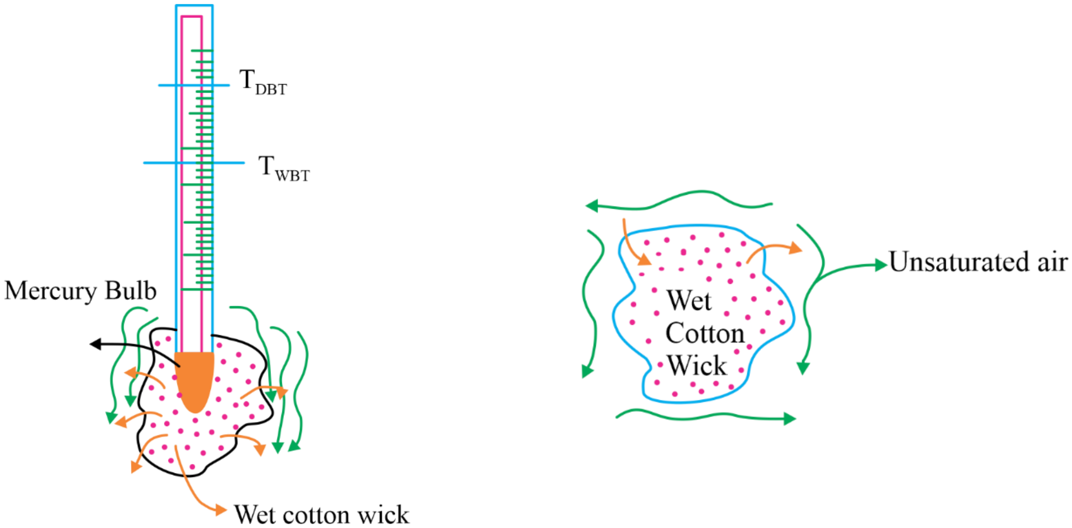


Fig.4.14 Thermometer with wet cotton wick

For unsaturated air, $T_{WBT} < T_{DBT}$

If air is saturated then there is no net Evaporation

In case of saturated air, $T_{DBT} = T_{WBT}$

Dew Point Temperature: (T_{DPT})

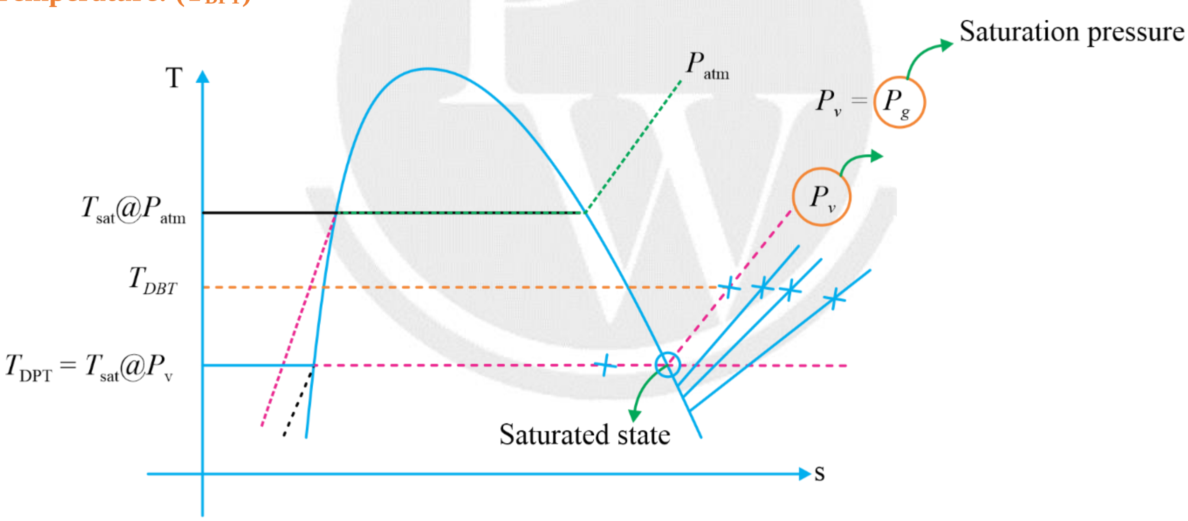


Fig.4.15 Finding Dew point temperature on T-s diagram

During condensation, the air is in its saturated state.

T_{DPT} is the temperature at which the first droplet gets formed.

In case of saturated air,

$T_{DBT} = T_{DPT}$

$\therefore$ In case of saturated air, $T_{DBT} = T_{WBT} = T_{DPT}$

4.8.5 Adiabatic Saturation Temperature

The temperature achieved by the air undergoing saturation under adiabatic condition is called Adiabatic Saturation Temperature.

