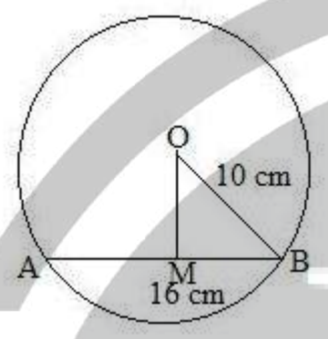
Question 1.

Solution:

Let *AB* be the chord of the given circle with centre *O* and a radius of 10 cm.
Then *AB* =16 cm and *OB* = 10 cm

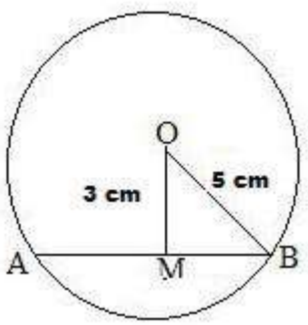


From *O*, draw *OM* perpendicular to *AB*.
We know that the perpendicular from the centre of a circle to a chord bisects the chord.
∴ *BM* = (162) cm=8 cm
In the right ΔOMB, we have:
$OB^2 = OM^2 + MB^2$ (Pythagoras theorem)
⇒ $10^2 = OM^2 + 8^2$
⇒ $100 = OM^2 + 64$
⇒ $OM^2 = (100 - 64) = 36$
⇒ $OM = \sqrt{36}$ cm=6 cm
Hence, the distance of the chord from the centre is 6 cm.

Question 2.

Solution:

Let *AB* be the chord of the given circle with centre *O* and a radius of 5 cm.
From *O*, draw *OM* perpendicular to *AB*.
Then *OM* = 3 cm and *OB* = 5 cm

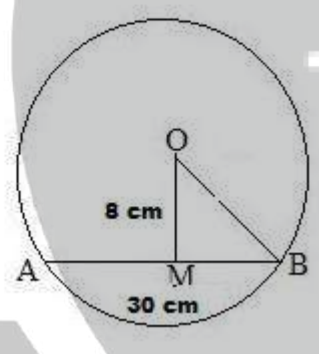


From the right Δ*OMB*, we have:
$OB^2 = OM^2 + MB^2$ (Pythagoras theorem)
⇒ $5^2 = 3^2 + MB^2$
⇒ $25 = 9 + MB^2$
⇒ $MB^2 = (25 - 9) = 16$
⇒ $MB = \sqrt{16} cm = 4$ cm
Since the perpendicular from the centre of a circle to a chord bisects the chord, we have:
$AB = 2 \times MB = (2 \times 4)$ cm = 8 cm
Hence, the required length of the chord is 8 cm.

Question 3.

Solution:

Let *AB* be the chord of the given circle with centre *O*. The perpendicular distance from the centre of the circle to the chord is 8 cm.
Join *OB.*
Then *OM* = 8 cm and *AB* = 30 cm

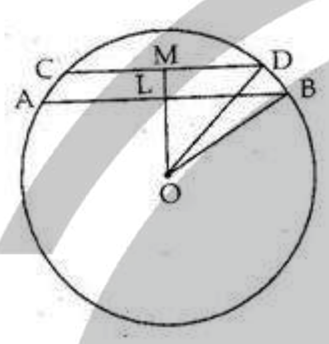


We know that the perpendicular from the centre of a circle to a chord bisects the chord.
∴ $MB = (AB/2) = (30/2)$ cm = 15 cm
From the right Δ*OMB*, we have:
$OB^2 = OM^2 + MB^2$
⇒ $OB^2 = 8^2 + 15^2$
⇒ $OB^2 = 64 + 225$
⇒ $OB^2 = 289$
⇒ $OB = \sqrt{289} cm = 17$ cm
Hence, the required length of the radius is 17 cm.

Question 4.

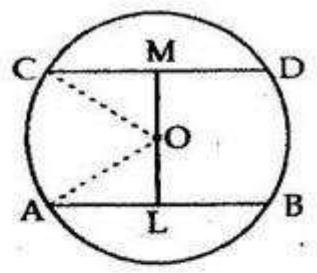
Solution:

We have:
(i)Let *AB* and *CD* be two chords of a circle such that *AB* is parallel to *CD* on the same side of the circle.
Given: *AB* = 8 cm, *CD* = 6 cm and *OB* = *OD* = 5 cm
Join *OL* and *OM*.



