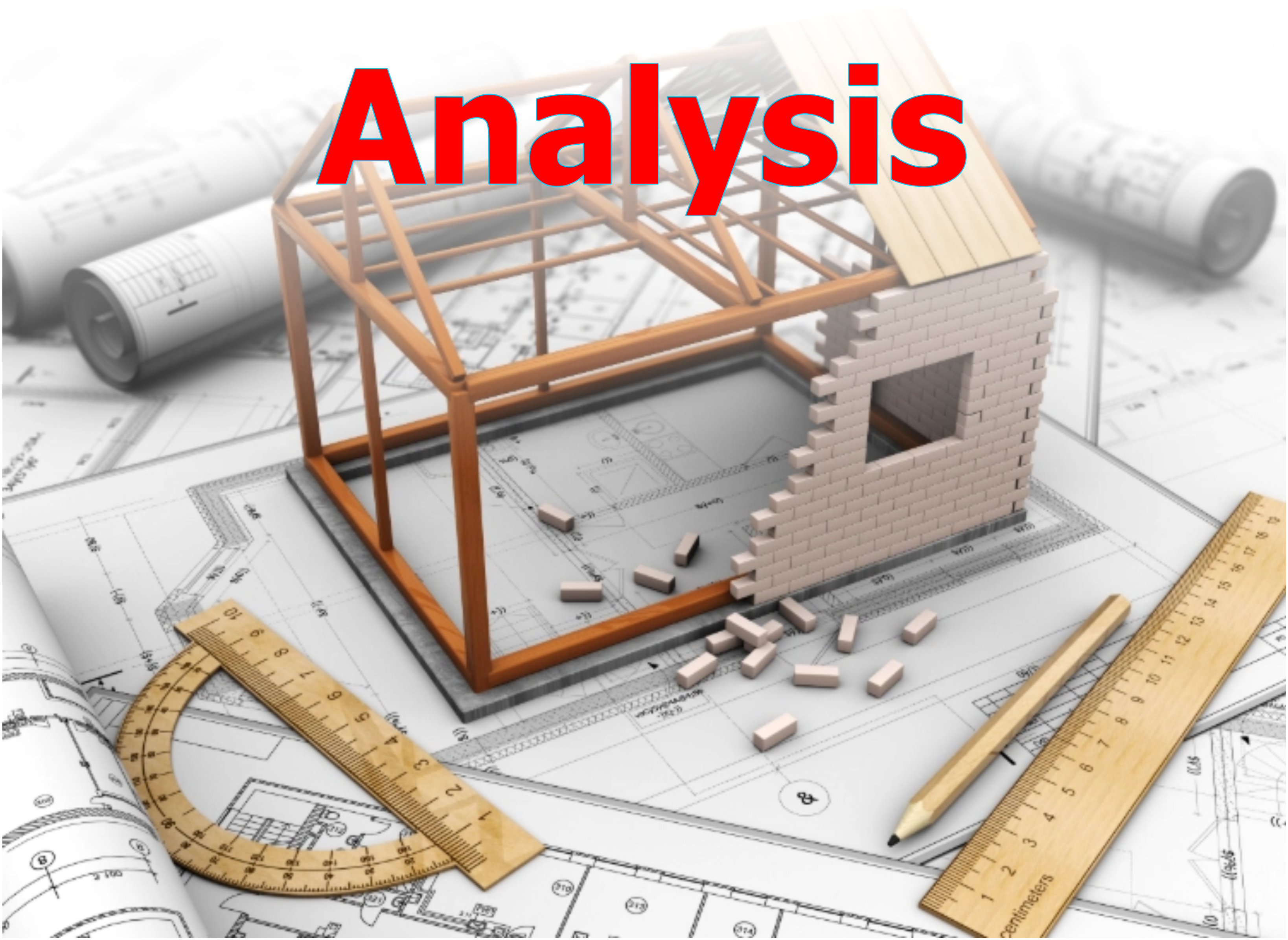
Structure Analysis



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STRUCTURE ANALYSIS

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1 INTRODUCTION TO STRUCTURE

1.1. Structure

An elastic body is termed as structure when it provides resistance against deformation due to the load acting over it.

1.1.1. Mechanism

In a mechanism no resistance is setup against deformation.

Assumption

- Elastic body
- Homogenous
- Isotropic
- Continuous Solid Structure
- Principal of Super position is valid.

Validity of Super position Principle

- Displacement is small.
- Elastic material and linear structural response. Load vs deformation is a straight line.
- Suppose are unyielding.
- Not valid for sender columns.

1.1.2. Classification of Structures

- Skeletal — e.g., Roof truss, building frames.
- Surface — e.g., Slabs and shells
- Solid — e.g., massive foundation

Skeletal Structure

- **Pin Jointed :** develop axial forces, external load @ joint, members are straight.
- **Rigid Jointed:** Angle between the member meeting @ joint remain unchanged, Resist external forces by developing bending moment, shear force, axial force, twisting moments.

Plane Frame	Space Frame
All member and forces are in one plane.	All members don't lie on one plane.
Axial	Axial
Shear	Biaxial Shear
Bending Moment	Biaxial Moment
	Twisting Moment

1.1.3. Poissons Ratio (μ)

Defined as the ratio of lateral strain to linear strain.

Concrete = 0.15
Steel = 0.33
Cork = 0
Isotropic material = 0.25
μ_{max} = 0.5 Ideal Elastic incompressible meter.

Bernoulli's Assumption

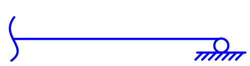
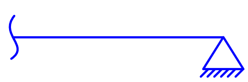
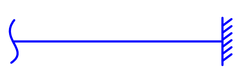
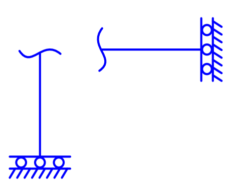
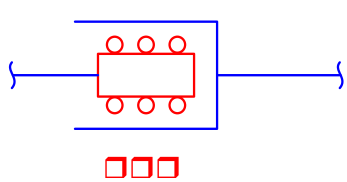
A plane section which are normal to the neutral axis before bending remain plane even after bending.
It means strain varies linearly over the cross-section.

Validity of Bernoulli's

- Valid for elastic, limit, ultimate theories.
- Valid for prismatic and non-prismatic.
- Valid for shallow beams.

1.2. Reaction

Reaction is the resistance against deformation. **Numbers (Planar Structure)**

- Free end  0
- Roller end  1
- Hinged Support  2
- Fixed Support  3
- Shear hinged – Horizontal & Vertical  2
- Damper  Horizontal Release

2 DETERMINACY AND INDETERMINACY

The structure that can be analysed by the equations of static equilibrium alone.

2.1. Equation of Static Equilibrium

2D Planar structure

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M = 0$$

3D Space Structure

$$\Sigma F_x = 0 \qquad \Sigma m_x = 0$$
$$\Sigma F_y = 0 \qquad \Sigma m_y = 0$$
$$\Sigma F_z = 0 \qquad \Sigma m_z = 0$$

Statically indeterminate structure are the one which cannot be found by equations of static equilibrium.

2.1.1. Degree of Static Indeterminacy

Can be termed as equations in addition to static equilibrium equation required to completely analyse the structure.

D_S = Number of unknown forces in member or at support – equation of static equilibrium available.

$$D_S = D_{Si} + D_{Se}$$

D_S → Total, D_{Si} → Internal, D_{Se} → External

2.1.2. Support Reaction

Plane

– Fix $\quad R_x,\ R_y,\ M_z$ Reactions

$\quad \delta_x,\ \delta_y,\ \theta_z$ Restrains

– Pin $\quad R_x,\ R_y,$ Reactions

$\quad \delta_x,\ \delta_y,$ Restrains

– Roller $\quad R_y$ Reactions

$\quad \delta_y$ Restrains

Space Structure

– Fix R_x, R_y, R_z, M_x, M_y, M_z Reactions

– Pin R_x, R_y, R_z Reactions

– Roller R_y Reactions

2.1.3. External Indeterminacy

$$D_{se} = R - 3 \quad \text{Plane}$$

$$= R - 6 \quad \text{Space}$$

2.1.4. Internal Indeterminacy

$$D_{si} = m - (2j - 3) \quad \text{Truss plane}$$

$$= m - (3j - 6) \quad \text{Truss space}$$

$$= 3C \quad \text{Rigid plane frame}$$

$$= 6C \quad \text{Rigid space frame}$$

j = No. of Joints

C = No. of cuts required for obtaining open configuration.

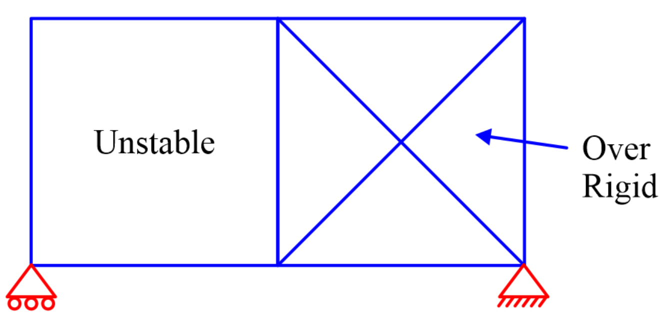
m = No. of members

Simplified Formula of D_S

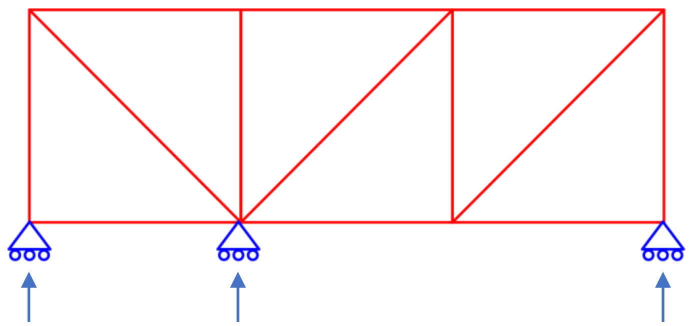
- Plane truss $D_S = (m + R) - 2j$
- Space truss $D_S = (m + R) - 3j$
- Rigid plane frame $D_S = (3m + R) - 3j$
- Rigid space frame $D_S = (6m + R) - 6j$

2.2. Truss (Static Indeterminacy)

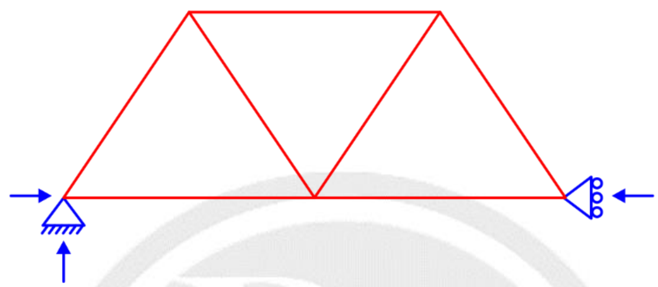
- Every joint is a Hinge Joint.
- Each joint has 2 nos. of equation (planar)
- Internal indeterminacy should be checked for individual loop. e.g.,



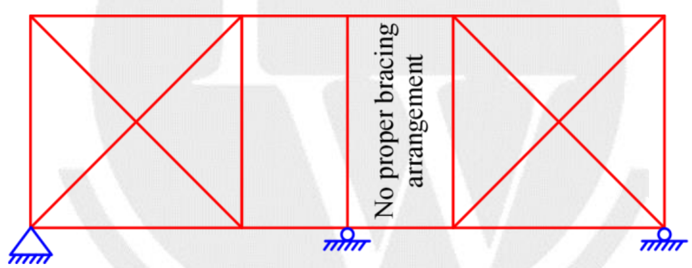
- All support reactions should not be parallel as may lead to unstability e.g.,



- All support reactions should not be concurrent as may lead to unstability. e.g.,



- If a truss is unstable we never discuss SD, or SID.
- If in any truss are appreciable deformation which can be due to no proper bracing it makes the structure unstable. e.g.,



Simple Truss

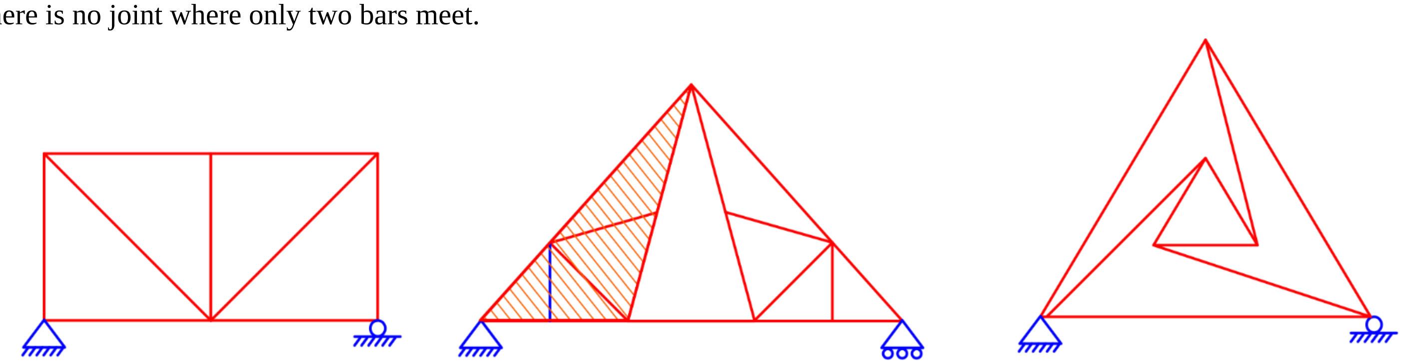
In a triangle when two bar and one joint are progressively added to form a truss.

Compound Truss

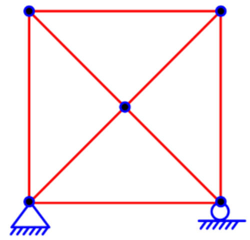
Two simple truss connected by a set of joints and bars.

Complex Truss

There is no joint where only two bars meet.



- Truss having members which crossover each other or member that serves as side for more than 2 triangle are likely to be indeterminate.



Difference between SD & SI

	Statically Determinate	Statically Indeterminate
(1)	Equilibrium equation sufficient to analyse	Insufficient
(2)	BM independent of material	Dependent
(3)	BM is independent of sections Area	Dependent
(4)	Stresses are not caused due to temperature change & lack of fit.	Stresses caused

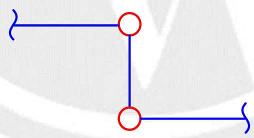
2.3. Frames/Beams

2.3.1. Internal Pin/Hinge

Pin cannot transmit moment from one part to other. Thus, provides extra condition $\Sigma M = 0$.

Internal Link

Bar with pin @ each end



incapable of transmitting moment as well as horizontal force.

Two additional conditions are, $\Sigma M = 0$, $\Sigma H = 0$

Loading Type

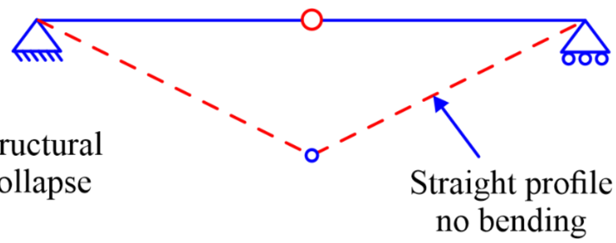
- General Loading has both vertical and horizontal component.
- Vertical loading condition is important for beams but not frames.

Open tree like Structure Concept used for Finding Indeterminacy

- trees have only one root
- trees cannot have closed looped branch.

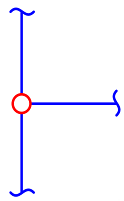
Unstable or Deficient Structure

If there are not sufficient number of Restraint the structure under go Rigid Body movement upon application of a small displacement. e.g.,

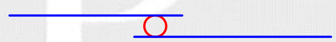


2.4. Restraining Members/Joint

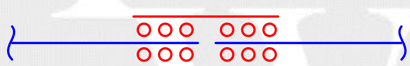
The concept relates to making structure completely rigid and then analyzing it for indeterminacy.

- Plane frame = $(m' - 1)$  m' = no. of members joining the Hinged Joint.
- Space frame = $3(m' - 1)$

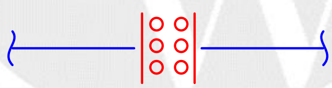
  Concept = Rotation of Members
- R_H & M are the restrains required to make rigid.



- R_H are the restrains required to make rigid.



- R_v is restraint required to make rigid.



2.4.1. Rigid Frames

In a plane frame, every member carries 3 forces. (BM, SF, Axial)

$$\text{Total no. of unknown} = \underset{\text{member}}{3m} + \underset{\text{Support reaction}}{R}$$

At each joint equilibrium equation = $3j$

$$\Sigma f_x = 0$$

$$\Sigma f_y = 0$$

$$\Sigma M = 0$$

$$D_S = 3m + R - 3j$$

$$D_S = 3m + R - 3j - \Sigma(m' - 1)$$

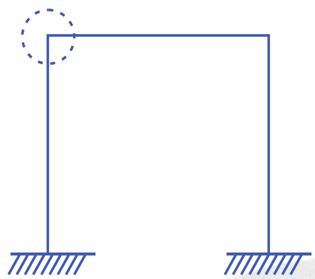
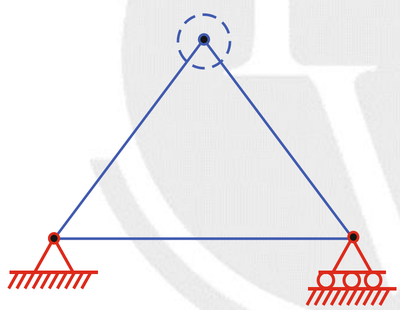
where m' is the number of members @ hinge it.

