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NCERT Solutions for Class 10 Maths Chapter 8 PDF

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NCERT Solutions for Class 10 Maths Chapter 8 PDF

NCERT Solutions for Class 10 Maths Chapter 8 Introduction to Trigonometry

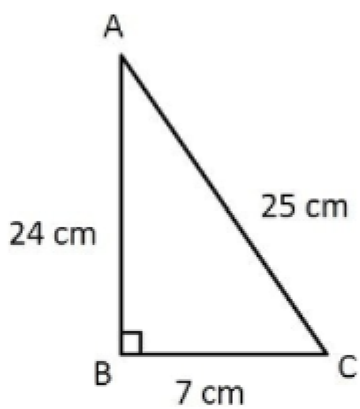
Exercise 8.1

1. In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine :
(i) sin A, cos A
(ii) sin C, cos C

Answer:

Let us draw a right angled triangle ABC, right angled at B.

Using Pythagoras theorem,



$AC^2 = AB^2 + BC^2$

[By using Pythagoras theorem]

$AC^2 = 24^2 + 7^2$

$AC^2 = 576 + 49$

$AC^2 = 625$

$AC = \sqrt{625}$

AC= 25

(i) Hypotenuse (AC) = 25

By definition,

sin A = $\frac{Perpendicular\ side\ opposite\ to\ angle\ A}{Hypotenuse}$

sin A = $\frac{BC}{AC}$

sin A = $\frac{7}{25}$

And,

cos A = $\frac{Base\ side\ adjacent\ to\ angle\ A}{Hypotenuse}$

cos A = $\frac{AB}{AC}$

cos A = $\frac{24}{25}$

sin C = $\frac{Perpendicular\ side\ opposite\ to\ angle\ C}{Hypotenuse}$

sin C = $\frac{AB}{AC}$

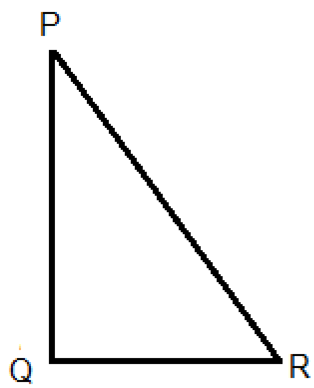
sin C = $\frac{24}{25}$

And,

(ii) cos C = $\frac{Base\ side\ adjacent\ to\ angle\ C}{Hypotenuse}$

$$\text{Cos C} = \frac{BC}{AC} = \frac{7}{25},$$

2. In adjoining figure, find tan P – cot R.



Answer:

using Pythagoras law we got

$QR^2 = PR^2 - PQ^2$

$\Rightarrow QR = \sqrt{PR^2 - PQ^2}$

$\Rightarrow QP = \sqrt{(13)^2 - (12)^2}$

$\Rightarrow QP = \sqrt{169 - 144}$

$\Rightarrow QP = \sqrt{25}$

$\Rightarrow QP = 5$

now

tan P - Cot R

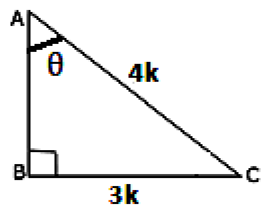
= 5/12 - 5/12

= 0

3. If sin A =3/4, calculate cos A and tan A.

Answer:

Given: A triangle ABC in which

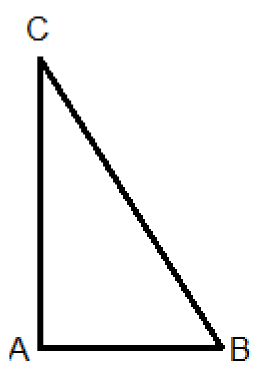


B =90

We know that sin A = BC/AC = 3/4
Let BC be 3k and AC will be 4k where k is a positive real number.
By Pythagoras theorem we get,
$AC^2 = AB^2 + BC^2$
$(4k)^2 = AB^2 + (3k)^2$
$16k^2 - 9k^2 = AB^2$
$AB^2 = 7k^2$
AB = $\sqrt{7}$ k
cos A = AB/AC = $\sqrt{7}$ k/4k = $\sqrt{7}/4$
tan A = BC/AB = 3k/$\sqrt{7}$ k = $3/\sqrt{7}$

4. Given 15 cot A = 8, find sin A and sec A.

Answer:



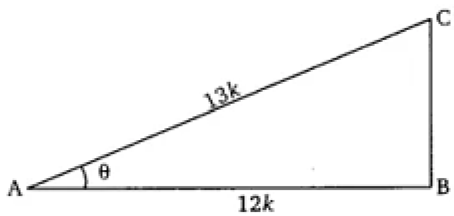
Let ΔABC be a right-angled triangle, right-angled at B.
We know that cot A = AB/BC = 8/15 (Given)
Let AB be 8k and BC will be 15k where k is a positive real number.
By Pythagoras theorem we get,
$AC^2 = AB^2 + BC^2$

$AC^2 = (8k)^2 + (15k)^2$
$AC^2 = 64k^2 + 225k^2$
$AC^2 = 289k^2$
AC = 17 k
sin A = BC/AC = 15k/17k = 15/17
sec A = AC/AB = 17k/8 k = 17/8

5. Given sec θ = 13/12, calculate all other trigonometric ratios.

Answer:

Consider a triangle ABC in which



Let AB = 12k and AC = 13k

Then, using Pythagoras theorem,

(AC)2 = (AB)2 + (BC)2

(13 *k*)2 = (12 *k*)2 + (BC)2

169 *k*2 = 144 *k*2 + BC2

25 *k*2 = BC2

BC = 5 *k*

(I) Sin $\theta = \frac{BC}{AC}$

$= \frac{5k}{13k} = \frac{5}{13}$

(ii) Cos θ = $\frac{AB}{AC}$

$= \frac{12k}{13k} = \frac{12}{13}$

(iii)Tan θ = $\frac{BC}{AB}$

= $\frac{5k}{12k}$ = $\frac{5}{12}$

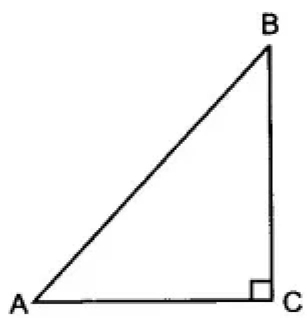
(iv) Cot θ = $\frac{AB}{BC}$

= $\frac{12k}{5k}$ = $\frac{12}{5}$

(v) Cosec θ = $\frac{AC}{BC}$

= $\frac{13k}{5k}$ = $\frac{13}{5}$

6. If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.



Answer:

cosA=cos

In a right angled triangle ABC,

cos A = $\frac{AC}{AB}$ and cos B = $\frac{BC}{AB}$

$\because$ cos A = cos B

$\frac{AC}{AB} = \frac{BC}{AB}$

$\therefore$ AC = BC

We have, opposite sides of equal angles are equal.

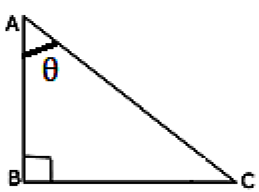
Therefore, In a right angled triangle ABC

But $\angle A = \angle B = 45^\circ$

7. If cot θ =7/8, evaluate :
(i)(1+sin θ)(1-sin θ)/(1+cos θ)(1-cos θ)
(ii) $\cot^2\theta$

Answer:

Consider a triangle ABC



$$\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$$

given = cot θ = $\frac{7}{8}$

Taking the numerator, we have

$(1+\sin\theta)(1-\sin\theta) = 1 - \sin^2\theta$ **[Since, $(a+b)(a-b) = a^2 - b^2$]**

Similarly,

$(1 + \cos\theta)(1 - \cos\theta) = 1 - \cos^2\theta$

We know that,

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow 1 - \cos^2\theta = \sin^2\theta$

And,

$1 - \sin^2 \theta = \cos^2 \theta$

Thus,

$(1 + \sin \theta)(1 - \sin \theta) = 1 - \sin^2 \theta = \cos^2 \theta$

$(1 + \cos \theta)(1 - \cos \theta) = 1 - \cos^2 \theta = \sin^2 \theta$

$\Rightarrow \frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$

$= \cos2 \theta / \sin2 \theta$

$= (\frac{\cos \theta}{\sin \theta})^2$

And, we know that

$(\frac{\cos \theta}{\sin \theta}) = \cot \theta$

$\frac{(1+\sin \theta)(1-\sin \theta)}{(1+\cos \theta)(1-\cos \theta)}$

$= (\cot \theta)^2$

$= (\frac{7}{8})^2$

(ii)

Given,

$\cot \theta = \frac{7}{8}$

So, by squaring on both sides we get

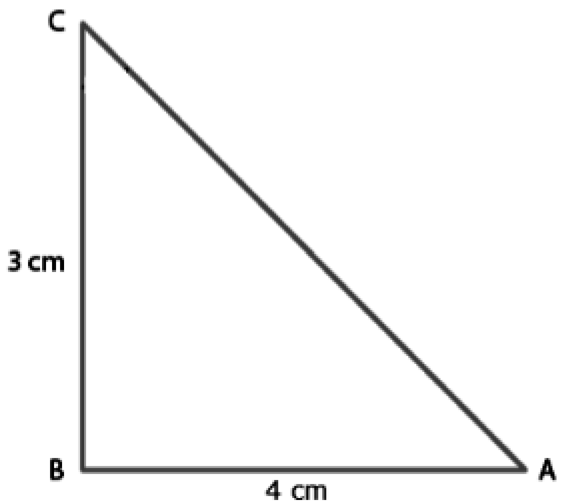
$(\cot \theta)^2 = (\frac{7}{8})^2$

$\therefore \cot^2 \theta = \frac{49}{64}$

8. If 3cot A = 4/3 , check whether $(1-\tan^2 A)/(1+\tan^2 A) = \cos^2 A - \sin^2 A$ or not.

Answer:

Consider a triangle ABC AB=4cm, BC= 3cm



3 cot A = 4

⇒ cot A = $\frac{4}{3}$

By definition,

tan A = $\frac{1}{Cot\ A}$ = $\frac{1}{(\frac{4}{3})}$

⇒ tan A = $\frac{3}{4}$

Thus, Base side adjacent to ∠A = 4

Perpendicular side opposite to ∠A = 3

In ΔABC, Hypotenuse is unknown

Thus, by applying Pythagoras theorem in ΔABC

We get

$AC^2 = AB^2 + BC^2$

$AC^2 = 4^2 + 3^2$

$AC^2 = 16 + 9$

$AC^2 = 25$

AC = 5

Hence, hypotenuse = 5

Now, we can find that

sin A = $\frac{opposite\ side\ to\ \angle A}{Hypotenuse}$ = $\frac{3}{5}$

And,

cos A = $\frac{adjacent\ side\ to\ \angle A}{Hypotenuse}$ = $\frac{4}{5}$

Taking the LHS,

L.H.S = $\frac{1-\tan^2 A}{1+\tan^2 A}$

Putting value of tan A

We get,

L.H.S= $\frac{1-(\frac{3}{4})^2}{1+(\frac{3}{4})^2}$

And $\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-(\frac{3}{4})^2}{1+(\frac{3}{4})^2}$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

