JEE Mains (12th)

Sample Paper -V

DURATION : 180 Minutes

M. MARKS : 300

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(1)	31.	(2)	61.	(4)
2.	(4)	32.	(2)	62.	(2)
3.	(3)	33.	(4)	63.	(4)
4.	(1)	34.	(1)	64.	(2)
5.	(2)	35.	(2)	65.	(3)
6.	(2)	36.	(1)	66.	(1)
7.	(2)	37.	(2)	67.	(2)
8.	(2)	38.	(2)	68.	(4)
9.	(4)	39.	(3)	69.	(3)
10.	(3)	40.	(3)	70.	(4)
11.	(2)	41.	(4)	71.	(2)
12.	(3)	42.	(1)	72.	(4)
13.	(1)	43.	(4)	73.	(2)
14.	(3)	44.	(1)	74.	(1)
15.	(1)	45.	(2)	75.	(3)
16.	(3)	46.	(3)	76.	(2)
17.	(1)	47.	(1)	77.	(4)
18.	(3)	48.	(4)	78.	(1)
19.	(1)	49.	(1)	79.	(2)
20.	(4)	50.	(3)	80.	(2)
21.	(1)	51.	(5)	81.	(0)
22.	(4)	52.	(5)	82.	(5)
23.	(5)	53.	(4)	83.	(12)
24.	(6)	54.	(12)	84.	(33)
25.	(1)	55.	(7)	85.	(11)
26.	(5)	56.	(4)	86.	(5)
27.	(7)	57.	(9)	87.	(2)
28.	(1)	58.	(4)	88.	(5)
29.	(4)	59.	(9)	89.	(16)
30.	(4)	60.	(8)	90.	(43)

PHYSICS

1. **(1)**

$M_1 = 10$ g $M_2 = 1$ kg
$L_1 = 10$ cm $L_2 = 1$ m
$T_1 = 0.1$ s $T_2 = 1$ s
$n_1 = 1$ $n_2 = ?$

The dimensional formula of force is [MLT^{-2}].

$\therefore\ a = 1, b = 1, c = -2$

$$n_2 = n_2\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

$$= 1\left(\frac{10\text{ g}}{1\text{ kg}}\right)^1\left(\frac{10\text{ cm}}{1\text{ m}}\right)^1\left(\frac{0.1\text{ s}}{1\text{ s}}\right)^{-2}$$

$$= 1\left(\frac{10^{-2}\text{ kg}}{1\text{ kg}}\right)^1\left(\frac{10^{-1}\text{ m}}{1\text{ m}}\right)^1\left(\frac{0.1\text{ s}}{1\text{ s}}\right)^{-2}$$

$$= \frac{1\times10^{-2}\times10^{-1}}{10^{-2}} = 10^{-1} = 0.1$$

Hence, the unit of force in a given system will equivalent to 0.1 N.

2. **(4)**

Given: $\upsilon = 24\times10^6$ Hz, $R = 0.60$ m

We know that, $R = \frac{mv}{qB}$

$\therefore\quad B = \frac{mv}{qR}$ …(i)

where $v = \omega R = 2\pi\upsilon R$

$\Rightarrow v = 2\pi\times24\times10^6\times0.60 = 9.04\times10^7\ \text{ms}^{-1}$

From (i), we get

$$B = \frac{(3.34\times10^{-27})(9.04\times10^7)}{1.6\times10^{-19}\times0.60} = 3.2\text{ T}$$

3. **(3)**

Let u be the initial speed and θ is the angle of the projection.

As per question, Speed at the maximum height

$$v_H = u\cos\theta = \frac{\sqrt{3}}{2}u$$

$$\therefore\ \cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = 30^\circ$$

Range, $R = \frac{u^2\sin2\theta}{g}$

Maximum height, $H = \frac{u^2\sin^2\theta}{2g}$

As $R = PH$ (Given)

$$\therefore\ \frac{u^2\sin2\theta}{g} = P\frac{u^2\sin^2\theta}{2g}$$

$$\Rightarrow 2\sin\theta\cos\theta = \frac{P}{2}\sin^2\theta \Rightarrow \tan\theta = \frac{4}{P}$$

or $P = \frac{4}{\tan30^\circ} = 4\sqrt{3}$

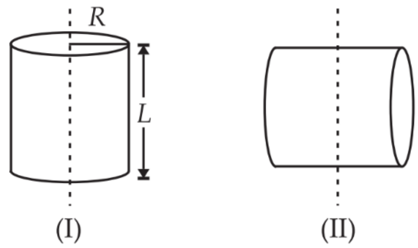
4. **(1)**

Equator, $\theta = 0^\circ$, $B_H = B_e\cos\theta = 5\cos0^\circ = 5$

At poles, $\theta = 90^\circ$, $V = B_e\sin\theta = 5\sin90^\circ = 5$.

Thus, total intensity of earth's magnetic field at poles is also 5 units, but in the vertical direction. At equator, value is the same, but the direction is horizontal.

5. **(2)**



(I) Moment of inertia of a cylinder about an axis passing through its centre and normal to its circular face = $\frac{MR^2}{2}$

(II) Moment of inertia of the same cylinder about an axis passing through its centre and perpendicular to its length = $M\left(\frac{L^2}{12}+\frac{R^2}{4}\right)$

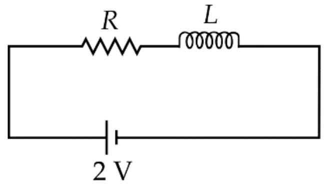
According to the question,

$$\frac{ML^2}{12}+\frac{MR^2}{4} = \frac{MR^2}{2}\ \text{ or } L = \sqrt{3}R$$

6. **(2)**

The current at any instant is given by

$$\Rightarrow\ I = I_0(1-e^{-Rt/L})$$



$$\Rightarrow\ \frac{I_0}{2} = I_0(1-e^{-Rt/L})$$

or $\frac{1}{2} = (1-e^{-Rt/L})$

or $e^{-RT/L} = \frac{1}{2}$ or $\frac{Rt}{L} = \ln 2$

$$\therefore\ t = \frac{L}{R}\ln2 = \frac{300\times10^{-3}}{2}\times0.693$$

$= 150\times0.693\times10^{-3} = 0.10395$ s = 0.1 s

7. **(2)**

Let v be the velocity of projection of the body from the surface of the earth.