If $\dot{m}_f \rightarrow$ Rate of evaporation then make up water is also supplied at the rate of $\dot{m}_f$.

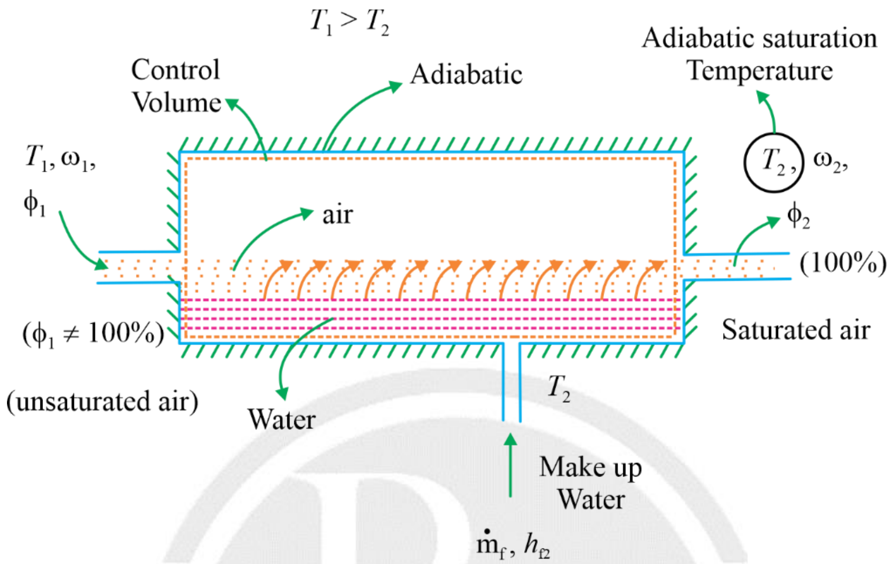


Fig.4.16 An adiabatic control volume to achieve adiabatic saturation temperature

Mass flow rate of water that is getting evaporated.

$\Rightarrow \dot{m}_f = (\omega_2 - \omega_1)\dot{m}_a$

Energy Balance:

$$E_{in} = E_{out}$$
$$\Rightarrow \dot{m}_{in}h_{in} + \dot{m}_f h_{f_2} = \dot{m}_{out}.h_{out}$$
$$\Rightarrow \dot{m}_a h_{in} = (\omega_2 - \omega_1)\dot{m}_a h_{f_2} = \dot{m}_a.h_{out}$$
$$\Rightarrow h_{in} + (\omega_2 - \omega_1).h_{f,_2} = h_{out} \qquad h_{out}|_{\substack{water\\vapour}} = h_{g,_2}$$
$$\Rightarrow c_P.T_1 + \omega_1(2500.9 + 1.82T_1) + (\omega_2 - \omega_1).h_{f,_2} = c_P T_2 + \omega_2(2500.9 + 1.82T_2)$$
$$\Rightarrow c_P T_1 + (\omega_1 = \omega_2)(2500.9) + (\omega_2 - \omega_1)h_{f,_2} = T_2(c_P + 1.82\omega_2)$$
$$\Rightarrow T_2 = \frac{c_P T_1 + (\omega_2 - \omega_1)(h_{f,_2} - 2500.9)}{(c_p + 1.82\omega_2)}$$

4.9 Psychrometric Chart

The plot that depicts the variation of specific humidity with variation of dry bulb temperature.

Constant specific volume lines are steeper than the constant T_{WBT} lines.