Take L.C.M of both numerator and denominator;

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{\frac{16-9}{16}}{\frac{16+9}{16}}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{7}{25}$$

Thus, LHS = $\frac{7}{25}$

Now, taking RHS

R.H.S = $\cos^2 A - \sin^2 A$

Putting value of sin A and cos A

R.H.S= $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}^2\right)$

$$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}^2\right)$$

$$\cos^2 A - \sin^2 A = \frac{16}{25} - \frac{9}{25}$$

$$\cos^2 A - \sin^2 A = \frac{16-9}{25}$$

$$\cos^2 A - \sin^2 A = \frac{7}{25}$$

Therefore,

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$$

Hence Proved

9. In triangle ABC, right-angled at B, if tan A =1/√3 find the value of:

(i) sin A cos C + cos A sin C
(ii) cos A cos C – sin A sin C

Answer:

Consider a triangle ABC in which

$\tan A = \frac{1}{\sqrt{3}}$

$\frac{P}{B} = \frac{1}{\sqrt{3}}$

Let $P = k$ and $B = k\sqrt{3}$

Now by Pythagoras theorem

$P^2 + B^2 = H^2$

H=2k

.

(i)

$$SinACosC + CosASinC = \left(\frac{P}{H}\right)\left(\frac{B}{H}\right) + \left(\frac{B}{H}\right)\left(\frac{P}{H}\right)$$
$$= \left(\frac{BC}{AC}\right)\left(\frac{BC}{AC}\right) + \left(\frac{AB}{AC}\right)\left(\frac{AB}{AC}\right)$$
$$= \frac{k^2}{4k^2} + \frac{3k^2}{4k^2} = 1$$

(ii)

$$CosACosC - SinASinC = \left(\frac{P}{H}\right)\left(\frac{P}{H}\right) + \left(\frac{B}{H}\right)\left(\frac{B}{H}\right)$$
$$= \left(\frac{BC}{AC}\right)\left(\frac{AB}{AC}\right) - \left(\frac{AB}{AC}\right)\left(\frac{BC}{AC}\right)$$
$$= 0$$

10. In Δ PQR, right-angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

Answer:

Given that, PR + QR = 25 , PQ = 5
Let PR be x. ∴ QR = 25 - x
By Pythagoras theorem ,
PR2 = $PQ^2 + QR^2$
$x^2 = (5)2 + (25 - x)^2$
$x^2 = 25 + 625 + x^2 - 50x$
50x = 650
x = 13
∴ PR = 13 cm

QR = (25 - 13) cm = 12 cm
sin P = QR/PR = 12/13
cos P = PQ/PR = 5/13
tan P = QR/PQ = 12/5

11. State whether the following are true or false. Justify your answer.
(i) The value of tan A is always less than 1.
(ii) sec A = 12/5 for some value of angle A.
(iii) cos A is the abbreviation used for the cosecant of angle A.
(iv) cot A is the product of cot and A.
(v) sin θ = 4/3 for some angle θ.

Answer:

i) False.
In ΔABC in which ∠B = 90º,
AB = 3, BC = 4 and AC = 5
Value of tan A = 4/3 which is greater than.
The triangle can be formed with sides equal to 3, 4 and hypotenuse = 5 as it will follow the Pythagoras theorem.
$AC^2 = AB^2 + BC^2$
$5^2 = 3^2 + 4^2$
25 = 9 + 16
25 = 25

(ii) True.
Let a ΔABC in which ∠B = 90º,AC be 12k and AB be 5k, where k is a positive real number.
By Pythagoras theorem we get,
$AC^2 = AB^2 + BC^2$
$(12k)^2 = (5k)^2 + BC^2$
$BC^2 + 25k^2 = 144k^2$
$BC^2 = 119k^2$
Such a triangle is possible as it will follow the Pythagoras theorem.

(iii) False.
Abbreviation used for cosecant of angle A is cosec A.cos A is the abbreviation used for cosine of angle A.

(iv) False.
cot A is not the product of cot and A. It is the cotangent of ∠A.

(v) False.
sin θ = Height/Hypotenuse
We know that in a right angled triangle, Hypotenuse is the longest side.
∴ sin θ will always less than 1 and it can never be 4/3 for any value of θ.

Exercise 8.2

1. Evaluate the following :

(i) sin 60° cos 30° + sin 30° cos 60°

(ii) 2 $\tan^2$45° + $\cos^2$30° – $\sin^2$60°

(iii) cos 45°/(sec 30° + cosec 30°)

(iv) (sin 30° + tan 45° – cosec 60°)/(sec 30° + cos 60° + cot 45°)

(v) (5$\cos^2$60° + 4$\sec^2$30° - $\tan^2$45°)/($\sin^2$30° + $\cos^2$30°)

Answer:

(i) sin 60° cos 30° + sin 30° cos 60°

cos 60° = 1/2, cos 30° = √3/2, sin 60° = √3/2, sin 30° = 1/2

Substituting all the values in the given expression,

(1/2 × √3/2) + (√3/2 × 1/2)

= √3/4 + √3/4

= 2√3/4

= √3/2

As we know that:

cos 60° = 1/2, cos 30° = √3/2, sin 60° = √3/2, sin 30° = 1/2

Substituting all the values in the given expression,

(1/2 × √3/2) + (√3/2 × 1/2)