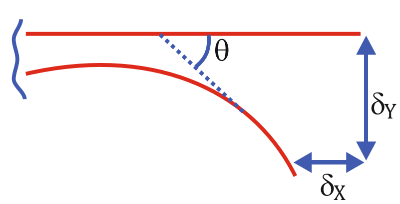
$$D_S = 6m + R - 6j - \Sigma 3(m' - 1)$$

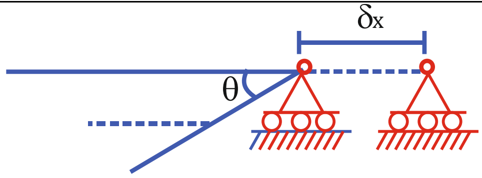
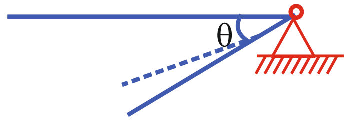
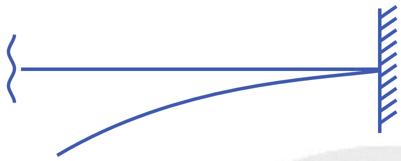
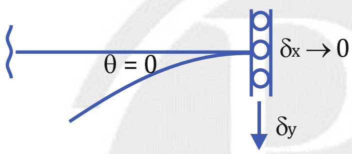
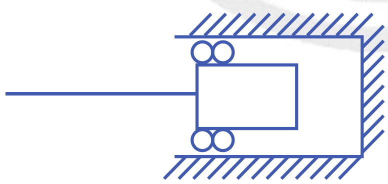
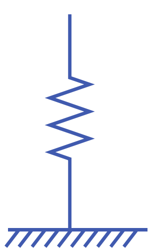
Denote by D_K.

- Also called as degrees of freedom (DOF)
- Kinematic indeterminacy:

The no of unknown joint displacements is called degrees of freedom

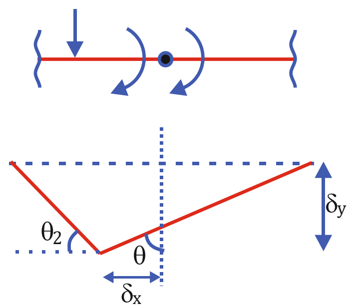
	Types of joints	Degrees of freedom
1	Rigid joint of a plane frame  **Note:** At a rigid joint, the included angle remains the same before and after displacement	$3\,(\delta_x, \delta_y, \theta)$.
2	Rigid joint of a space frame	6 (3 rotations θ_{xy}, θ_{yz}, θ_{xz} and 3 translations $\delta_x, \delta_y \,\&\, \delta_z$)
3	Pin joint of a plane frame  **NOTE:** As moments are not present, the design rotations are not considered in trusses.	
4	Pin joint of a space frame	$3\,(\delta_x, \delta_y, \delta_z)$

	Types of support	Degrees of Freedom
1	Free end. 	$3\,(\delta_x, \delta_y, \theta)$
2	Roller support.	$2\,(\theta, \delta_x)$.

	 	
3	Hinged/Pinned support  	1 (θ)
4	Joined support 	0
5	Vertical shear hinge  	1 (δ_y)
6	Horizontal shear hinge 	1 (δ_x)
7	Damper support 	1 (δ_x).
8	Spring support  **Note:** Reactions will resist displacements. Vertical, $dv = 0$ Horizontal reaction, $\delta h = 0$ Moment reaction, $\theta = 0$	2 (θ, δ_x).

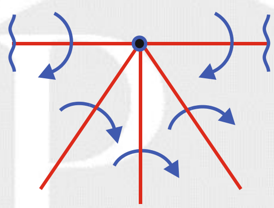
2.3. Effect of force release on D.O.F.

1. Internal moment hinge.



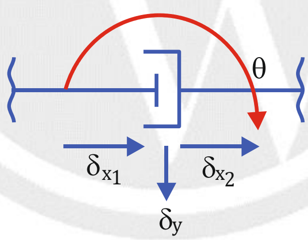
Note: Each member connected to a hinge can have its own notation, in addition to δx and δy.

Example:

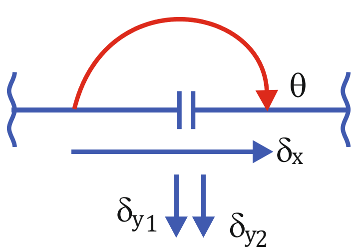


5 notations and 2 translations.

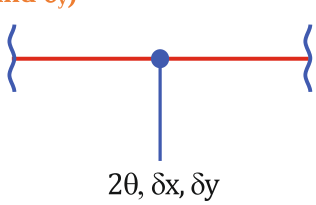
2. Horizontal shear releases.



3. Vertical, shear release



4. (2 rotations - θ_1 and θ_2 2 translations - δ_x and δ_y)



4. D.O.F

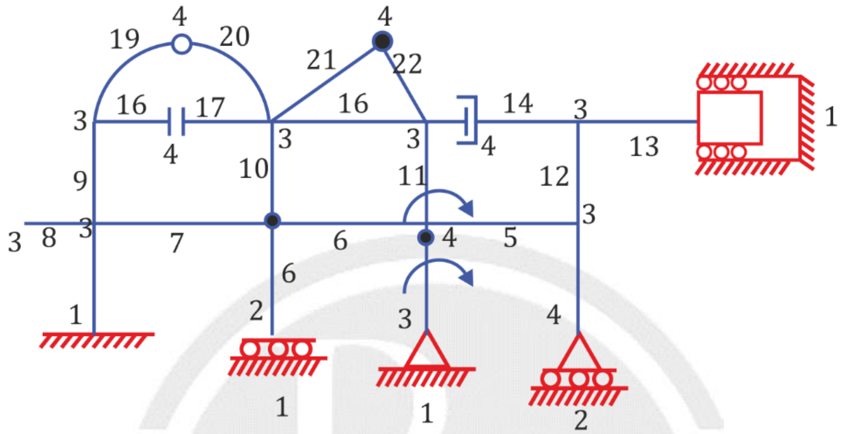
(2 horizontal trans- δ_{x1} and δ_{y1} vertical trans - δ_y and θ.)

4. D.O.F

(δ_{y1}, δ_{y2}, δx & θ).

D_k of rigid jointed plane frame:

D_k of rigid jointed plane Frame:



D_k = 52 (Considering axial deformations)

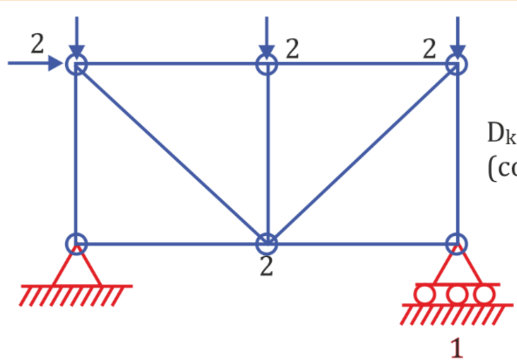
For a rigid joint with infinite member, there is only single rotation for a hinged joint, there will be infinite rotations:

Note: Practically the axial deformations of members or rigid jointed structures are negligire.
Assume axial deformation of all members are neglected then Dk = 52 – total no. of members.
= 52 – 22 = 30 (neglection axial deformations)

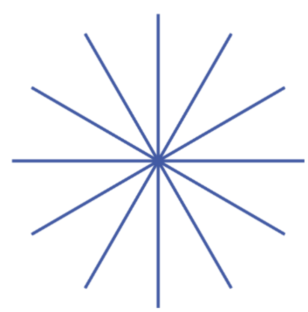
Axial deformation neglected or
members are rigid or
members are stiff or
members incotenoible

} neglect axial deformation

D_K of Pin jointed Plane Frames:



D_k = 9
(count the no: of members)



Note: Rotations are not considered in trusses the only possible D.O.F in trusses are axial deformations. Hence the equation of neglecting AD do not arise in pin-jointed trusses

Formula for D_K:

$D_K = NJ - C$ where

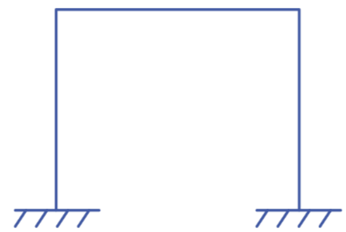
N = 3 ; rigid jointed plane frame

N = 6: rigid jointed space frame

N = D.O.F at a joint

J = no of joints

C = compatibility equations



N = 2; pin jointed plane frame

N = 3 ; pin jointed space frame

J = 4 (supports are also considered as jointed)

O = Reactions, if actual deformations consider

= *m* + r; if axial deformations are neglect

Where *m* → no of members.

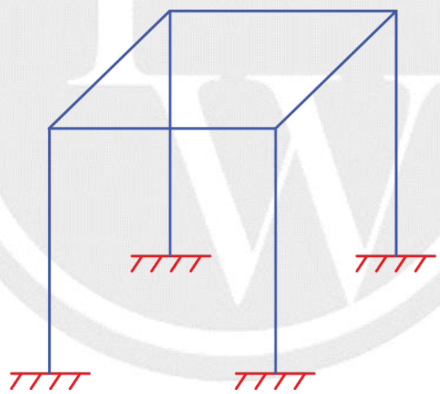
$D_K = 3 \times 4 - 6 = 6$

$D_K = 6 \times 4$ (joints)

= 24 (considering AD)

$D_K = 24 - 8$ (member)

= 16 (neglecting AD).



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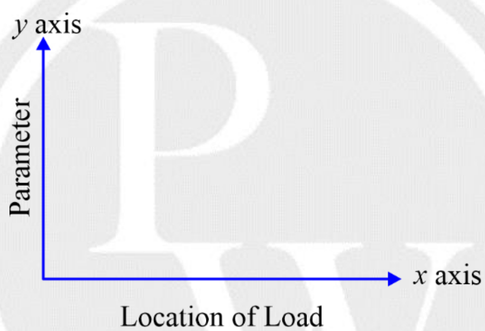
3 INFLUENCE LINE DIAGRAM

3.1. Introduction

ILD –Definition & Diagrams

It is the graphical representation of variation of following parameter - reaction, shear force, bending moment etc. as a unit load moves from one end of the structure to other end of the structure.



ILD – Value of Parameter

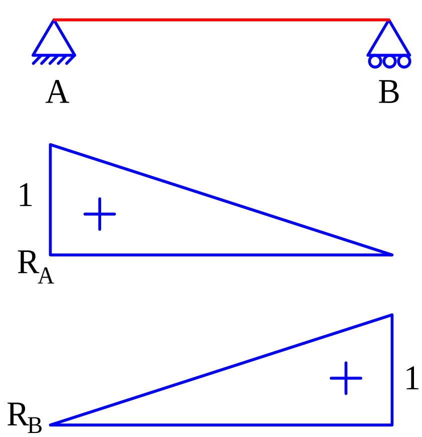
How to calculate value of Parameter (Reaction, Shear, Moment)due to external load using ILD

Value of Parameter = ∑(Point Load x Ordinate of ILD at the location of load)

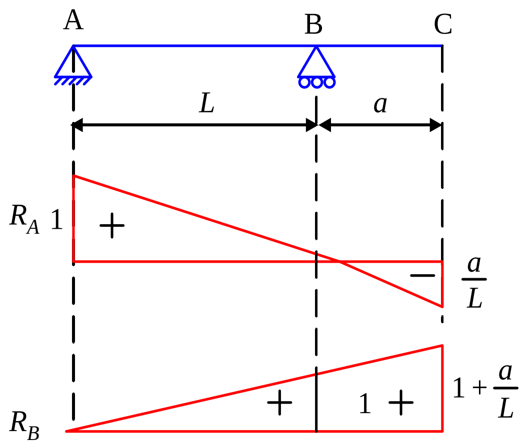
+

∑ (Intensity of UDL x Area of ILD under the loading zone)

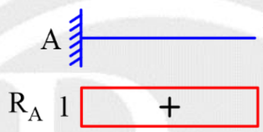
ILD for Reaction of Simply Supported Beam



ILD for Reaction of Overhanging Beam

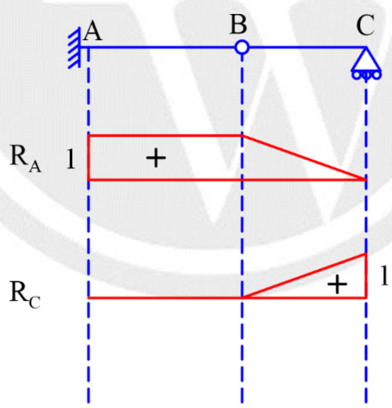


ILD for Support Reaction of a Cantilever

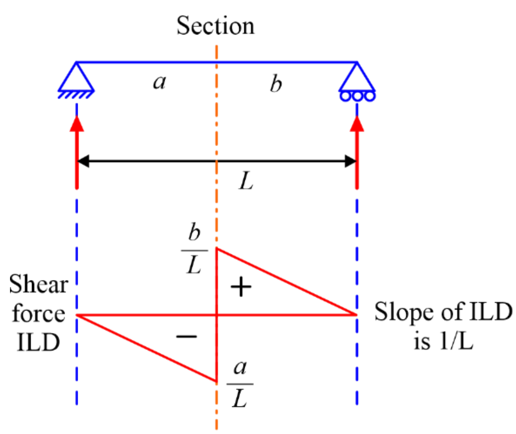


ILD –Beams with Internal Hinge

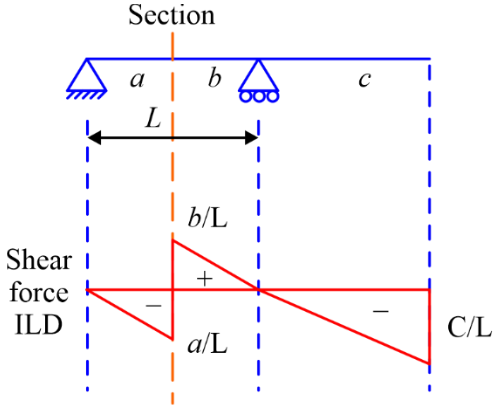
In case Multiplan beam connected by means of a internal hinge, the ILD changes its direction or slope at the hinge location or point.

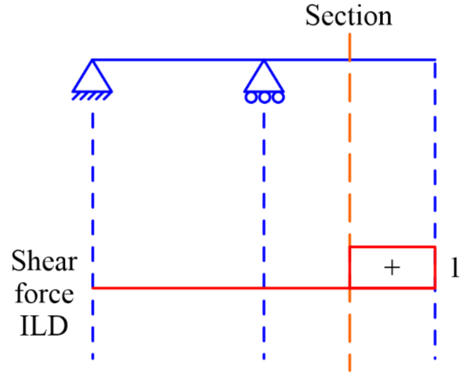


ILD – Beam Shear Force

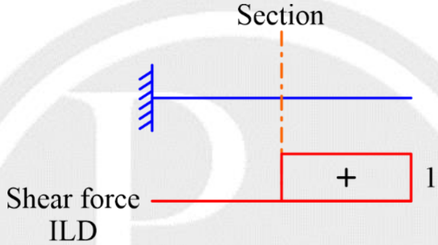


3.2. Overhanging Beam





Cantilever



3.2.1. ILD – Truss Bridge

Through

- Vehicle travel inside the bridge
- Loading point are the lower joints

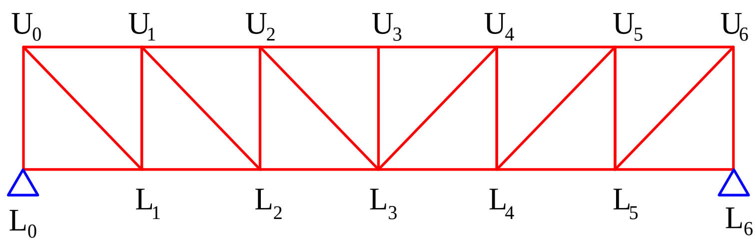
Deck

- Vehicle travel over the Bridge
- Loading points are the Upper joints

Trough

Truss Bridge

Member – Upper Chord, Lower Chord, Diagonal , Vertical Member



Upper Chord = Compression

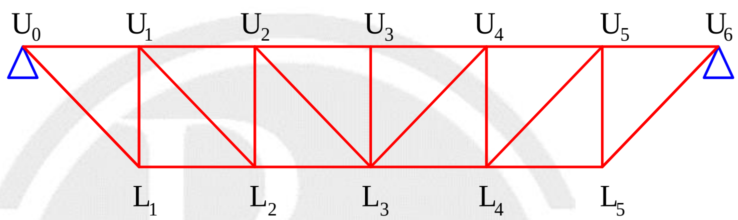
Lower Chord = Tension

Diagonal = Compression & Tension

Vertical = Compression & Tension

Deck Truss Bridge

Member – Upper Chord, Lower Chord, Diagonal , Vertical Member



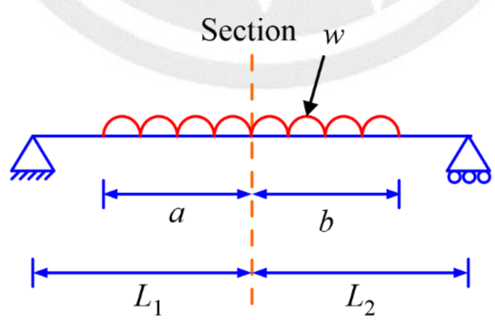
3.2.2. ILD – Rolling Loads

Maximum Bending Moment at a given section:

- For UDL

$$\frac{wa}{L_1} = \frac{w_b}{L_2}$$

- Average load on left of section = Average load on right of section

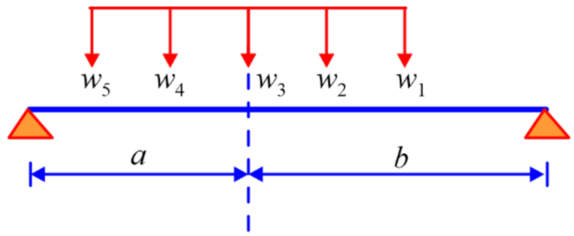


Maximum Bending Moment at a given section:

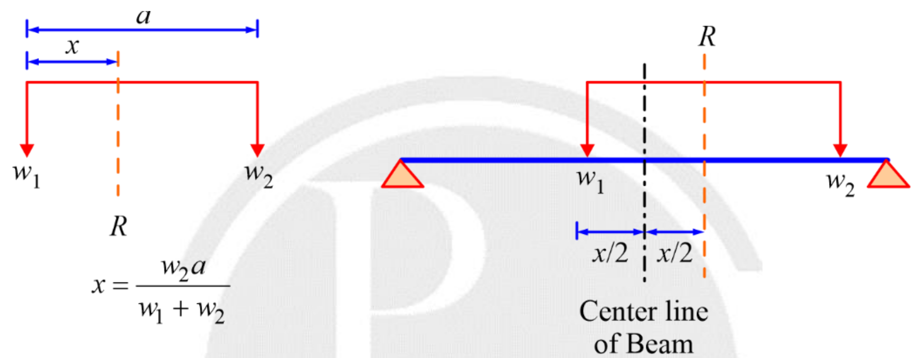
For wheel loads

'Maximum' BM at a given section of a beam occurs on a section when the average load on the left side of section minus the average load on right side of section **changes sign** when the point load passes over the section

Maximum Bending Moment at a section due to Rolling Point Loads



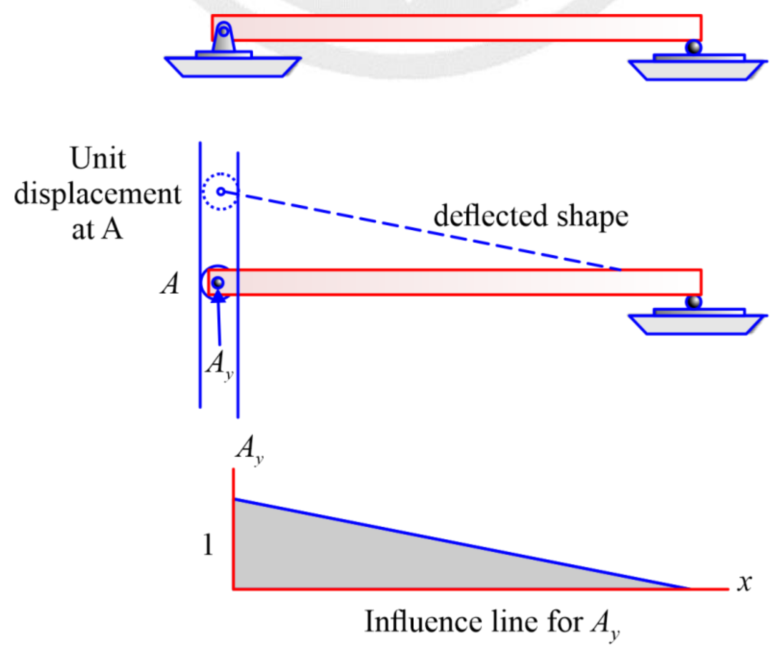
Maximum Bending Moment under a chosen wheel load: The BM under a chosen load of a wheel load system will be a maximum, when the load system is so placed on the girder that the chosen load and the resultant of all the wheel, loads are equidistant from the middle point of the girder.



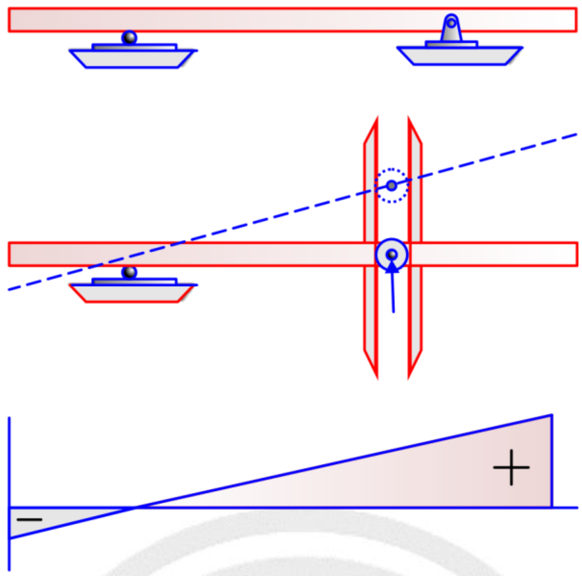
ILD – Müller-Breslau Principle

The **Müller-Breslau principle** is a method to determine influence lines. The **principle** states that the influence lines of an action (force or moment) assumes the scaled form of the deflection displacement.

Simple Supported beam

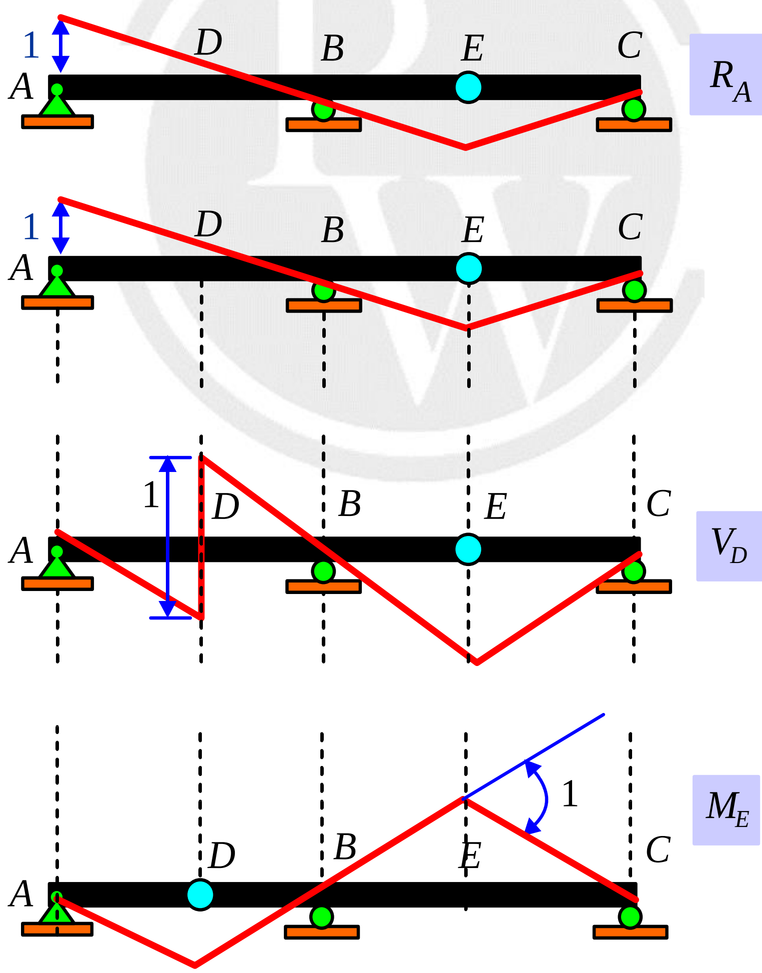


Over Hanging beam



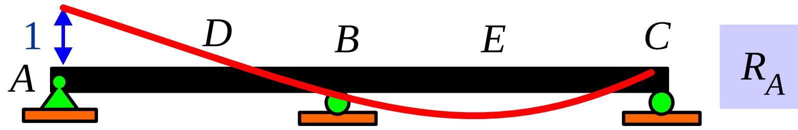
Determinate Beam

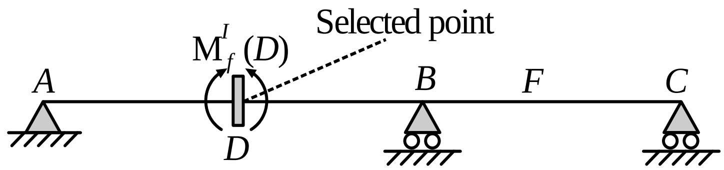
Straight Profiles

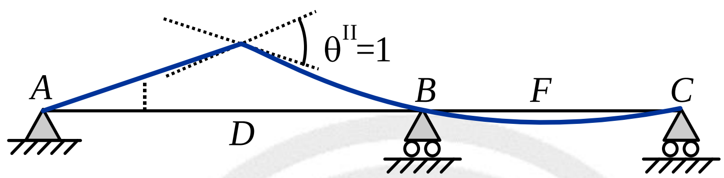


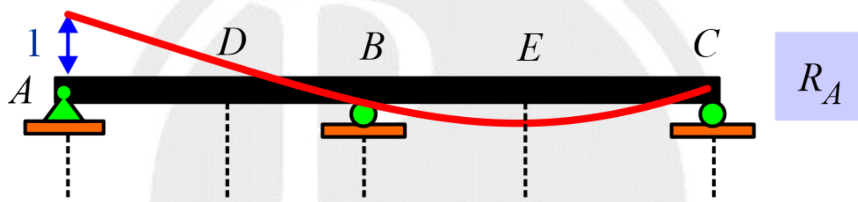
Indeterminate Beam

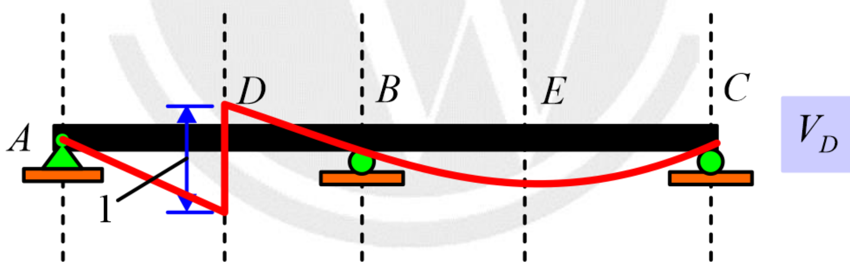
Curved Profiles

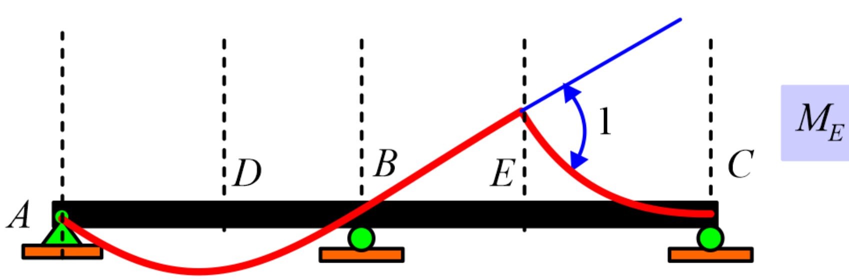


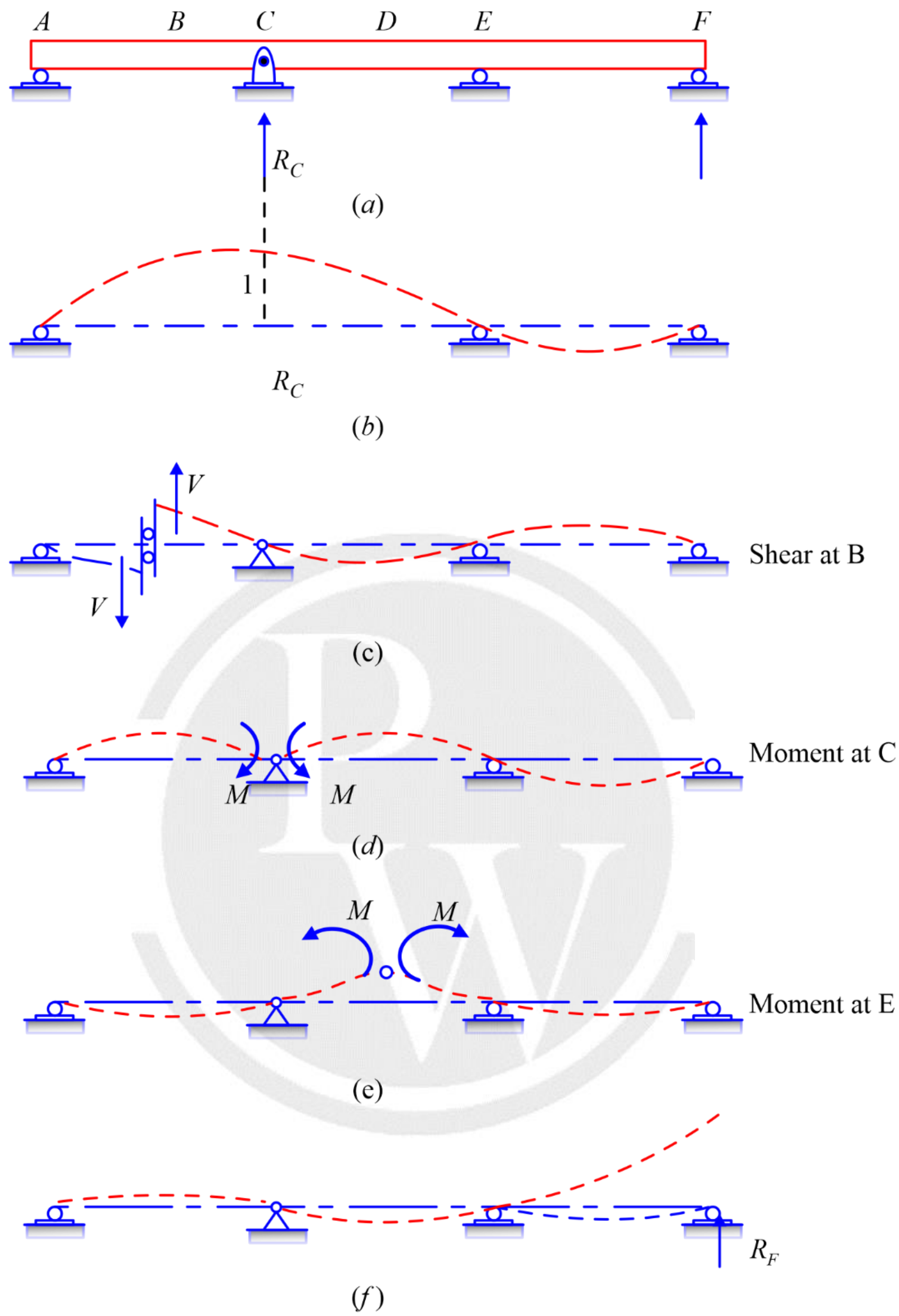




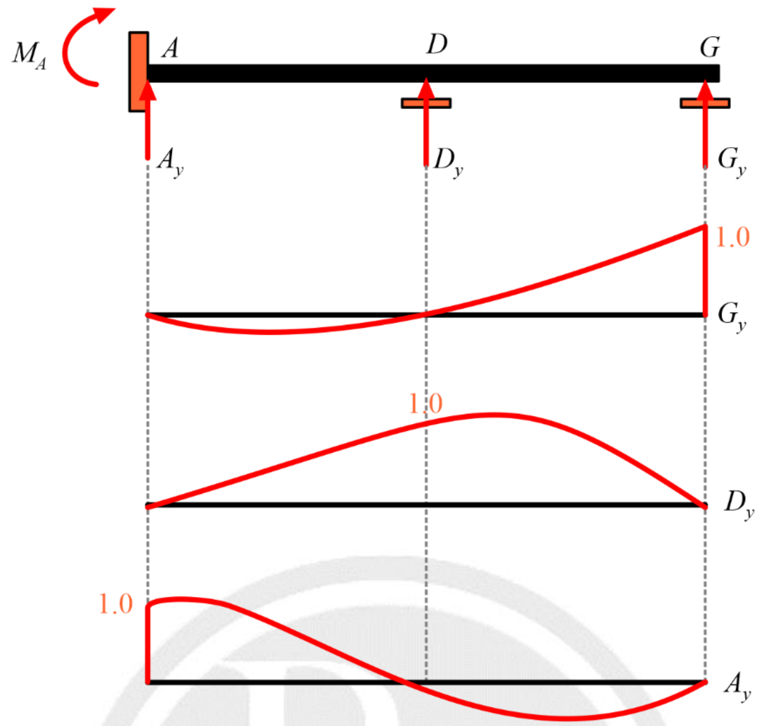


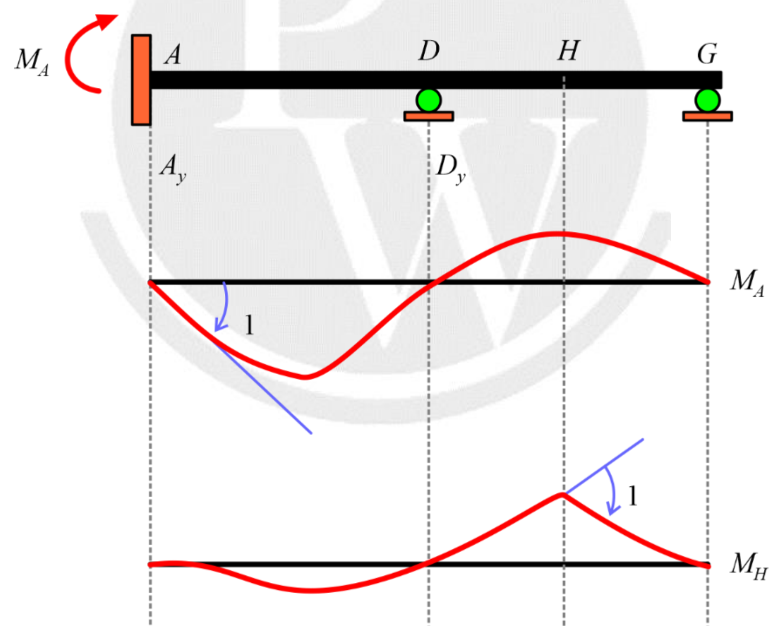






A
B
C
D
E
F
R_C
(a)
1
R_C
(b)
V
V
Shear at B
(c)
M
M
Moment at C
(d)
M
M
Moment at E
(e)
R_F
(f)


M_A
A
D
G
A_y
D_y
G_y
1.0
G_y
1.0
D_y
1.0
A_y


M_A
A
D
H
G
A_y
D_y
M_A
1
1
M_H

❑❑❑

4 MOMENT DISTRIBUTION METHOD

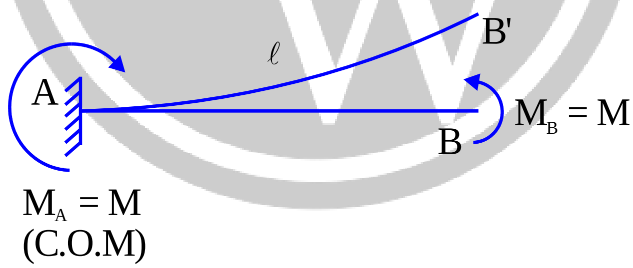
4.1. Introduction

1. Assumptions in moment distribution Method

- The effect of Axial forces and axial deformations are Neglected in moment distribution method.
- Clockwise and moments are taken as +ve and Anticlockwise end moment are taken a –ve {This sign convention is used while distributing the ends moments only}. In this case we will not see weather the end moment are sagging the beam or Hogging the Beam.
- Sagging BM is taken as +ve and Hogging BM is taken as –ve. The sign convention is used while drawing BM dig. only.