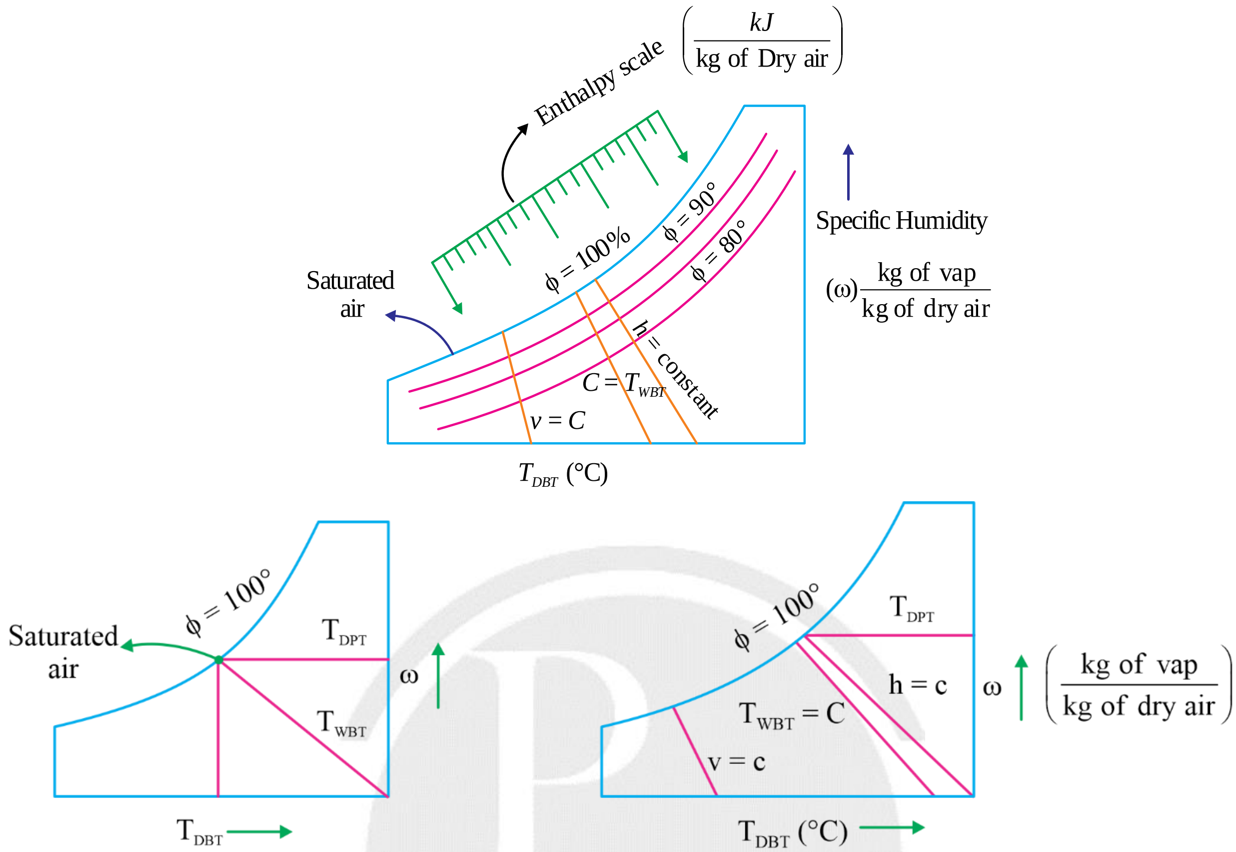


Fig. 4.17 Psychrometric chart

4.9.1 Basic Psychrometric Processes

(i) Sensible Heating:

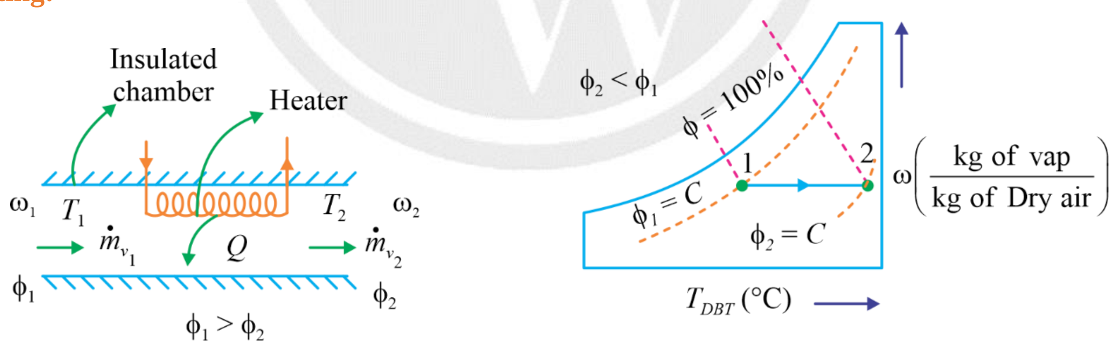


Fig.4.18 Arrangement for sensible heating and its representation on Psychrometric chart

In case of sensible heating,

$\omega_1 = \omega_2$ $\qquad\qquad$ $\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$

$\phi_2 < \phi_1$ $\qquad\qquad$ $\dot{m}v_1 = \dot{m}v_2 = \dot{m}v$

For the sections 1-2:

$$\dot{m}_{\text{in}} h_{\text{in}} + Q = \dot{m}_{\text{out}} \cdot h_{\text{out}}$$

$$\Rightarrow \dot{m}_a . h_{in} + \dot{Q} = \dot{m}_a . h_{out}$$

$$\Rightarrow \dot{Q} = \dot{m}_a (h_{out} - h_{in})$$

(ii) Heating with humidification:

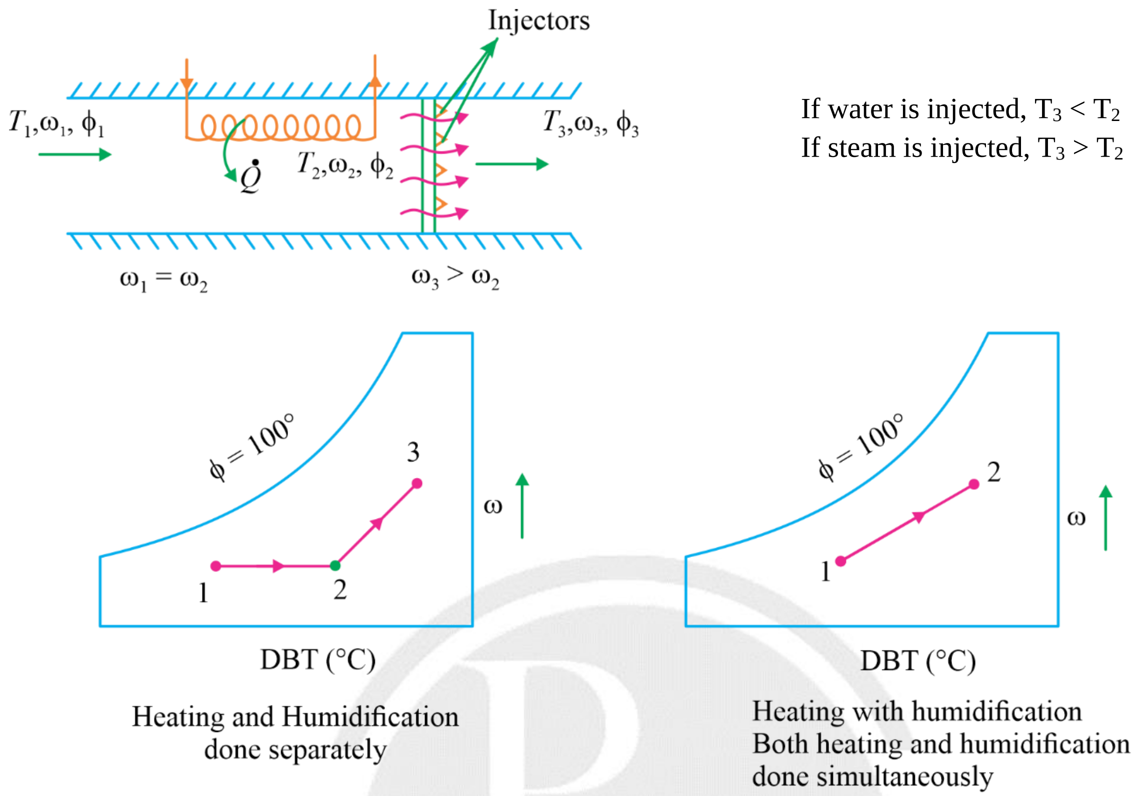


Fig. 4.19 Arrangement for heating with humidification and its representation on Psychrometric chart

(iii) Sensible Cooling:

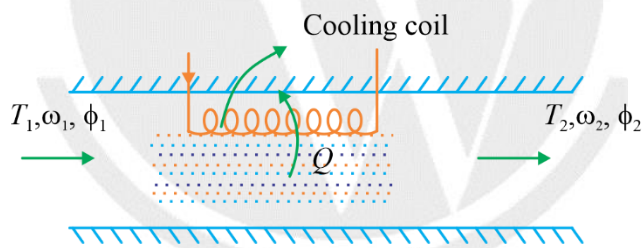


Fig.4.20 Arrangement for sensible cooling

In Sensible cooling

$(\omega_1 = \omega_2),\ (T_2 < T_1)\ and\ (\phi_2 > \phi_1)$

(iv) Cooling with dehumidification:

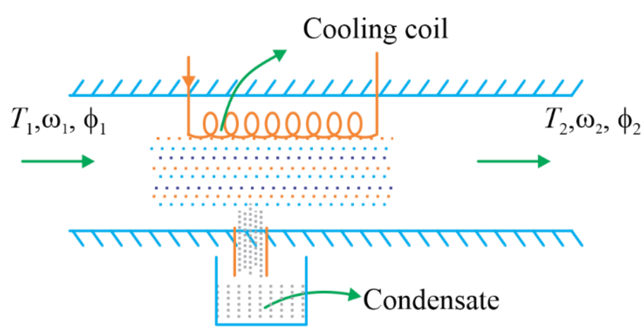


Fig. 4.21 Arrangement for cooling with dehumidification

$T_2 < T_1$.
$\omega_2 < \omega_1$.
In general, $\phi_2 > \phi_1$.
$\dot{m}_{v,1} = \omega_1 \times \dot{m}_a$
$\dot{m}_{v,2} = \omega_2 \times \dot{m}_a$
Rate of condensation $= (\omega_1 - \omega_2)\dot{m}_a$