= √3/4 + √3/4

= 2√3/4

= √3/2

(ii) 2 $\tan^2$45° + $\cos^2$30° – $\sin^2$60°

2 tan45*tan45 + cos30*cos30 - sin60*sin60
= 2(1)(1) + cos(90-60)*cos(90-60) - sin60*sin60
= 2 + sin60*sin60 - sin60*sin60
= 2

Solution : 2

(iii) cos 45°/(sec 30° + cosec 30°)

(cos 45°)/(sec 30° + cosec 30°)

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}\left(2 + 2\sqrt{3}\right)} = \frac{\sqrt{3}}{2\sqrt{2} + 2\sqrt{6}}$$

$$= \frac{\sqrt{3}\left(2\sqrt{6} - 2\sqrt{2}\right)}{\left(\left(2\sqrt{6}\right) + 2\sqrt{2}\right)\left(2\sqrt{6} - 2\sqrt{2}\right)}$$

$$= \frac{2\sqrt{3}\left(\sqrt{6} - \sqrt{2}\right)}{\left(2\sqrt{6}\right)^2 - \left(2\sqrt{2}\right)^2} = \frac{2\sqrt{3}\left(\sqrt{6} - \sqrt{2}\right)}{24 - 8} = \frac{2\sqrt{3}\left(\sqrt{6} - \sqrt{2}\right)}{16}$$

$$= \frac{\sqrt{18} - \sqrt{6}}{8} = \frac{3\sqrt{2} - \sqrt{6}}{8}$$

(iv) (sin 30° + tan 45° – cosec 60°)/(sec 30° + cos 60° + cot 45°)

$$\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$$

$$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= \frac{\sqrt{3} + 2\sqrt{3} - 4}{4 + \sqrt{3} + 2\sqrt{3}}$$

$$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4}$$

$$= \frac{27 - 12\sqrt{3} - 12\sqrt{3} + 16}{27 - 16}$$

$$= \frac{43 - 24\sqrt{3}}{11}$$

(v) $(5\cos^2 60° + 4\sec^2 30° - \tan^2 45°)/(\sin^2 30° + \cos^2 30°)$

$$\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$$

$$\frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{15+64-12}{12}}{\frac{4}{4}} = \frac{67}{12}$$

2. Choose the correct option and justify your choice :

(i) $2\tan 30°/1+\tan^2 30°$ =

(A) sin 60° (B) cos 60° (C) tan 60° (D) sin 30°

(ii) $1-\tan^2 45°/1+\tan^2 45°$ =

(A) tan 90° (B) 1 (C) sin 45° (D) 0

(iii) $\sin^2 A = 2 \sin A$ is true when A =

(A) 0° (B) 30° (C) 45° (D) 60°

(iv) $2\tan 30°/1-\tan^2 30°$ =
(A) cos 60° (B) sin 60° (C) tan 60° (D) sin 30°

Answer:

(i) $2\tan 30°/1+\tan^2 30°$ =
(A) sin 60° (B) cos 60° (C) tan 60° (D) sin 30°

By substituting the values of given trigonometric ratios in the above equation, we get

$= 2 \times (1/\sqrt{3})/1 + (1/\sqrt{3})^2$

$= 2 \times (1/\sqrt{3})/(1 + 1/3)$

$= (2/\sqrt{3}) / (4/3)$

$= 6/4\sqrt{3}$

= $= \sqrt{3}/2$

(ii) $1-\tan^2 45°/1+\tan^2 45°$ =

(A) tan 90° (B) 1 (C) sin 45° (D) 0

By substituting the values of given trigonometric ratios for tan 45°.

$= 1 - (1)^2/1 + (1)^2$

$= (1 - 1)/(1 + 1)$

$= 0/2$

$= 0$

(iii) $\sin^2 A = 2 \sin A$ is true when A =

(A) 0° (B) 30° (C) 45° (D) 60°

0^0

Sin 0° = 0 and Sin 2 x 0° = Sin 0° = 0

Sin 30° = ½ But Sin 2 x 30° = Sin 60° is not equal to Sin 30°. The same holds true for sin 45°.

(iv) $2\tan 30° / 1 - \tan^2 30°$ =

(A) cos 60° (B) sin 60° (C) tan 60° (D) sin 30°

By substituting the values of given trigonometric ratios for tan 30°, we get

$= 2 \times (1/\sqrt{3}) / 1 - (1/\sqrt{3})^2$

$= (2/\sqrt{3}) / (1 - 1/3)$

$= (2/\sqrt{3}) / (2/3)$

$= \sqrt{3}$

Out of the given option only tan 60° = √3.

3. If tan (A + B) = √3 and tan (A – B) = 1/√3; 0° < A + B ≤ 90°; A > B, find A and B.

Answer:

⇒ tan (A + B) = tan 60°

⇒ (A + B) = 60° ... (i)

tan (A – B) = 1/√3

⇒ tan (A - B) = tan 30°
⇒ (A - B) = 30° ... (ii)

Adding (i) and (ii), we get

A + B + A - B = 60° + 30°

2A = 90°

A= 45°

Putting the value of A in equation (i)

45° + B = 60°

⇒ B = 60° - 45°

⇒ B = 15°

Thus, A = 45° and B = 15° ……….(i)

4. State whether the following are true or false. Justify your answer.

(i) sin (A + B) = sin A + sin B.
(ii) The value of sin θ increases as θ increases.
(iii) The value of cos θ increases as θ increases.
(iv) sin θ = cos θ for all values of θ.
(v) cot A is not defined for A = 0°.