2. For cantilever Beam, C.O.M = – M {–ve because applied moment & C.O.M are opposite to each other}

(Applied Moment)



3. Carry Over Factor

It is the Ratio of carry over Moment and Applied moment

Case (1) : When for end is fixed $C.O.F = \frac{C.O.M}{\text{Applied moment}} = \frac{M/a}{M} = \boxed{\frac{1}{2}}$ **

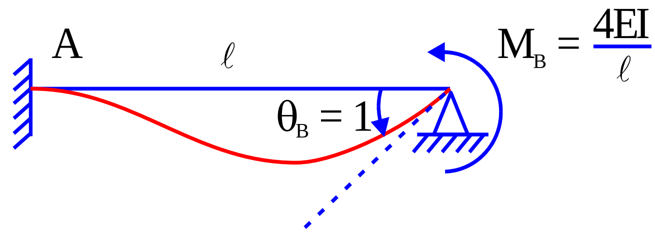
Case (2) : When for end is hinged = $C.O.F. = \frac{0}{M} = \boxed{0}$ **

Case (3) : For cantilever beam = $C.O.F. = \frac{-M}{M} = \boxed{-1}$ **

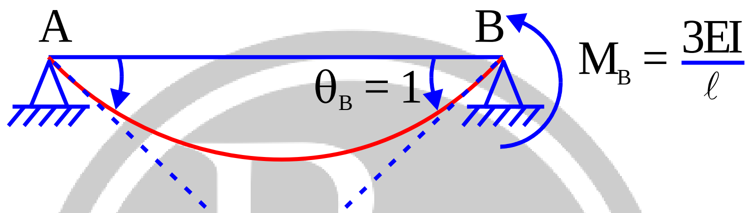
4. Stiffness Factor

It is the moment required to produce unit rotation

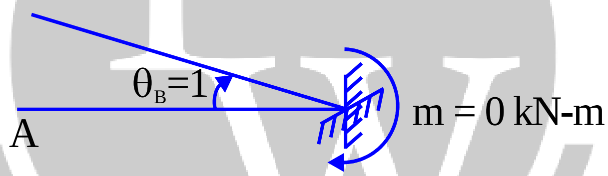
Case (1) : When for end is fixed stiffness factor = s = $\frac{4EI}{\ell}$



Case (2) : When for end is hinged, s = $\frac{3EI}{\ell}$



Case (3) : When for end is free, stiffness factor = s = 0



5. Relative Stiffness (K)

It is the ratio of moment of inertia and length of the member.

Case (1) : When for end is fixed, relative stiffness $K = \frac{I}{\ell}$

Case (2) : When for end is hinged, relative stiffness $K = \frac{3}{4}\frac{I}{\ell}$

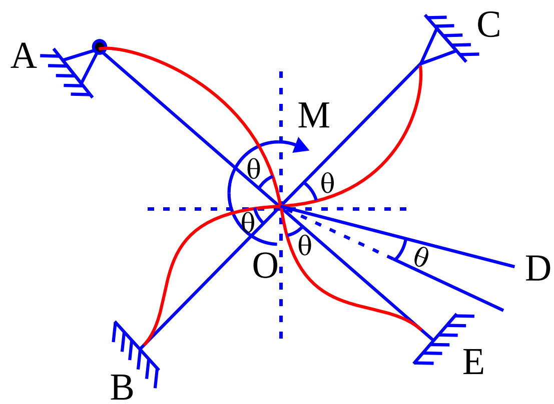
$$\frac{4EI}{\ell} \rightarrow \frac{I}{\ell}$$

$$\frac{3EI}{\ell} \rightarrow \frac{3}{4}\frac{I}{\ell}$$

Case (3) It for end is free $\boxed{K = 0}$ {Because s = 0}

6. Distribution Factor

It is the ratio in which the applied moment M is shared by the members meeting at any rigid joint.



'0' is a Rigid joint {After application of loads, angle between member remain same}

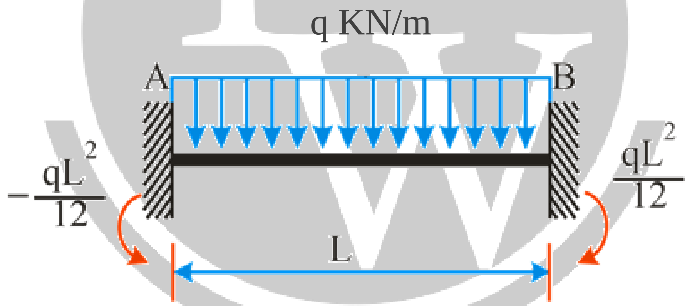
$$D.F = \frac{K}{\Sigma K}$$

7. Note:

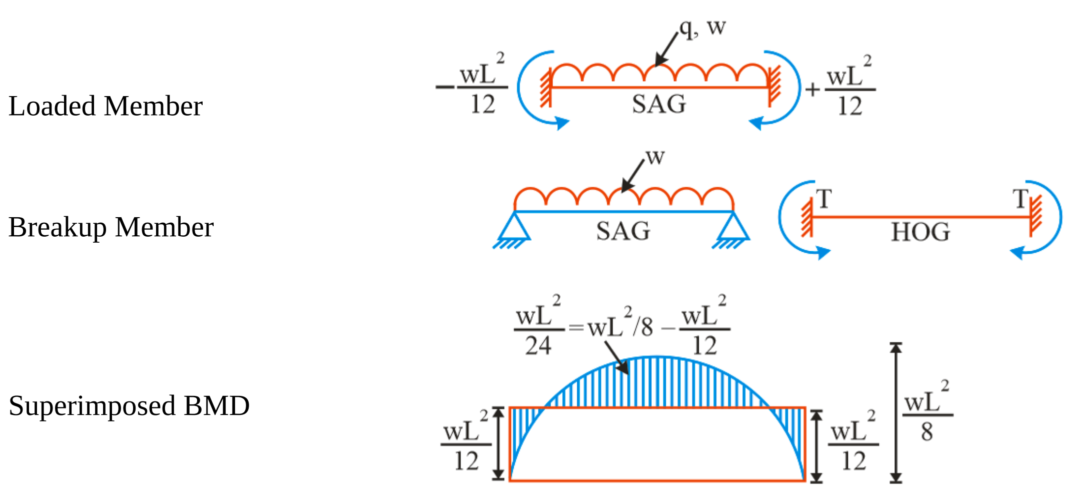
(1) The sum of the distribution factor of all members meeting oat a Rigid Joint = 1

(2) The concept of Distribution factor is Applicable to member metting at a Rigid Joint only.

4.2. MDM – Standard Case

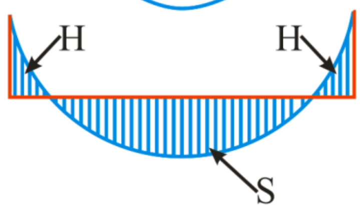


Bending Moment Diagram and Deflected Shape



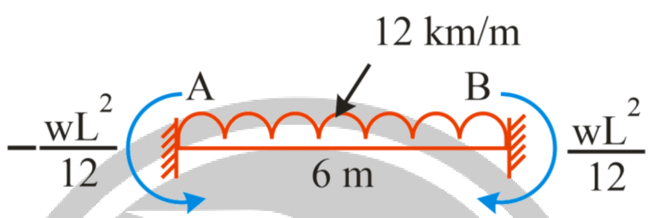
Deflected Shape

Actual BMD on Tension Side



Bending Moment Diagram and Deflected Shape

Example:



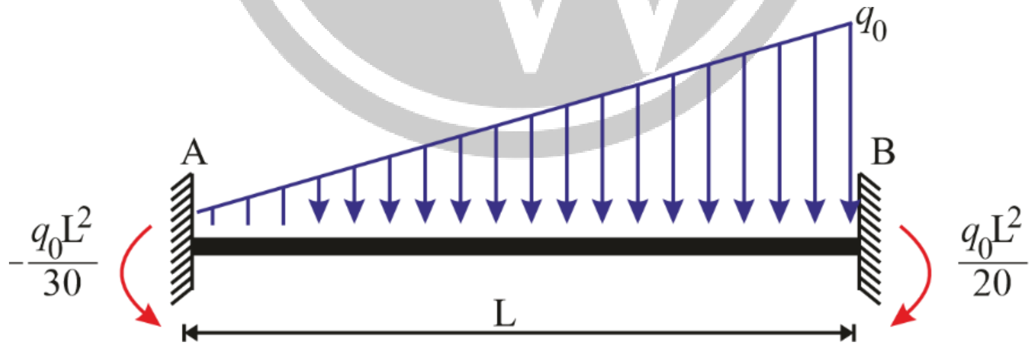
$$M_{AB} = \frac{-12\times 6^2}{12} \Rightarrow -36 \text{ kNm}$$

M_{BA} = + 36 kNm

4.3. MDM – Standard Cases 4

Example:

$M_{BA} > M_{AB}$

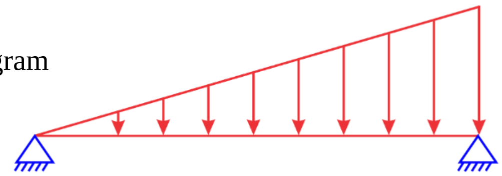


$$M_{AB} = \frac{-q\cdot L^2}{30}$$

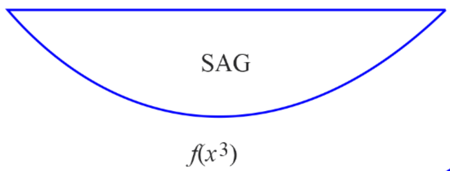
$$M_{BA} = \frac{+qL^2}{20}$$

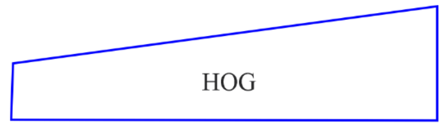
Bending Moment Diagram and Deflected Shape

Loading diagram

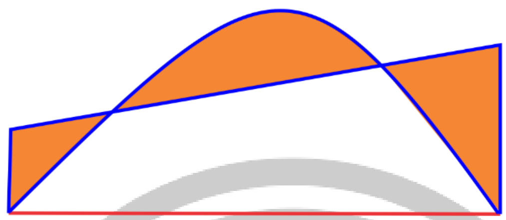


BMD



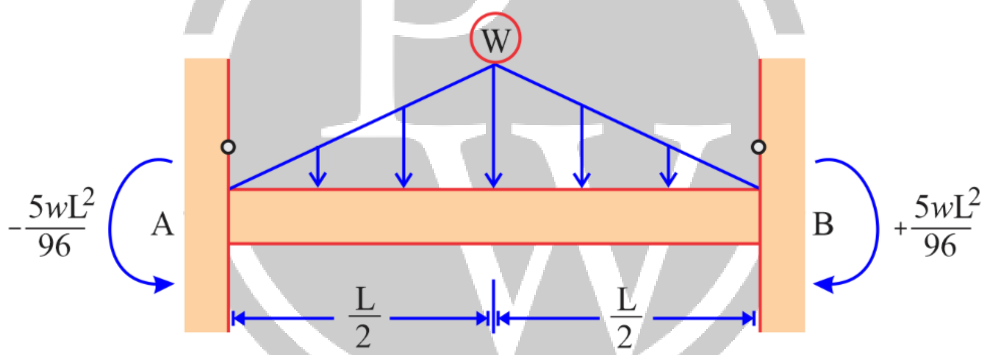


Superimposed BMD



Example:

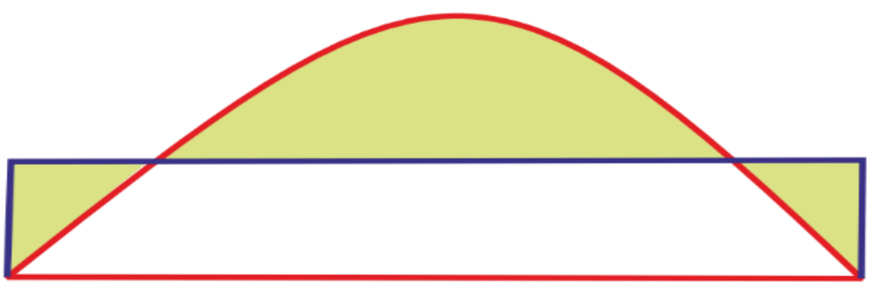
Standard Case



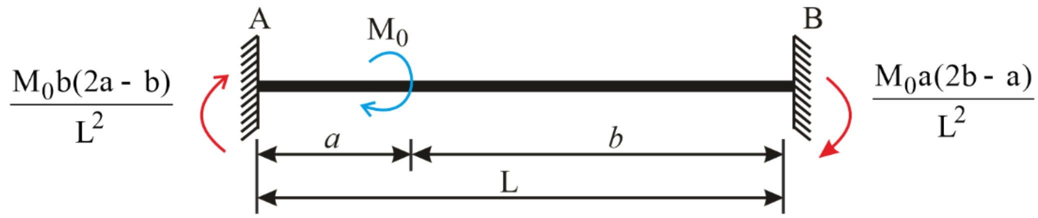
$$M_{AB} = -\frac{5}{96}wL^2$$

$$M_{BA} = +\frac{5wL^2}{96}$$

Bending Moment Diagram



4.4. MDM – Standard Cases 5

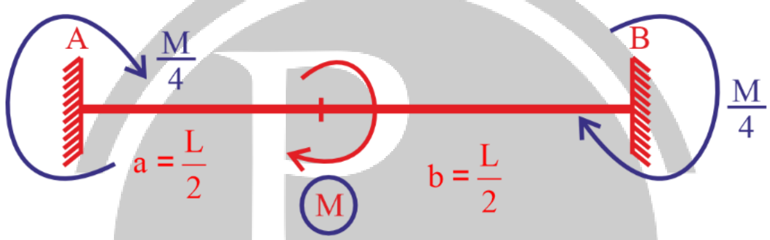


$$M_{AB} = \frac{M_0 b(2a - b)}{L^2}$$

$$M_{BA} = \frac{M_0 a(2b - a)}{L^2}$$

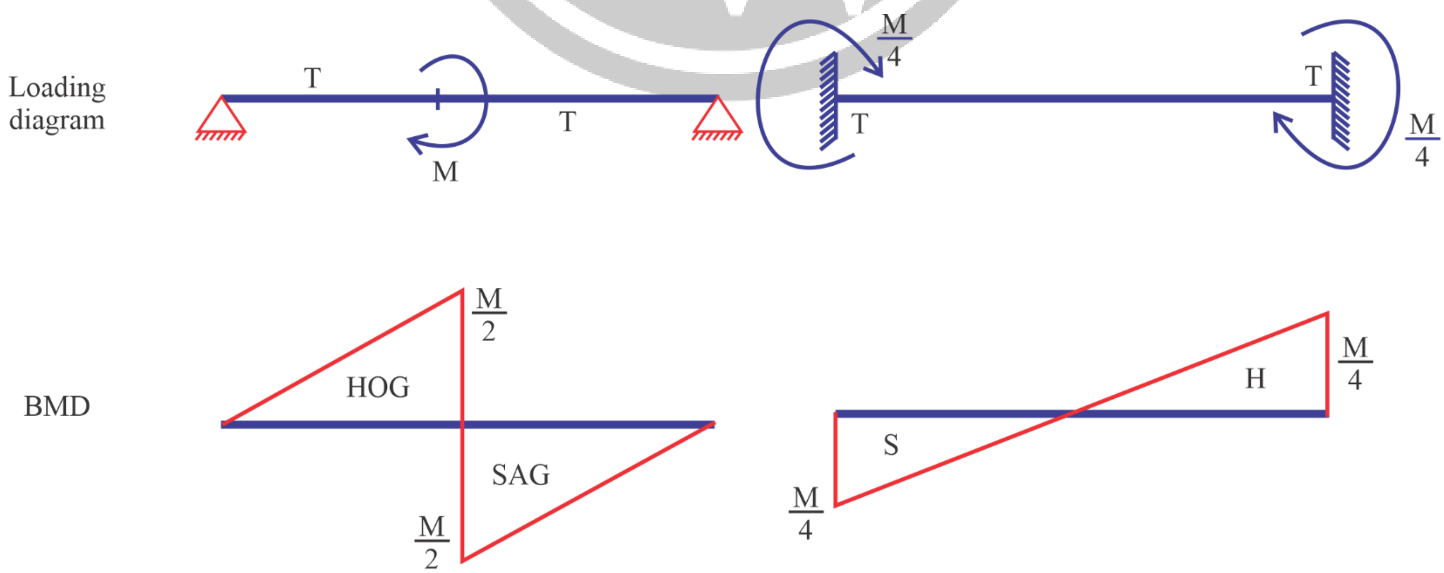
Example:

when $a = b = L/2$

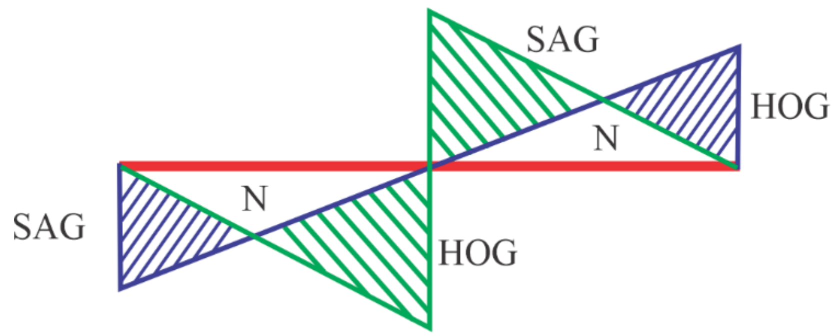


$$M_{AB} = \frac{M.b(2a - b)}{L} = \frac{M.\frac{L}{2}\left[L - \frac{L}{2}\right]}{L} = \frac{M}{4}$$