According to conservation of energy, we get

$$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{(R+h)}$$

$$\Rightarrow\ v^2 - \frac{2GM}{R} = -\frac{2GM}{(R+h)}$$

As per questions,

$$v=\frac{3}{4}v_e=\frac{3}{4}\sqrt{\frac{2GM}{R}}\quad\left(\because\ v_e=\sqrt{\frac{2GM}{R}}\right)$$

$$\therefore\ \left(\frac{3}{4}\sqrt{\frac{2GM}{R}}\right)^2-\frac{2GM}{R}=-\frac{2GM}{(R+h)}$$

$$\Rightarrow\ \frac{9}{16}\left(\frac{2GM}{R}\right)-\frac{2GM}{R}=-\frac{2GM}{(R+h)}$$

$$\Rightarrow\ \frac{9}{16R}-\frac{1}{R}=-\frac{1}{(R+h)}$$

$$\Rightarrow\ \frac{7}{16R}=-\frac{1}{R+h}\Rightarrow 16R=7R+7h$$

$$\therefore\ h=\frac{9}{7}R$$

8. (2)

9. (4)

For 16 g of helium, $\mu_1=\frac{16}{4}=4$

For 16 g of oxygen, $\mu_2=\frac{16}{32}=\frac{1}{2}$

For mixture of gases,

$$C_V=\frac{\mu_1 C_{V_1}+\mu_2 C_{V_2}}{\mu_1+\mu_2}\ \text{where } C_V=\frac{f}{2}R$$

$$C_P=\frac{\mu_1 C_{P_1}+\mu_2 C_{P_2}}{\mu_1+\mu_2}\ \text{where } C_P=\left(\frac{f}{2}+1\right)R$$

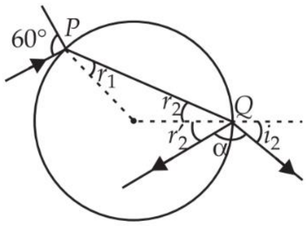
For helium, $f = 3$, $\mu_1 = 4$

For oxygen, $f = 5$, $\mu_2 = \frac{1}{2}$

$$\therefore\ \frac{C_P}{C_V}=\frac{\left(4\times\frac{5}{2}R\right)+\left(\frac{1}{2}\times\frac{7}{2}R\right)}{\left(4\times\frac{3}{2}R\right)+\left(\frac{1}{2}\times\frac{5}{2}R\right)}=\frac{47}{29}=1.62.$$

10. (3)

For refraction at P, $\frac{\sin 60°}{\sin r_1}=\sqrt{3}$



$$\Rightarrow \sin r_1=\frac{1}{2}\Rightarrow r_1=30°$$

Since $r_2=r_1\ \therefore r_2=30°$

For refraction at Q, $\frac{\sin r_2}{\sin i_2}=\frac{1}{\sqrt{3}}$

$$\Rightarrow\ \frac{\sin 30°}{\sin i_2}=\frac{1}{\sqrt{3}}\Rightarrow\ \sin i_2=\frac{\sqrt{3}}{2}$$

$i_2=60°$

11. (2)

All the non-zero digits are significant. The trailing zero (s) in a number with decimal point are significant. Power of 10 is irrelevant for the determination of significant figures. Hence, 4.8000 × 104 has 5 significant figures.

All the non-zero digits are significant. All the zeros between two non-zero digits are significant, no matter where the decimal point is. The trailing zero(s) in a number with decimal point are significant. Hence, 48000.50 has 7 significant figures.

12. (3)

In first case, potential gradient, $K=\frac{\varepsilon_0}{l}$ where ε_0 is the emf of the battery in potentiometer circuit. As per question

$$\varepsilon=\frac{Kl}{5}=\frac{\varepsilon_0}{l}\times\frac{l}{5}=\frac{\varepsilon_0}{5}$$

In second case, length of potentiometer wire

$$=l+\frac{l}{2}=\frac{3l}{2}$$

Potential gradient, $K'=\frac{\varepsilon_0}{3l/2}=\frac{3\varepsilon_0}{3l}$

If l' is the new balancing length, then

$$\varepsilon=\frac{\varepsilon_0}{5}=\frac{2\varepsilon_0}{3l}\times l'\ \text{or}\ l'=\frac{3}{10}l$$

13. (1)

Time taken by body A, t_1 = 5 s

Acceleration of body A = a_1

Time taken by body B, t_2 = 5 – 2 = 3 s

Acceleration of body B = a_2

Distance covered by first body in 5th second after its start,

$$S_5=u+\frac{a_1}{2}(2t_1-1)=0+\frac{a_1}{2}(2\times5-1)\frac{9}{2}a_1$$

Distance covered by the second body in the 3rd second after its start,

$$S_3=u+\frac{a_2}{2}(2t_2-1)=0+\frac{a_2}{2}(2\times3-1)=\frac{5}{2}a_2$$

Since $S_5=S_3$

$$\therefore\ \frac{9}{2}a_1=\frac{5}{2}a_2$$

or $a_1:a_2=5:9$

14. (3)

Magnetic moment, M = IA

$$M=\frac{qv}{2\pi r}\times\pi r^2\left(q\times\frac{\omega}{2\pi}\right)\pi r^2=\frac{1}{2}q\omega r^2$$

Angular momentum,

$L = mvr = m(\omega r)r = m\omega r^2$

$$\therefore\ \frac{M}{L}=\frac{\frac{1}{2}q\omega r^2}{m\omega r^2}=\frac{1}{2}\frac{q}{m}$$

15. **(1)**

Here, M = 2000 kg, m = 10 g = 0.01 kg

Force on car = rate of change of momentum of bullets

$F = nmv = 10 \times 0.01 \times 500 = 50$ N

$a = \frac{F}{M} = \frac{50}{2000} = 0.025 \text{ m s}^{-2}$

16. **(3)**

In case of a solenoid as $B = \mu_0 nI$,

$\phi = B(nlS) = \mu_0 n^2 lSI$ and hence

$$L = \frac{\phi}{I} = \mu_0 n^2 lS = \mu_0 \frac{N^2}{l} S \qquad \left(as\ n = \frac{N}{l}\right)$$

When N and l are doubled, then

$$L' = \mu_0 \frac{(2N)^2}{2l} S = 2\mu_0 \frac{N^2}{l} S = 2L$$

i.e., inductance of the solenoid will be doubled.

17. **(1)**

Let the mass of the unexploded bomb be $5m$.

It explodes into the two pieces of masses m and $4m$ respectively.

Initial momentum of the unexploded bomb

$5m\left(40\hat{i} + 50\hat{j} - 25\hat{k}\right)$

After explosion, momentum of the smaller piece = $m\vec{v}_1 = m\left(200\hat{i} + 70\hat{j} + 15\hat{k}\right)$

and momentum of the larger piece = $4m\vec{v}_2$

where $\vec{v}_1$ and $\vec{v}_2$ are the velocities of the two pieces respectively.