Answer:

(i) False.
Let A = 30° and B = 60°, then
sin (A + B) = sin (30° + 60°) = sin 90° = 1 and,
sin A + sin B = sin 30° + sin 60°
= 1/2 + √3/2 = 1+√3/2

(ii) True.
sin 0° = 0
sin 30° = 1/2
sin 45° = 1/√2
sin 60° = √3/2
sin 90° = 1
Thus the value of sin θ increases as θ increases.

(iii) False.
cos 0° = 1
cos 30° = √3/2
cos 45° = 1/√2

cos 60° = 1/2
cos 90° = 0
Thus the value of cos θ decreases as θ increases.

(iv) True.
cot A = cos A/sin A
cot 0° = cos 0°/sin 0° = 1/0 = undefined.

Exercise 8.3

1. Evaluate :
(i) sin 18°/cos 72° (ii) tan 26°/cot 64° (iii) cos 48° – sin 42° (iv) cosec 31° – sec 59°

Answer:

(i) sin 18°/cos 72°
= sin (90° - 18°) /cos 72°
= cos 72° /cos 72° = 1

(ii) tan 26°/cot 64°
= tan (90° - 36°)/cot 64°
= cot 64°/cot 64° = 1

(iii) cos 48° - sin 42°
= cos (90° - 42°) - sin 42°
= sin 42° - sin 42° = 0

(iv) cosec 31° - sec 59°
= cosec (90° - 59°) - sec 59°
= sec 59° - sec 59° = 0

2. Show that :
(i) tan 48° tan 23° tan 42° tan 67° = 1
(ii) cos 38° cos 52° – sin 38° sin 52° = 0

Answer:
(i) tan 48° tan 23° tan 42° tan 67°
= tan (90° - 42°) tan (90° - 67°) tan 42° tan 67°
= cot 42° cot 67° tan 42° tan 67°
= (cot 42° tan 42°) (cot 67° tan 67°) = 1×1 = 1

(ii) cos 38° cos 52° - sin 38° sin 52°
= cos (90° - 52°) cos (90°-38°) - sin 38° sin 52°
= sin 52° sin 38° - sin 38° sin 52° = 0

3. If tan 2A = cot (A – 18°), where 2A is an acute angle, find the value of A.

Answer:
tan 2A = cot (A- 18°)
⇒ cot (90° - 2A) = cot (A -18°)
Equating angles,
⇒ 90° - 2A = A- 18° ⇒ 108° = 3A
⇒ A = 36°

4. If tan A = cot B, prove that A + B = 90°.

Answer:
tan A = cot B
⇒ tan A = tan (90° - B)
⇒ A = 90° - B
⇒ A + B = 90°

5. If sec 4A = cosec (A – 20°), where 4A is an acute angle, find the value of A.

Answer:
sec 4A = cosec (A - 20°)
⇒ cosec (90° - 4A) = cosec (A - 20°)
Equating angles,
90° - 4A= A- 20°
⇒ 110° = 5A
⇒ A = 22°

6. If A, B and C are interior angles of a triangle ABC, then show that
sin (B+C/2) = cos A/2

Answer:

In a triangle, sum of all the interior angles

$A + B + C = 180°$

$\Rightarrow B + C = 180° - A$

$\Rightarrow (B+C)/2 = (180°-A)/2$

$\Rightarrow (B+C)/2 = (90°-A/2)$

$\Rightarrow \sin (B+C)/2 = \sin (90°-A/2)$

$\Rightarrow \sin (B+C)/2 = \cos A/2$

7. Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Answer:

$\sin 67° + \cos 75°$
$= \sin (90° - 23°) + \cos (90° - 15°)$
$= \cos 23° + \sin 15°$

Exercise 8.4

1. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

We know that

$\text{cosec}^2 A = 1 + \cot^2 A$

$$\frac{1}{\cos ec^2 A} = \frac{1}{1+\cot^2 A}$$

$$\sin^2 A = \frac{1}{1+\cot^2 A}$$

$$\sin A \; = \pm \frac{1}{\sqrt{1+\cot^2 A}}$$

$\sqrt{1+\cot^2 A}$ will always be positive as we are adding two positive quantities.

Therefore

$$\sin A \; = \frac{1}{\sqrt{1+\cot^2 A}}$$

we know that

$$\tan A = \frac{\sin A}{\cos A}$$

However

$$\cot A = \frac{\cos A}{\sin A}$$

Therfore tan A = 1/cot A

Also $\sec^2 A = 1+\tan^2 A$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

2. Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer:

$\sin^2 A + \cos^2 A = 1$

$\text{cosec}^2 A = 1 + \cot^2 A$

$\sec^2 A = 1 + \tan^2 A$

We know that,

$\cos A = 1/\sec A$ .....Equation (1)

Also,

$\sin^2 A + \cos^2 A = 1$ (trigonometric identity)

$\sin^2 A = 1 - \cos^2 A$ (By transposing)

Using value of cos A from Equation (1) and simplifying further

$\sin A = \sqrt{}1 - (1 / \sec A)^2$

$= \sqrt{}(\sec^2 A - 1) / \sec^2 A$

$= \sqrt{(\sec^2 A - 1)} / \sec A$ ....Equation (2)

$\tan^2 A + 1 = \sec^2 A$ (Trigonometric identity)

$\tan^2 A = \sec^2 A - 1$ (By transposing)

$\tan A = \sqrt{(\sec^2 A - 1)}$ ... Equation (3)

cot A = cosA/sinA

$= (1/\sec A) / [\sqrt{(\sec^2 A - 1)}/\sec A]$ .....(By substituting the values from Equations (1) and (2))

$= 1 / \sqrt{(\sec^2 A - 1)}$

cosec A = 1/sin A

$= \sec A / \sqrt{(\sec^2 A - 1)}$ (By substituting from Equation (2) and simplifying)

