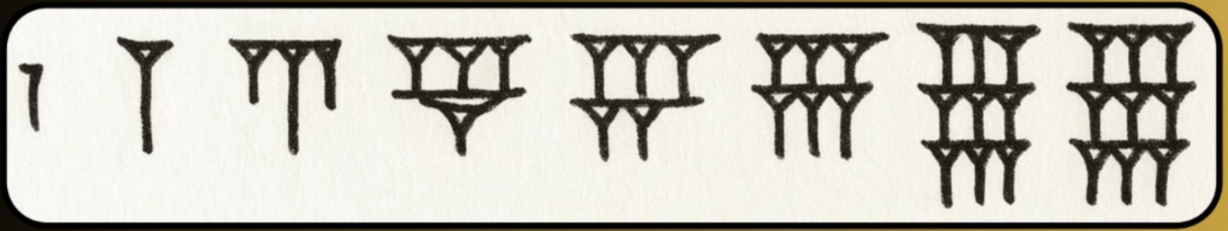
Class 8th

A STORY OF NUMBERS

Reema's Curiosity Sparks a Journey



- A curious girl discovers a paper with strange symbols.
- The markings belonged to the **Mesopotamians**, an ancient civilization from 4000 years ago.

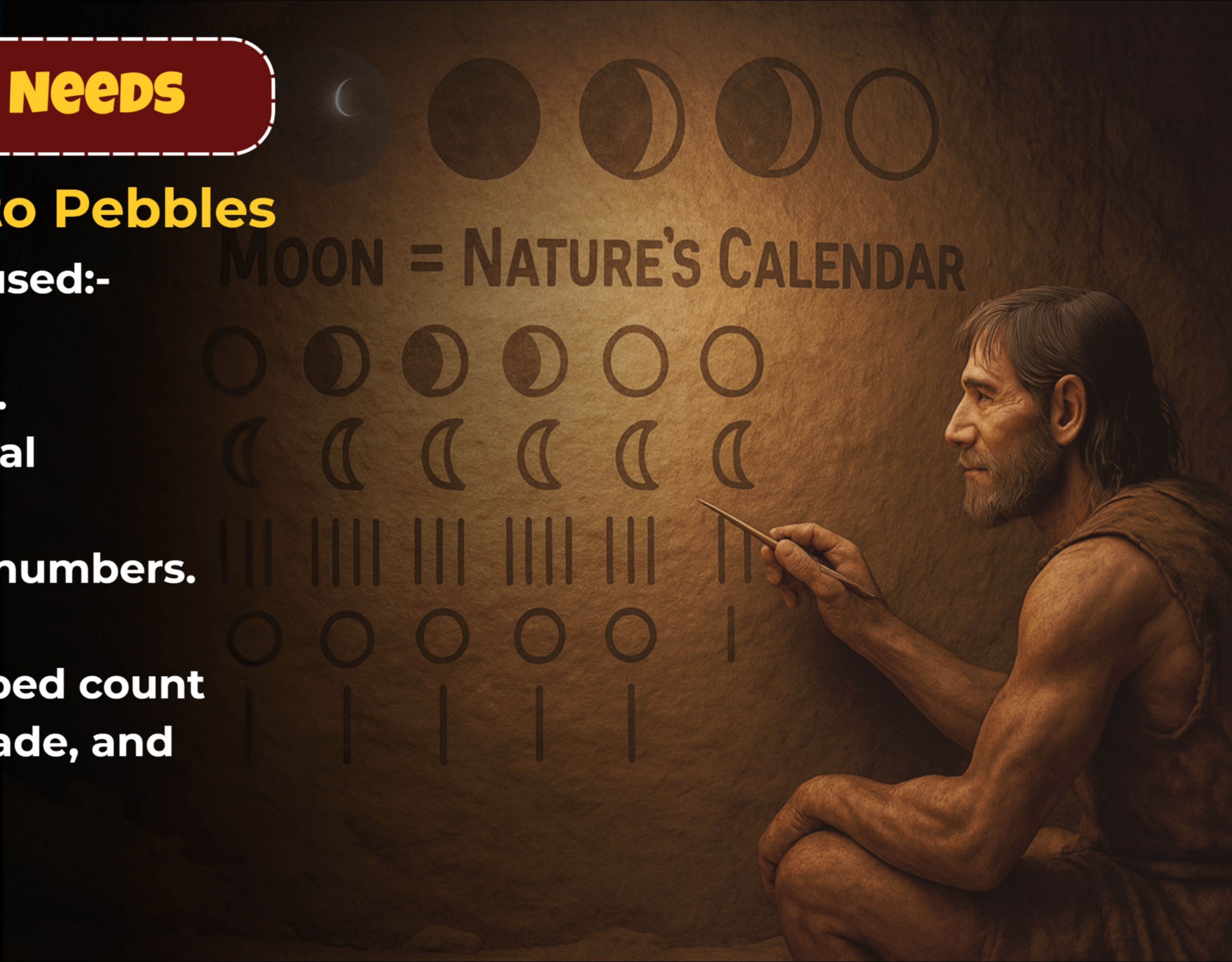
EARLY COUNTING NEEDS

From Fingers to Pebbles

Early humans first used:-

- Fingers to count.
- Stones as physical counters.
- Sticks to record numbers.

These methods helped count animals, manage trade, and measure time.



HISTORY OF NUMBERS

Hindu Number System

- Modern age numbers (0 to 9) were developed in India, around 2000 years ago
- Bakhshali manuscript (3rd century CE)
- Aryabhata (499 CE) was the first mathematician.
- Transmitted to Arab World (800 CE) - Al Khwarizmi
- Transmitted to Europe and Africa (1100 CE).
- 17 Century - Adopted by the world

Algorithm

The Mechanism of Counting

How do we ensure that all cows have returned safely after grazing?

Do we have fewer cows than our neighbour

If there are fewer, how many more cows would we need so that we have the same number of cows as our neighbour

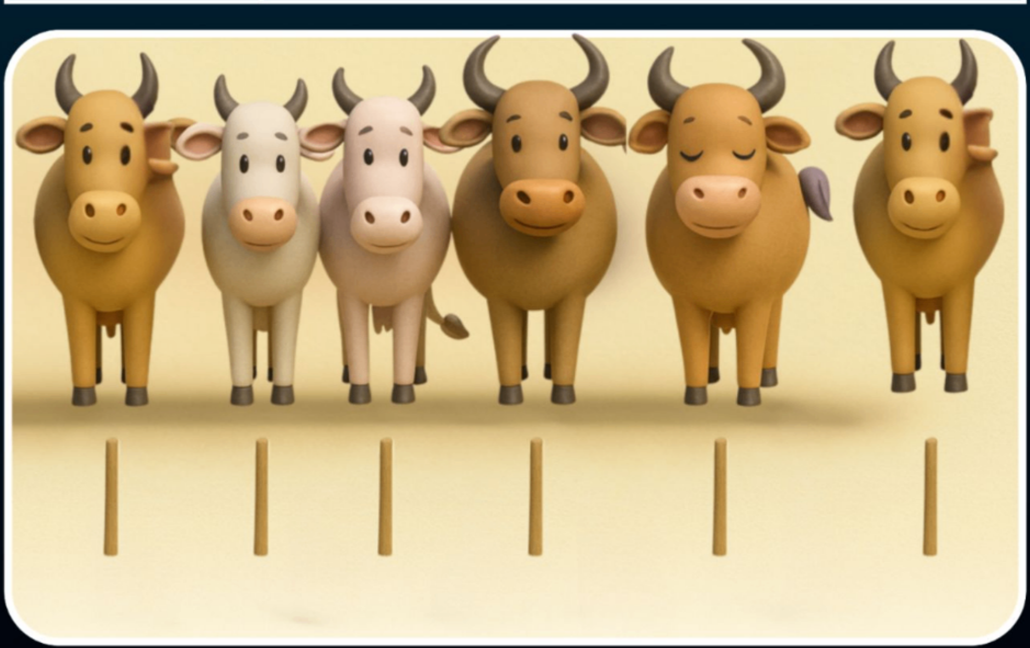
COUNTING WITH STICKS

Method 1

- One stick for one cow — each cow in the herd gets a matching stick.
- Collection of sticks shows the total herd size.
- Comparison helps check if any cows are missing.
- A simple counting method using physical objects.

This is the foundation of counting.

Number	Its Representation (using sticks)
1	|
2	||
3	|||
4	||||
5	|||||
.	



SOUNDS AS NUMBERS

Method 2

- Early humans used sequences of sounds (like letters a–z) to count.
- Each object matched a sound, creating a one-to-one system.
- Limitation: With only 26 English letters, counting cannot exceed 26 objects.

Number	Its Representation (using sounds or names)
1	a
2	b
3	c
4	d
5	e
.	.
26	z

NUMBER SYSTEM

Standard sequence

- To count a collection, we need a standard sequence (objects, names, or symbols).
- This sequence must have a fixed order.
- We call such a sequence a **number system**.
- Counting is done by making a one-to-one mapping between objects and the sequence.
- Objects are counted by following the order of the sequence.

EARLY SYMBOL SYSTEMS

→ Roman Number System

Method 3

- Symbol-based systems like Roman numerals used fixed characters like I, V, X, etc.
- Worked well for basic representation of numbers.
- Limitation: Inefficient for large numbers.
- Not suitable for complex calculations or operations.

50 → L
100 → C
500 → D
1000 → M

Number	1	2	3	4	5	6	7	8	9	10
Representation using symbols	I	II	III	IV	V	VI	VII	VIII	IX	X
Number	11	12	13	14	15	16	17	18	19	20
Representation using symbols	XI	XII	XIII	XIV	XV	XVI	XVII	XVIII	XIX	XX