4.9.2 Representation of all the processes in Psychrometric Chart

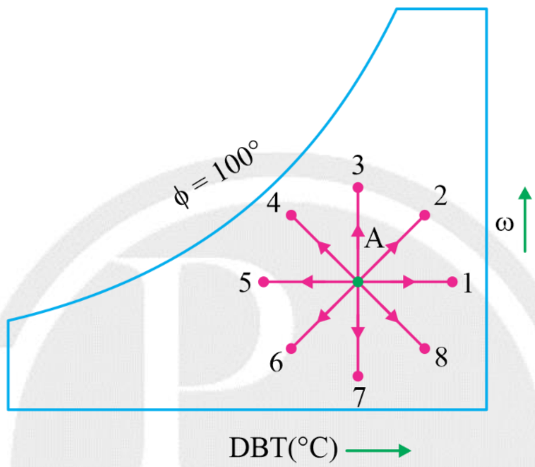


Fig.4.22 Representation of all processes on psychrometric chart

A – 1 → Sensible Heating

A – 2 → Heating with humidification

A – 3 → Humidification

A – 4 → Cooling with humidification (Occurs in air washer)

A – 5 → Sensible cooling.

A – 6 → Cooling with dehumidification

A – 7 → Dehumidification.

A – 8 → Heating with dehumidification.

(Chemical Dehumidification)

4.10 Adiabatic mixing of two moist air streams

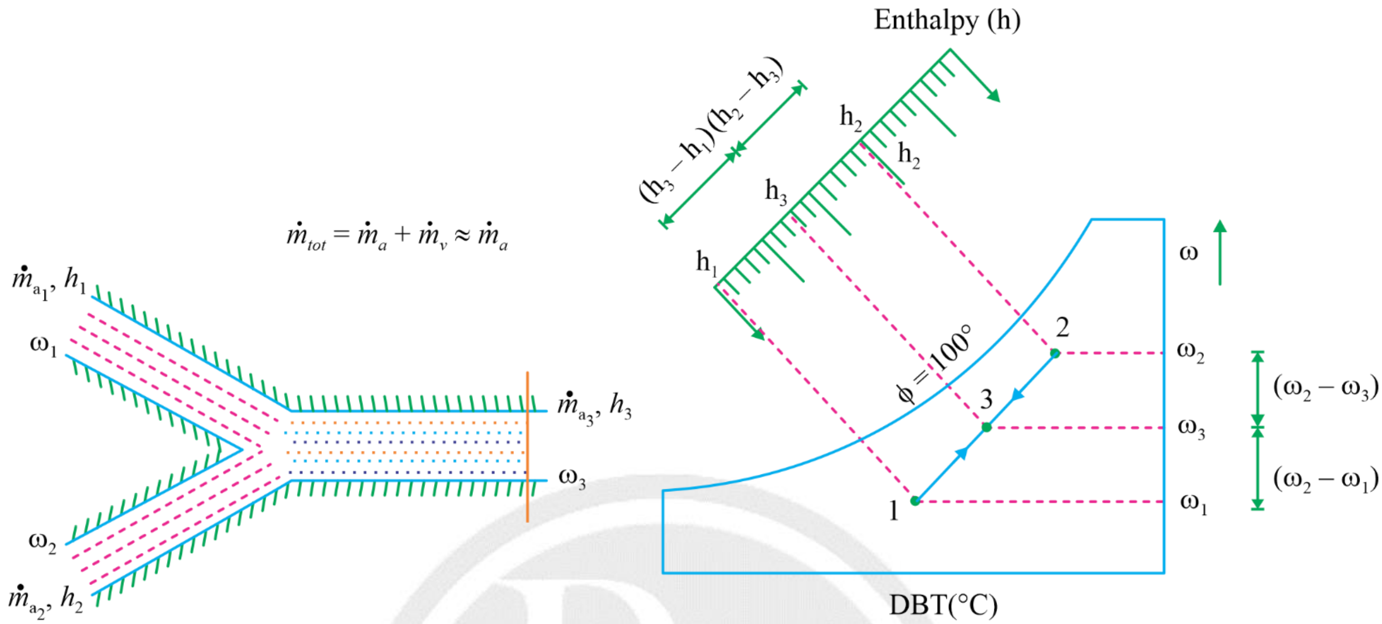


Fig.4.23 Mixing of two streams of moist air and representation of different states on psychrometric chart

For air $\rightarrow \dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a_3}$