According to law of conservation of momentum, we get

$$5m\left(40\hat{i} + 50\hat{j} - 25\hat{k}\right) = m\left(200\hat{i} + 70\hat{j} + 15\hat{k}\right) + 4m\vec{v}_2$$

$$4m\vec{v}_2 = 5m\left(40\hat{i} + 50\hat{j} - 25\hat{k}\right) - m\left(200\hat{i} + 70\hat{j} + 15\hat{k}\right)$$

$$\vec{v}_2 = \frac{1}{4}(180\hat{j} - 140\hat{k}) = 45\hat{j} - 35\hat{k}$$

18. **(3)**

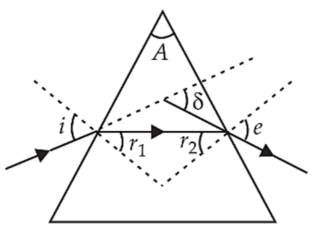
For normal incidence,

$i = 0°$, $r_1 = 0°$

As $r_1 + r_2 = A$

$\therefore\ r_2 = A - r_1 = 30°$

As $\mu = \frac{\sin e}{\sin r_2}$



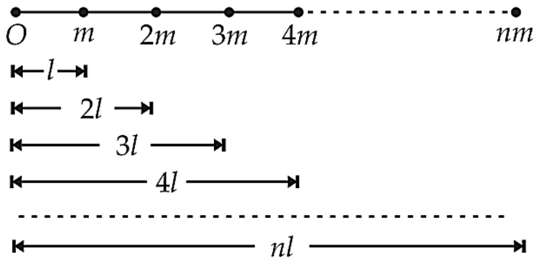
$\therefore\ \sin e = \mu \sin r_2$

$= \sqrt{2} \sin 30° = \frac{1}{\sqrt{2}}$

$e = 45°$

$\delta = i + e - A = 0° + 45° - 30° = 15°$.

19. **(1)**



The distance of centre of mass of given configuration of the particles from the fixed point O is

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + ... + m_nx_n}{m_1 + m_2 + m_3 + ... + m_n}$$

$$X_{CM} = \frac{(m)(l) + (2m)(2l) + ... + (nm)(nl)}{m + 2m + 3m + ... + nm}$$

$$= \frac{ml[1^2 + 2^2 + 3^2 + ... + n^2]}{m[1 + 2 + 3 + ... + n]}$$

$$= \frac{\frac{(l)(n)(n+1)(2n+1)}{6}}{\frac{(n)(n+1)}{2}} = \frac{(2n+1)l}{3} \text{ cm}$$

20. **(4)**

$\Delta x = \frac{D(\mu - 1)t}{d}$. Also $\Delta x = \frac{nD\lambda}{d}$.

$\therefore\ \frac{D(\mu-1)t}{d} = \frac{nD\lambda}{d}$ or $(\mu - 1)t = n\lambda$

or $\frac{t_1}{t_2} = \frac{n_1}{n_2} \Rightarrow t_2 = \frac{n_2 t_1}{n_1}$

or $t_2 = \frac{20 \times 4.8}{30} = 3.2$ nm

21. **(1)**

Let $\lambda_A = \lambda\ \therefore\ \lambda_B = 2\lambda$

If N_0 is total number of atoms in A and B at $t = 0$, then initial rate of disintegration of $A = \lambda N_0$, and initial rate of disintegration $B = 2\lambda N_0$

As $\lambda_B = 2\lambda_A\ \therefore\ T_B = \frac{1}{2} T_A$

i.e., Half life of B is half the half life of A.

After one half life of A

$$\left(-\frac{dN}{dt}\right)_A = \frac{\lambda N_0}{2}$$

Equivalently, after two half lives of B

$$\left(-\frac{dN}{dt}\right)_B = \frac{2\lambda N_0}{4} = \frac{\lambda N_0}{2}$$

After $n = 1$ *i.e.*, one half life of

$$A\left(-\frac{dN}{dt}\right)_A = -\left(\frac{dN}{dt}\right)_B$$

22. **(4)**

Force on 10 kg mass = 10 × 12 = 120 N

The mass of 10 kg will pull the mass of 20 kg in the backward direction with a force of 120 N.

∴ Net force on mass 20 kg = 200 – 120 = 80 N

So, acceleration for 20 kg mass

$= \frac{80\text{ N}}{20\text{ kg}} = 4\text{ ms}^{-2}$

23. **(5)**

Loss in elastic potential energy = Gain in kinetic energy

$$\frac{1}{2}k(l-l_0)^2 = \frac{1}{2}mv^2$$

$$v = (l-l_0)\sqrt{\frac{k}{m}} = \left(\frac{l_0}{\cos 37°} - l_0\right)\sqrt{\frac{k}{m}}$$

$$= l_0\left[\frac{1}{\cos 37°} - 1\right]\sqrt{\frac{k}{m}} = (0.1)\left[\frac{5}{4} - 1\right]\sqrt{\frac{80}{2\times10^{-3}}}$$

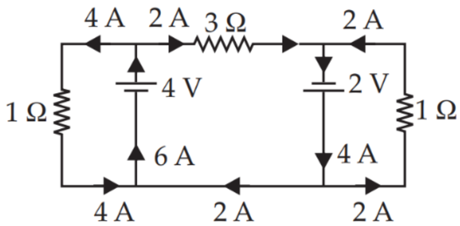
$$= 0.1 \times \frac{1}{4} \times 2 \times 10^2 = 5\text{ ms}^{-1}$$

24. **(6)**

Current through 1 Ω resistor parallel to 4 V battery = 4 A

Current through 1 Ω resistor parallel to 2 V battery = 2 A

Current through 3 Ω resistor = $\frac{4+2}{3} = 2$ A



∴ Current through 4 V battery = 4 + 2 = 6 A

25. **(1)**

The velocity of longitudinal wave is,

$$v_L = \sqrt{\frac{Y}{\rho}}$$

The velocity of transverse wave is

$$v_T = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\pi r^2 \rho}}$$

$$\frac{v_L}{v_T} = \sqrt{\frac{Y}{\rho} \times \frac{\pi r^2 \rho}{T}} = \sqrt{\frac{Y}{T/\pi r^2}} = \sqrt{\frac{Y}{\text{Stress}}}$$

$$\text{Stress} = \frac{Y}{(v_L/v_T)^2} = \frac{1\times10^{11}}{(100)^2} = 1\times10^7\ \text{Nm}^{-2}$$

Hence, $x = 1$.

26. **(5)**

The capacitance of a parallel plate capacitor in air is given by

$$C = \frac{\varepsilon_0 A}{d}$$

By introducing a slab of thickness t, the new capacitance C′ becomes

$$C' = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{K}\right)}$$

The charge (Q = CV) remains the same in both the cases.

Hence,

$$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t\left(1 - \frac{1}{K}\right)} \quad \text{or} \quad d = d' - t\left(1 - \frac{1}{K}\right)$$

Here, $d' = d + 2.4 \times 10^{-3}$ m, t = 3 mm = 3×10^{-3} m Substituting these values, we get

$$d = d + (2.4\times10^{-3}) - 3\times10^{-3}\left(1 - \frac{1}{K}\right)$$

$$\text{or } (2.4\times10^{-3}) = 3\times10^{-3}\left(1 - \frac{1}{K}\right)$$

Solving it, we get $K = 5$

27. **(7)**

Here, radius of sphere R = 1 cm = 1×10^{-2} m Work function, W = 4.7 eV

Energy of incident radiation

$$= \frac{hc}{\lambda} = \frac{1240\text{ eV nm}}{200\text{ nm}} \quad (\text{Take } hc = 1240\text{ eV nm})$$

= 6.2 eV

According to Einstein's photoelectric equation

$$\frac{hc}{\lambda} = W + eV_s$$

6.2 eV = 4.7 eV + eV_s

V_s = 1.5 V

The sphere will stop emitting photoelectrons, when the potential on its surface becomes equal to 1.5 V.

$$\therefore \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = 1.5$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Ne}{R} = 1.5$$

where N = Number of photoelectrons emitted, e = charge of each electron.

$$N = \frac{1.5 \times R}{\frac{1}{4\pi\varepsilon_0} \times e} = \frac{1.5\times1\times10^{-2}}{9\times10^9 \times 1.6\times10^{-19}}$$

$$N = \frac{15}{16} \times \frac{1}{9} \times 10^8 = \frac{5}{48} \times 10^8$$

$$N = \frac{50}{48} \times 10^7 = 1.04 \times 10^7 \qquad \therefore Z = 7$$

28. **(1)**

Activity, A = λN

$$A = \frac{1}{\tau}N \qquad \left(\text{As } \lambda = \frac{1}{\tau}\right)$$

where τ is the mean life time.