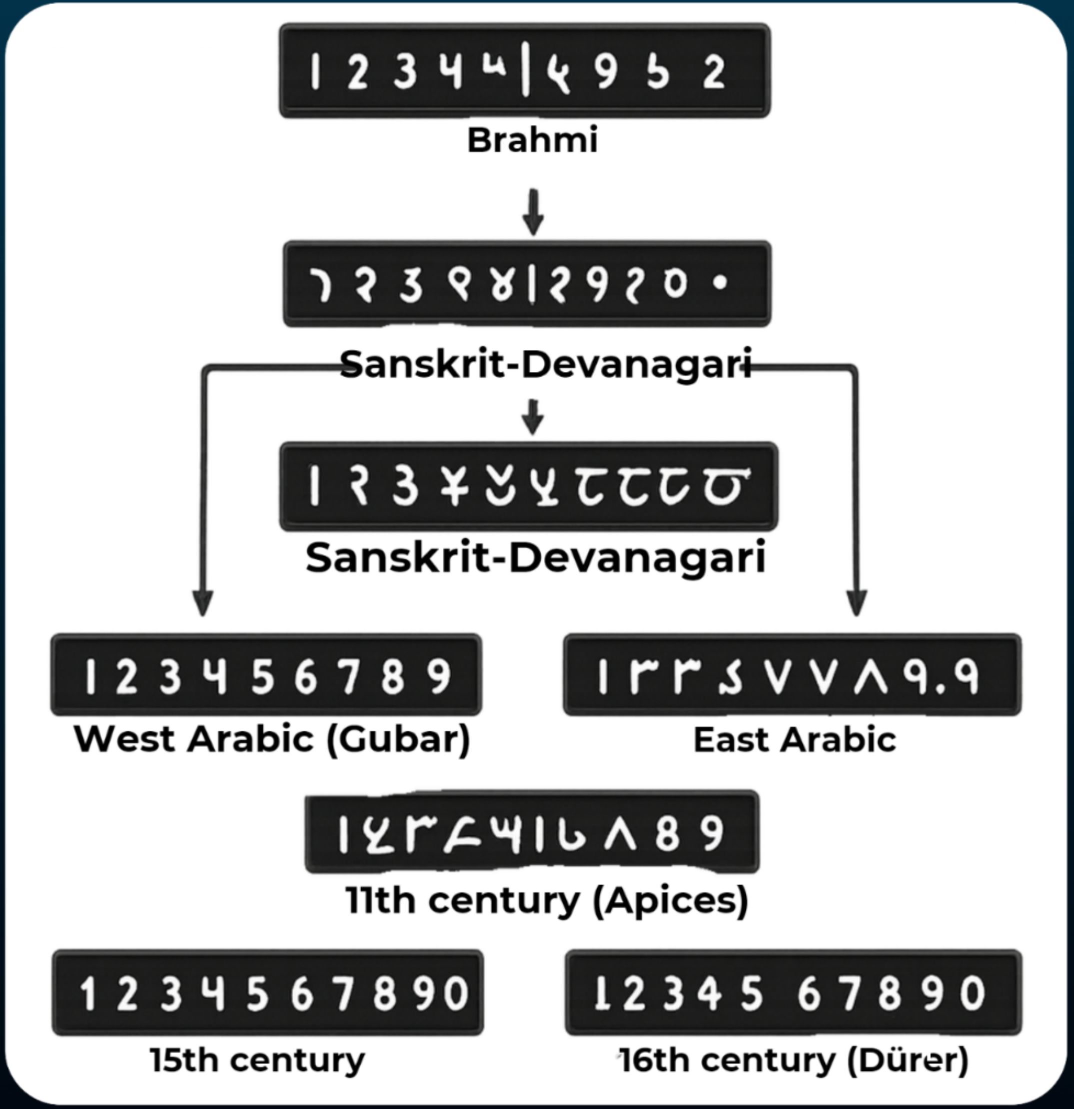
WHAT ARE NUMERALS?

Symbols That Represent Numbers

Numerals are written symbols used to denote numbers.

Examples:

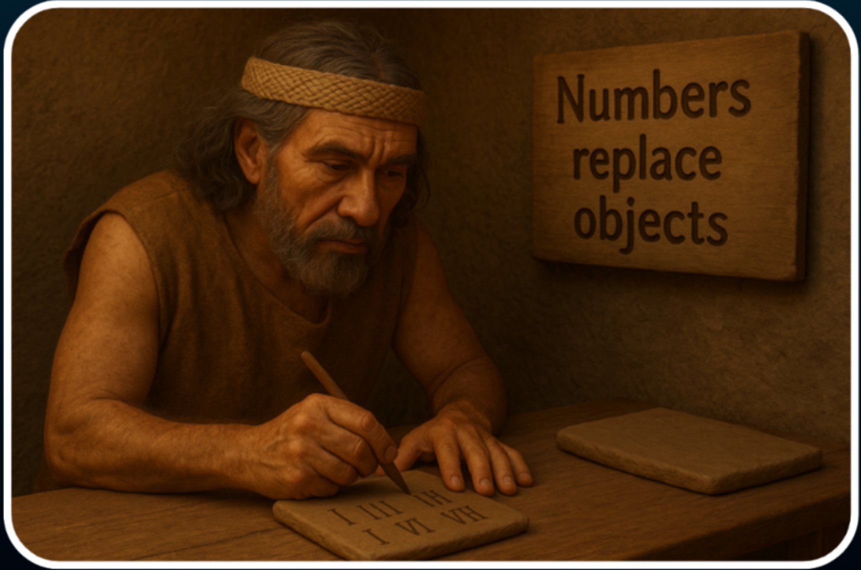
- 3 (Hindu-Arabic),
- III (Roman),
- ३ (Devanagari).

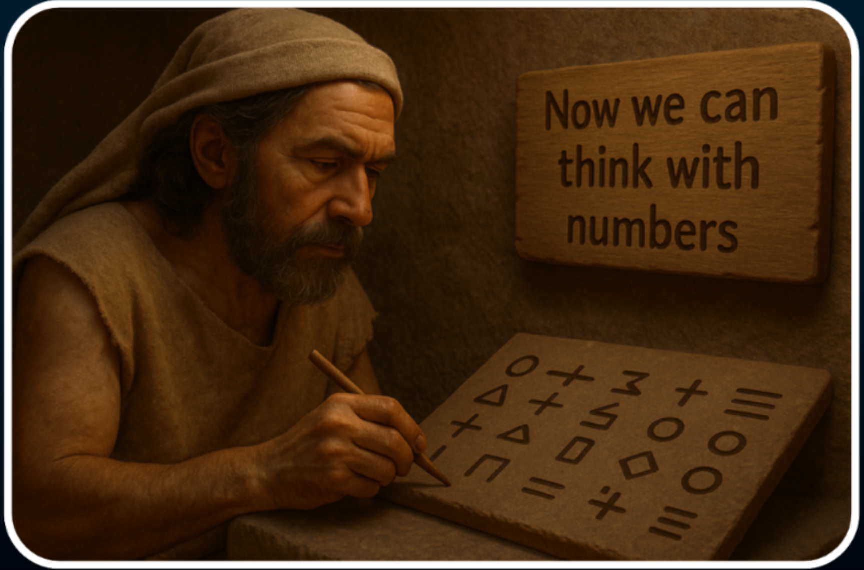


EVOLUTION TOWARD ABSTRACTION

From Counting Tools to Mental Math

- Over time, humans moved beyond physical counters (sticks, stones, marks).
- Shifted to abstract thinking using numerals.
- Developed systems for calculations and more complex operations.

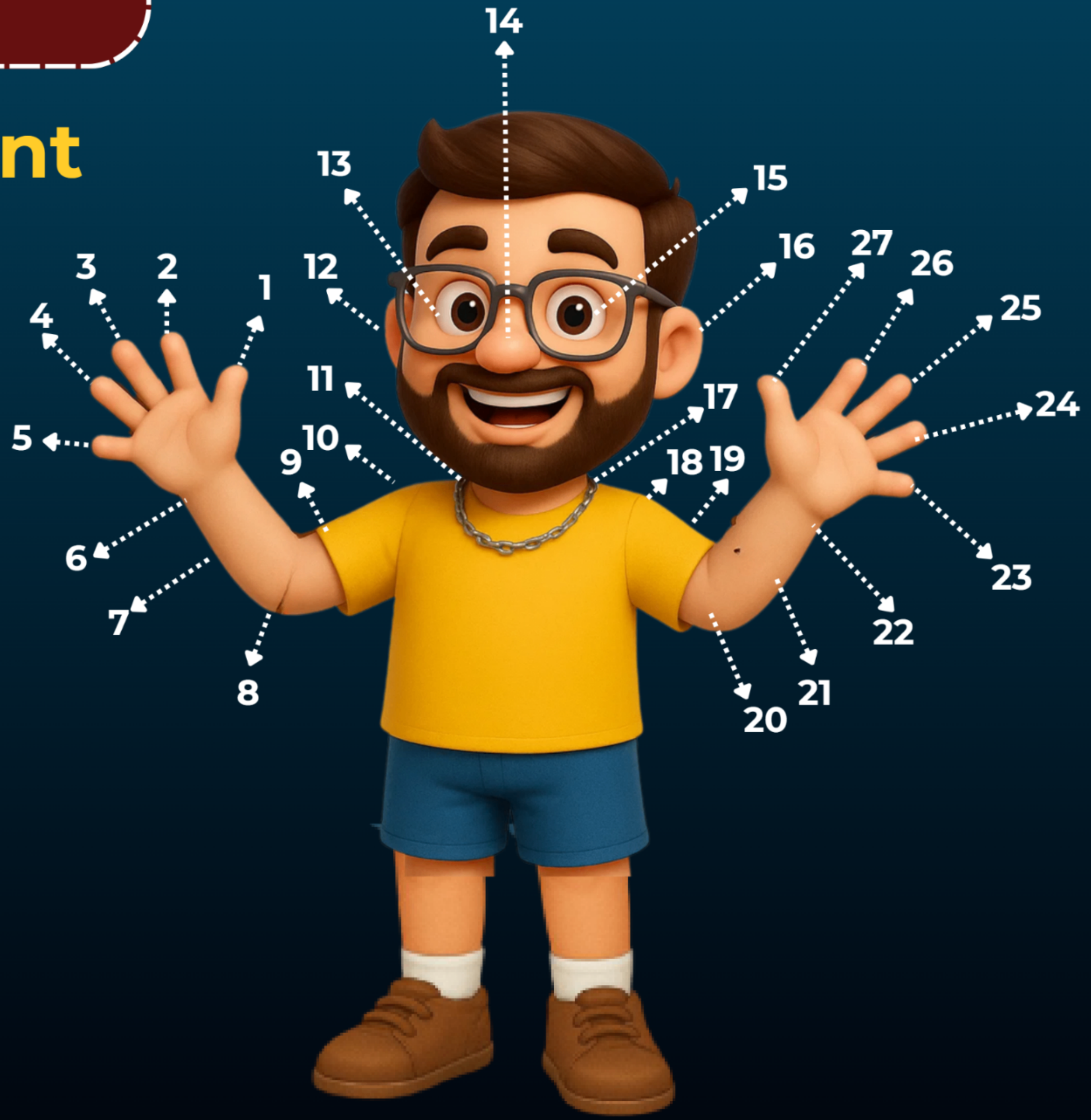




USE OF BODY PARTS

Using the Human Body to Count

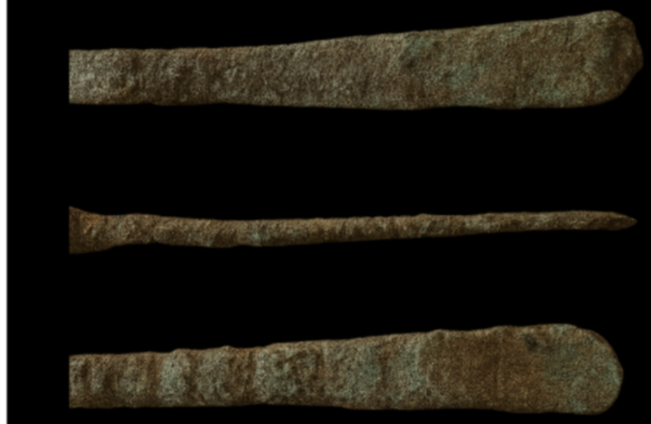
- Early societies counted using fingers, toes, knuckles, and body parts.
- Some tribes extended counting to the whole body.
- This allowed them to count up to 20 or more.