For water vapour $\rightarrow \dot{m}_{v_1} + \dot{m}_{v_2} = \dot{m}_{v_3}$

$\Rightarrow \omega_1 . \dot{m}_{a_1} + \omega_2 . \dot{m}_{a_2} = \omega_3 (\dot{m}_{a_1} + \dot{m}_{a_2})$

$\Rightarrow (\omega_1 - \omega_3)\dot{m}_{a_1} . = (\omega_3 - \omega_2)\dot{m}_{a_2}$

$$\boxed{\frac{\dot{m}_{a,1}}{\dot{m}_{a,2}} = \frac{\omega_3 - \omega_2}{\omega_1 - \omega_3} = \frac{h_3 - h_2}{h_1 - h_3}}$$

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5 GAS COMPRESSORS

5.1 Introduction

A compressor is a device in which work is done on the gas, to raise its pressure.

Applications of Compressed air: Motor for tools, air brake for vehicles, servo Mechanisms etc.

Compressors
- +ive displacement machine → Reciprocating, Root's blower, Rotary.
- Non +ive displacement machine → Centrifugal compressors, Axial flow.

+ive displacement → Possess means to prevent undesired flow reversal. Here, work is transferred by virtue of hydrostatic force on boundary.

In non-+ive displacement→ Work is transferred by virtue of change of momentum of stream of fluid flowing over the blades.

5.1.1 Work of compression

Work for compression is same for both reciprocating and centrifugal compressor. {Expression is same}

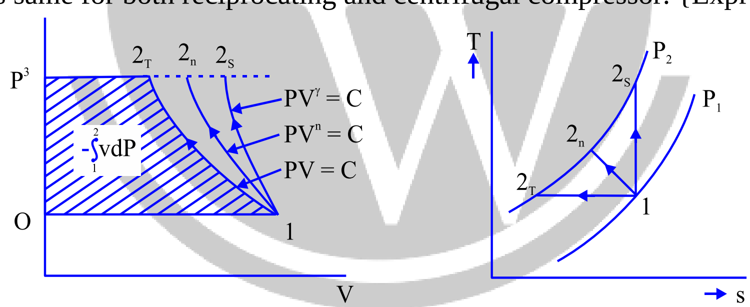
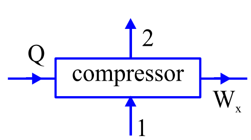


Fig. 5.1. Compression work in different processes

Steady flow energy equation for compressor is.



For reversible process; $Q = \Delta h - \int vdP$

Compression is adiabatic, then Q = O

$\Rightarrow$ $h_1 + Q = h_2 + W_X$ → (S.F.E.E for compressor)

$\Rightarrow$ $h_2 - h_1 = -w_x = \int vdp$ → Work required for compression

If compression is polytropic, $(PV^n = C) \Rightarrow V^n = \frac{P_1 V_1^n}{P} \Rightarrow V = \frac{P_1^{\frac{1}{n}}.V_1}{P^{\frac{1}{n}}}$

$$\therefore \qquad W_X = -\int vdP = \frac{-n}{n-1}.P_1V_1\left(\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right)$$

$$\therefore \qquad \text{Work required for compression} = \frac{n}{n-1}.P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

If process of compression is isothermal, then $W = P_1V_1 \ln\left(\frac{P_2}{P_1}\right)$. For any flow process, $W_{comp.}$ is denoted by Area of curve projected on to P-axis on a P-v diagram.

So, $PV^n = C \Rightarrow \frac{dP}{dV} = -n.\frac{P_1}{V_1} \rightarrow$ Slope of any point.

In general, $1 < n < \gamma$ and

For a given pressure ratio $\left(\frac{P_2}{P_1}\right)$; if 'W' denotes compression work,

$$W_{Isotherma} < W_{Polytropic} < W_{adiabatic}$$

Adiabatic efficiency of compressor; $\eta_s = \frac{h_{2s} - h_1}{W_C}$

Isothermal efficiency of compressor; $\eta_s = \frac{h_{2T} - h_1}{W_C}$

Minimum work of compression, with cooling is isothermal work.

Minimum work of compression, without cooling is Isentropic work.

5.2 Single stage Reciprocating Air Compressor

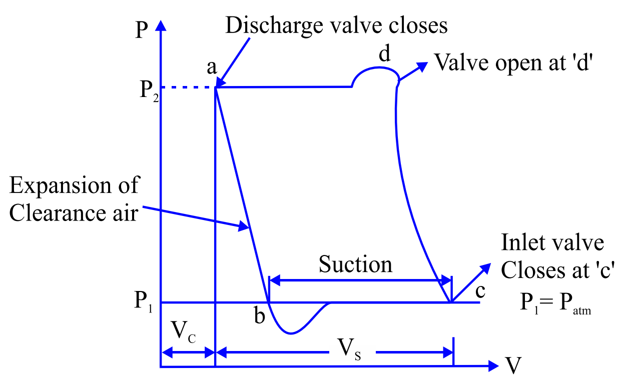


Fig. 5. 2. P-V diagram of single stage reciprocating air compressor

Compressor operates on 2-stroke cycle.