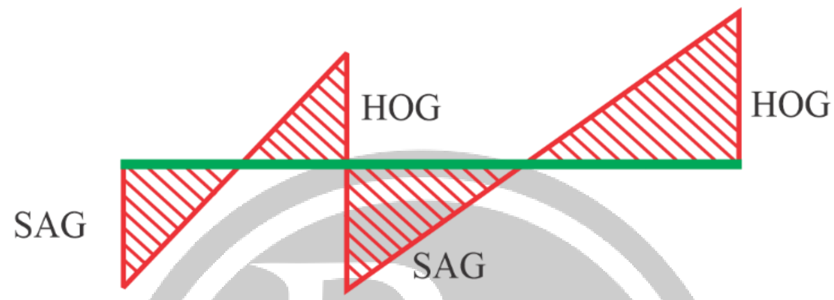
Bending Moment Diagram and Deflected Shape



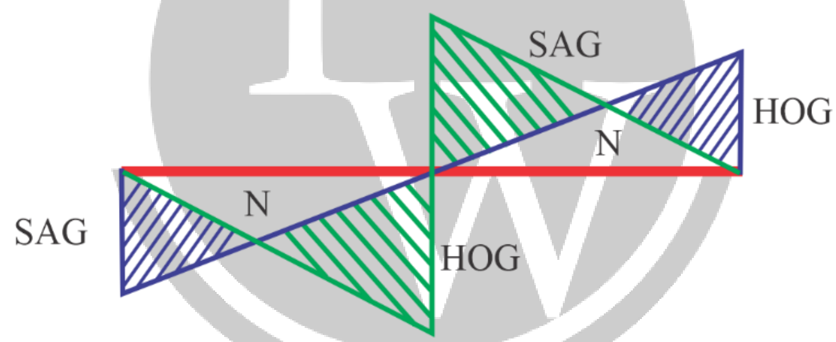
Resultant BMD after Superposition



Resultant BMD

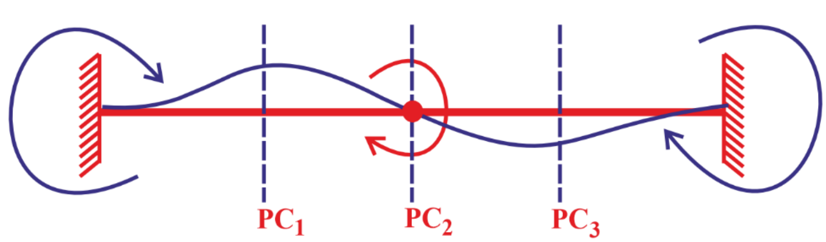


BMD on Tension Side



BMD by Super Position

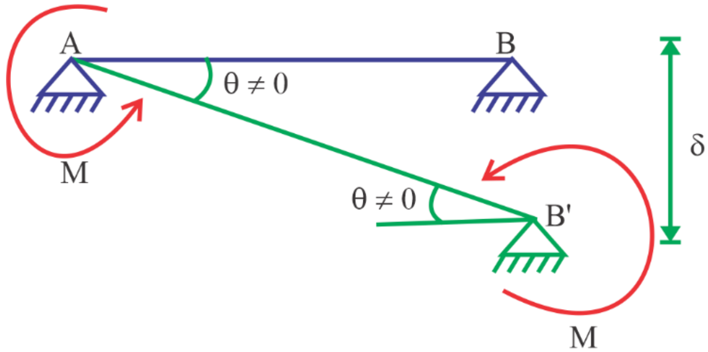
3 Point of Control Flexure and 4 Curvature



4.5. MDM – Standard Cases 6

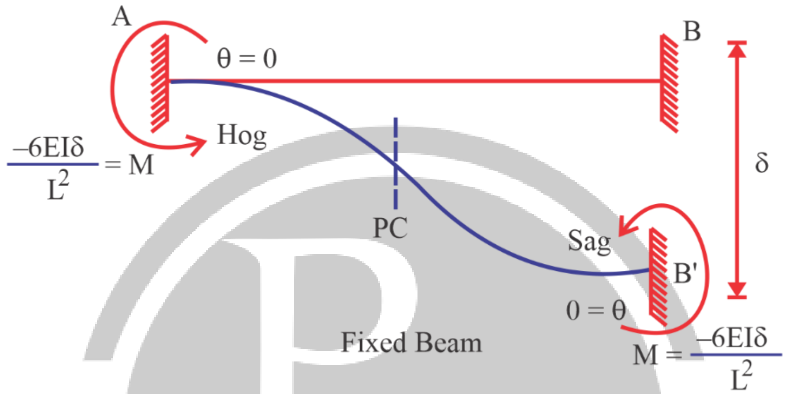
4.4.1. Sinking of Support

Relative to A, support B sinks by δ value.



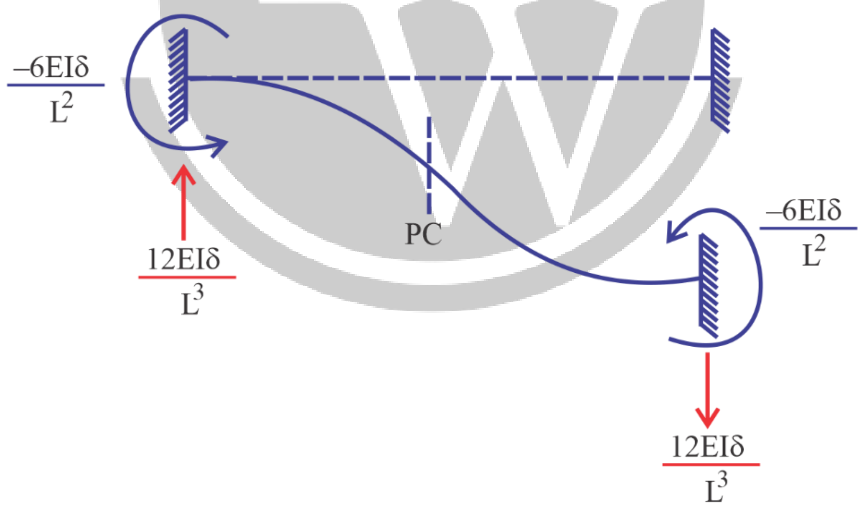
Simple Supported Beam

If A & B are made fixed θ = 0.

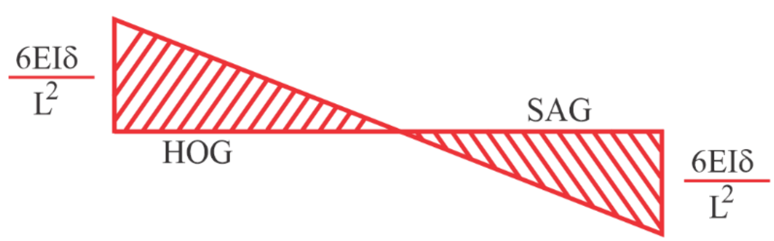


Fixed Beam

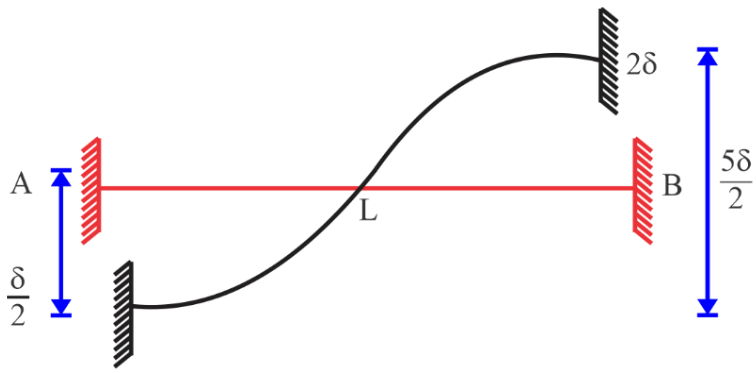
End Reaction



Bending Moment diagram



Example:

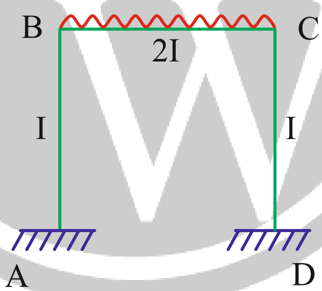


- A setells by $\frac{\delta}{2}$
- B rises by 2δ

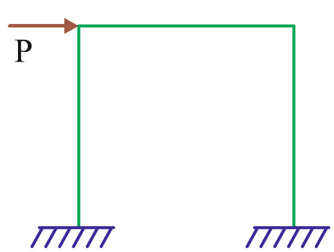
$$M_{AB} = \frac{-6EI\left(\frac{5}{2}\delta\right)}{L^2}, \qquad \text{Relative disp} = 2\delta + \frac{\delta}{2} = \frac{5}{2}\delta$$

4.5. Portal frame with sway – Moment distribution method

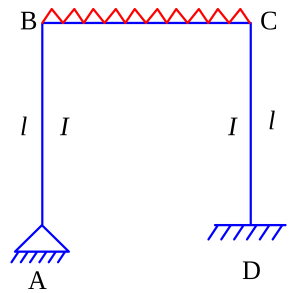
(1) If the structure and loading are symmetrical, then the frame will not in any direction.



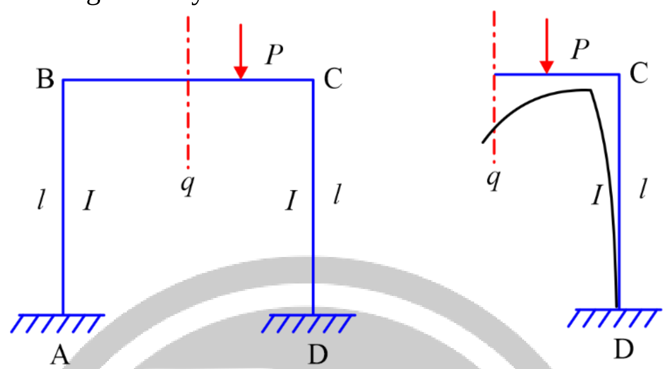
(2) (a) Pure sway (Due to 'P' frame sways right words)



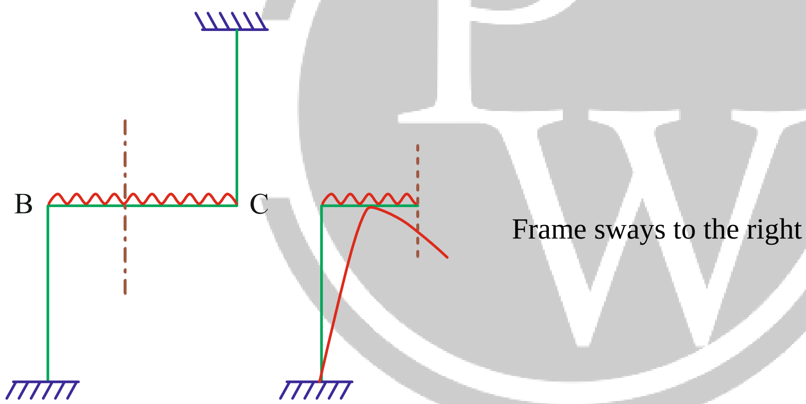
(b) In this case loading is symmetrical but structure is not symmetrical so, frame sways towards weak column side. i.e frame sway left.



(c) Structure is symmetrical loading is not symmetrical.



(d) Cut the frame at centre and check the deflected shape of the cut frame in that direction frame also sways. In the above frame sways to the left.



(3) Causes of side sway of frames:

(a) Unsymmetrical loading

(b) Unsymmetrical out line.

(c) Different end conditions of columns

(d) Non - uniform section of members

(e) Horizontal loading on columns

(f) Settlement of supports

(g) Combination of above.

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5 PLASTIC ANALYSIS

5.1. Topics

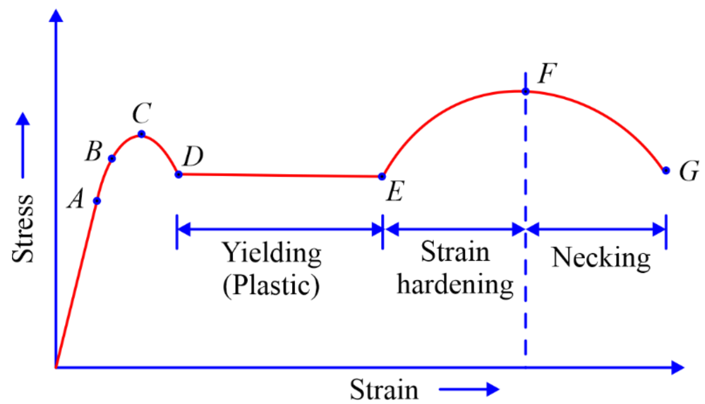
- Stress Strain Diagram
- Assumptions in plastic analysis
- Plastic Moment of a Section
- Shape factor
- Moment Curvature relation
- Determinacy and Indeterminacy
- Locations of plastic hinge
- Conditions in Plastic Analysis
- Mechanism in Structures
- Types of independent Mechanisms
- Theorems of Plastic Analysis
- Sequence of Plastic Hinge formation
- Collapse load determination
- Plastic Hinge length
- Load Factor

5.2. Introduction to Plastic Analysis

- Plastic design of a structure limits the structural usefulness of the material of the structure up to **ultimate load**.
- The load is found from the strength of steel in the **plastic range**.
- The method has its main application in steel structures as the strength of steel **beyond the yield stress** is fully utilized in this method.
- The method is **economical**
- The new steel code **IS 800 – 2007** utilizes this approach for Design

5.3. Stress Strain Curve - Plastic Analysis

- The concept of ductility forms the basis for the plastic theory.
- Mild steel is a ductile material.



5.3.1. Key Points of Mild Steel Curve

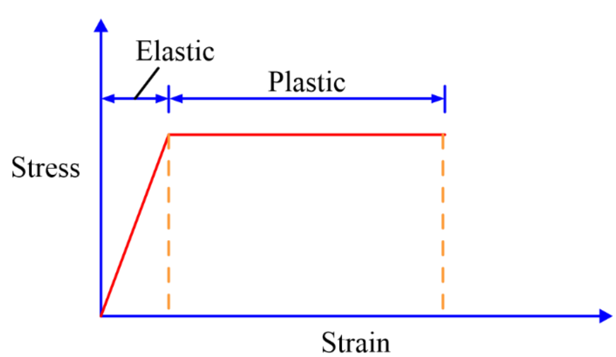
- Pt. A – Proportional limit
- Pt. B – Elastic limit
- Pt. C – upper yield point
- Pt. D – lower yield point
- DE zone of Plastic deformation
- Pt. F – ultimate stress point
- EF Zone - Strain Hardening
- Pt. G – Fracture point
- FG Zone – Necking

5.3.2. Elastic Vs. Plastic Theory

- **Elastic method** is based on **Hook's law.** Hence the structural usefulness of the material of the structure is limited to a stage when the stress in **extreme fiber reaches the yield stress** of material. The rest of the cross section remains unstressed, and the method do not take into account the strength beyond the yield stress point.
- **Plastic design** the structural usefulness if found from the **strength of steel in the plastic range**. Plastic Analysis is also known as **Ultimate load analysis**. The sections designed by this method are smaller in size.