TALLY MARKS

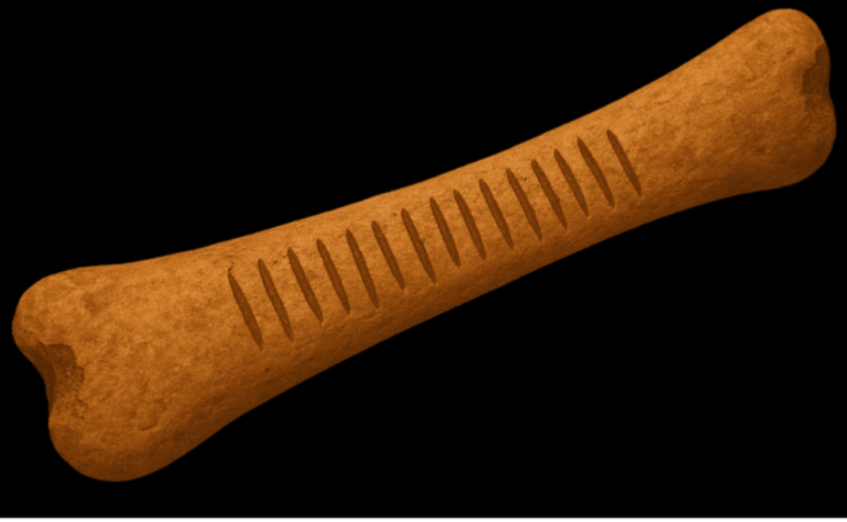
Recording with Lines

- Archaeologists found bones with tally marks dating back over 20,000 years.
- The Lebombo bone (South Africa) is an even older tally stick.
- It has 29 notches, estimated to be about 44,000 years old.
- Considered one of the oldest mathematical artefacts.
- May have been used as a tally stick or a lunar calendar.



Lebombo bone

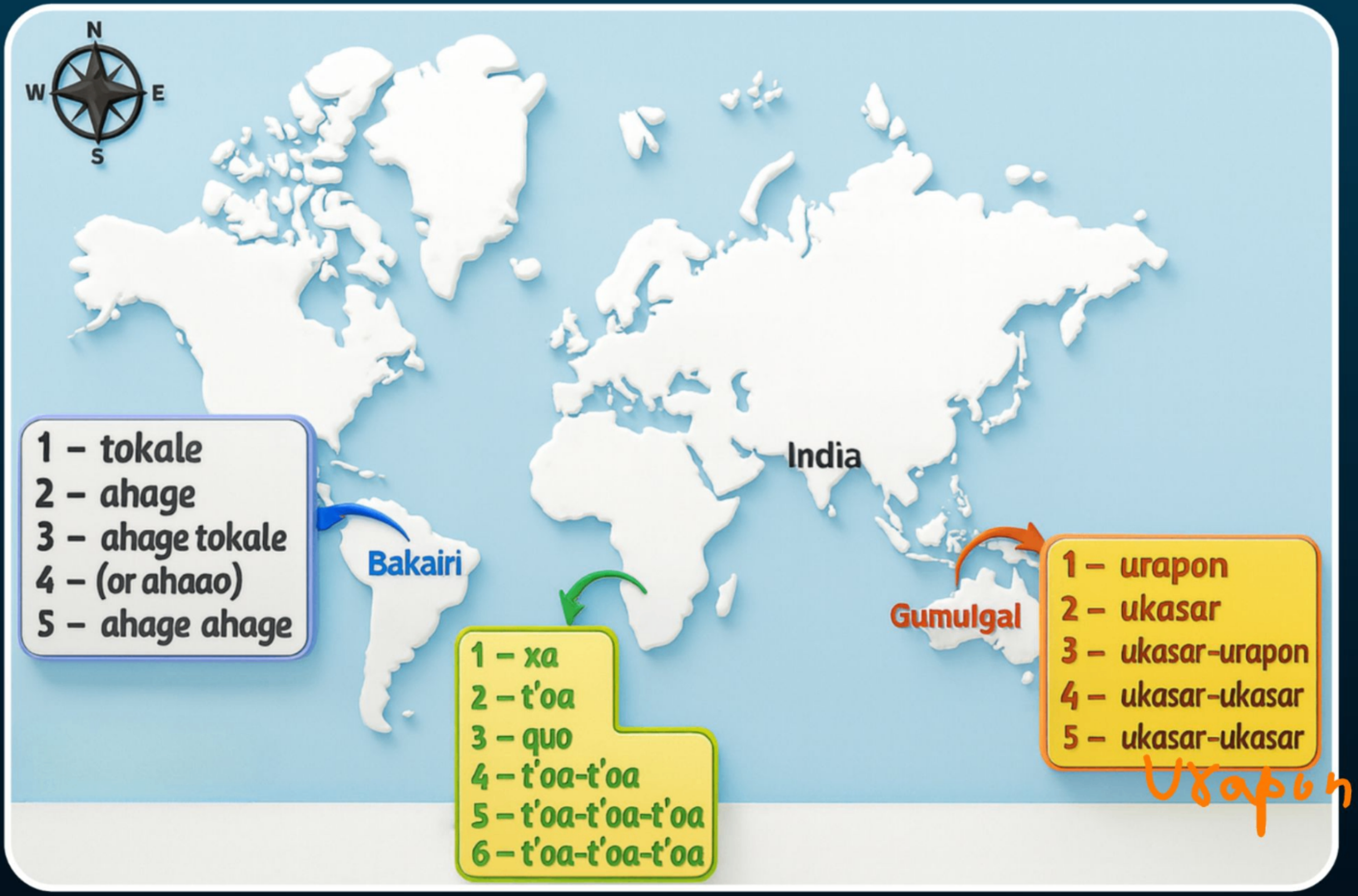
Ishango bone



Counting in Twos - Gumulgal People

Unique Counting Traditions

- The Gumulgal people of Australia had a unique counting method.
- They counted in pairs, treating "two" as a basic unit.
- Numbers were viewed as combinations of twos, not individual counts.



1 – tokale
2 – ahage
3 – ahage tokale
4 – (or ahaao)
5 – ahage ahage

1 – xa
2 – t’oa
3 – quo
4 – t’oa-t’oa
5 – t’oa-t’oa-t’oa
6 – t’oa-t’oa-t’oa

1 – urapon
2 – ukasar
3 – ukasar-urapon
4 – ukasar-ukasar
5 – ukasar-ukasar

Urapon

Gumulgal (Australia)

1 → Urapon

2 → Ukasar

3 → Ukasar Urapon

4 → Ukasar Ukasar

5 → Ukasar Ukasar Urapon

100

10,000

10 →

9 →

COUNTING IN TWOS - BUSHMEN

Natural Groupings in Counting

- The San (Bushmen) of Southern Africa had unique counting methods.
- They used body-based counting (fingers, body parts).
- Also practiced paired-object counting.
- Inspired by nature — e.g., animal legs, pairs of eyes.

Roman Numerals

The Symbols Behind the System

- Efficiency comes from grouping numbers into landmark sizes.
- A number is represented not by a single group size but by a sequence of group sizes.
- These landmark numbers act as reference points in counting.
- This marks an important breakthrough in the evolution of number systems.

Roman Numerals

The Symbols Behind the System

- Group the number into as many 10s (X) as possible.
- From the remainder, group into 5s (V).
- Finally, represent leftover numbers with 1s (I).

Example: The number 27 = 10 + 10 + 5 + 1 + 1

So, 27 in Roman numerals is XXVII.

Instead of representing 50 as XXXXX, a new symbol is given to it: L.

$$\underline{\underline{2}}\,\underline{\underline{7}} = 2 \times \underline{\underline{10}} + 1 \times 5 + 1 \times 2$$

Ten's Place

unit Place

XXVII

LANDMARK NUMBERS

Key Values in the Roman System

- The Roman numeral system introduced newer symbols for larger numbers.
- Each of these is treated as a landmark number.
- These landmark numbers serve as basic symbols for building other numbers.
- Examples of landmark numbers in the Roman system:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

ARITHMETIC CHALLENGES

The Problem with Roman Math

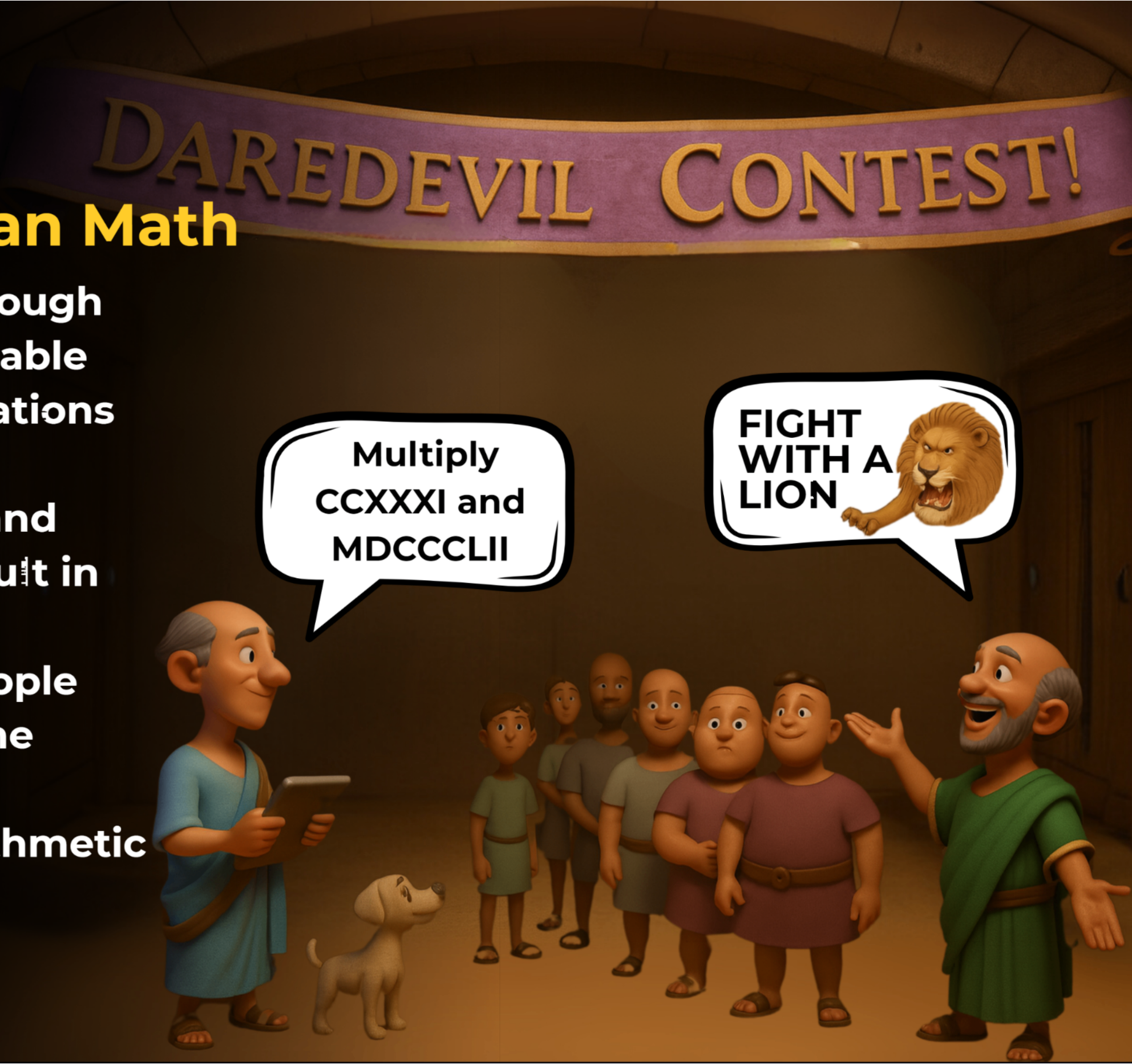
Roman numerals lacked zero and place value, making addition, subtraction, and multiplication difficult.

E.g., try adding XXVII + XIII vs 27 + 13.