$N = A\tau = (10^{10}$ decay/s$)(10^9$ s$) = 10^{19}$ atoms

Mass of the sample, $m = N \times$ (mass of 1 atom)

$= 10^{19} \times 10^{-25}$ kg $= 10^{-6}$ kg = 1 mg

29. **(4)**

According to Kepler's third law

$T^2 \propto r^3$

$$\therefore \frac{T_A^2}{T_B^2} = \frac{r_A^3}{r_B^3}$$

or $\frac{r_A}{r_B} = \left(\frac{T_A}{T_B}\right)^{2/3}$ = $(8)^{2/3}$ = 4 or $r_A = 4r_B$

30. **(4)**

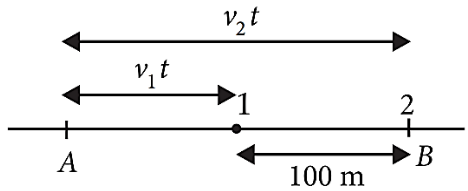
Let the velocities of car 1 and car 2 be v_1 m s^{-1} and v_2 m s^{-1}.

$\therefore$ Apparent frequencies of sound emitted by car 1 and car 2 as detected at end point are

$$\upsilon_1 = \frac{\upsilon_0 v}{v - v_1} \text{ and } \upsilon_2 = \frac{\upsilon_0 v}{v - v_2}$$

$\therefore$ 330 = or v_1 = 30 m s^{-1}

and 360 = $\frac{300 \times 330}{330 - v_2}$ or v_2 = 55 m s^{-1}



The distance between both the cars just when the 2nd car reaches point B(as shown in figure) is 100 m = $v_2 t - v_1 t$

$$t = \frac{100}{v_2 - v_1} = \frac{100}{55 - 30} = 4 \text{ s}$$

CHEMISTRY

31. **(2)**

Ist excitation potential i.e. $E_2 - E_1 = 15\text{eV}$

i.e. $$E_2 - E_1 = 15\text{eV} = -E_o\left(\frac{Z^2}{4} - \frac{Z^2}{1}\right)$$

$$\Rightarrow 15\text{eV} = E_0 Z^2 \times \frac{3}{4} \quad .....(i)$$

Now; $E_\infty - E_1 = E_0 Z^2 = 15\text{eV} \times \frac{4}{3} = 20\text{eV}$

Thus potential required =20 eV

32. **(2)**

Av. Momentum $= M_g \times \sqrt{\frac{8RT}{\pi M_g}} = \sqrt{\frac{8RTM_g}{\pi}}$

$$\therefore \frac{\text{Av. momentum of } O_2}{\text{Av. momentum of He}} = \sqrt{\frac{M_{O_2} \cdot T}{M_{He} \cdot T}} = \sqrt{\frac{32 \times T}{4 \times 300}} = 1$$

$\therefore 32T = 4 \times 300$

$$\therefore T = \frac{1200}{32} = 37.5^o C$$

33. **(4)**

Rate $\propto$ area of orifice

Let diagonal of square=area of circle=d

$$\frac{r_A}{r_B} = \frac{\pi\left(\frac{d}{2}\right)^2}{\left(\frac{d}{\sqrt{2}}\right)^2} = \frac{\pi}{2}$$

$$\frac{r_A}{r_B} = \frac{\pi}{2}$$

34. **(1)**

$$K_p = \frac{P_{PCl_3} \cdot P_{Cl_2}}{P_{PCl_5}}$$

$$= \frac{\left(\frac{2}{6} \times 3\right)\left(\frac{2}{6} \times 3\right)}{\left(\frac{2}{6} \times 3\right)} = 1 \text{ atm}$$

35. **(2)**

$$\underset{1-0.33}{N_2O_4} \rightleftharpoons \underset{0.66}{2NO_2}$$

$$X_{NO_2} = \frac{0.66}{1.33}; \quad X_{N_2O_4} = \frac{0.67}{1.33}$$

$$K_p = \frac{\left(\frac{0.66}{1.33}\right)^2 \times P_1^2}{\frac{0.67}{1.33} \times P_1} \quad ...(1)$$

$$\underset{1-0.4}{N_2O_4} \rightleftharpoons \underset{0.8}{1NO_2}$$

$$X_{NO_2} = \frac{0.8}{1.4}; \quad X_{N_2O_4} = \frac{0.6}{1.4}$$

$$K_p = \frac{\left(\frac{0.8}{1.4}\right)^2 \times P_2^2}{\frac{0.6}{1.4} \times P_2} \quad ...(2)$$

Equating equation (1) & (2)

$$\frac{P_1}{P_2} = \frac{0.8 \times 0.8 \times 1.33 \times 0.67}{0.66 \times 0.66 \times 0.6 \times 1.4} = 1.55$$

36. **(1)**

$[H^+] = [OH^-] = x$ (for pure water)

$[H^+][OH^-] = K_w$

$x^2 = 2.56 \times 10^{-14}$

$x = 1.6 \times 10^{-7}$

$pH = 7 - \log 1.6 = 6.81$

37. **(2)**

$\Delta T_b = 1.08 = K_b \times m \times 2$ $(i = 2 \text{ at B.P})$

$\Delta T_f = 1.8 = K_f \times m \times i$

$$\Rightarrow \frac{1.08}{1.8} = \frac{K_b}{K_f} \times \frac{2}{i} \Rightarrow i = \frac{0.3 \times 2 \times 1.8}{1.08} = 1$$

$\therefore$ $i = 1$ at F.P. so it behaves as non-electrolyte at F.P of solution.

38. (2)

$$-\frac{1}{4}\frac{dA}{dt}=\frac{-dB}{dt}=\frac{1}{2}\frac{dC}{dt}=\frac{1}{2}\frac{dD}{dt}$$

39. (3)

$$A \longrightarrow nB$$

t = 0 A_0

t = t $A_0 - x$ nx

At point of intersection [A] = [B]

$\Rightarrow A_0 - x = nx \qquad \Rightarrow x = \frac{A_0}{n+1}$

$\therefore [B] = nx = \frac{nA_0}{n+1}$

40 (3)

Theoretical

41. (4)

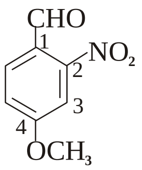
(I) Neutral with complete octete.

(II) Charge but complete octet.