The perpendicular from the centre of a circle to a chord bisects the chord.
∴ $LB = AB/2 = (8/2) = 4$cm
Now, in right angled Δ*BLO*, we have:
$OB^2 = LB^2 + LO^2$
⇒ $LO^2 = OB^2 - LB^2$
⇒ $LO^2 = 5^2 - 4^2$
⇒ $LO^2 = 25 - 16 = 9$
∴ $LO = 3$ cm
Similarly, $MD = CD/2 = (6/2) = 3$cm
In right angled Δ*DMO*, we have:
$OD^2 = MD^2 + MO^2$
⇒ $MO^2 = OD^2 - MD^2$
⇒ $MO^2 = 5^2 - 3^2$
⇒ $MO^2 = 25 - 9 = 16$
⇒ $MO = 4$ cm
∴ Distance between the chords = $(MO - LO) = (4 - 3)$ cm = 1 cm

(ii) Let *AB* and *CD* be two chords of a circle such that *AB* is parallel to *CD* and they are on the opposite sides of the centre.
Given: *AB* = 8 cm and *CD* = 6 cm
Draw $OL \perp AB$ and $OM \perp CD$.

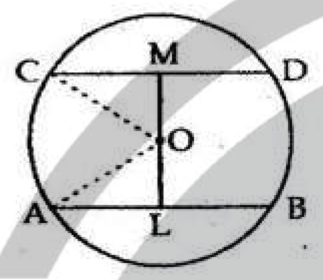


Join *OA* and *OC*.
$OA = OC = 5$ cm (Radii of a circle)
The perpendicular from the centre of a circle to a chord bisects the chord.
$\therefore AL = AB/2 = (8/2) = 4$cm
Now, in right angled ΔOLA, we have:
$OA^2 = AL^2 + LO^2$
$\Rightarrow LO^2 = OA^2 - AL^2$
$\Rightarrow LO^2 = 5^2 - 4^2$
$\Rightarrow LO^2 = 25 - 16 = 9$
$\therefore LO = 3$ cm
Similarly, $CM = CD/2 = (6/2) = 3$cm
In right angled Δ*CMO*, we have:
$OC^2 = CM^2 + MO^2$
$\Rightarrow MO^2 = OC^2 - CM^2$
$\Rightarrow MO^2 = 5^2 - 3^2$
$\Rightarrow MO^2 = 25 - 9 = 16$
$\therefore MO = 4$ cm
Hence, distance between the chords = $(MO + LO) = (4 + 3)$ cm = 7 cm

Question 5.

Solution:

Let *AB* and *CD* be two chords of a circle such that *AB* is parallel to *CD* and they are on the opposite sides of the centre.
Given: *AB* = 30 cm and *CD* = 16 cm
Draw $OL \perp AB$ and $OM \perp CD$.



Join *OA* and *OC*.
OA = *OC* = 17 cm (Radii of a circle)
The perpendicular from the centre of a circle to a chord bisects the chord.
∴ *AL*=(*AB*/2)=(30/2)=15 cm

AL=AB/2=(8/2)=4cm
Now, in right angled Δ*OLA*, we have:
$OA^2 = AL^2 + LO^2$
$\Rightarrow LO^2 = OA^2 - AL^2$
$\Rightarrow LO^2 = 17^2 - 15^2$
$\Rightarrow LO^2 = 289 - 225 = 64$
∴ *LO* = 8 cm
Similarly, *CM*=(*CD*2)=(162)=8 cm

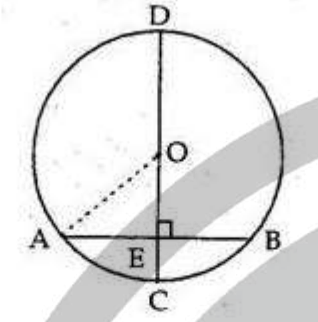
CM=CD/2=(6/2)=3cm
In right angled Δ*CMO,* we have:
$\Rightarrow OC^2 = CM^2 + MO^2$
$\Rightarrow MO^2 = OC^2 - CM^2$
$\Rightarrow MO^2 = 17^2 - 8^2$
$\Rightarrow MO^2 = 289 - 64 = 225$
∴ *MO* = 15 cm

Hence, distance between the chords = (*LO* + *MO*) = (8 + 15) cm = 23 cm

Question 6.