ABACUS

The Problem with Roman Math

- The Roman numeral system, though relatively efficient, was not suitable for performing arithmetic operations easily.
- Operations like multiplication and division were particularly difficult in the Roman system.
- To overcome this limitation, people used a calculating tool called the abacus.
- The **abacus** helped perform arithmetic operations more effectively.



THE HINDU-ARABIC ADVANTAGE

Place Value and Zero Revolution

The Hindu system introduced place value (units, tens, hundreds) and zero, simplifying math.

Show how 204 = 2 × 100 + 0 × 10 + 4 × 1.

1 → I
2 → II
3 → III
4 → IV
5 → V
6 → VI

$\boxed{\text{CCIV}}$

700 = 500+200
= DCC

1000 → M
100 → C
10 → X

1000 → M
900 → CM
800 → DCCC → D+c+c+c

Represent the following numbers in the Roman system.

(i) 1222 (ii) 2999 (iii) 302 (iv) 715 (V) 2367

→ H.W

1222

$\underset{M}{1\times1000}+\underset{C}{2\times100}+\underset{X}{2\times10}+\underset{I}{2\times1}$

$\boxed{MCCXXII}$

(iii) 302

$3\times100+0\times10+2\times1$

$\boxed{CCCII}$

(ii) 2999

$\underset{M}{2\times1000}+9\times100+9\times10+9\times1$

$\boxed{MMCMXCIX}$

(4) $715=7\times100+1\times10+5\times1$

$=\boxed{DCCXV}$

I → 1

V → 5

X → 10

C → 100

D → 500

M → 1000

$\overline{V}$ → 5000

$\overline{X}$ → 10000

IV → 4

IX → 9

XC → 90

LXXXX

Try adding the following numbers without converting them to Hindu numerals:

(a) CCXXXII + CCCCXIII

40

Cs → 6

XL ← Xs → 4

Is → 5 → V

5Cs → D → 500

$\boxed{\text{DCXLV}}$

How will you multiply two numbers given in Roman numerals, without converting them to Hindu numerals? Try to find the product of the following pairs of landmark numbers: V × L, L × D, V × D, VII × IX.

ⓘ $V \times \boxed{L}$

$\underline{\underline{V}} \to$ ⑤

5×50

250

$2^0 = 1 \to L$

$2^1 = 2 \to C$

$2^2 = 4 \to CC$

$4+1=5=V$

$CC+L$

$\underline{\underline{CCL}}$

7×9

$IX \to 1$

$XVIII \to 2$

$XXXVI \to 2^2$

$\underline{L} \times \underline{\underline{D}}$

$\swarrow$

$\underline{50}$

$\underline{\underline{50 \times 500}}$

$\underline{\underline{25000}}$

$2^0 = 1 \rightarrow D$

$2^1 = 2 \rightarrow M$

$2^2 = 4 \rightarrow MM \rightarrow 2000$

$2^3 = 8 \rightarrow M\overline{V} \rightarrow 4000$

$16 \rightarrow \overline{V}MMM \rightarrow 8000$

$32 \rightarrow \overline{X}\,\overline{V}M \rightarrow 16000$

$5000 \quad \overline{V}$

$10000 \quad \overline{X}$

$\overline{X} \rightarrow 1$

$\overline{V} \rightarrow 2 \rightarrow \overline{X}$

$M \rightarrow 5 \rightarrow \overline{V}$

$\overline{X}\,\overline{V}\,\overline{V}\,\overline{V}$

$50 = 32 + 16 + 2$

$= \overline{X}\,\overline{V}M + \overline{V}MMM + M$

$\underline{\underline{\boxed{\overline{X}\,\overline{X}\,\overline{V}}}}$

$V \times \boxed{D}$

$\downarrow$

5

$5 \times 500 = \underline{\underline{2500}}$

$2^0 = 1 \rightarrow D$

$2^1 = 2 \rightarrow M$

$2^2 = 4 \rightarrow MM$

$4+1$

$MM + D$

$\boxed{MMD}$

$D \rightarrow 500$

$V \rightarrow 5$

$X \rightarrow 10$

$L \rightarrow 50$

$C \rightarrow 100$

$M \rightarrow 1000$

A group of indigenous people in a Pacific island use different sequences of number names to count different objects. Why do you think they do this?

Each object (like coconuts, fish, canoes) has its own importance.

Different sequences make it clear what is being counted.

It reflects their culture and traditions.

It shows respect for nature and resources.

Consider the extension of the Gumulgal number system beyond 6 in the same way of counting by 2s. Come up with ways of performing the different arithmetic operations (+, –, ×, ÷) for numbers occurring in this system, without using Hindu numerals. Use this to evaluate the following:

(i) (ukasar-ukasar-ukasar-ukasar-urapon) + (ukasar-ukasar ukasar-urapon)

Ukasar → 2

Urapon → 1

7 Ukasar

2 Urapon → 1 Ukasar

$\boxed{\text{8 Ukasar}}$ → Ukasar – Ukasar – $\underline{\underline{\text{8 times}}}$

(ii) (ukasar-ukasar-ukasar-ukasar-urapon) – (ukasar-ukasar ukasar)
(iii) (ukasar-ukasar-ukasar-ukasar-urapon) × (ukasar-ukasar)
(iv) (ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar-ukasar) ÷ (ukasar-ukasar)

ii) Ukasar – Urapon

iii) 9 Ukasar X Ukasar

18 Ukasar

D = U – U – U – U – U – U – U – U
Divisor = U – U

D — D
1 → U-U-U-U-U-U-U-U — U-U
2 → U-U-U-U-U-U — U-U
3 → U-U-U-U — U-U
4 → U-U — U-U = 0
R = 0 , Q = 4 = Ukasar-Ukasar

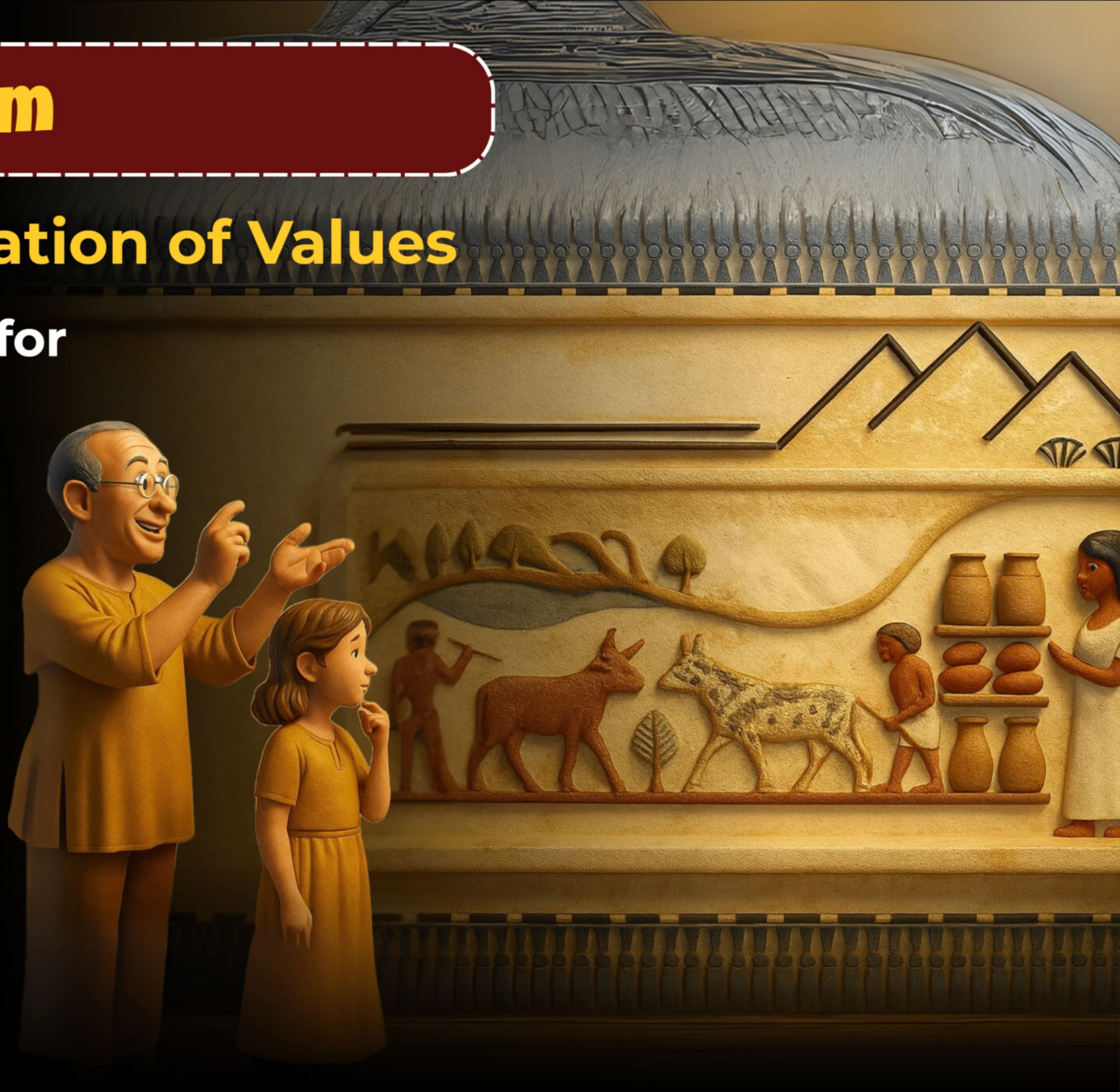
Identify the features of the Hindu number system that make it efficient when compared to the Roman number system.

The Hindu number system is efficient because it uses place value, zero, compact symbols, easy arithmetic, and only 10 digits, unlike the Roman system which is lengthy and awkward for calculations.

EGYPTIAN NUMBER SYSTEM

Hieroglyphic Representation of Values

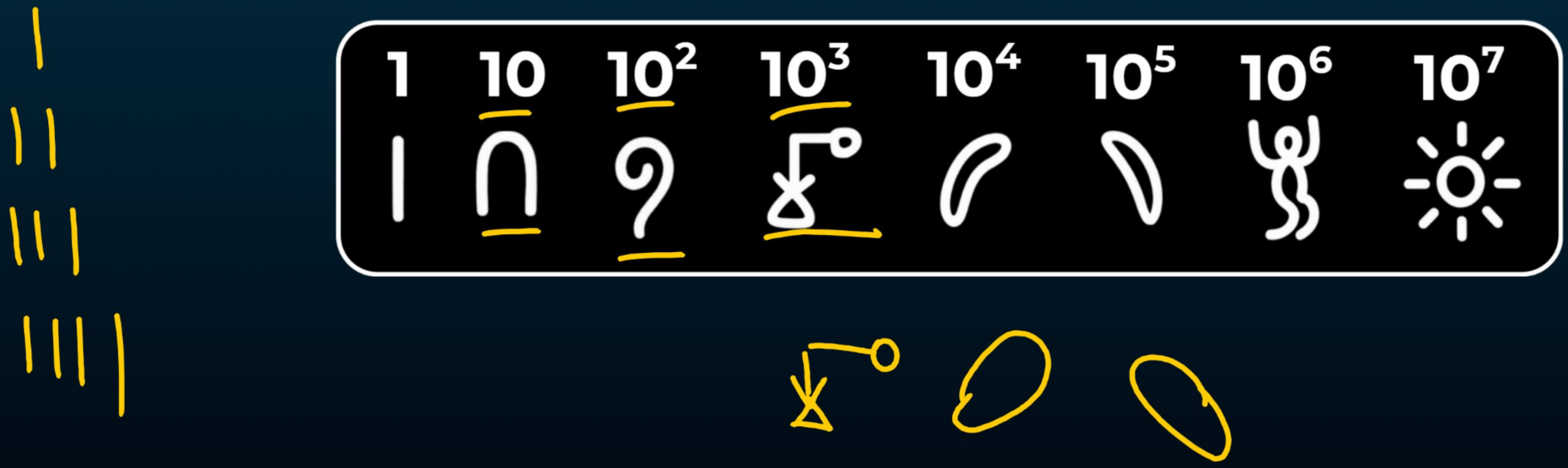
The Egyptians used symbols for powers of ten (e.g., 1, 10, 100), repeated as needed (e.g., 30 = three 10 symbols).