(III) Incomplete octet but negative charge on electronegative oxygen

(IV) Negative charge on electropositive carbon and also incomplete octete.

42. (1)



4-methoxy-2-nitro benzaldehyde

43. (4)

(1) ortho substituted benzoic acid, So more acidic than Benzoic acid.

(2) Sulphonic acid derivative, more acidic than benzoic acid.

(3) Picric acid (pK_a~1). Strong acid due to three electron withdrawing NO_2.

(4) Electron donating effect of –OH will decrease the acidic strength.

44. (1)

(I) Guanidine derivative

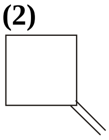
(II) Lone pair on sp^2 hybridized nitrogen

(III) Lone pair delocalised in Benzene ring

(IV) Ortho effect so basic strength less than (III)

So decreasing order of basic strength. I>II>III>IV

45. (2)



Molecular formula C_5H_8. Clearly molecular formula is different. So cannot be isomer.

46. (3)

Propagation step

$Cl^{\bullet} + CH_4 \rightarrow \dot{C}H_3 + HCl$ ….. (1A)

$\dot{C}H_3 + Cl_2 \rightarrow CH_3Cl + Cl^{\bullet}$ ……(1B)

47. (1)

According to Freundlisch

$\frac{x}{m} = KP^{\frac{1}{n}}$

$\log\frac{x}{m} = \log K + \frac{1}{n}\log P$

Slope $= \frac{1}{n}$

48. (4)

Most transition elements form interstitial compounds.

49. (1)

$Cu^{2+} + 2I^- \rightarrow CuI + \frac{1}{2}I_2$

Cu^{2+} ions on oxidize iodide to iodine.

50. (3)

Boron is a non metal.

51. (5)

$K\overset{+7}{Mn}O_4 \longrightarrow \overset{+2}{Mn}SO_4$

$Mn + 5e^- \longrightarrow Mn^{2+}$

52. (5)

$CuSO_4.5H_2O$

53. (4)

$$E = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\times3\times10^8}{498\times10^{-9}} = 3.99\times10^{-19}\ J$$

54. (12)

$MgCl_2.6H_2O$ & $CaCl_2.6H_2O$ can exist.

55. (7)

(1) 1 mole each with $-OH, -C\equiv CH$ and $-COCH_3$

(2) 2 mole each with COOEt and COCl Total = 7

56. (4)

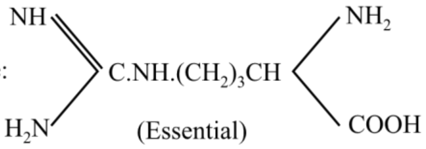
Nature of solvent, Nature of alkyl group in the substrate, Nature of nucelophile, Nature of leaving group

57. (9)

$$\frac{pka_2 + pka_3}{2} = \frac{8+10}{2} = 9$$

58. **(4)**

Arginine:



(Essential)

59. **(9)**

$MOH \rightleftharpoons M^+ + OH^-$

$[OH^-]\sqrt{K_{sp}} = \sqrt{1\times10^{-10}} = 10^{-5}$

pH = 9.

60. **(8)**

$FeCr_2O_4 \to Fe^{3+} + CrO_4^{2-}$

$Fe^{2+} \to Fe^{3+} + e^-$

$2Cr^{3+} \to 2\,CrO_4^{2-} + 6e^-$

n-factor = 7

Theoretical requirement = 7F

$x\times\frac{87.5}{100} = 7$

$x = \frac{700}{87.5} = 8F$

MATHEMATICS

61. **(4)**

AB = *BA*

$\Rightarrow \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix} = \begin{bmatrix}a & b\\c & d\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$

$\Rightarrow \begin{bmatrix}a+2c & b+2d\\3a+4c & 3b+4b\end{bmatrix} = \begin{bmatrix}a+3b & 2a+4b\\c+3d & 2c+4d\end{bmatrix}$

$a + 2c = a + 3b \Rightarrow c = \frac{3}{2}b$

$b + 2d = 2a + 4b$

$\Rightarrow 2a - 2d = -3b$

Now $\frac{3a-3d}{3b+c} = \frac{\frac{-9}{2}b}{3b+\frac{3}{2}b} = -1$

62. **(2)**

$\lim\limits_{x\to0^+}\frac{e^{(x^x-1)} - x^x - 1 + 1}{(x^x-1)^2(x^x+1)^2}$

$= \lim\limits_{x\to0^+} 1 + (x^x-1) + \frac{(x^x-1)^2}{2!} + ...1 - (x^x-1)$

$= \lim\limits_{x\to0^+}\frac{\frac{(x^x-1)^2}{2!} + \frac{(x^x-1)^3}{3!} + ...}{(x^x-1)^2(x^x+1)^2}$

$= \lim\limits_{x\to0^+}\frac{1}{2!}(1+.....)\frac{1}{(x^x+1)^2} = \frac{1}{2}\times\frac{1}{4} = \frac{1}{8}$

63. **(4)**

$k = x^2 - 4x + 5 = (x^2 - 4x + 4) + 1$

$= (x-2)^2 + 1$

$\therefore$ $\sin^{-1}k$, $\cos^{-1}k$ defined at $k = 1$ at which $x = 2$

So, given equation

$4 + 2a + \frac{\pi}{2} = 0$

$\Rightarrow a = -\left(\frac{\pi+8}{4}\right)$

64. **(2)**

$\alpha = \log_{10}15 = \log_{10}\left(\frac{30}{2}\right) = \log_{10}30 - \log_{10}2$

$1 + \log_{10}3 - \log_{10}2$ and $\beta = 4\log_{10}2$

$a(1+\log_{10}3 - \log_{10}2) + b(4\log_{10}2) + c$

$= \log_{10}Z$

Set $A = \{\log_{10}1, \log_{10}2, ...\log_{10}50\}$, no. of integers which are divisible 2, 3, 4, 5, 6, 8, 9, 10, 12, …….., 50.

65. **(3)**

$\int \sec^2 x(\sin x)^{-2010}\,dx - 2010$

$\int \frac{1}{(\sin x)^{2010}}\,dx$

$\frac{\tan x}{(\sin x)^{2010}} + 2010\int\frac{\tan x\cos x}{(\sin x)^{2011}}dx - 2010$

$\int\frac{1}{(\sin x)^{2010}}dx = \frac{(\tan x)}{(\sin x)^{2010}} + 2010$

$\int\frac{1}{(\sin x)^{2010}}dx - 2010\int\frac{1}{(\sin x)^{2010}}dx$

66. **(1)**

According to the question

$\frac{\vec{c}.(\hat{i}+\hat{j})}{|\vec{c}|\sqrt{2}} = \frac{(\hat{i}+\hat{j}).(\hat{j}+\hat{k})}{\sqrt{2}.\sqrt{2}}$

$\frac{2x+y}{\sqrt{x^2+(x+y)^2+y^2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow 2(4x^2+y^2+4xy)$