Solution:

CD is the diameter of the circle with centre *O* and is perpendicular to chord *AB*.
Join *OA*.

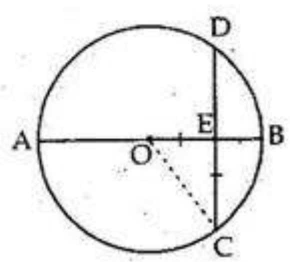


Given: $AB = 12$ cm and $CE = 3$ cm
Let $OA = OC = r$ cm (Radii of a circle)
Then $OE = (r - 3)$ cm
Since the perpendicular from the centre of the circle to a chord bisects the chord, we have:
$AE = (AB/2) = (12/2)$ cm $= 6$ cm
Now, in right angled ΔOEA, we have:
$\Rightarrow OA^2 = OE^2 + AE^2$
$\Rightarrow r^2 = (r - 3)^2 + 6^2$
$\Rightarrow r^2 = r^2 - 6r + 9 + 36$
$\Rightarrow r^2 - r^2 + 6r = 45$
$\Rightarrow 6r = 45$
$\Rightarrow r = (45/6)$ cm $= 7.5$ cm
$\Rightarrow r = 7.5$ cm
Hence, the required radius of the circle is 7.5 cm.

Question 7.

Solution:

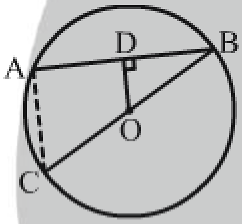
AB is the diameter of the circle with centre *O,* which bisects the chord *CD* at point *E*.
Given: $CE = ED = 8$ cm and $EB = 4$ cm
Join *OC*.



Let $OC = OB = r$ cm (Radii of a circle)
Then $OE = (r - 4)$ cm
Now, in right angled ΔOEC, we have:
$OC^2 = OE^2 + EC^2$ (Pythagoras theorem)
$\Rightarrow r^2 = (r - 4)^2 + 8^2$
$\Rightarrow r^2 = r^2 - 8r + 16 + 64$
$\Rightarrow r^2 - r^2 + 8r = 80$
$\Rightarrow 8r = 80$
$\Rightarrow r = (80/8)$ cm $= 10$ cm
$\Rightarrow r = 10$ cm
Hence, the required radius of the circle is 10 cm.

Question 8.

Solution:

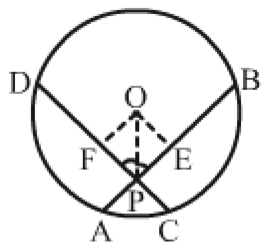


Given: *BC* is a diameter of a circle with centre *O* and $OD \perp AB$.
To prove: *AC* parallel to *OD* and $AC = 2 \times OD$
Construction: Join *AC*.
Proof:

We know that the perpendicular from the centre of a circle to a chord bisects the chord.
Here, $OD \perp AB$
D is the mid point of *AB.*
i.e., $AD = BD$
Also, *O* is the mid point of *BC*.
i.e., $OC = OB$
Now, in ABC
D is the mid point of *AB* and *O* is the mid point of *BC.*
According to the mid point theorem, the line segment joining the mid points of any two sides of a triangle is parallel to the third side and equal to half of it.
i.e., $OD \parallel AC$ and $OD=12AC$
$\therefore AC = 2 \times OD$

Question 9:

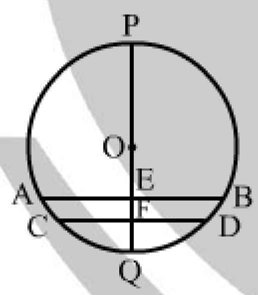
Solution:



Given: *O* is the centre of a circle in which chords *AB* and *CD* intersect at *P* such that *PO* bisects ∠*BPD.*
To prove: *AB* = *CD*
Construction: Draw *OE* ⊥ *AB* and *OF* ⊥ *CD*
Proof: In Δ*OEP* and Δ*OFP,* we have:
∠*OEP* = ∠*OFP* (90° each)
OP = *OP* (Common)
∠*OPE* = ∠*OPF* (âˆµ OP bisects ∠BPD)
Thus, Δ*OEP* ≅ Δ*OFP* (AAS criterion)
⇒ *OE* = *OF*
Thus, chords *AB* and *CD* are equidistant from the centre *O*.
⇒ *AB* = *CD* (âˆµ Chords equidistant from the centre are equal)
∴ *AB* = *CD*

Question 10.

Solution:



Given: *AB* and *CD* are two parallel chords of a circle with centre *O*. *POQ* is a diameter which is perpendicular to *AB*.
To prove: *PF* ⊥ *CD* and *CF* = *FD*
Proof:
AB || *CD* and *POQ* is a diameter.
∠*PEB* = 90° (Given)
∠*PFD* = ∠*PEB* (âˆµ *AB* || *CD,* Corresponding angles)
Thus, *PF* ⊥ *CD*
∴ *OF* ⊥ *CD*
We know that the perpendicular from the centre to a chord bisects the chord.
i.e., *CF* = *FD*
Hence, *POQ* is perpendicular to *CD* and bisects it.

Question 11.

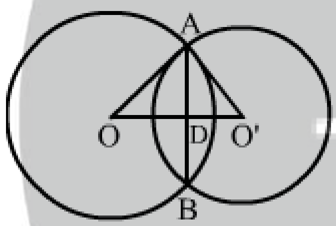
Solution:

Given: Two distinct circles
To prove: Two distinct circles cannot intersect each other in more than two points.
Proof: Suppose that two distinct circles intersect each other in more than two points.
∴ These points are non-collinear points.

Three non-collinear points determine one and only one circle.
∴ There should be only one circle.
This contradicts the given, which shows that our assumption is wrong.
Hence, two distinct circles cannot intersect each other in more than two points.

Question 12.

Solution:

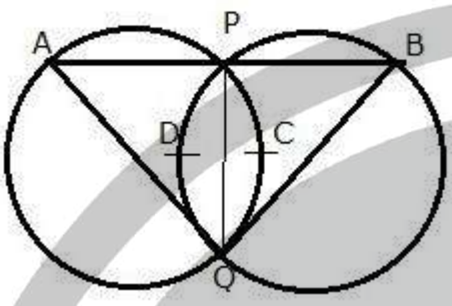


Given: $OA = 10$ cm, $O'A = 8$ cm and $AB = 12$ cm
$AD = (AB/2) = (12/2) = 6$ cm
Now, in right angled ΔADO, we have:
$OA^2 = AD^2 + OD^2$
$\Rightarrow OD^2 = OA^2 - AD^2$
$= 10^2 - 6^2$
$= 100 - 36 = 64$
$\therefore OD = 8$ cm
Similarly, in right angled $\Delta ADO'$, we have:
$O'A^2 = AD^2 + O'D^2$
$\Rightarrow O'D^2 = O'A^2 - AD^2$
$= 8^2 - 6^2$
$= 64 - 36$
$= 28$
$\Rightarrow O'D = \sqrt{28} = 2\sqrt{7}$ cm
Thus, $OO' = (OD + O'D)$
$= (8 + 2\sqrt{7})$ cm
Hence, the distance between their centres is $(8 + 2\sqrt{7})$cm

Question 13.

Solution:

Given: Two equal circles intersect at point *P* and *Q*.
A straight line passes through *P* and meets the circle at points *A* and *B.*
To prove: *QA* = *QB*
Construction: Join *PQ*.

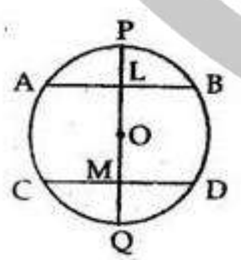


Proof:
Two circles will be congruent if and only if they have equal radii.
Here, *PQ* is the common chord to both the circles.
Thus, their corresponding arcs are equal (if two chords of a circle are equal, then their corresponding arcs are congruent).
So, arc *PCQ* = arc *PDQ*
∴ $\angle QAP = \angle QBP$ (Congruent arcs have the same degree in measure)
Hence, *QA* = *QB* (In isosceles triangle, base angles are equal)

Question 14.