WHAT IS A NUMBER BASE?

Foundation of Counting Systems

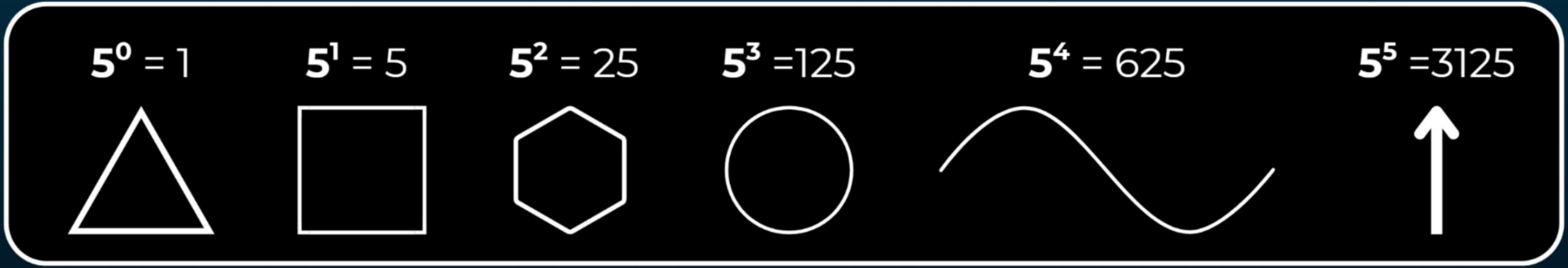
A number base defines how many unique digits are used before "carrying over." In base-10,
we count: 0–9, then move to 10.



BASE-10 VS BASE-5

Counting in Different Worlds

Base-10 uses ten digits (0–9), base-5 uses five (0–4). After 4 in base-5, we write 10 (like base-10 does after 9).



$5^0 = 1$

$5^1 = 5$

$5^2 = 25$

$5^3 = 125$

$5^4 = 625$

$5^5 = 3125$

ARITHMETIC IN BASE-N

How Addition Works in Different Bases

Show how to add numbers in base-5 or base-10 using simple examples.

For example: Base-5: 13 + 12 = 30 (in base-5).

WHY BASES MATTER

From Culture to Computation

Number bases appear in various systems:

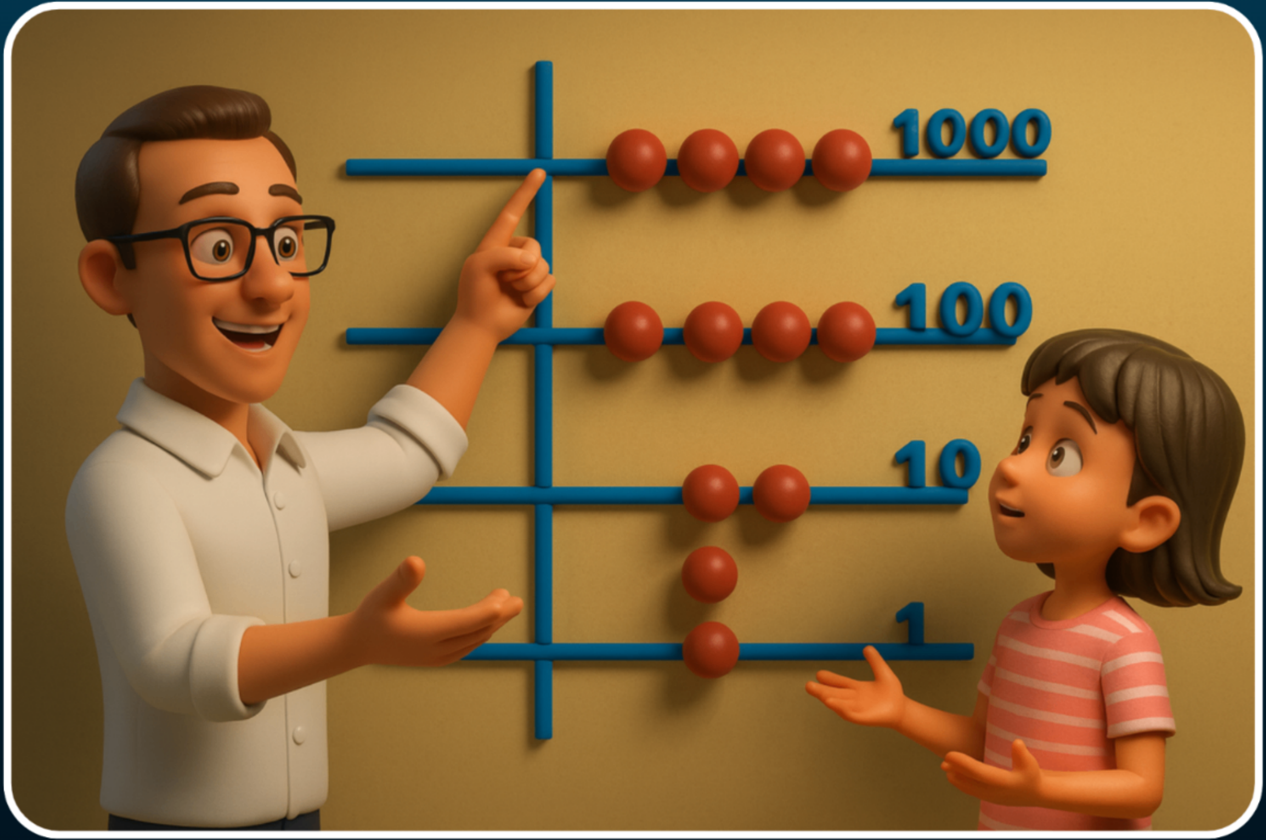
Base-2 (binary in computers),

Base-60 (time),

Base-12 (dozen).

ABACUS

- By the 11th century, people using Roman numerals also adopted a calculating device called the abacus.
- This abacus was based on the decimal system.
- It was constructed as a board with lines (as shown in the figure).
- The first line represented the value 1.
- Each successive line represented the next power of 10 (10, 100, 1000, etc.).



Represent the following numbers in the Egyptian system: 10458, 1023, 2660, 784, 1111, 70707.

10458

$1\times10^4+0\times10^3+4\times10^2+5\times10^1+8\times10^0$

1023

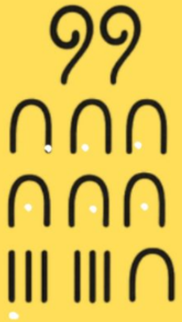
$1\times10^3+0\times10^2+2\times10^1\times3\times10^0$

70707

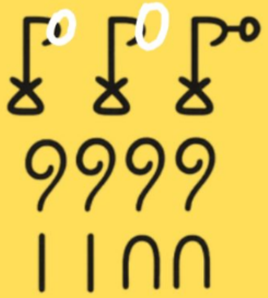
$7\times10^4+\underline{0\times10^3}+7\times10^2+\underline{0}\times10^1+7\times1$

What numbers do these numerals stand for?

(i)



(ii)



1
10
100
1000
10000

$5^0 = 1$
$5^1 = 5$
$5^2 = 25$
$5^3 = 125$
$5^4 = 625$

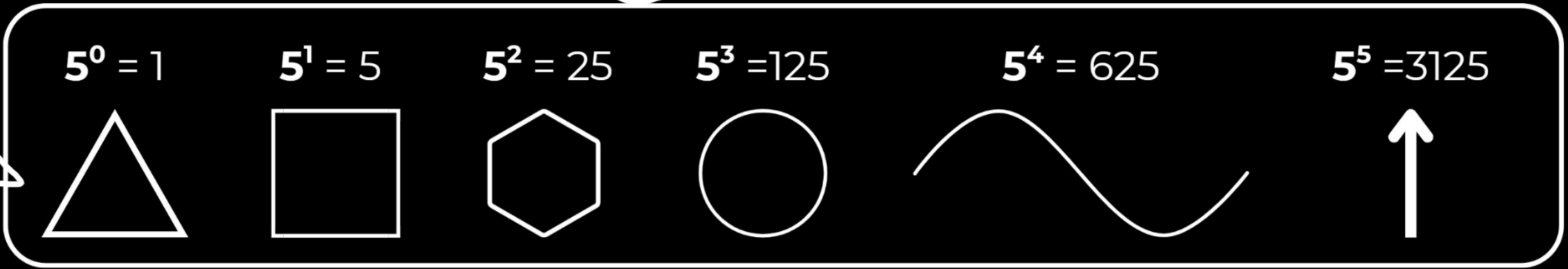
200+70+6
276

3000
400
20
2

3422

Write the following numbers in the above base-5 system using the symbols

15, 50, 137, 293, 651.



$250+25+15+3$

$15 = 5^1+5^1+5^1$

$=$

$293 = 5^3\times2+5^2\times1+5^1\times$

$= 250$

$50 = 5^2+5^2$

$=$

$137 = 125+5+5+2$

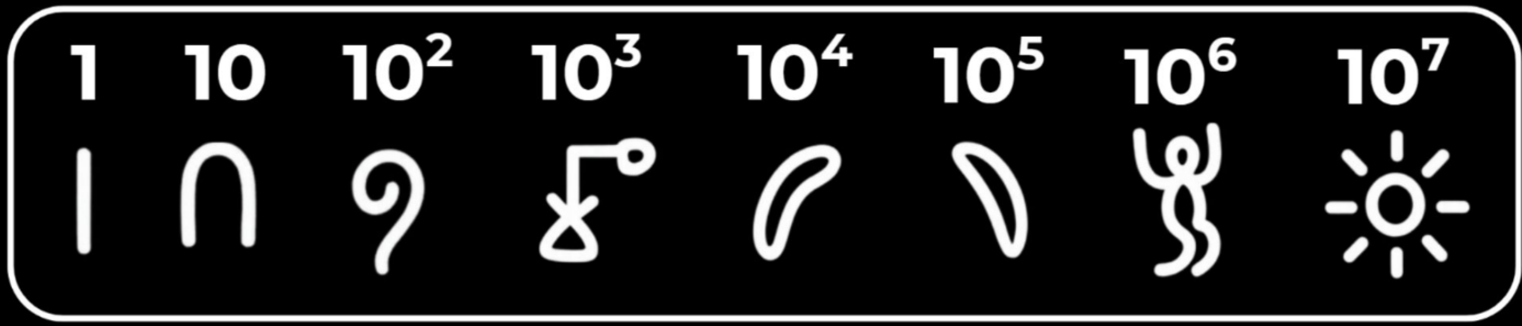
$= 5^3+5^1+5^1+2\times5^0$

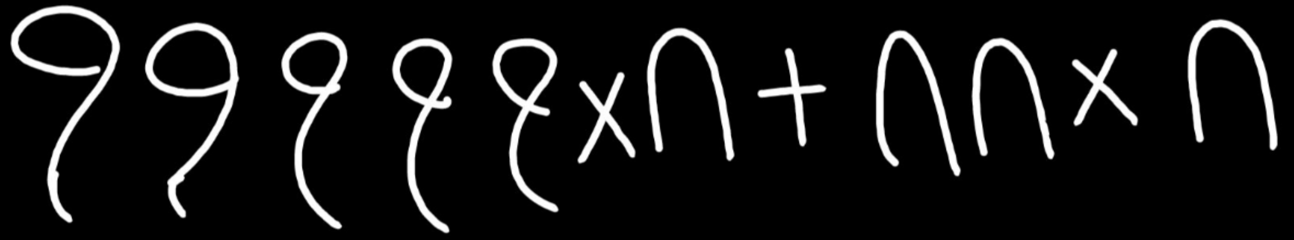
$=$

$5^3=125$

find the following products —

522×10





$5\times10^{2}\times10^{1}$

5×10^{3}

$\boxed{5220}$

|

Is there a number that cannot be represented in our base-5 system above? Why or why not?