$= x^2 + (x^2+y^2+2xy) + y^2$

$\Rightarrow 6x^2 + 6xy = 0$

$\Rightarrow x = 0, x = -y$

$\vec{c} = x(\hat{i}+j) + y(j+\hat{k}) = x(\hat{i}-k)$

Now $\vec{c}\cdot j = \frac{1}{\sqrt{2}}(\hat{i}-\hat{k})\cdot j$

67. **(2)**

$B=-A^{-1}BA \Rightarrow AB=-AA^{-1}BA$

$= -IBA$

$AB = -BA$

$\Rightarrow AB + BA = 0$ …..(i)

Now $(A + B)(A + B)$

$= A^2 + B^2 + AB + BA$

$= A^2 + B^2$

68. **(4)**

Now $\theta = \sin^{-1}\sqrt{\frac{(\sqrt{3}-1)^2}{8}} + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sec^{-1}(\sqrt{2})$

$= \sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) + \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sec^{-1}(\sqrt{2})$

$= \frac{\pi}{12} + \frac{\pi}{6} + \frac{\pi}{4} = \frac{\pi + 2\pi + 3\pi}{12} = \frac{\pi}{2}$

$\because \sin^{-1}\frac{\sqrt{3}-1}{2\sqrt{2}} = \frac{\pi}{12}$

69. **(3)**

$f(x) = \int_x^2 \frac{dy}{\sqrt{1+y^3}}$

$\Rightarrow f'(x) = 0 - \frac{1}{\sqrt{1+x^3}}$

$\therefore \int_0^2 xf(x)dx = \left(f(x)\cdot\frac{x^2}{2}\right)_0^2 - \int^2 f'(x)\cdot\frac{x^2}{2}dx$

$= 2f(2) + \frac{1}{2}\int_0^2 \frac{x^2}{\sqrt{1+x^3}}dx$

$(\because f(2) = 0)$

$= 0 + \frac{1}{6}\int \frac{3x^2}{\sqrt{1+x^3}}dx$

$= \frac{1}{6}\left[(1+x^3)^{1/2}\right]_0^2$

$= \frac{1}{3}(3-1) = 2/3$

70. **(4)**

$\Delta APA'(a\cos\theta, b\sin\theta) = \left(\sqrt{2}\frac{1}{\sqrt{2}}, 2\sqrt{2}\frac{1}{\sqrt{2}}\right)$

$P(1, 2)$

Equation of tangent:

$\frac{x(1)}{2} + \frac{y(2)}{8} = 1$

$\Rightarrow \frac{x}{2} + \frac{y}{4} = 1$

$A(2,0), B(0,4)$

Similarly, $m_T = -2$

$m_N = 1/2$. Equation of normal at $P(1, 2)$ is

$y - 2 = \frac{1}{2}(x-1) \Rightarrow x - 2y + 3 = 0$

$A'(-3,0)$

$\Delta_1 = \frac{1}{2}\sqrt{(2+3)^2}\,2 = 5$ sq unit $B'\left(0, \frac{3}{2}\right)$

$\Delta_2 = \frac{1}{2}\times\frac{5}{2}\times 1 = \frac{5}{9}$ sq unit

Now, $\frac{\Delta_1}{\Delta_2} = \frac{5}{5}\times 4 = 4:1$

71. **(2)**

$1 = \int \frac{\cos x/2 - \sin x/2}{2\cos^2 x/2} e^{-x/2} dx$

Put $-x/2 = t \Rightarrow -dx = 2dt$

$I = -\int e^t(\sec t + \sec t \tan t)dt = -e^t \sec t + c$

$= -e^{-x/2}\sec x/2 + c$

72. **(4)**

$S = 1(\alpha_1 + \alpha_{2008}) + 2(\alpha_2 + \alpha_{2007}) + 3(\alpha_3 + \alpha_{2006}) +$ … $+ 2007(\alpha_{2007} + \alpha_2) + 2008(\alpha_{2008} + d)$

$S = 2009(\alpha_1 + \alpha_{2008}) + 2009(\alpha_2 + \alpha_{2009}) + ...$

$= 2009(1 + \alpha_1 + \alpha_2 + \alpha_3 + ... + \alpha_{2008}) - 1$

= 2009 (Sum of 2009th roots –1)

= 2009 (0 – 1) = – 2009

73. **(2)**

$\theta = \tan^{-1}\left|\frac{y}{x}\right|$

When z lies on 2nd quadrant then

$z = \pi - \tan^{-1}\left|\frac{y}{x}\right|$

74. **(1)**

$S = 1 + 3 + 6 + 10 + ...$

$S = 1 + 3 + 6.......$

$0 = 1 + 2 + 3 + 4 + 5 + ... - S_n$

$\Rightarrow S_n = \frac{n(n+1)}{2}$

$S_n = 2a^{-1} + \frac{n(n+1)}{2}a = 1 + \frac{n(n+1)}{2}.2$

Now $T_n = \cot^{-1}(1 + n(n+1)) = \tan^{-1}\frac{1}{1+n(n+1)}$

$T_n = \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$= \tan^{-1}(n+1) - \tan^{-1}n$

$T_1 = \tan^{-1}2 - \tan^{-1}1$

$T_2 = \tan^{-1}3 - \tan^{-1}2$

$T_3 = \tan^{-1}4 - \tan^{-1}3$

$T_n = \tan^{-1}(n+1) - \tan^{-1}n$

$T_1 + T_2 + \to + T_n = \tan^{-1}(n+1) - \tan^{-1}1$

$n \to \infty = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

75. **(3)**

$x = y = 0$

$f(1) = \left(\sqrt{f(0)} + \sqrt{f(0)}\right)^2 = (1+1)^2 = 2^2$

$x = 1, y = 0$

$f(1+1) = \left(\sqrt{f(1)} + \sqrt{f(0)}\right)^2 = 3^2$

$f(2+1) = (3+1)^2$

$f(3+1) = (4+1)^2$

$f(x-1+1) = (1+x)^2 \Rightarrow f(x) = (1+x)^2$

76. **(2)**

$p'(x) = \lambda(x-2)(x-4)$

$= \lambda\left(x^2 - 6x + 8\right)$

$\Rightarrow P'(0) = 8\lambda$

Integrating: $p(x) = \lambda\left(\frac{x^3}{3} - 3x^2 + 8x\right) + \mu$

$p(2) = \lambda\left(\frac{8}{3} - 12 + 16\right) + \mu = 8$

$\Rightarrow \frac{20\lambda}{3} + \mu = 8$ .....(ii)

Also, $p(4) = 1 \Rightarrow \lambda\left(\frac{64}{3} - 48 + 32\right) + \mu = 1$

$\Rightarrow \frac{16}{3}\lambda + \mu = 1$ .....(iii)

(ii) – (iii) $\Rightarrow 7 = \frac{4\pi}{3} \Rightarrow \lambda = \frac{21}{4}$

$p'(0) = 8^2 \frac{21}{4} = 42$

77. **(4)**

$\tan^{-1}\left(\frac{2}{9}\right) + \tan^{-1}\frac{4}{(33)} + \tan^{-1}\frac{8}{129} + ...\infty$

$= \tan^{-1}\frac{2}{1+4.2} + \tan^{-1}\frac{4}{1+8.4} + \tan^{-1}\frac{8}{1+16.8} + ...\infty$