3. Evaluate :

(i) $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°)$
(ii) sin 25° cos 65° + cos 25° sin 65°

Answer:

(i) $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°)$

$$\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^\circ 73^\circ}$$

$$= \frac{[\sin(90^\circ - 27^\circ)]^2 + \sin^2 27^\circ}{[\cos(90^\circ - 73^\circ)]^2 + \cos^2 73^\circ}$$

$$= \frac{[\cos 27^\circ]^2 + \sin^2 27^\circ}{[\sin 73^\circ]^2 + \cos^2 73^\circ}$$

$$= \frac{\cos^2 27^\circ + \sin^2 27^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$$

= 1/1 (As $\sin^2 A + \cos^2 A = 1$)

= 1

(ii) sin 25° cos 65° + cos 25° sin 65°

sin25° cos65° + cos25° sin65°

=(sin 25°) {cos(90°-25°)}+cos 25°{sin(90°-25)}

=(sin 25°)(sin 25°) + (cos 25°)(cos 25°)

= $\sin^2 25^\circ + \cos^2 25^\circ$

= 1 (As $\sin^2 A + \cos^2 A = 1$)

4. (i) 9 $\sec^2 A$ - 9 $\tan^2 A$ =

(A) 1 (B) 9 (C) 8 (D) 0

(ii) (1 + tan θ + sec θ) (1 + cot θ - cosec θ)

(A) 0 (B) 1 (C) 2 (D) - 1

(iii) (secA + tanA) (1 - sinA) =

(A) secA (B) sinA (C) cosecA (D) cosA

(iv) 1+$\tan^2 A$/1+$\cot^2 A$ =

(A) $\sec^2 A$ (B) -1 (C) $\cot^2 A$ (D) $\tan^2 A$

Answer:

(i) **(i) 9 $\sec^2 A$ - 9 $\tan^2 A$ =**

(A) 1 (B) 9 (C) 8 (D) 0

$\sin^2 A + \cos^2 A = 1$

$\text{cosec}^2 A = 1 + \cot^2 A$

$\sec^2 A = 1 + \tan^2 A$

(i) $9 \sec^2 A - 9 \tan^2 A$

$= 9 (\sec^2 A - \tan^2 A)$

$= 9 \times 1$ [By using the identity, $1 + \sec^2 A = \tan^2 A$]

$= 9$

Thus, option (B) is the correct answer.

(ii) (1 + tan θ + sec θ) (1 + cot θ - cosec θ)

(A) 0 (B) 1 (C) 2 (D) - 1

L.H.S $= (1 + \cot\theta - \text{cosec}\,\theta)(1 + \tan\theta + \sec\theta)$

$$= \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$

$$= \left(\frac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)\left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)$$

$$= \frac{1}{\sin\theta\cos\theta}\begin{pmatrix}\sin\theta\cos\theta + \sin^2\theta + \sin\theta + \cos^2\theta \\ + \sin\theta\cos\theta + \cos\theta - \cos\theta - \sin\theta - 1\end{pmatrix}$$

$$= \frac{1}{\sin\theta\cos\theta}\left(2\sin\theta\cos\theta + \left(\sin^2\theta + \cos^2\theta\right) - 1\right)$$

$$= \frac{1}{\sin\theta\cos\theta}(2\sin\theta\cos\theta + 1 - 1)$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$= 2$

$=$ R.H.S

(iii) (secA + tanA) (1 - sinA) =

(A) secA (B) sinA (C) cosecA (D) cosA

(secA + tanA) (1 − sinA)

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

= cosA

Hence, alternative cosA is correct

(iv) 1+tan²A/1+cot²A =

(A) sec²A (B) -1 (C) cot²A (D) tan²A

$$\text{LHS} = \left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \frac{\sec^2 A}{\text{cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

$$\text{RHS} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$$

$$= \left(\frac{1-\tan A}{\frac{\tan A - 1}{\tan A}}\right)^2 = \left(\frac{1-\tan A}{\tan A - 1} \times \tan A\right)^2 = (-\tan A)^2 = \tan^2 A$$

LHS = RHS.

5. Prove the following identities, where the angles involved are acute angles for which the

expressions are defined.

(i) (cosec θ - cot θ)^2 = (1-cos θ)/(1+cos θ)

(ii) cos A/(1+sin A) + (1+sin A)/cos A = 2 sec A

(iii) tan θ/(1-cot θ) + cot θ/(1-tan θ) = 1 + sec θ cosec θ

[Hint : Write the expression in terms of sin θ and cos θ]

(iv) (1 + sec A)/sec A = sin^{2}A/(1-cos A)

[Hint : Simplify LHS and RHS separately]

(v) (cos A–sin A+1)/(cos A+sin A–1) = cosec A + cot A,using the identity cosec^{2}A = 1+cot^{2}A.

(vi) √1 + sin A/1 - sin A = sec A+ tan A

(vii) (sin θ - 2sin^{3}θ)/(2cos^{3}θ-cos θ) = tan θ

(viii) (sin A + cosec A)^2 + (cos A + sec A)^2 = 7+tan^{2}A+cot^{2}A

(ix) (cosec A – sin A)(sec A – cos A) = 1/(tan A+cotA)

[Hint : Simplify LHS and RHS separately]

(x) (1+tan^{2}A/1+cot^{2}A) = (1-tan A/1-cot A)^2 = tan^{2}A

Answer:

(i) (cosec θ - cot θ)^2 = (1-cos θ)/(1+cos θ)

L.H.S. = (1 – cos θ)/(1 + cos θ)

= (1 – cos θ)/(1 + cos θ) × (1 – cos θ)/(1 – cos θ)

= (1 – cos θ)^2/(1 – cos^2 θ)

= (1 – cos θ)^2/sin^2 θ

= [1 – cos θ/sin θ]^2

= [1 /sin θ – cos θ/sin θ]^2

= [cosec θ - cot θ]^2

= [- (cot θ - cosec θ)]^2

= (cot θ – cosec θ)^2

= R.H.S.