Compute the landmark numbers of a base-7 system. In general, what are the landmark numbers of a base-n system?

$7^0 \rightarrow$ ϩ

$7^1 \rightarrow a$

$7^2 \rightarrow n$

$7^3 \rightarrow y$

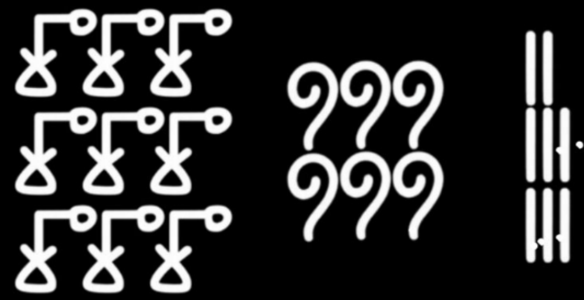
$7^4 \rightarrow m$

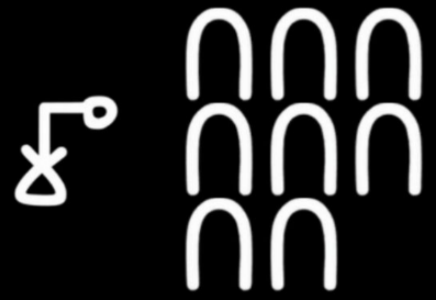
50

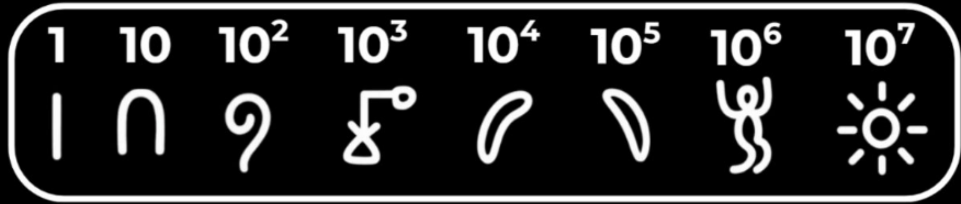
$7^2 + 7^0$

$\boxed{n\text{ϩ}}$

Add the following Egyptian numerals:

(i)  and 

(ii)  and 



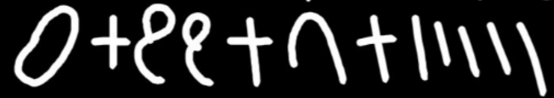
$9\times10^3+11\times10^2+15\times10^0$

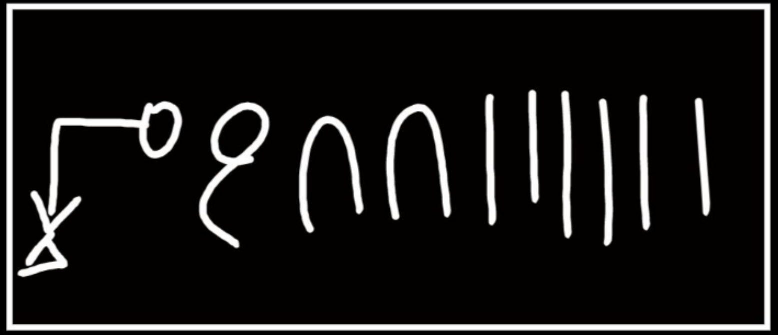
$9\times10^3+10\times10^2+1\times10^2+10+5$

$\underline{9\times10^3}+\underline{10^3}+10^2+10+5$

$10\times10^3+10^2+10+5$

10^4+10^2+10+5





Add the following numerals that are in the base-5 system that we created:

○ ⬡ ⬡ □ △ △ + ○ ○ ○ ⬡ □ □ △ △

$5^0 = 1$	$5^1 = 5$	$5^2 = 25$	$5^3 = 125$	$5^4 = 625$	$5^5 = 3125$
△	□	⬡	○	∿	↑

5^0 △
5^1 □
5^2 ⬡
5^3 ○
5^4 ∿

$5^3+5^2+5^1+5^0+5^0$

$4\times5^3+3\times5^2+3\times5^1+4\times5^0$

○ ○ ○ ○ ⬡ ⬡ ⬡ □ □ □ △ △ △ △

What is any landmark number multiplied by following products Each landmark number is a power of 10 and so multiplying it with 10 increases the power by 1, which is the next landmark number.

(i) 𓎆 x 𓎆

10×10

100

𓍢

(ii) 𓍢 x 𓎆

100×10

1000

𓆼

(iii) 𓆼 x 𓎆

1000×10

10^4

𓂭

(iv) 𓂭 x 𓎆

$10^4 \times 10$

10^5

𓆐

What is any landmark number multiplied by following products

(i) 𓎆 x 𓍢

10×100

1000

𓆼

(ii) 𓍢 x 𓍢

100×100

10^4

𓂭

(iii) 𓆼 x 𓍢

1000×100

10^5

𓆐

(iv) 𓂭 x 𓍢

$10^4 \times 10^2$

10^6

𓁨

Find the following products — Thus, the product of any two landmark numbers is another landmark number!

(i) 𓎆 x 𓂭 (ii) 𓍢 x 𓆼 (iii) 𓆼 x 𓆼 (iv) 𓂭 x 𓁨

PLACE VALUE SYSTEM

The Mesopotamian Number System

- In the beginning, the number system of **ancient Mesopotamia** used different symbols for **different landmark numbers**.
- Later, this system developed into a base-60 system, known as the **sexagesimal system**.
- The sexagesimal system allowed for a **very efficient way of representing numbers**.

60

PLACE VALUE: A GAME-CHANGER

What Is a Place Value System?

In a place value system, the position of each digit determines its value (e.g., 375 = 3 × 100 + 7 × 10 + 5 × 1).
This reduces the need for endless symbols and simplifies operations.

MESOPOTAMIAN NUMBER SYSTEM

Early Use of Place Value – Without Zero

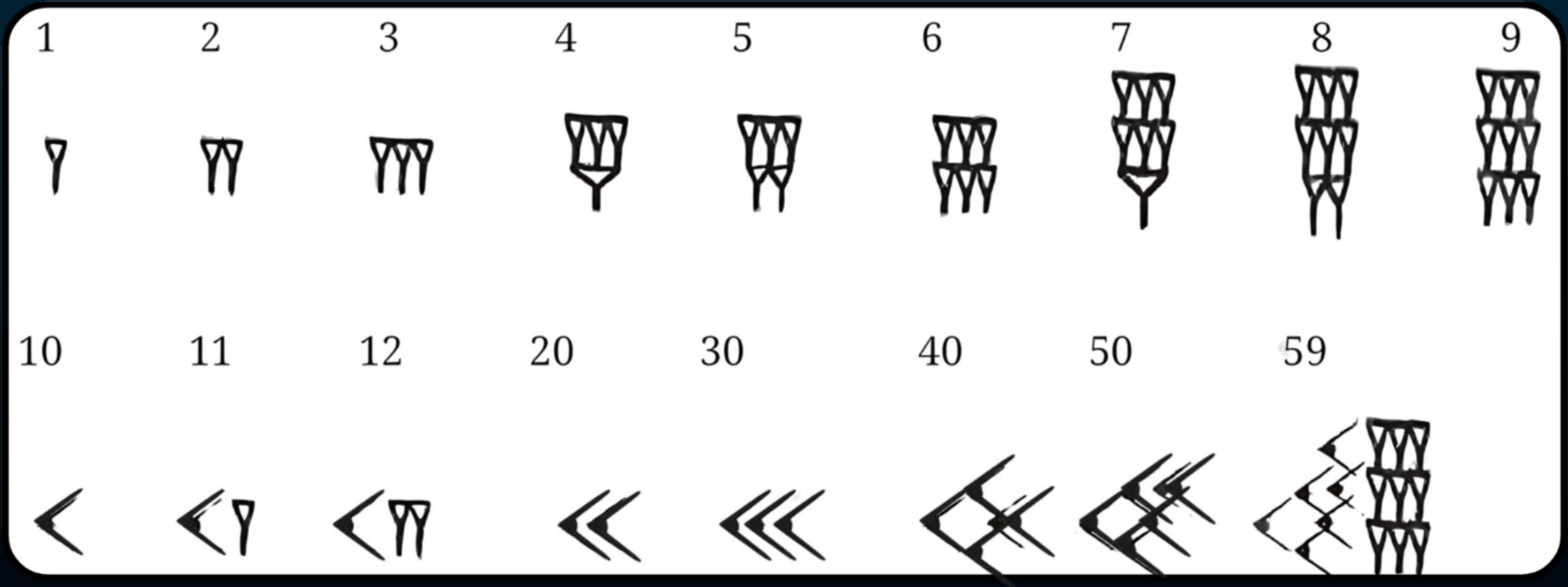
The influence of the Mesopotamian sexagesimal system, also known as the Babylonian number system, can be seen even now in our units of time measurements — 1 hour = 60 minutes and 1 minute = 60 seconds

This system used the symbol 𒁹 for 1 and 𒌋 for 10.

$60^1 = \boxed{1}$

$60^2 = \boxed{2}$

$60^3 = \boxed{3}$



MESOPOTAMIAN BASE-60 SYSTEM

The First Place Value Idea

- Mesopotamians used a base-60 (sexagesimal) system. Symbols for 1s, 10s, 60s, 3600s, etc., were arranged positionally.
- Used blanks (and later placeholder symbols) for empty places.

Let us represent the number 640 in this system

Mesopotamian

640

$60^1 = 60$

$60^2 = 3600$

60) 640 (10
$- 600$
40

$640 = 10 \times 60 + 1 \times 40$

= 

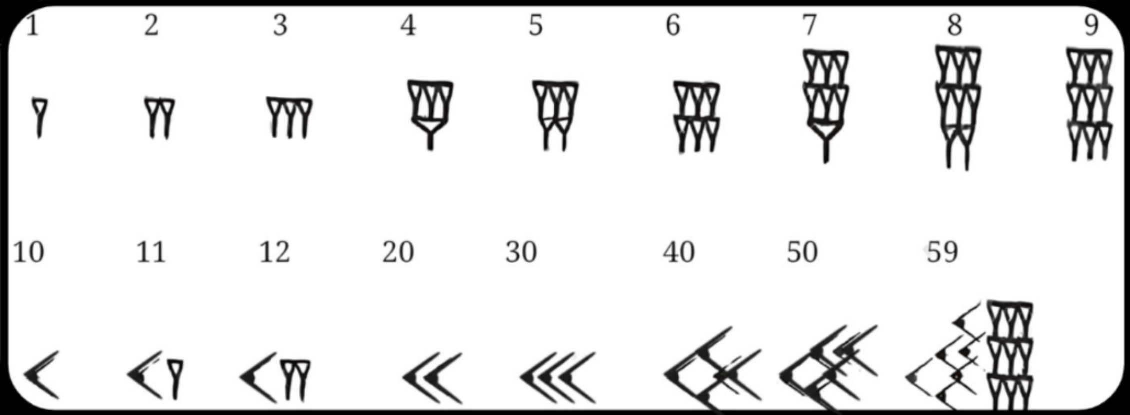
Let us try another number — 7530.