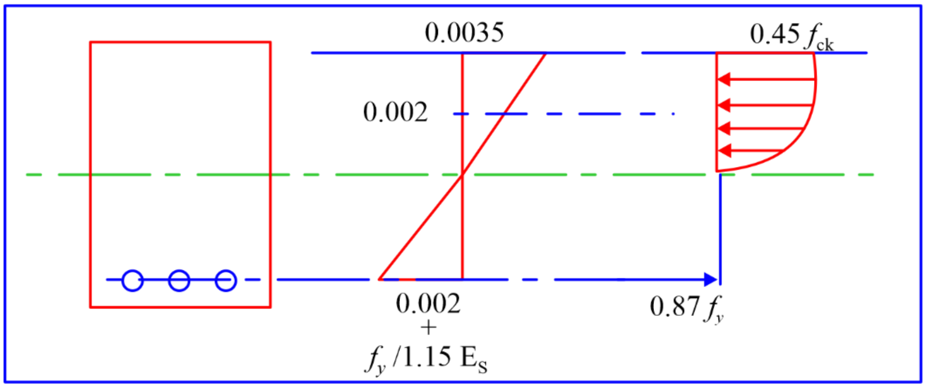
Important Note:

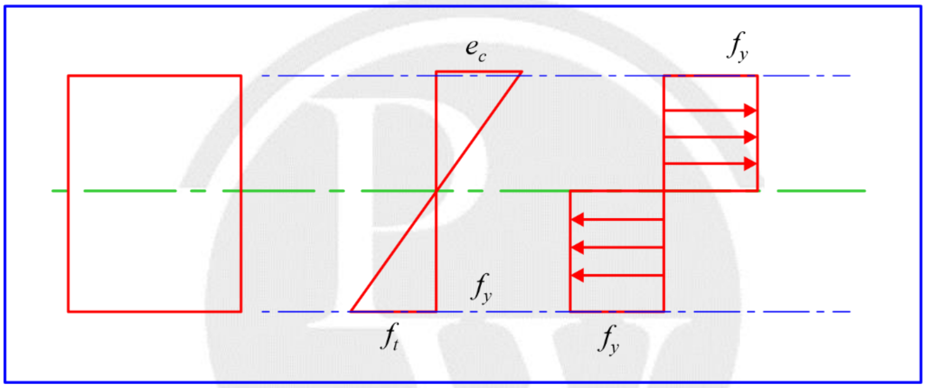
- Strain controlled
- Strain hardening and necking is neglected.
- Lower yield point is considered.
- Elasto Plastic curve forms the basis of design



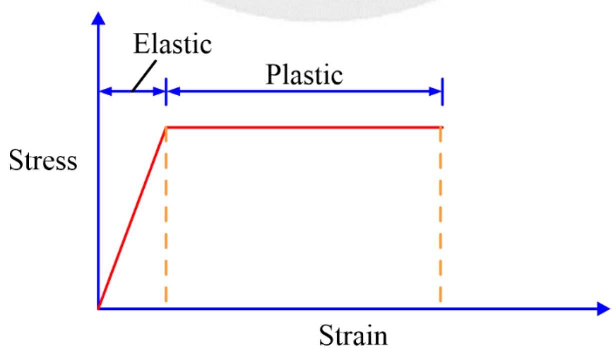
5.4. Assumptions - Plastic Analysis

5.4.1. Bernoulli's Assumption.





- Axial and shear deformations are neglected
- The cross section must be symmetrical w.r.t. the plane of loading
- The stress strain relationship is assumed to be **bi-linear** i.e. it consists of 2 straight lines.
- The material is **homogeneous**, and **isotropic** in both elastic and plastic state

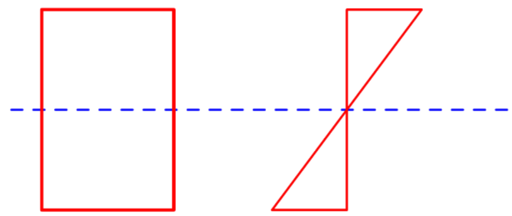


5.4.2. Applicability

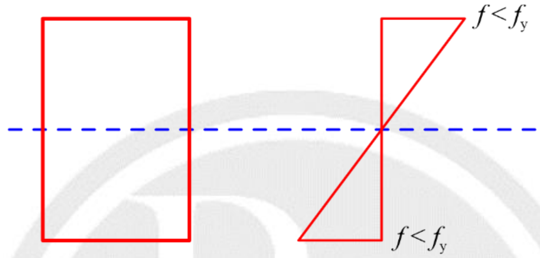
- Plastic theory is not suitable for structures subjected to impact and fatigue
- High tensile steel does not posses a defined yield point and the horizontal yielding part does not exist in the stress strain diagram hence its use is not applicable
- The Plastic theory is not applicable in brittle material but in RCC it can be applied with limitation to rotation exceptionally.

5.5. Plastic Moment - Plastic Analysis

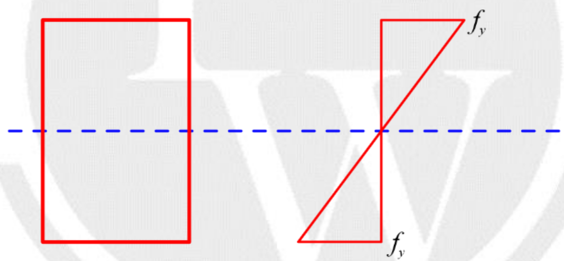
Bending strain distribution because **of Bernoulli's assumption**



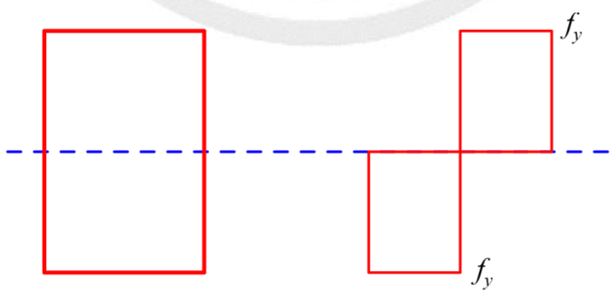
Bending stress (linear because of **Hook's law (Safe Moment)**



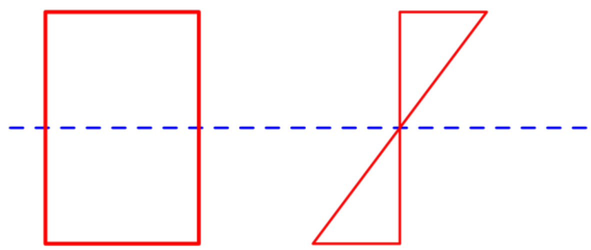
Bending stress diagram at yielding **(Yield Moment**)



Bending stress diagram at plastic state **(Plastic Moment)**



Bending strain distribution at **collapse condition (linear)**



5.6. Shape Factor- Plastic Analysis

- Shape factor represents **"Reserved Strength** " of beam section beyond yield moment to reach plastic state
- The more S.F. implies more reserve strength **beyond yield moment** to each plastic state

5.6.1. Shape factor for Different c/s

- Rectangular Section 1.5
- Circular Section 1.7
- I section 1.14
- H section 1.5
- Diamond section 2.0
- Triangular section 2.34

5.7. Moment Curvature Relation - Plastic Analysis

Curvature is proportional to M , i.e. as the moment increases curvature also increases

$$\frac{d^2y}{dx^2} = \frac{d\theta}{dx} = \frac{1}{R}$$

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R}$$

Where : M = Moment at Section

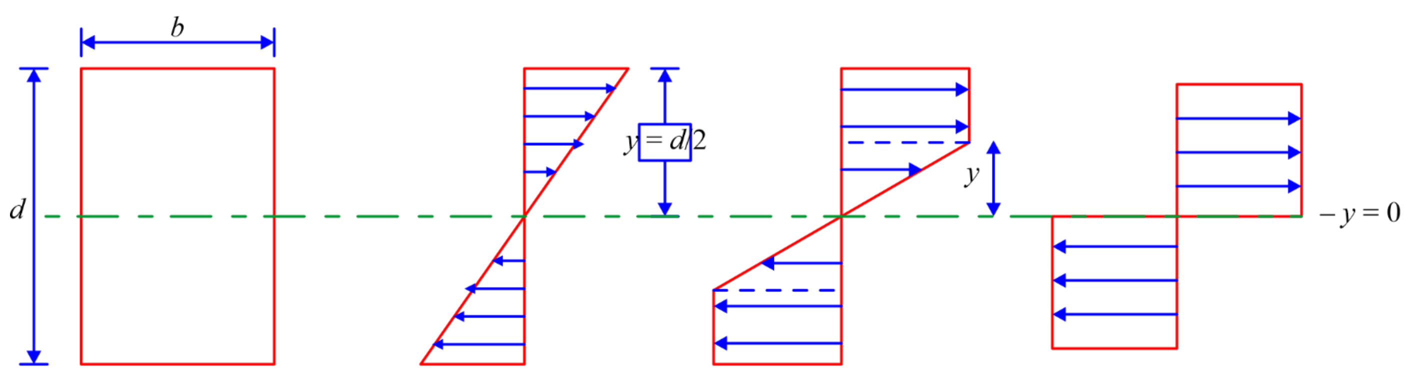
I = Moment of Inertia

Y = distance from NA to extreme fibre of elastic state

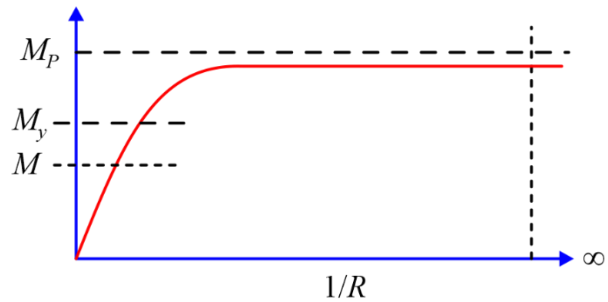
R = radius of curvature of bent up beam

1/R = curvature i.e rate of change of slope $\alpha \frac{1}{y}$

At fully plastic state curvature is infinity,



5.7.1. Moment Curvature Graph



Conclusion

- Mp depends on the area of c/s and distribution of the area.
- The most efficient c/s is I - section (because for a given c/s area, centroidal distance are maximum.

5.8. Indeterminacy - Plastic Analysis

How is Indeterminacy helpful in studying Plastic Analysis

Considering the planar structure we know that there are 3 Equations of Equilibrium under **general loading case.**

$$\sum F_x = 0, \sum Fy = 0, \sum Mz = 0$$

Also for the planar structure as **Beam** subjected to **vertical loading** there are only 2 Equations of Equilibrium.

$$\sum F_y = 0, \sum M_z = 0, \textbf{ horizontals are neglected}$$

In Regards to supports

- Roller has one Normal Reaction
- Hinged or Pinned has total of two reactions in Vertical and Horizontal
- Fixed support has total of three reactions in vertical , horizontal and moment
- **Internal Hinge** has zero moment at the location and if present in sufficient number it make the structure unstable. It gives an additional equation due to which the indeterminacy of the structure decreases
- **Plastic hinge** has Plastic Moment at the location and if present in sufficient number it make the structure unstable

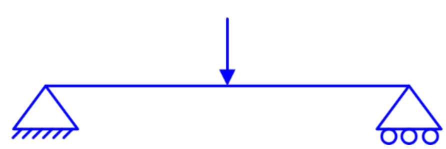
Static indeterminacy of any structure is given as

$$SI = \text{Reactions} - \text{Equilibrium equation} - \text{nos.of Hinge}$$

$$\text{If SI} < 0 \text{ Unstable, SI} = 0 \text{ Determinate, SI}>0 \text{ Indeterminate}$$

- If a structure is statically stable then the number of additional hinges (Plastic) required for the complete collapse of the structure is

$$N = SI + 1$$



5.9 Plastic Hinge Location

- Under the Point or Concentrated load
- At the location of maximum moment
- At the change of cross section
- At location where material properties change
- At fixed or continuous support pts.

5.9.1. Condition - Plastic Analysis

There are 3 conditions in plastic equilibrium

- Equilibrium condition
- Mechanism condition
- Yield condition

Equilibrium Condition

$$\sum F_x = 0, \sum F_y = 0, \quad \sum M_z = 0$$

Mechanism condition

At collapse, sufficient no. of plastic hinges must be developed so that a part or entire structure must transform in to a mechanism leading to its collapse.

Number of mechanism = Probable Location – Static Indeterminacy

Yield Condition

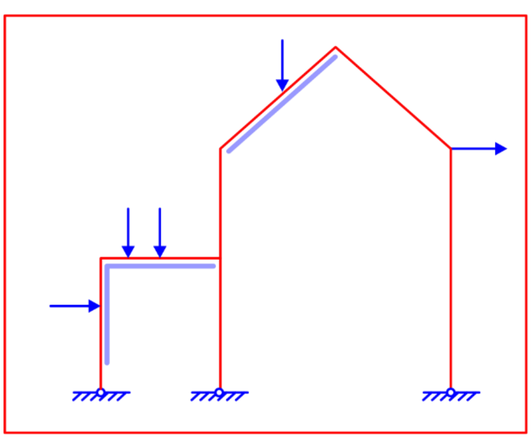
At collapse the bending moment at any section should not exceed the plastic moment capacity i.e. $\boldsymbol{M \leq M_p}$

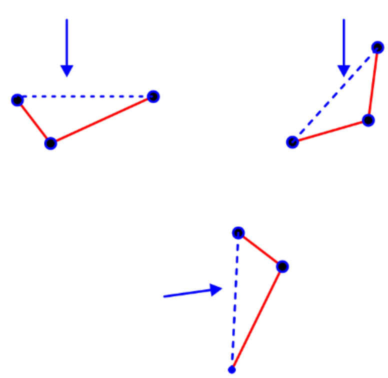
5.10. Mechanism - Plastic Analysis

- Beam mechanisms
- Sway mechanisms
- Gable mechanism
- Joint mechanism

5.10.1. Beam Mechanisms

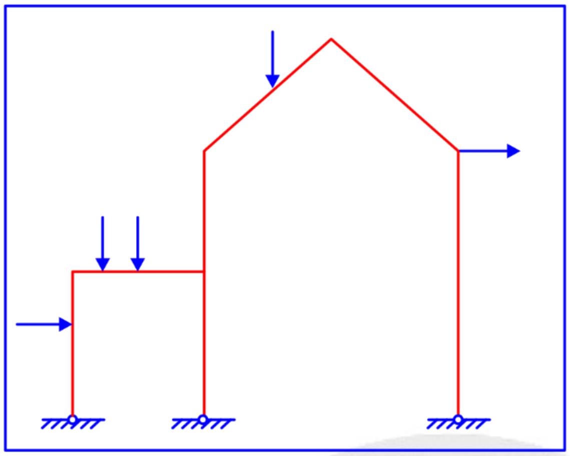
Takes place in simply supported, continuous, fixed beams etc.

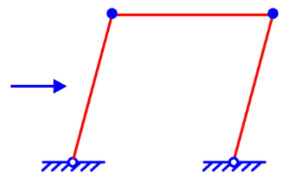




5.10.2. Sway Mechanisms

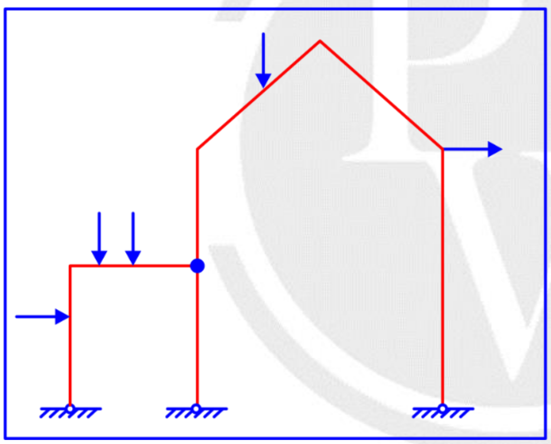
Takes place in Frames due to drifting of the column top joints.

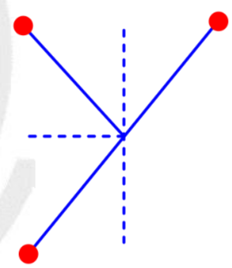




5.10.3. Joint Mechanism

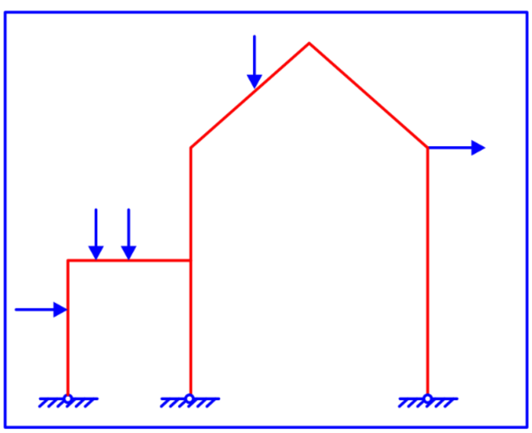
Occurs where **more than two structural members** meet. Plastic hinge is formed in all the members at that joint

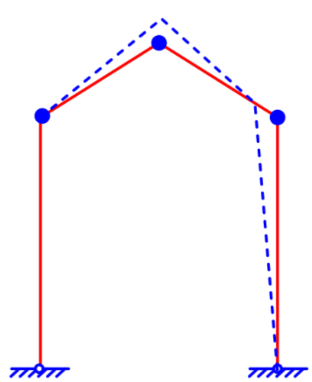




5.10.4. Gable Mechanism

Columns spread more at the top than at the base. This occurs in Gable frames of a warehouse or gable frames.





5.11. Theorems - Plastic Analysis

Static Theorem

- This theorem is based on the principles of statics
- It utilizes two conditions that are Equilibrium and Yield
- It is a lower bound theorem which states that for any distribution of moments the frame should be safe and admissible under the value of load W, where W is less than or equal to collapse load Wu
- Hence $W \le W_u,\ M \le M_p$

Kinematic Theorem

- It is based on the concept of work done and energy absorbed
- It utilizes two conditions that are Equilibrium and Mechanism
- It is a upper bound theorem which states that for a given frame subjected to load W, the W is found to correspond to any assumed mechanism, must be either greater or equal to the collapse load Wu
- Hence $W \ge W_u$

5.11.1. Static Theorem Procedure

- Draw the Normal BMD for the load on member
- Identify the locations where Plastic Moment capacity can be reached in a sequence.
- Relate the load W_u with the last plastic hinge location in terms of the bending moment M_p

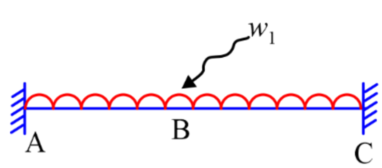
5.11.2. Kinematic Theorem Procedure

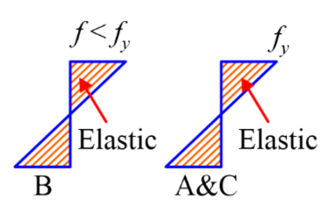
- Locate the possible location of plastic hinges
- Determine the number of possible independent and combined mechanisms
- Write the equation of equilibrium by the principal of virtual work i.e.