Stroke-1 (a-c): From a to b; air in the clearance volume expands and at 'b' pressure of air inside < $P_{atm.}$ So, suction begins and this suction of air into cylinder continues till 'c'. where $P = P_{atm}$.

Stroke-2 (c-a): Compression follows until the pressure in the cylinder is more than that in the receiver. Outlet valve opens at d and Air is delivered for rest of the stroke.

It is seen, the effect of air in clearance volume is to reduce the quantity of air drawing into piston, during the suction. So, clearance volume is made as small as possible.

Areas above P_2 and below P_1 is the work done for physical pressure drop. This work is called valve loss.

5.2.1 For Idealized machine

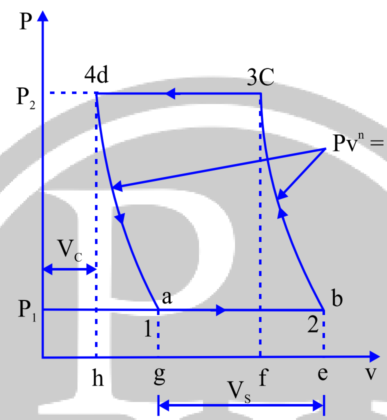


Fig. 5.3. Idealized P-V diagram of single stage reciprocating air compressor

Volume of air drawn during suction = $V_b - V_a$.

Mass of gas in clearance volume doesn't have any effect on compression work.

5.2.2 Volumetric Efficiency of Reciprocating Compressor

The ratio of actual volume of gas taken into cylinder during suction stroke to the swept volume (V_S) of piston is η_{vol}.

$\therefore \eta_{vol} = \frac{mv_1}{V_s}$ where m→ Mass of gas, v_1 → Specific volume at inlet.

$$\therefore \eta_{vol} = \frac{V_2 - V_1}{V_S} = \frac{V_C + V_S - V_1}{V_S} = 1 + \frac{V_C}{V_S} - \frac{V_1}{V_S}$$

Let C = Clearance ratio = $\frac{\text{Clearance Volume}}{\text{Swept Volume}} = \frac{V_C}{V_S}$

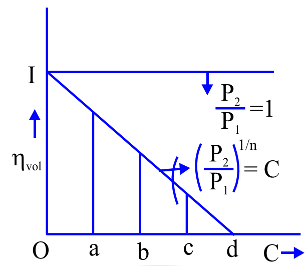
$$\therefore \quad \eta_{Vd} = 1 + C - \frac{V_1}{V_C} \times \frac{V_C}{V_S} = 1 + C - C\left(\frac{V_1}{V_C}\right)$$

Here $P_1V_1^n = P_2V_4^n \Rightarrow V_1 = V_4\left(\frac{P_2}{P_1}\right)^{1/n} = V_c.\left(\frac{P_2}{P_1}\right)^{1/n}$

$\therefore\ \eta_{vol} = 1 + C - C\left(\dfrac{P_2}{P_1}\right)^{1/n}$ $\boxed{\therefore\ \eta_{vol} = 1 + C - C\left(\dfrac{P_2}{P_1}\right)^{1/n}}$

Since $\left(\dfrac{P_2}{P_1}\right) > 1$; η_{vol} decrease as C increase and η_{vol} decrease as $\left(\dfrac{P_2}{P_1}\right)$ increase

5.2.3 Effect of Clearance on Volumetric Efficiency



If clearance volume increase,

η_{vol} & m decrease

$$W_{comp} \neq f(C)$$

So, for a given pressure ratio, $\eta_{vol} = 0$ when $C_{max} = \dfrac{1}{\left(\dfrac{P_2}{P_1}\right)^{1/n} - 1}$

$$\therefore\ \eta_{vol} = 0 \text{ when } C = C_{max} \ \&\ C_{max} = \frac{1}{\left(\dfrac{P_2}{P_1}\right)^{1/n} - 1}$$