Represent the following numbers in the Mesopotamian system —
(i) 63 (ii) 132 (iii) 200 (iv) 60 (v) 3605

$60 \times 1 + 3$

$132 = 120 + 12$

$2 \times 60 + 12$



$200 = 3 \times 60 + 20$

=

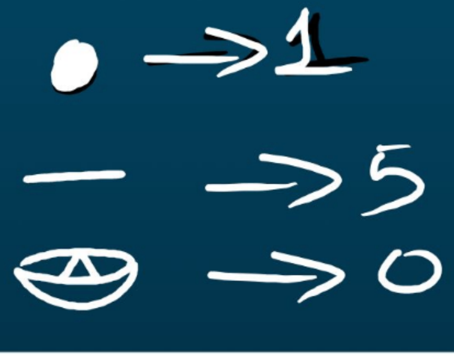
$60 =$

3605

$60^2 = 3600$

$1 \times 3600 + 5$

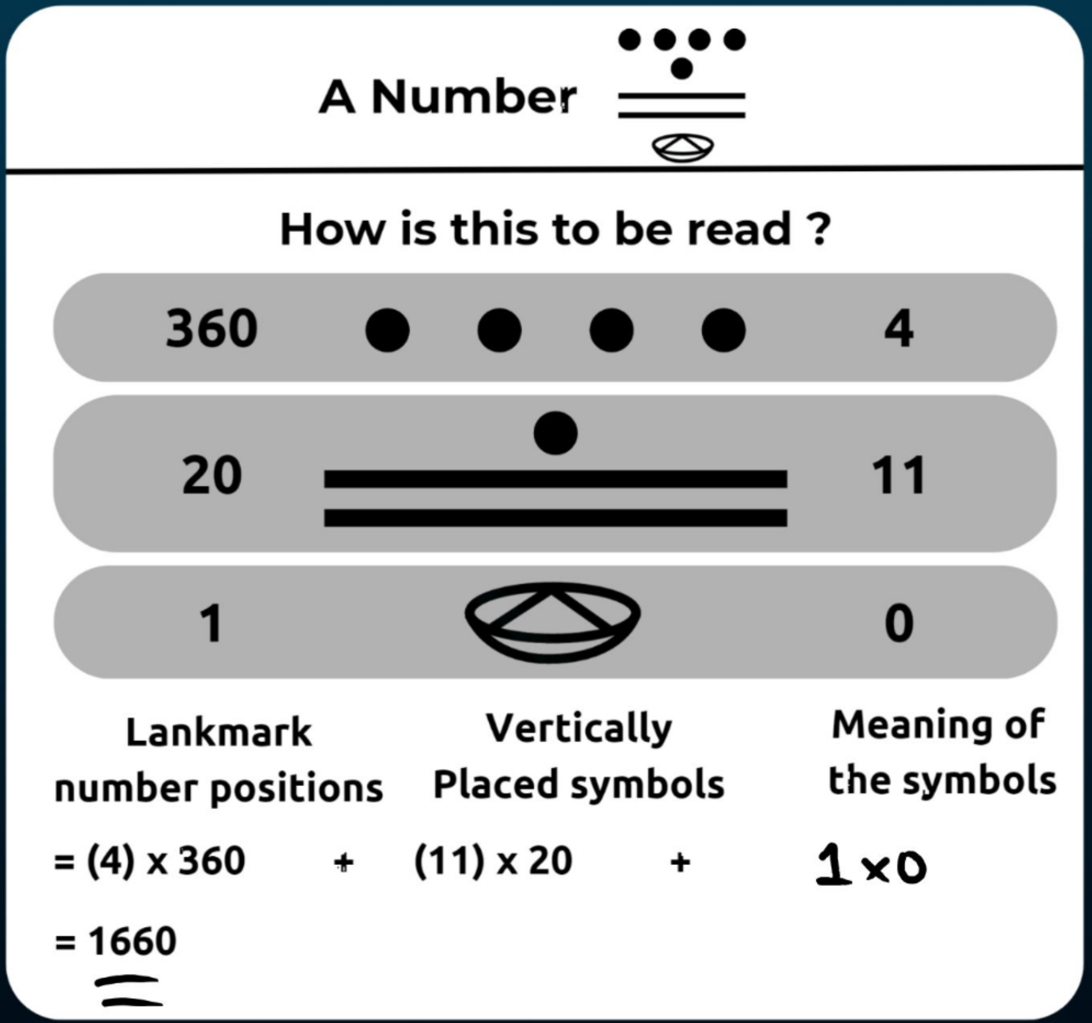
MAYAN NUMBER SYSTEM



Base-20 and Early Use of Zero

"Symbols in the Mayan Number system are placed vertically to represent a number."

- Symbols for different landmark numbers were written one below the other.
- The lowest row of symbols represented the number of 1s.
- The row above represented the number of 20s.
- The next row above represented the number of 360s.
- This pattern continued for higher landmark numbers.



= (4) x 360 + (11) x 20 + 1×0

= 1660

$20^0 = 1$

$20^1 = 20$

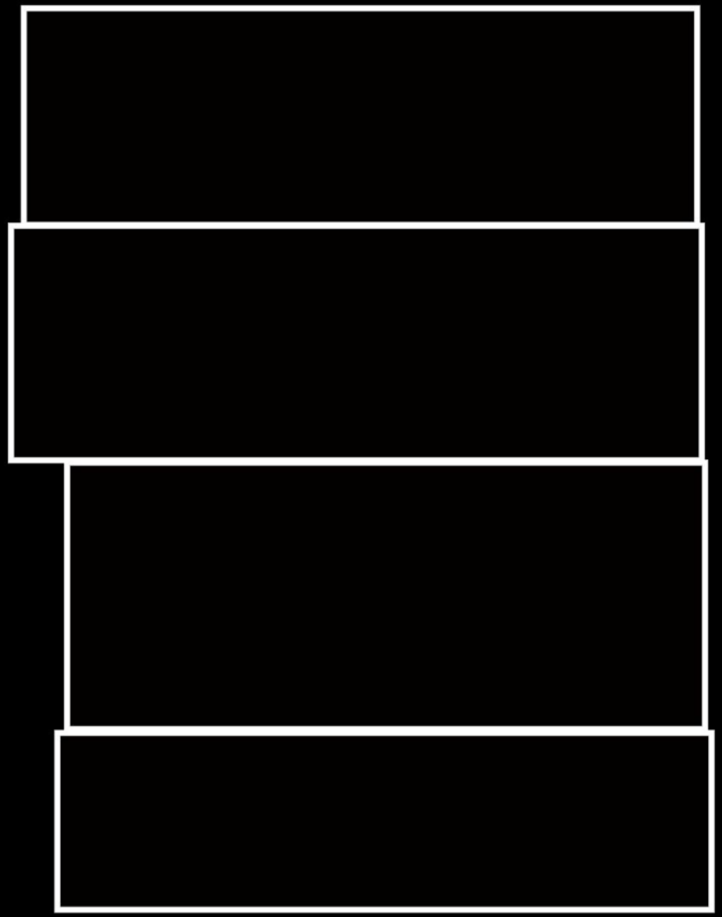
$20^1 \times 18 = 360$

$20^2 \times 18 = 7200$

$20^3 \times 18 =$

5 → —

1 → •



$20^2 \times 18 = 7200$

$20^1 \times 18 = 360$

$20^1 = 20$

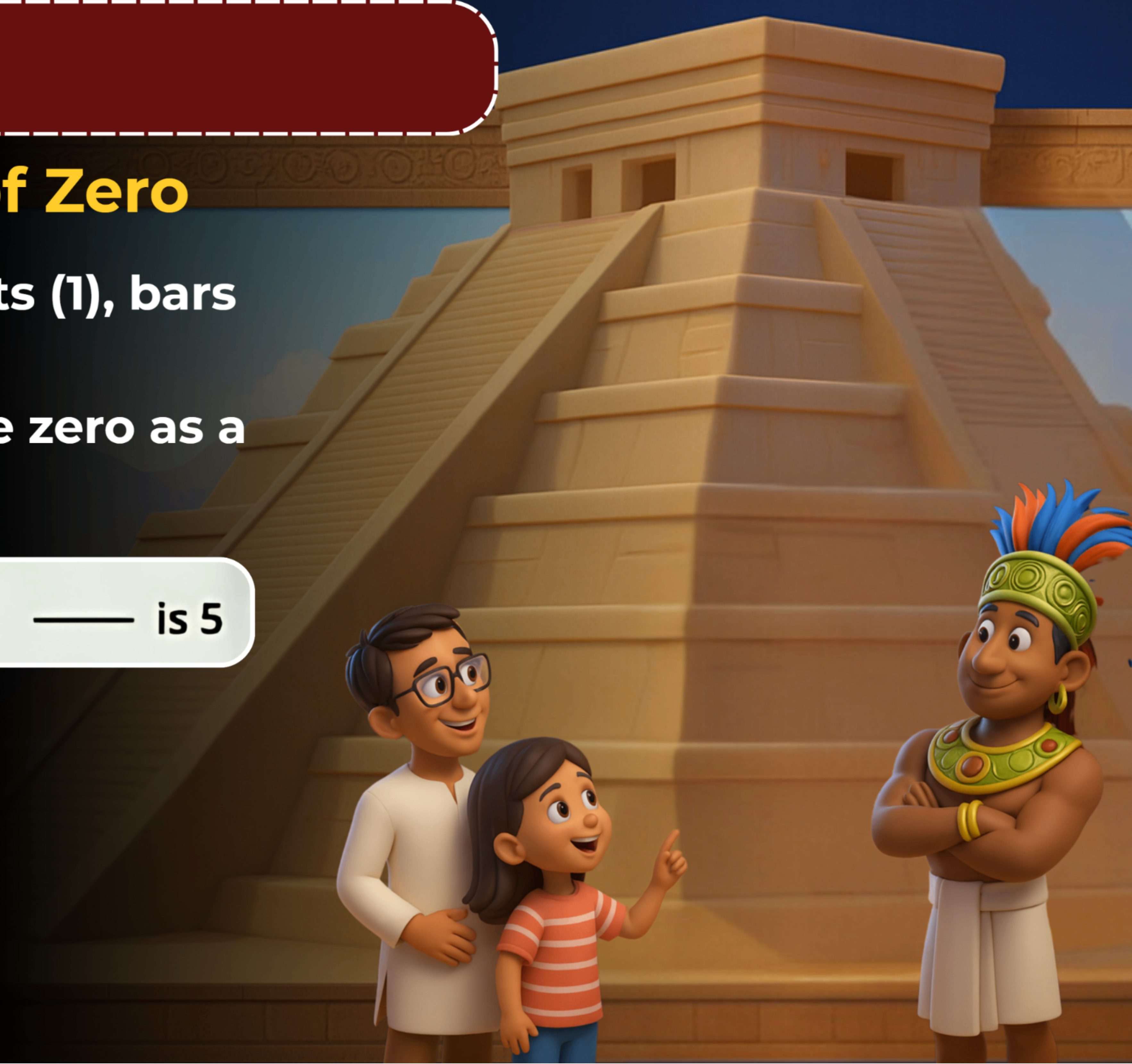
$20^0 = 1$

MAYAN NUMBER SYSTEM

Base-20 and Early Use of Zero

- The Mayan system used dots (1), bars (5), and a shell symbol (0).
- It was one of the first to use zero as a placeholder.