Work done = Energy stored

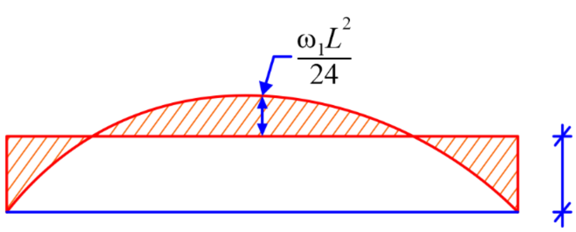
5.12. Sequence of Plastic Hinge Formation

(1) Load w_1





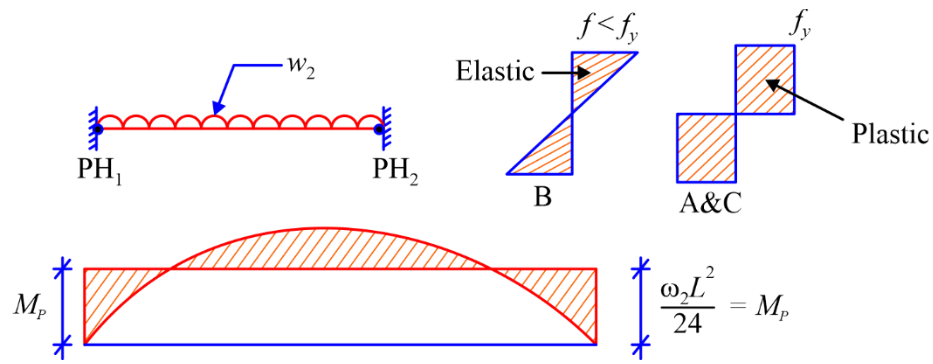
Bending Stress diagram



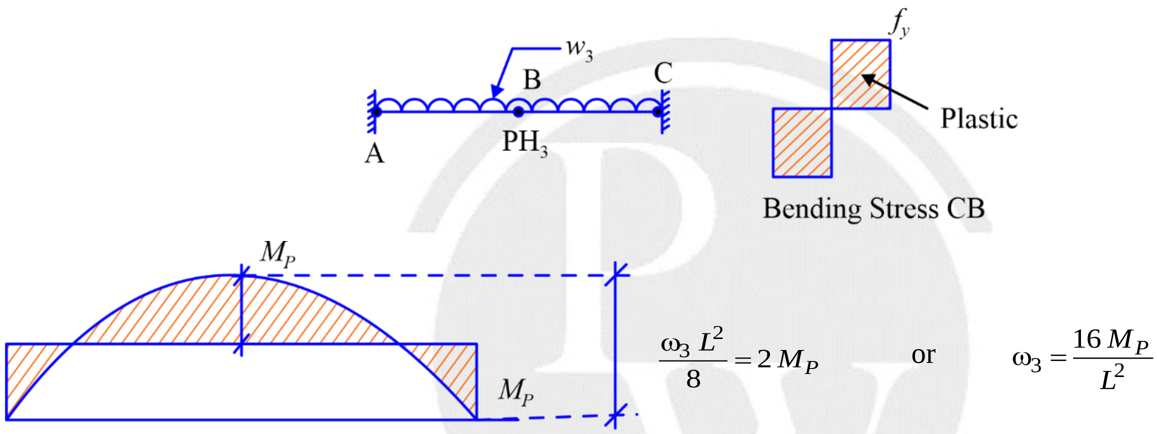
Bending Moment Diagram

$$\frac{\omega_1 L^2}{12} \cdot M_y \quad \text{or} \quad \omega_1 = \frac{12 M_y}{L^2}$$

(2) $w_2 > w_1$ Load Increases



(3) $w_3 > w_2$ Load Increased



Hence, If

$$\omega_1 = 10 \text{ kN/m}$$

Then find ω_2 and ω_3 for Rectangular Section.

(1) $$\frac{\omega_2}{\omega_1} = \frac{12 M_P/L^2}{12 M_y/L^2} = \frac{M_P}{M_y} = S_f = 1.5$$

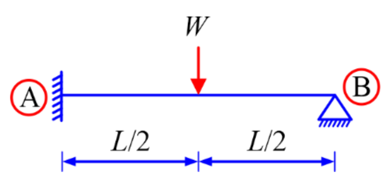
$\Rightarrow$ $$\omega_2 = 1.5, \ \omega_1 = 15 \text{ kN/m}$$

(2) $$\frac{\omega_3}{\omega_2} = \frac{16 M_P/L^2}{12 M_P/L^2} = \frac{4}{3} \quad \Rightarrow \quad \omega_3 = \frac{4}{3} \times 15 = 20 \text{ kN/m.}$$

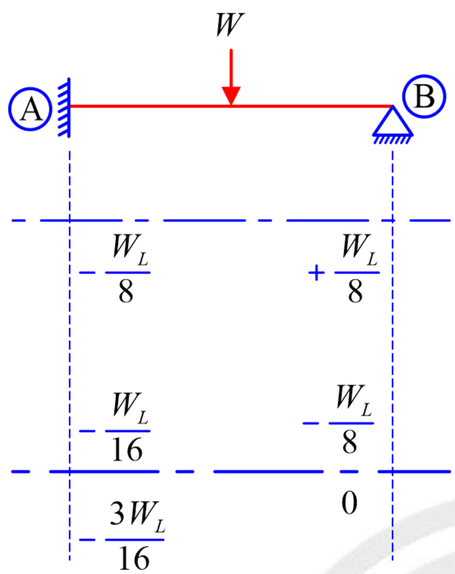
Ques. Find the Reaction at the Propped.

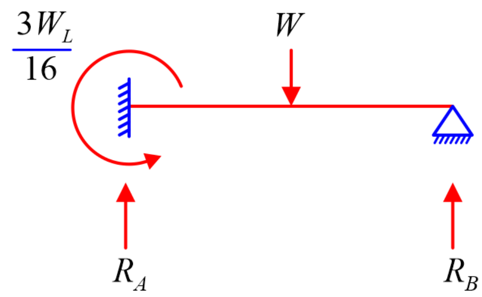
(a) Elastic Condition

(b) Collapse or Plastic Condition



Elastic Condition





Taking $\Sigma M = 0$ about (A)

$$-\frac{3W_L}{16} + \frac{W \cdot L}{2} - R_3 . L = 0$$

Hence, $$R_B = \frac{5}{16}W$$

Plastic Condition

Considering the equilibrium of right side of beam.

$$\Sigma M = 0 \quad \text{about P.H.}$$

$$+M_P - R_B \cdot \frac{L}{2} = 0$$

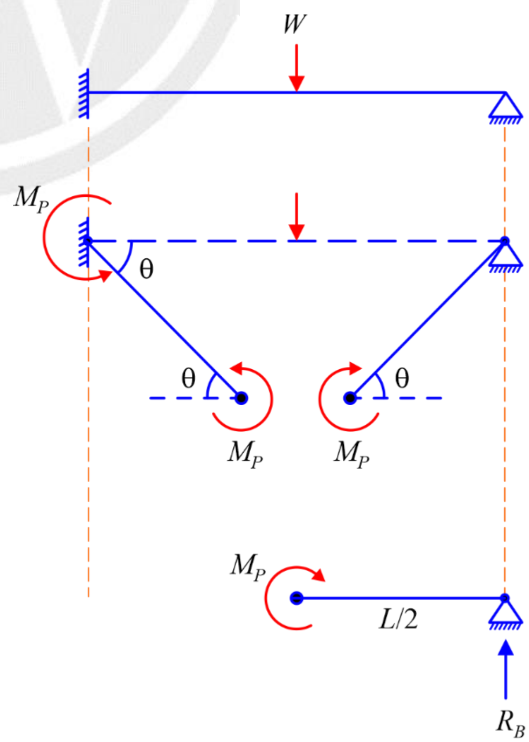
$\Rightarrow$ $$R_B = \frac{2M_P}{L}$$

From standard case,

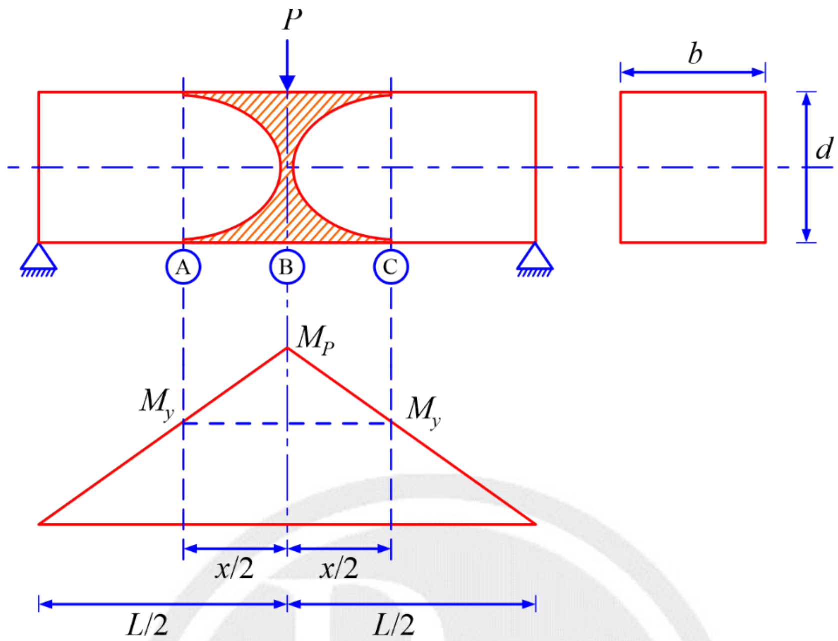
We know, $$W = \frac{6M_P}{L}$$

Hence, $$R_B = \frac{2}{L} \cdot \frac{W_L}{6} = \frac{W}{3}$$

$$R_B = \frac{W}{3}$$



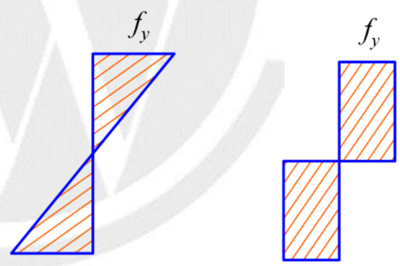
5.13. Length of Plastic Hinge for Rectangular Section



5.13.1. Bending Stress Diagram

$$M_P = \frac{P \cdot L}{4}$$

For Rectangular Section, $\frac{M_P}{M_y} = \text{SF} = 1.5$



From Bending Moment Dia.

$$\frac{M_P}{L/2} = \frac{M_y}{\frac{L}{2} \cdot \frac{x}{2}}$$

$$\Rightarrow \quad (L - x) = \frac{2}{3} L$$

$$\Rightarrow \quad x = \frac{L}{3}$$

Therefore, the hinge length of the plastic zone in equal to 1/3rd of the span.

$$L_f = \frac{P_u}{P_w} = \frac{M_P}{M}$$

P_u = Collapse Load

P_w = Working Load

M_P = Plastic Moment

M = Service Moment

Hence, $$L_f = \frac{f_y \cdot Z_p}{f \cdot Z_e} = \frac{f_y}{f} \cdot S_f$$

$\Rightarrow$ $$L_f = (\text{Fos}) \cdot S_f$$

$$\frac{f_u}{f} = \text{Fos in elastic design.}$$

$$f = \text{permissible bending stress}$$
$$= 0.66 f_y \text{ [As per 15800 – 1984]}$$

5.14. Collapse Diagram - Plastic Analysis

5.14.1. Collapse diagram due to plastic hinge formation

Rigid body motion is considered i.e. structure becomes unstable and Mechanism has formed.

Hence to form a Mechanism in a statically indeterminate structure the **nos. of plastic hinge** required is equal to

$$\boldsymbol{N = SI + 1}$$

There are 3 types of structural collapse based on number of plastic hinges formed

- **Partial collapse :** the number of plastic hinges formed is less than that required for complete collapse. Hence only a part of the structure becomes unstable
- **Complete collapse :** the number of plastic hinges formed is SI + 1
- **Over Complete collapse :** the number of plastic hinges formed is greater than that required for complete collapse. Hence there are chances of multiple mechanism occurring simultaneously.

Plastic Hinge Length

- Zone of yielding at which infinite rotations may take place.
- Large changes of slope occur over small length of member at a position
- The zone acts as if it was hinged with a constant moment Mp.
- The length of the yielded zone is called the hinge length.
- Plastic hinges are formed first at the sections subjected to greatest deformation(curvature).

Load Factor - Plastic Analysis

- It is defined as the ratio of collapse load to the working load
- It can also be defined as the product of the factor of safety and shape factor
- In practice a load factor varying from 1.7 to 2.0 is assumed depending upon the engineer's judgment
- IS800 – 2007 is silent on Load factor.
- Load factor has been defined in IS800 – 1984
- The prime function of load factor is to ensure that the structure is safe under working condition.

$$L_f = \frac{P_u}{P_w} = \frac{M_P}{M}$$

Hence, $$L_f = \frac{f_y \cdot Z_p}{f \cdot Z_e} = \frac{f_y}{f} \cdot S_f$$

⇒ $$L_f = (\text{Fos}) \cdot S_f$$

P_u = Collapse Load

P_w = Working Load

M_P = Plastic Moment

M = Service Moment

$\frac{f_u}{f}$ = Fos in elastic design.

f = permissible bending stress

= $0.66 f_y$ [As per 15800 – 1984]

Example

Load factor for Rectangular Section,

⇒ $$L_f = \left(\frac{f_y}{0.66\ f_y}\right) \times 1.5 = 2.26$$

Load factor for I section,

⇒ $$L_f = \left(\frac{f_y}{0.66\ f_y}\right) \times 1.14 = 1.70$$

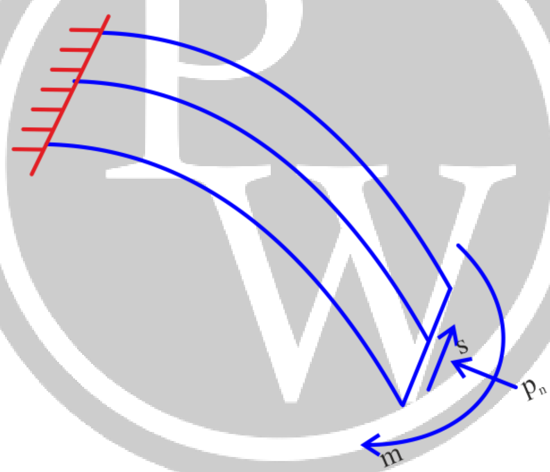
❑❑❑

6 ARCHES & CABLES

6.1. Properties of Fluid

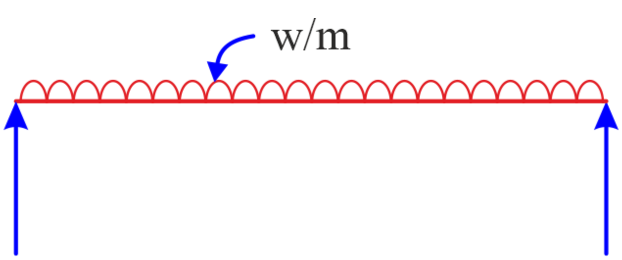
An Anch is a curved beam in vertical plane

- Design forces in an Arch:
- Rn : normal thrust or axial compression
- S : Radial shear force
- M : Bending moment

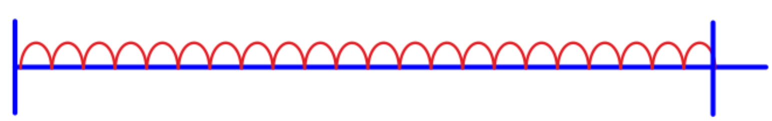


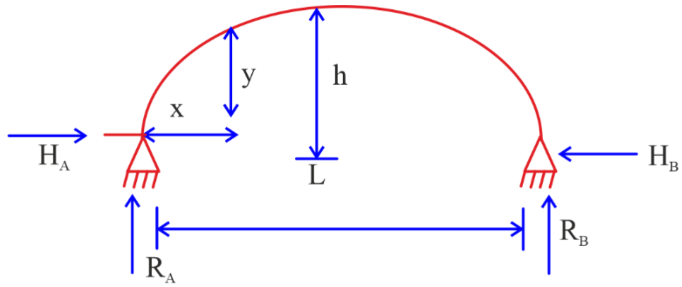
Advantages of Arches compared to SSB.

For a SSB:



For an Arch:





$$m_x = R_A x - wx\frac{x}{2}$$

$$m_{beam} = R_A x - \frac{wx^2}{2}$$

$$m_x = M_{arch} = \left(R_A.x - \frac{wx^2}{2}\right) - H_A y = M_{beam} - H_A y$$

$$M_{arch} = M_{beam} - H_{moment}$$

(a) An arch is economical for long spans compared to SSB

(b) The horizontal reaction developed of the support of each will reduce the net moment compared to that of SSB.

Note:

Arches are primarily suby to axial compression. Hence stone which strong in axial compression were used in olden days for contraction of arches.