$= \tan^{-1}\left(\frac{4-2}{1+4.2}\right) + \tan^{-1}\left(\frac{8-4}{1+8.4}\right) + \tan^{-1}\left(\frac{16-8}{1+16.8}\right) + ...\infty$

$= \tan^{-1}4 - \tan^{-1}2 + \tan^{-1}8 - \tan^{-1}4 + \tan^{-1}16 - \tan^{-1}8 + ...$

$Sum = \tan^{-1}2^{n+1} - \tan^{-1}2 \Rightarrow S_\infty = \frac{\pi}{2} - \tan^{-1}2$

78. **(1)**

p	q	$\sim p$	$p \Leftrightarrow q$	$\sim p \wedge (p \Leftrightarrow q)$	$\sim A$
T	T	F	T	F	T
T	F	F	F	F	T
F	T	T	F	F	T
F	F	T	T	T	F

$\sim[\sim p \wedge (p \Leftrightarrow q)] = pvq$

79. **(2)**

$SD = \sqrt{\frac{\sum x_i^2}{9} - \left(x^-\right)^2}$

$= \sqrt{\frac{\sum(x_i-5)^2}{9} - \left(\frac{\sum(x_i-5)}{9}\right)^2}$

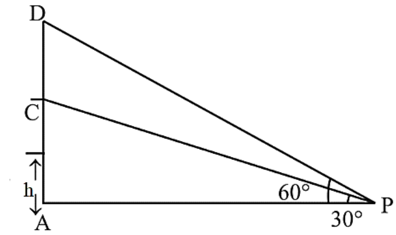
$SD = \sqrt{5-1}$ $\left[\because \sum_{i=1}^{9}\frac{(x_i-5)^2}{9} = 5, \frac{\sum(x_i-5)}{9} = 1\right]$

80. **(2)**

$\frac{dx}{dt} = (2t+3)$

$dx = (2t+3)dt$

$x = t^2 + 3t + c$ ....(i)



$BC = 1^2 + 3.(1) + c = 4 + c$

$BD = 3^2 + 3.3 + c = 18 + c$

$= \tan 30° = \frac{h+4+c}{AP}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h+4+c}{AP}$ ....(ii)

$\tan 60° = \frac{h+14+c}{AP}$

$\sqrt{3} = \frac{h+14+c}{AP}$ ...(iii)

from (ii) & (iii)

$\frac{14}{AP} = \tan 60° - \tan 30° = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\Rightarrow AP = 7\sqrt{3}$

81. **(0)**

$T_r = \cot^{-1}\left(2r^2\right) = \tan^{-1}\left(\frac{1\times 2}{2\times 2r^2}\right)$

$= \tan^{-1}\frac{2}{1+4r^2-1} = \tan^{-1}\left(\frac{2}{1+(2r-1)(2r+1)}\right)$

$$T_r=\tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r-1)(2r+1)}\right)$$
$$T_r=\tan^{-1}(2r+1)-\tan^{-1}(2r-1)$$
$$T_1=\tan^{-1}3-\tan^{-1}1$$
$$T_2=\tan^{-1}5-\tan^{-1}3$$
$$T_3=\tan^{-1}7-\tan^{-1}5$$
$$T_n=\tan^{-1}(2n+1)-\tan^{-1}(2n-1)$$
$$T_1+T_2+....+T_n=\tan^{-1}(2n+1)-\tan^{-1}1$$
$$S_\infty=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$$
$$[S_\infty]=\left[\frac{\pi}{4}\right]=0$$

82. (5)

$(16-x),(20-3x),\leftarrow$ Natural, No.

$(2x-1),(4\times-5),\leftarrow W$

We know that,

$(16-x)\ge(2x-1),(20-3x)\ge(4x-5)$

$x = 2, 3$

When $x = 2$: $f(x)={}^{14}C_3+{}^{14}C_3=728$

at $x = 3$, $f(x)={}^{13}C_5+{}^{11}C_4=1617$

Now $\frac{2345}{469}$

83. (12)

$$\frac{(x+y+z)^2}{2}=\frac{x^3+y^3+z^3}{3} \quad \text{...(i)}$$

Given, we know that,

$$\frac{a\alpha+b\beta+c\gamma}{3}\ge\left(\frac{a+b+c}{3}\right).\left(\frac{\alpha+\beta+\gamma}{3}\right) \quad \text{....(ii)}$$

Again

$$\frac{x^3+y^3+z^3}{3}\ge\left(\frac{x+y+z}{3}\right)\left(\frac{x^2+y^2+z^2}{3}\right) \quad \text{...(iii)}$$
$$\frac{x^2+y^2+z^2}{3}\ge\left(\frac{x+y+z}{3}\right)\left(\frac{x+y+z}{3}\right) \quad \text{.....(iv)}$$

From ((iii) & (iv)

$$\frac{x^3+y^3+z^3}{3}\ge\left(\frac{x+y+z}{3}\right)^3$$
$$\Rightarrow x+y+z\le\frac{27}{2} \quad \text{...(v)}$$

If $x + y + z = 13$ not satisfy

$x + y + z = 12$

Maximum value = 12

84. (33)

Area of ellipse = πab sq units

$$(x-r)^2+y^2=r^2$$
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, P(a\cos\theta, b\sin\theta)$$
$$r^2(a\cos\theta-r)^2+(b\sin\theta)^2 \quad \text{...(i)}$$

Normal passes through $(r, 0)$

$$a\frac{r}{\cos\theta}-b\frac{0}{\sin\theta}=a^2-b^2=a^2e^2\Rightarrow\cos\theta=\frac{r}{ae^2}$$

Putting $\cos\theta=\frac{r}{ae^2}$ in (i) we get,

$$b^2=\frac{r^2}{e^2}\Rightarrow b=\frac{r}{e}$$

Also, $b^2=a^2(1-e^2)\Rightarrow a^2=\frac{b^2}{1-e^2}$

Now Area = $\pi ab=\frac{\pi b^2}{\sqrt{1-e^2}}$

$$A=\frac{\pi r^2}{e^2\sqrt{1-e^2}}$$
$$\frac{dA}{de}=0\Rightarrow 3e^2=2\Rightarrow e^2=\frac{2}{3}\Rightarrow e=\sqrt{\frac{2}{3}}$$

85. (11)

$$g(x)=2\sin x+\tan x-\frac{3x}{\pi}+1$$
$$g'(x)=2\cos x+\sec^2 x-\frac{3}{\pi}>0, \forall x\in\left[\frac{\pi}{6},\frac{\pi}{3}\right]$$

$g(x)$ is monotonically increasing,

$$g\left(\frac{\pi}{6}\right)=2\sin\frac{\pi}{6}+\tan\frac{\pi}{6}-\frac{3}{\pi}.\frac{\pi}{6}+1$$
$$=1+\frac{1}{\sqrt{3}}-\frac{1}{2}+1$$
$$g\left(\frac{\pi}{6}\right)=\frac{3}{2}+\frac{1}{\sqrt{3}}\Rightarrow f\left(\frac{\pi}{6}\right)=l\text{n}\left(\frac{3}{2}+\frac{1}{\sqrt{3}}\right)$$