Solution:

Given: *AB* and *CD* are two chords of a circle with centre *O*. Diameter *POQ* bisects them at points *L* and *M*.
To prove: *AB* || *CD*
Proof: *AB* and *CD* are two chords of a circle with centre *O.* Diameter *POQ* bisects them at *L* and *M*.

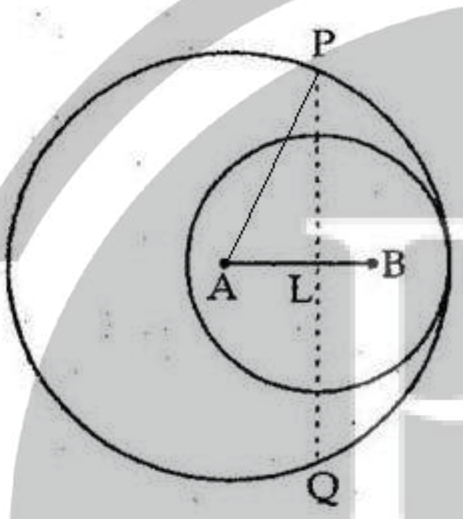


Then $OL \perp AB$
Also, $OM \perp CD$
∴ $\angle ALM = \angle LMD = 90^o$
Since alternate angles are equal, we have:
AB|| *CD*

Question 15.

Solution:

Two circles with centres *A* and *B* of respective radii 5 cm and 3 cm touch each other internally.
The perpendicular bisector of *AB* meets the bigger circle at *P* and *Q.*
Join *AP*.



Let *PQ* intersect *AB* at point *L*.
Here, *AP* = 5 cm
Then *AB* = (5 - 3) cm = 2 cm

Since *PQ* is the perpendicular bisector of *AB,* we have:
AL=(*AB/2*)=(2/2)=1 cm

Now, in right angled Δ*PLA*, we have:

$AP^2 = AL^2 + PL^2$

$\Rightarrow PL^2 = AP^2 - AL^2$

$= 5^2 - 1^2$

$= 25 - 1 = 24$

⇒ PL=√24=√26cm

Thus *PQ = 2 × PL*

= (2×2√6)=4√6cm

Hence, the required length of *PQ* is 4√6cm

Question 16.

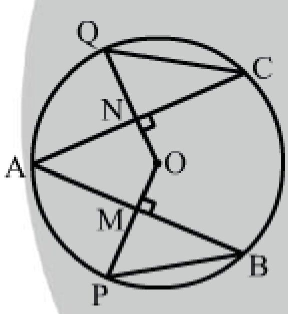
Solution:

We have:
OB = OC, ∠BOC = ∠BCO = y
External ∠OBA = ∠BOC + ∠BCO = (2y)
Again, OA = OB, ∠OAB = ∠OBA = (2y)
External ∠AOD = ∠OAC + ∠ACO
Or x = ∠OAB + ∠BCO
Or x = (2y) + y = 3y
Hence, x = 3y

Question 17.

Solution:

Given: AB and AC are chords of the circle with centre O. AB = AC, OP ⊥ AB and OQ ⊥ AC

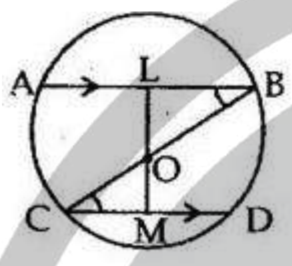


To prove: PB = QC
Proof:
AB = AC (Given)
⇒ 12AB=12AC
The perpendicular from the centre of a circle to a chord bisects the chord.
∴ MB = NC ...(i)
Also, OM = ON (Equal chords of a circle are equidistant from the centre)
and OP = OQ (Radii)
⇒ OP - OM = OQ - ON
∴ PM = QN...(ii)
Now, in ΔMPB and ΔNQC, we have:
MB = NC [From (i)]
∠PMB = ∠QNC [90° each]
PM = QN [From (ii)]
i.e., ΔMPB ≅ ΔNQC (SAS criterion)
∴ PB = QC (CPCT)

Question 18.