6.2. Classification of Arches

1. Based on Shape

(a) Parabolic

(b) Semi-circular

(c) Segmental

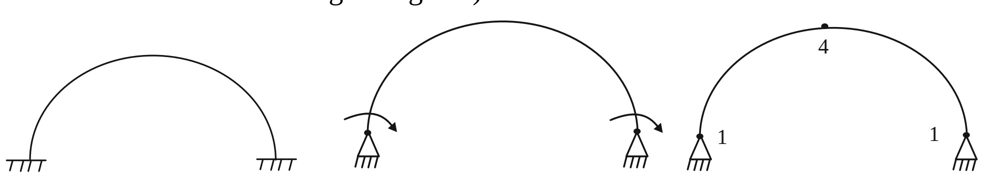
2. Based on number of hinges (or D_s) :

(a) Fixed arches ($D_s = 3$, $D_k = 0$)

(b) Two hinged arches ($D_s = 1$, $D_k = 2$)

(c) Three hinged arches ($D_s = D$, $D_k = 6$ considering AD

= 4 neglecting AD)



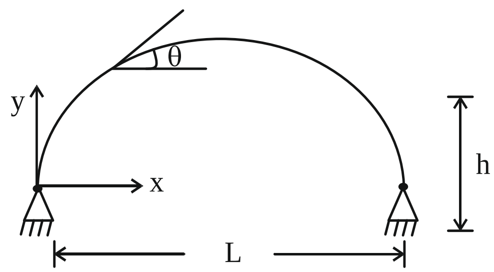
Parabolic Arches

$$y = \frac{4h}{l^2}x(1-x)$$

(one of the support as origin)

$$\tan\theta = \frac{dy}{dx} = \frac{4h}{1^2}(1-2x)$$

$$\frac{x^2}{y} = \text{const. (Crown as origin)}$$

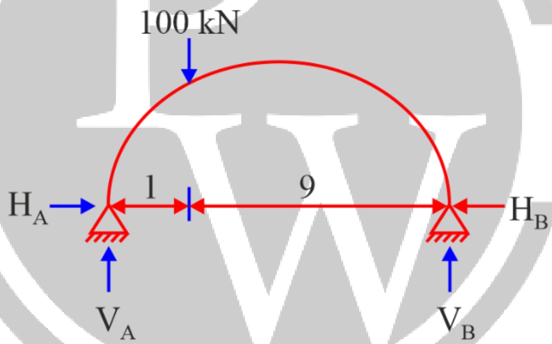


Calculation of Reactions at support of Arches

(a) Supports are at same level

To calculate vertical reactions, if the supports are at same level, analysis is similar to that of a SSB.

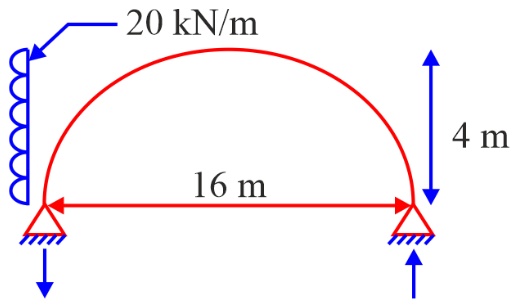
$$\Sigma M_A = 0$$
$$10 V_B = 100 \times 1$$
$$V_B = 10 \text{ KN} \ \& \ V_A = 90 \text{ KN}$$



Horizontal reaction is not influencing the vertical reaction as their line of action through the support

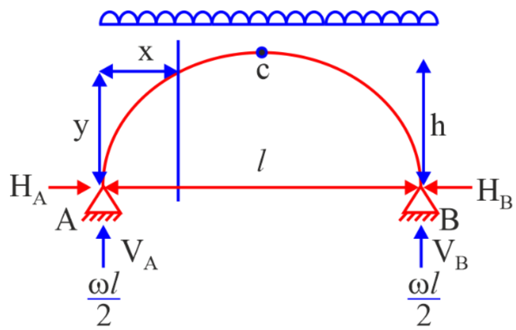
$$\Sigma M_A = 0$$

∴ $$16 VB = 20 \times 4 \times 2$$
$$VB = 10 \text{ KN} (\uparrow)$$
$$VA = 10 \text{ KN} (\downarrow)$$



6.3. Calculation of Horizontal Reactions

(a) Three Hinged Arches. – parabolic arch suby. to wall throughout



Apply, $\Sigma M_C = 0$ (from right)

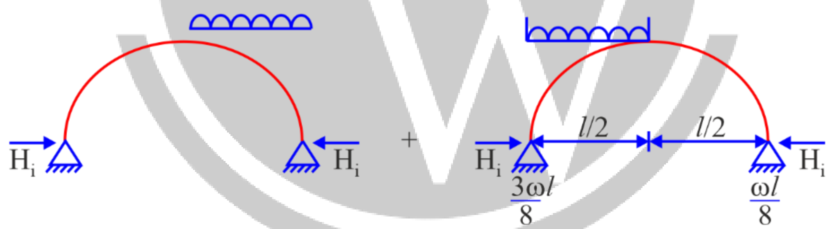
$$\Rightarrow \quad H_B \times h + \frac{wl}{2} \times \frac{l}{4} = \frac{wl}{2} \times \frac{1}{2}$$

$$H_B = \frac{wl^2}{8j}$$

$H_A = H_B = H$ (no other horizontal forces)

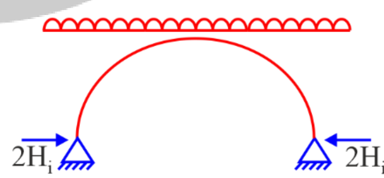
$$M_x = \frac{wl}{2} \times x - \frac{wl^2}{8h} \times y - \frac{wx^2}{2} = \frac{wlx}{2} - \frac{wl^2}{8h}\left(\frac{4h}{l^2} x(1-x)\right) - \frac{wx^2}{2} = 0$$

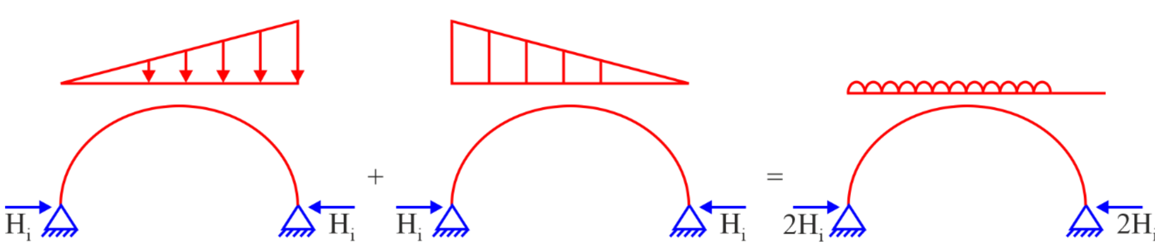
$M_x = 0$



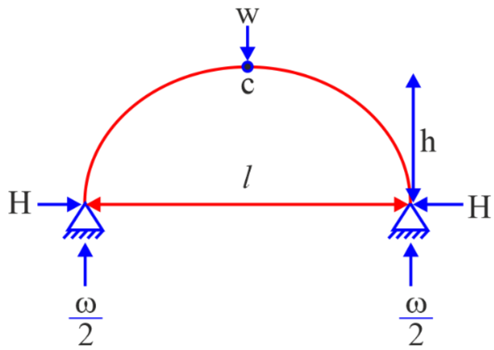
$$2H_i = \frac{wl^2}{8h}$$

$$H_i = \frac{wl^2}{16h}$$





$$2H_i = \frac{wl^2}{8h} \qquad H_i = \frac{wl^2}{16h}$$



$$\Sigma m_c = 0 \Rightarrow \frac{W}{2} \times \frac{l}{2} = H \times h$$

$$\boxed{H = \frac{Wl}{4h}}$$

6.4. ILD for 3-hinged Arches

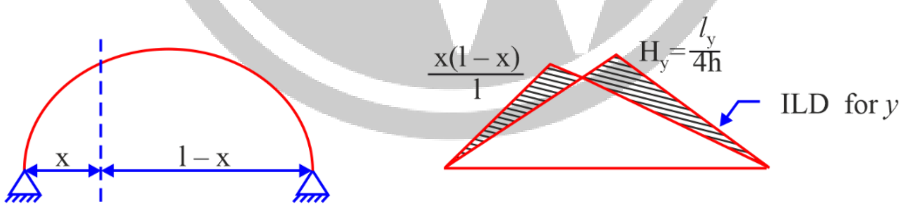
(a) ILD for Horizontal Thrust

When unit to At support horizontal thrust = 0

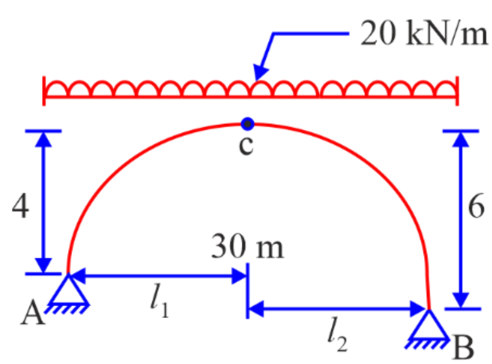
(a) ILD for M_x

$$M_{arch} = M_{beam} - Hy$$

x & y are the coordinated of the choosen section where ILD is to be drawn for BM.



(b) Supports at different level



Calculate reaction the supports.

(1) There hinged parabolic Unsymmetric Arches

Step 1: Calculate horizontal distance of Ac & BC

We known $\frac{x^2}{y}$ = const. (For parabolic arch wrt crown as origin)

$$\frac{x}{\sqrt{y}} = \text{const}$$

$$\frac{l_1}{\sqrt{h_1}} = \frac{l_2}{\sqrt{h_2}} = \text{const.} = \frac{l_1 + l_2}{\sqrt{h_1} + \sqrt{h_2}} = \frac{l}{\sqrt{h_1} + \sqrt{h_2}}$$

$$\frac{l_1}{\sqrt{L_1}} = \frac{30}{\sqrt{L_1} + \sqrt{b}}$$

$\Rightarrow$ $$l_1 = \frac{60}{2 + \sqrt{6}} = 13.48 \text{ m}$$

l_2 = 16.51 m

As supports are not at same level, we cannot calculate vertical reactions by treating like a SSB initially

Apply ΣM_c = 0 (from left)

$$R_A \times 13.48 = 20 \times \frac{13.48^2}{2} + 4H$$

R_A = 0.3 H + 134.9

Apply ΣM_c = 0 (from right)

$$R_B \times 16.51 = 20 \times \frac{16.51^2}{2} + 6H$$

RB = 165.1 + 0.363 H

Apply ΣV = 0 for the entire arch,

$R_A + R_B$ = 20 × 30

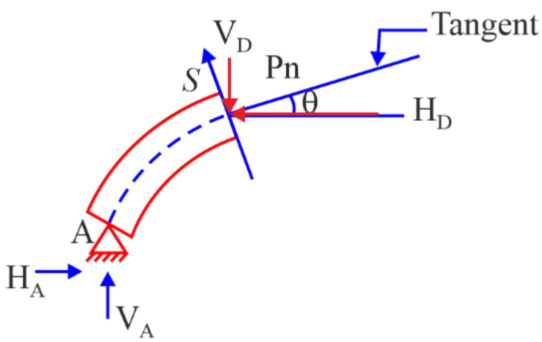
H = 455.2 KN

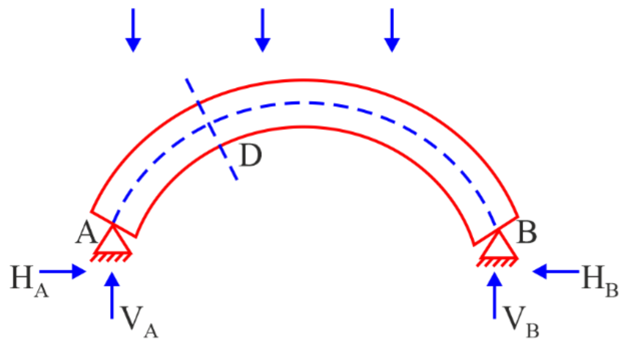
R_A = 269.64 KN

R_B = 330.36 KN

6.5. Radial shear & Normal Thrust

Consider free body diagram of part AD.





V_D → net vertical reaction at D

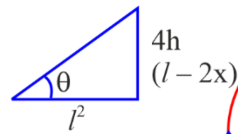
H_D → net horizontal reaction at D

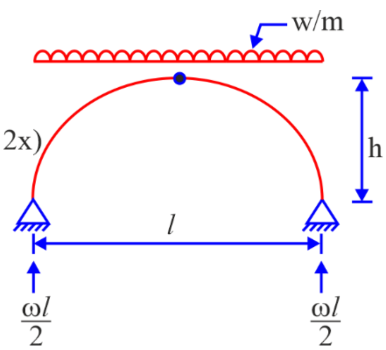
θ → angle b/w the tangent at D and horizontal

$p_n \rightarrow$ normal thrust or axial compression
$S \rightarrow$ radial SF.

- p_n is the resultant of H_D & V_D resolved in the direction of p_n
  $p_n = H_D \cos\theta + V_D \sin\theta$
- Radial shear, S is the resultant of H_D & V_D in the direction of s.
  $S = H_D \sin\theta - V_D \cos\theta$
- Prove that the shear force at any section of a 3-hinged parabolic arch subjected at well throughout is zero.

For parabolic arch at any section,





$$\frac{dy}{dx} = \frac{4h}{l^2}(1-2x) = \tan\theta$$

$$\sin\theta = \frac{4h(1-2x)}{\sqrt{l^4 + 16h^2(1-2x)^2}}$$

$$\cos\theta = \frac{l^2}{\sqrt{l^4 + lbh^2(1-2x)^2}}$$

$$S = H_D \sin\theta - V_D \cos\theta$$

$$= \frac{wl^2}{8h} \times \frac{4h(1-2x)}{\sqrt{l^4 + lbh^2(1-2x)^2}} - \frac{wl}{2}\frac{l^2}{\sqrt{l^4 + lbh^2(1-2x)^2}} + wx.\frac{l^2}{\sqrt{l^4 + 16h^2(1-2x)^2}}$$

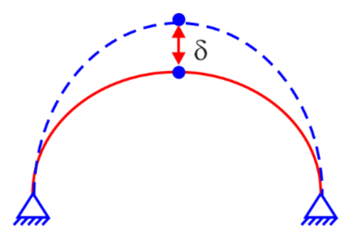
$$= 0$$

6.6. Effect of Temperature on 3 hinged Arches

As 3 hinged arch is statically determinate, no thermal stresses are developed. We know stresses depend upon BM. At a section. No stresses means no change in the moment of 3-hinged arch due to temperature change.

$$M = F_Z \Rightarrow F = \frac{M}{Z}$$

$$\partial = \left(\frac{l^2 + 4h^2}{4h}\right)\alpha T$$



As T ↑, y ↑, H ↓

$$M_{arch} = M_{beam} - Hy$$

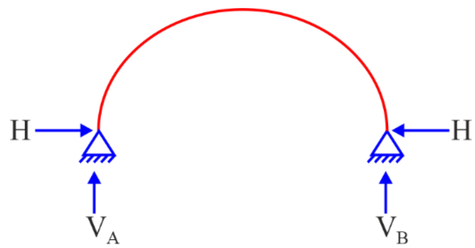
$\frac{dH}{H} = \frac{-dh}{h}$ –ve indicates that H and h vary in apposite directions.

A cantilever suby to temperature change uniformly or temperature gradient $\left(\frac{dT}{dy} = 0\right)$ zero, then the cantilever is free to elongate.

Here no resistance against deformation or no resistance against strain. No resistance means no strains

6.6.1. Two Hinged Arches

Assume supports of two hinged arch will not yield laterally. According to cartigliands theorem, if no deformation, assuming horizontal reaction as redundant,



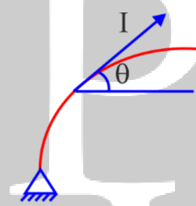
$$\frac{\partial U}{\partial H}=0\left(\frac{\partial U}{\partial R}=\partial\right)$$

$\Rightarrow$ $$H=\frac{\int M_y ds}{\int y^2 ds}\ ;\ M = \text{beam moment}$$

$H=\dfrac{\int M_y ds}{\int y^2 ds}$ is useful for arches like 3–hinged arch with udl throughout. For unsymmetrical loads, numerator and denominator of above equation are not integrable.

In order to analyse it is assumed that, $I = I_o \sec\theta$ at any section I where IO is moment of inertia at the crown with this assumption,

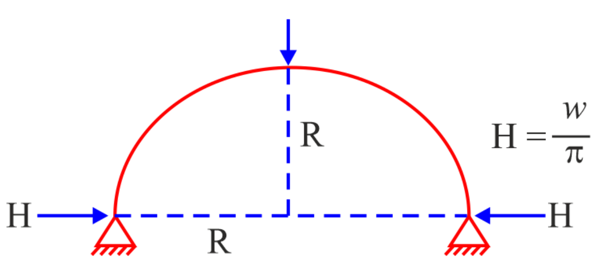
$$H=\frac{\int M\cdot y dx}{\int y^2 dx}$$



ILD for 2 hinged Arch:

Two hinged semi circular Arches

(1) Point load at Crown.



(2) Temperature effect on 2 hinged Arches

M_{arch}	$= M_{beam}$	$- Hy$
(change)	(const)	(changes)

As there is no change at the crown, y won't change

But H changes

As T ↑, H↑

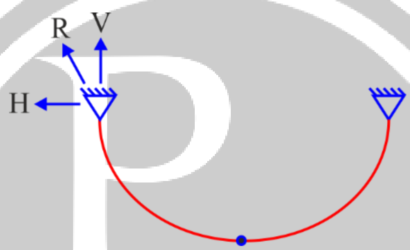
If temperature increases, H increases. If temperature increases, no change in the value of rise. Hence temperature will try to push the supports out. But they will not. In this process, H will increase. As H↑, Hy increases. M_{arch} decreases

Effects of Rib Shorbening in 2 hinged Arches

The effect of normal thrust in the arch is to shorten the rib of the arch and thus release part of horizontal thrust.

6.7. Cables

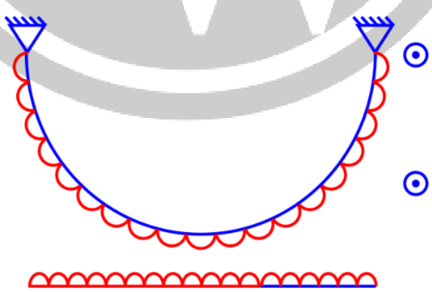
Assumptions:



(1) Cable is flexible

BM @ every point is zero.

(1) Self weight is neglected



- Load along the horizontal span-Shape of cable is parabola
- Udl is along the curve-shape of cable is catenary
- In chain surveying also, correction to sag is catenary

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