SYMBOLS  is 0 • is 1 —— is 5



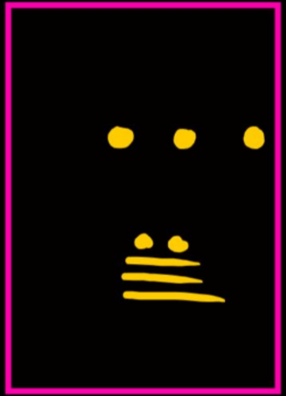
Represent the following numbers using the Mayan system:

(i) 77 (ii) 100 (iii) 361 (iv) 721

77

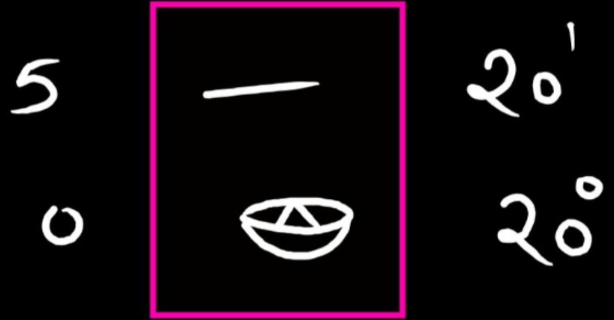
3×20

$+ 17 \times 1$



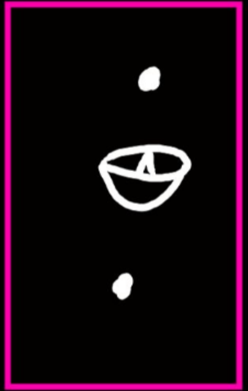
100

$5 \times 20^1 + 0 \times 20^0$



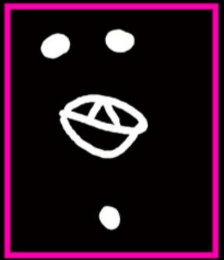
361

$1 \times 360 + 0 \times 20^1 + 1 \times 20^0$



721

$2 \times 360 + 0 \times 20^1 + 1 \times 20^0$



Chinese Rod Numerals

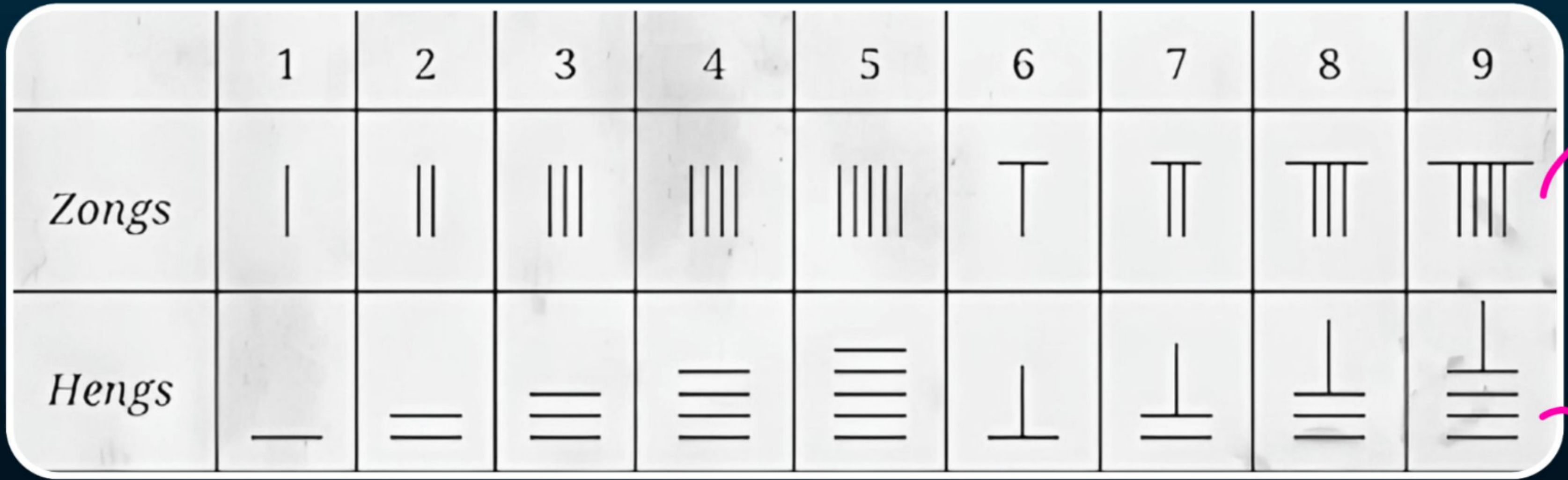
Place Value Without Zero

- The Chinese used two number systems — a written system for recording quantities, and a system making use of rods for performing computations.
- The numerals in the rod-based number system are called rod numerals.

Chinese Rod Numerals

Place Value Without Zero

- Ancient Chinese used rod numerals placed on counting boards.
- The value changed based on position, but there was no symbol for zero — instead, an empty space was used.



	1	2	3	4	5	6	7	8	9
Zongs	𝍠	𝍡	𝍢	𝍣	𝍤	𝍥	𝍦	𝍧	𝍨
Hengs	𝍩	𝍪	𝍫	𝍬	𝍭	𝍮	𝍯	𝍰	𝍱

Unit
Hund
Ten Tho

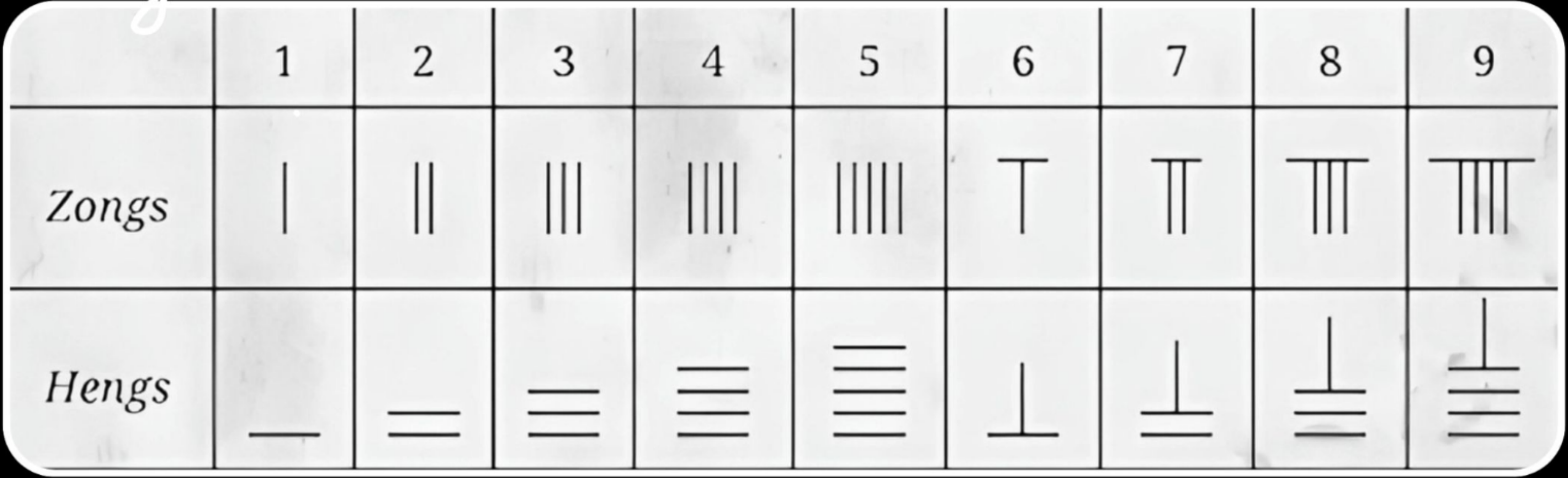
Ten
Thousand
Lakh

The zongs represent units, hundreds, tens of thousands, etc., and the hengs tens, thousands, hundreds of thousands, etc.

Read and Solve the following number representation

A Number = ⊤ ≡ ||||

Hengs Zongs Hengs

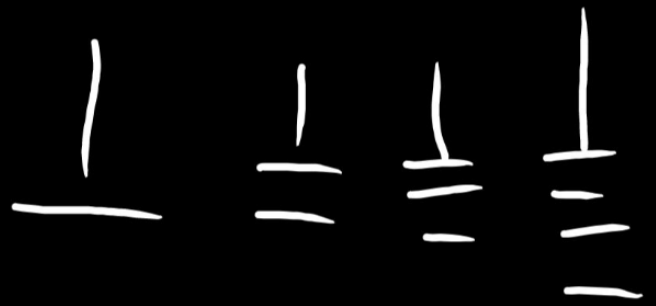


	1	2	3	4	5	6	7	8	9
Zongs	|	||	|||	||||	|||||	⊤	⊤|	⊤||	⊤|||
Hengs	—	=	≡	— — — —	— — — — —	⊥	⊥ —	⊥ =	⊥ ≡

4527

≣ ||||| = ⫪

2634



HINDU NUMERALS: THE GLOBAL STANDARD

Origins in Ancient India

- Developed around 2000 years ago, using 10 digits (0–9).
- First full usage of place value + zero as a digit.

WHAT MADE IT UNIQUE

Simplicity with Power

Each position = power of 10.

Digits occupy positions based on value (e.g., 375 = 3×100 + 7×10 + 5×1).

Zero used both as placeholder and number.

ZERO AS A NUMBER

Not Just an Empty Space

In India, zero was mathematically defined by Aryabhata (as a digit) and Brahmagupta (as a number with operations). Enabled algebra and negative numbers.

RING OF NUMBERS

Foundation for Modern Math

- By defining zero and negative numbers, Indian mathematicians created a closed set under +, –, × — known today as a ring in algebra.
- Combines: finite symbols, positional value, zero placeholder, efficient operations. No ambiguity in reading or writing numbers.

GLOBAL IMPACT OF HINDU NUMERALS

Everyday and Scientific Use

Used in science, tech, finance, time, computing. Without it, modern progress would stall.

$$5234 = 5\times1000 + 2\times100 + 3\times10 + 4\times1$$

EVOLUTION OF IDEAS IN NUMBER REPRESENTATION

1. Count in groups of a single number.

ukasar-ukasaur-urapon

2. Group using landmark numbers.

I V X L C M'...

3. Choosing powers of a number as landmark numbers — the idea of a base.

$1 \quad 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad \ldots$

4. Using positions to denote the landmark numbers — the idea of place value system.

1 7 2 9 $1 \times 1000 + 7 \times 100 + 2 \times 10 + 9 \times 1$

5. The idea of 0 as a positional digit and as a number.


THANK YOU