Solution:

Given: *BC* is a diameter of a circle with centre *O. AB* and *CD* are two chords such that *AB* || *CD.*
TO prove: *AB* = *CD*
Construction: Draw *OL* ⊥ *AB* and *OM* ⊥ *CD.*

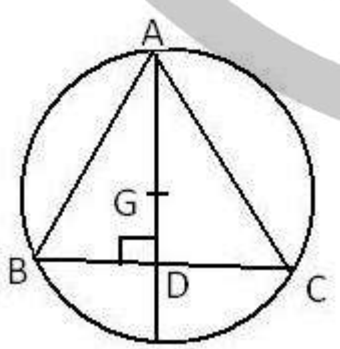


Proof:
In Δ*OLB* and Δ*OMC*, we have:
∠*OLB* = ∠*OMC* [90° each]
∠*OBL* = ∠*OCD* [Alternate angles as *AB* || *CD*]
OB = *OC* [Radii of a circle]
∴ Δ*OLB* ≅ Δ*OMC* (AAS criterion)
Thus, *OL* = *OM* (CPCT)
We know that chords equidistant from the centre are equal.
Hence, *AB* = *CD*

Question 19.

Solution:

Let Δ*ABC* be an equilateral triangle of side 9 cm.
Let *AD* be one of its median.



Then, *AD* ⊥ *BC* (Δ*ABC* is an equilateral triangle)
Also, *BD*=(*BC/2*)=(9/2)=4.5cm

In right angled ΔADB, we have:

$AB^2 = AD^2 + BD^2$

$\Rightarrow AD^2 = AB^2 - BD^2$

$\Rightarrow AD = AB^2 - BD^2$

$= \sqrt{(9)2} - \sqrt{(9/2)2}$cm

$= 93\sqrt{2}$cm

In the equilateral triangle, the centroid and circumcentre coincide and *AG* : *GD* = 2 : 1.

Now, radius = $AG = 23AD$

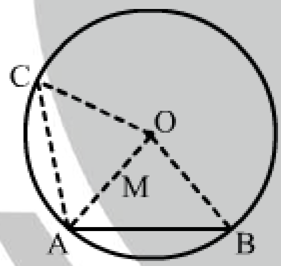
$\Rightarrow AG = (2/3 \times 9\sqrt{3}/2) = 3\sqrt{3}$cm

∴ The radius of the circle is $3\sqrt{3}$cm

Question 20.

Solution:

Given: *AB* and *AC* are two equal chords of a circle with centre *O.*



To prove: ∠*OAB* = ∠*OAC*

Construction: Join *OA, OB* and *OC*.

Proof:

In Δ*OAB* and Δ*OAC*, we have:

AB = *AC* (Given)

OA = *OA* (Common)

OB = *OC* (Radii of a circle)

∴ Δ *OAB* ≅ Δ *OAC* (By SSS congruency rule)

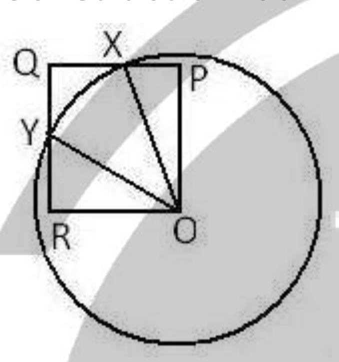
⇒ ∠*OAB* = ∠*OAC* (CPCT)

Hence, point *O* lies on the bisector of ∠*BAC.*

Question 21.

Solution:

Given: *OPQR* is a square. A circle with centre *O* cuts the square at *X* and *Y.*
To prove: $QX = QY$
Construction: Join *OX* and *OY*.



Proof:
In ΔOXP and ΔOYR, we have:
$\angle OPX = \angle ORY$ (90° each)
$OX = OY$ (Radii of a circle)
$OP = OR$ (Sides of a square)
$\therefore \Delta OXP \cong \Delta OYR$ (BY RHS congruency rule)
$\Rightarrow PX = RY$ (By CPCT)
$\Rightarrow PQ - PX = QR - RY$ (*PQ* and *QR* are sides of a square)
$\Rightarrow QX = QY$