Now $f\left(\frac{\pi}{3}\right)=\ln\left(\sqrt{3}+\sqrt{3}-1+1\right)=l\text{n}2\sqrt{3}$

So $a+b=\ln\left(\frac{3}{2}+\frac{1}{\sqrt{3}}\right)+l\text{n}2\sqrt{3}$

$$=l\text{n}\left(\left(\frac{3}{2}+\frac{1}{\sqrt{3}}\right).2\sqrt{3}\right)$$
$$=l\text{n}\left(3\sqrt{3}+2\right)=l\text{n}\ (7.2)$$
$$1<(a+b)<2\Rightarrow[a+b]=1$$

$11[a+b]=11$

86. (5)

$$f(x)=\sin(l\text{n}x)-\cos(l\text{n}x) \quad x>0$$
$$=\sqrt{2}\sin\left(l\text{n}x-\frac{\pi}{4}\right)$$
$$f'(x)=\frac{\sqrt{2}}{x}\cos\left(l\text{n}x-\frac{\pi}{4}\right)\left[\cos=\sin\left(\frac{\pi}{2}-x\right)\right]$$
$$f'(x)=\frac{\sqrt{2}}{x}\sin\left(\frac{\pi}{4}+l\text{n}x\right)$$
$$f'(x)\ge0\Rightarrow\sin\left(\frac{\pi}{4}+l\text{n}x\right)\ge0$$
$$\Rightarrow0\le\left(\frac{\pi}{4}+l\text{n}x\right)\le\pi \quad \Rightarrow-\frac{\pi}{4}\le l\text{n}x\le3\frac{\pi}{4}$$
$$e^{-\pi/4}\le ce^{\frac{3\pi}{4}}$$
$$\mu-\lambda=\pi\Rightarrow-5\cos(\mu-\lambda)=-5(-1)=5$$

87. **(2)**

$$I=\frac{1}{2}\int\frac{2\sin 3x.\cos 5x-2\sin 3x.\cos 4x}{2\sin 3x.\cos 3x+\sin 3x}dx$$

$$=\frac{1}{2}\int\frac{(\sin 8x-\sin 2x)-(\sin 7x-\sin x)}{\sin 6x+\sin 3x}dx$$

$$=\frac{1}{2}\int\frac{2\sin\frac{9x}{2}.\cos\frac{7x}{2}-2\sin\frac{9x}{2}.\cos\frac{5x}{2}}{2\sin\frac{9x}{2}.\cos\frac{3x}{2}}dx$$

$$=\frac{1}{2}\int\frac{\cos\frac{7x}{2}-\cos\frac{5x}{2}}{\cos\frac{3x}{2}}dx\ =-\frac{1}{2}\times 2\int\frac{\sin 3x.\sin\frac{x}{2}}{\cos\frac{3x}{2}}dx$$

$$=-2\int\frac{\sin\frac{3x}{2}.\cos\frac{3x}{2}.\sin\frac{x}{2}}{\cos\frac{3x}{2}}dx=-\int(\cos x-\cos 2x)\,dx$$

$$=\int(\cos 2x-\cos x)dx=\frac{\sin 2x}{2}-\sin x-C$$

88. **(5)**

$$f(x)=3\cos^4 x+10\cos^3 x+6\cos^2 x-3$$

$$\Rightarrow f'(x)=12\cos^3 x(-\sin x)+30\cos^2 x(-\sin x)+12(-\sin x)\cos x$$

$$=-3\sin 2x(2\cos^2 x+5\cos x+2)$$

$$=2\cos^2 x+4\cos x+\cos x+2$$

$$=2\cos x(\cos x+2)+1(\cos x+2)$$

$$=-3\sin 2x(2\cos x+1)(\cos x+2)$$

$$f'(x)=0\Rightarrow\sin 2x=0\Rightarrow x=0,\frac{\pi}{2},\pi$$

$$\cos x=\frac{-1}{2}\Rightarrow x=+\frac{2\pi}{3}$$

+   –   +   –   +
0   π/2   2π/3   π

$$f(x)\in\left(0,\frac{\pi}{2}\right)\cup\left(\frac{2\pi}{3},\pi\right)$$

$$p=2,r=3\Rightarrow p+r=5$$

89. **(16)**

$$\left(n^2-1\right)^3=(n+1)^3(n-1)^3$$

$$3n^2+1=\frac{1}{2}\left(6n^2+2\right)=\frac{1}{2}\left[(n+1)^3-(n-1)^3\right]$$

$$S=\sum_{n=2}^{\infty}\frac{3n^2+1}{\left(n^2-1\right)^3}=\frac{1}{2}\sum_{r=0}^{\infty}\frac{\left(6n^2+2\right)}{\left(n^2-1\right)^3}$$

$$=\frac{1}{2}\sum_{r=0}^{\infty}\left[\frac{(n+1)^3-(n-1)^3}{(n+1)^3(n-1)^3}\right]$$

$$=\frac{1}{2}\sum_{r=0}^{\infty}\left(\frac{1}{(n-1)^3}-\frac{1}{(n+1)^3}\right)$$

$$T_1=\frac{1}{2}\left(\frac{1}{1^3}-\frac{1}{3^3}\right)$$

$$T_2=\frac{1}{2}\left(\frac{1}{2^3}-\frac{1}{4^3}\right)$$

$$T_3=\frac{1}{2}\left(\frac{1}{3^3}-\frac{1}{5^3}\right)$$

. .
. .
. .
. .
. .

$$S=\frac{1}{2}\left(1+\frac{1}{8}\right)=\frac{9}{16}$$

90. **(43)**

$${}^{3n}C_n=(2n+1)(2n+2)(2n+3).....(2n+n)$$

$${}^{2n}C_n=(n+1)(n+2)(n+3)....(n+n)$$

$$\frac{A}{B}=\lim_{n\to\infty}\left(\frac{(2n+1)(2n+2)-(2n+n)}{(n+1)(n+2)(n+1)}\right)^{1/n}$$

$$\log\left(\frac{A}{B}\right)=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\ln\left(\frac{2n+r}{n+r}\right)$$

$$\frac{1}{n}\underset{n\to\infty}{v}\sum_{r=1}^{n}ln\left[\left(\frac{2+\frac{r}{n}}{1+\frac{r}{n}}\right)\right]$$

$$\log\left(\frac{A}{B}\right)=\int_0^1 ln\left(\frac{2+x}{1+x}\right)dx$$

$$=\int_0^1 ln(2+x)dx-\int_0^1 ln(1+x)dx$$

$$\left((x+2)\log(x+2)-(x+2)\right)_0^1-\left[(1+x)\log(1+x)-(1+x)\right]_0^1$$

$$=\left((3\log 3-3)-(2\log 2-2)\right)-(2\log 2-2)-(0-1)$$

$$\log\left(\frac{A}{B}\right)=\log 27-\log 16=\log\frac{27}{16}$$

$$\frac{A}{B}=\frac{27}{16}\Rightarrow A=27,B=16$$