5.2.4 Effect of Pressure Ratio on Volumetric Efficiency

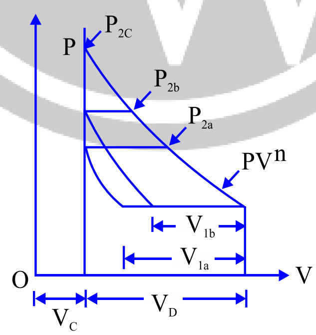


Fig. 5. 4. Effect of pressure ratio on volumetric efficiency

As $\left(\dfrac{P_2}{P_1}\right)$ increase; η_{vol} decrease

The max. pressure ratio $\left(\dfrac{P_{2\,max}}{P_1}\right)$ attainable for a reciprocating compressor cylinder is limited by the clearance 'C'

$$\boxed{\therefore\ \left(\frac{P_{2\,max}}{P_1}\right) = \left(1 + \frac{1}{C}\right)^n \ \{\text{When } \eta_{vol} = 0\}}$$

Compressor Displacement Volume, $V = \frac{\pi}{4} d^2 L$

Induction volume rate/volume flow rate = $V = \frac{\pi}{4} d^2 L \cdot \left(\frac{N}{60}\right)$

$\downarrow$ Where $N \rightarrow$ r.p.m.

For single acting compressor. $\boxed{\left(I.P. = \frac{P_m . LANK}{60} kW \right)}$

5.3 Multi Stage Compression

For compressing to high pressure, it is advantageous to do in multi stage.
The compression for min. work requires the compression to be isothermal.

But $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \Rightarrow T_2$ increase as $\left(\frac{P_2}{P_1}\right)$ increase and also η_{vol} decrease as $\left(\frac{P_2}{P_1}\right)$ increase

For these factors when $P_2 >> P_1$; mutli-stage is preferred.

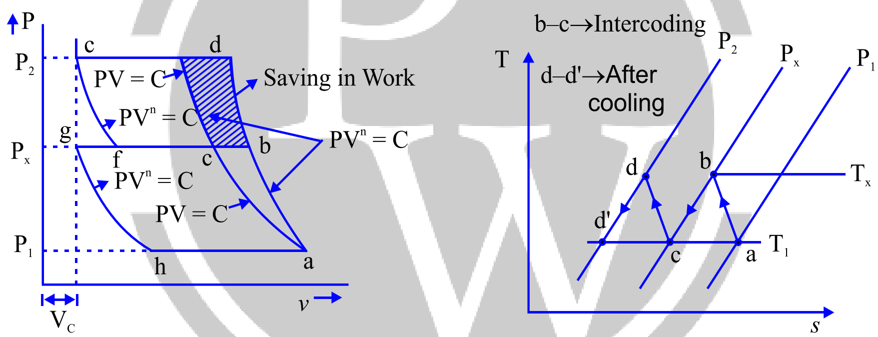


Fig. 5.4 P-v and T-s diagram of multistage compression

5.3.1 Perfect Intercooling

The exiting gas from intercooler at T_x is cooled completely to the original temperature 'T_1'. ($b - c \rightarrow$ Intercooling)

The total work for compression in two stages per kg of gas is given by

$$W_C = \frac{n}{n-1} rT_1 \left[\left(\frac{P_x}{P_1}\right)^{\frac{n-1}{n}} - 1 \right] + \frac{n}{(n-1)} rT_1 \left[\left(\frac{P_2}{P_x}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{n}{(n-1)} rT_1 \left[\left(\frac{P_x}{P_1}\right)^{\frac{n-1}{n}} + \left(\frac{P_2}{P_x}\right)^{\frac{n-1}{n}} - 2 \right]$$

Here P_1, T_1 & P_2 are fixed. Only P_x is variable.

$\therefore$ For optimum value of minimum work, $\frac{dW_c}{dP_x} = O$

$\Rightarrow \quad P_x = \sqrt{P_2P_2} \quad \Rightarrow \quad T_2 = T_x$

So, for $(W_c)_{min}$; Pressure Ratio in L.P. stage = Pressure Ratio in H.P. Stage.

$$\therefore \quad (W_c)_{min} = \frac{2.nrT_1}{n-1}\left\{\left[\frac{P_2}{P_1}\right]^{\frac{n-1}{2n}} - 1\right\}$$

Heat rejected in Intercooler, $Q_{bc} = c_P.[T_x - T_1].\frac{kJ}{kg}$

For perfect Intercooling with 'N' stage compression.

Optimum Pressure Ratio in each stage = $\frac{P_x}{P_1} = \left(\frac{P_d}{P_s}\right)^{1/N}$

Minimum work of compression, $\boxed{W_C = \frac{N.n\,rT_1}{n-1}\left\{\left(\frac{P_d}{P_s}\right)^{\frac{n-1}{nN}} - 1\right\}}$

5.3.2 Advantage of Multi-stage Compression

(i) Increased overall η_{vol}.

(ii) Leakage losses are reduced considerably.

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