(ii) cos A/(1+sin A) + (1+sin A)/cos A = 2 sec A

LHS = Cos A/(1+ sin A) + (1+ sin A)/cos A

$$= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$$

$$= \frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{\cos A(1+\sin A)}$$

$$= \frac{1+1+2\sin A}{\cos A(1+\sin A)}$$

$$= \frac{2(1+\sin A)}{\cos A(1+\sin A)}$$

$$= \frac{2}{\cos A} = 2\sec A$$

(iii) tan θ/(1-cot θ) + cot θ/(1-tan θ) = 1 + sec θ cosec θ

[Hint : Write the expression in terms of sin θ and cos θ]

$$\text{LHS} = \frac{\tan\theta}{(1-\cot\theta)} + \frac{\cot\theta}{(1-\tan\theta)}$$

$$= \frac{\tan\theta}{\left(1-\frac{\cos\theta}{\sin\theta}\right)} + \frac{\cot\theta}{\left(1-\frac{\sin\theta}{\cos\theta}\right)}$$

$$= \frac{\sin\theta\tan\theta}{(\sin\theta-\cos\theta)} + \frac{\cos\theta\cot\theta}{(\cos\theta-\sin\theta)}$$

$$= \frac{\sin\ \theta\ \times \frac{\sin\theta}{\cos\theta}\cos\theta \times \frac{\cos\theta}{\sin\theta}}{(\sin\theta-\cos\theta)}$$

$$= \frac{\frac{\sin^2\theta}{\cos\theta}\ \frac{\cos^2\theta}{\sin\theta}}{(\sin\theta-\cos\theta)}$$

$$\sin^3\theta - \cos^3\theta$$

$= \frac{\sin^3\theta - \cos^3\theta}{\cos\theta\sin\theta(\sin\theta - \cos\theta)}$

$= \frac{(\sin\theta - \cos\theta)\left(\sin^2\theta + \sin\theta\cos\theta + \cos^2\theta\right)}{\cos\theta\sin\theta(\sin\theta - \cos\theta)}$

$= \frac{1 + \sin\theta\cos\theta}{\cos\theta\sin\theta}$

$= \frac{1}{\cos\theta\sin\theta} + \frac{\sin\theta\cos\theta}{\cos\theta\ \sin\theta}$

$= \frac{1}{\cos\theta\sin\theta} + \frac{\sin\theta\cos\theta}{\cos\theta\sin\theta}$

$= \sec\theta\cos ec\ \theta + 1$

$= 1 + \sec\theta\ \cos ec\ \theta$

=RHS

(iv) (1 + sec A)/sec A = $\sin^2$A/(1-cos A)

[Hint : Simplify LHS and RHS separately]

We know that, $\sin^2\theta + \cos^2\theta = 1$

$\frac{1 + \sec\theta}{\sec\theta} = \frac{1 + \frac{1}{\cos\theta}}{\frac{1}{\cos\theta}}$

$= \frac{\frac{\cos\theta + 1}{\cos\theta}}{\frac{1}{\cos\theta}}$

$= \frac{1 + \cos\theta}{1}$

Multiplying the numerator and denominator by $(1 - \cos\theta)$ we have

$$\frac{1+\sec\theta}{\sec\theta} = \frac{(1+\cos\theta)(1-\cos\theta)}{1-\cos\theta}$$

$$= \frac{1-\cos^2\theta}{1-\cos\theta}$$

$$= \frac{\sin^2\theta}{1-\cos\theta}$$

(v) (cos A–sin A+1)/(cos A+sin A–1) = cosec A + cot A,using the identity $\text{cosec}^2 A = 1+\cot^2 A$

L.H.S = cos A - sin A + 1/(cos A + sin A - 1)

Diving both numerator and denominator by sin A

= [cos A/sin A - sin A/sin A + 1/sin A]/[cos A/sin A + sin A/sin A - 1/sin A]

We know that the trigonometric functions,

cot (x) = cos(x)/sin (x) = 1/tan (x)

cosec(x) = 1/sin (x)

We get,

⇒ (cot A - 1 + cosec A) / (cot A + 1 - cosec A)

⇒ cot A - (1 - cosec A) / cot A+ (1 - cosec A)

We know that, $1 + \cot^2 A = \text{Cosec}^2 A$

Hence multiplying [cot A - (1 - cosec A)] in numerator and denominator

$= [(\cot A) - (1 - \text{cosec } A) \times (\cot A) - (1 - \text{cosec } A)]/[(\cot A) + (1 - \text{cosec } A) - \times (\cot A) - (1 - \text{cosec } A)]$

$= [\cot A - (1 - \text{cosec } A)]^2/[(\cot A)^2 - (1 - \text{cosec} A)^2]$

$= [\cot^2 A + (1 - \text{cosec} A)^2 - 2\cot A(1 - \text{cosec} A)]/[\cot^2 A - (1 + \text{cosec}^2 A - 2\text{cosec } A)]$

$= (\cot^2 A + 1 + \text{cosec}^2 A - 2\text{cosec } A - 2\cot A + 2\cot A \text{cosec} A)/(\cot^2 A - (1 + \text{cosec}^2 A - 2\text{cosec } A))$

$= (2\text{cosec}^2 A + 2\cot A \text{ cosec } A - 2\cot A - 2\text{cosec} A)/(\cot^2 A - 1 - \text{cosec}^2 A + 2\text{cosec} A)$

$= 2\text{cosec } A(\text{cosec } A + \cot A) - 2(\cot A + \text{cosec } A)/(\cot^2 A - \text{cosec}^2 A - 1 + 2\text{cosec } A)$

$= (\text{cosec } A + \cot A)(2\text{cosec } A - 2)/(-1 - 1 + 2\text{cosec } A)$

$= (\text{cosec } A + \cot A)(2\text{cosec } A - 2)/(2\text{cosec } A - 2)$

$= \text{cosec } A + \cot A$

= R.H.S

(vi) $\sqrt{1 + \sin A/1 - \sin A} = \sec A + \tan A$

LHS = $1 + \sin A/(1 - \sin A)$ .....(1)

Multiplying and dividing by $(1 + \sin A)$

$\Rightarrow (1 + \sin A)(1 + \sin A/1 - \sin A)(1 + \sin A)$

$= (1 + \sin A)^2/(1 - \sin^2 A)$ $[a^2 - b^2 = (a - b)(a + b)]$

$= (1 + \sin A)/1 - \sin^2 A$

$= 1 + \sin A/\cos^2 A$

$= 1 + \sin A/\cos A$

$= 1/\cos A + \sin A/\cos A$

$= \sec A + \tan A$

= R.H.S

(vii) $(\sin\theta - 2\sin^3\theta)/(2\cos^3\theta - \cos\theta) = \tan\theta$

L.H.S = $(\sin\theta - 2\sin^3\theta)/(2\cos\theta - \cos\theta)$

Taking Sin θ and Cos θ common in both numerator and denominator respectively.

$\sin\theta\,(1 - 2\sin^2\theta)/\cos\theta\,(2\cos^2\theta - 1)$

By Identity $\sin^2 A + \cos^2 A = 1$ hence, $\cos^2 A = 1 - \sin^2 A$ and substituting this in the above equation,

$\Rightarrow \sin\theta\,(1 - 2\sin^2\theta)/\cos\theta\,\{2(1 - \sin^2\theta) - 1\}$

$= \sin\theta\,(1 - 2\sin^2\theta)/\cos\theta(2 - 2\sin^2\theta - 1)$

$= \sin\theta\,(1 - 2\sin^2\theta)/\cos\theta(1 - 2\sin^2\theta)$

$= \sin\theta/\cos\theta$

$= \tan\theta$

= R.H.S

(viii) $(\sin A + \text{cosec } A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

L.H.S =-(sin A + cosec A)² + (cos A + sec A)²

By using $(a + b)^2 = a^2 + 2ab + b^2$

⇒ $\sin^2 A + \text{cosec}^2 A + 2\sin A \,\text{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$

By rearranging and using sec A = 1/cos A and cosec A = 1/sin A

⇒ $(\sin^2 A + \cos^2 A) + (\text{cosec}^2 A + \sec^2 A)+ 2 \sin A (1/\sin A) +\text{-}2\cos A (1/\cos A)$

Hence $(\sin^2 A + \cos^2 A) = 1$, $\text{cosec}^2 A = (1 + \cot^2 A)$ and $(\sec^2 A\text{-} \tan^2 A) = 1$

⇒ $1 + 1 + \cot^2 A + 1 + \tan^2 A + 2 + 2$

$= 7 + \tan^2 A + \cot^2 A$

= R.H.S

(ix) (cosec A – sin A)(sec A – cos A) = 1/(tan A+cotA)

[Hint : Simplify LHS and RHS separately]

L.H.S = (cosec A - sin A)(sec A - cos A) .....Equation (1)

We know that the trigonometric functions,

sec (x) = 1/cos (x)

cosec (x) = 1/sin (x)

By substituting the above relations in Equation (1)

⇒ (1/sin A - sin A)(1/cos A - cos A)

= $(1 - \sin^2 A)/\sin A \times (1 - \cos^2 A)/\cos A$

= $\cos^2 A \sin^2 A/\sin A \cos A$

= sin A cos A/1

= $\sin A \cos A/(\sin^2 A + \cos^2 A)$ $[(\sin^2 A + \cos^2 A) = 1]$

= $1/\sin^2 A + \cos^2 A$ [Dividing numerator and denominator by (sin A cos A)]

= $1/ [(\sin^2 A/\sin A \cos A) + (\cos^2 A/\sin A \cos A)]$

= 1/[(sin A/cos A) + (cos A/sin A)]

= 1/(tan A + cot A)

= RHS

(x) $(1+\tan^2 A/1+\cot^2 A) = (1-\tan A/1-\cot A)^2 = \tan^2 A$

Taking LHS, $(1 + \tan^2 A)/(1 + \cot^2 A)$

$= \sec^2 A/\text{cosec}^2 A$

$= (1/\cos^2 A)/(\sin^2 A)$

$= (1/\cos^2 A) \times (\sin^2 A/1)$

$= \tan^2 A$

= RHS

Taking,-$(1 - \tan A/1 - \cot A)^2$

$= [(1 - \tan A)/(1 - 1/\tan A)]^2$

$= [(1 - \tan A)/(\tan A - 1)/\tan A]^2$

$= ((1 - \tan A) \times \tan A(\tan A - 1))^2$

$=(-\tan A)^2$

$= \tan^2 A$

= R.H.S

Hence, L.H.S = R.H.S.