CBSE Board Paper Solution - 2020

Class	:	X
Subject	:	Mathematics (Standard) - Theory
Set	:	1
Code No	:	30/5/1
Time allowed	:	3 Hours
Maximum Marks	:	80 Marks

General instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper comprises four sections – A, B, C and D. This question paper carries 40 question All questions are compulsory

(ii) Section A : Question Numbers 1 to 20 comprises of 20 question of one mark each.

(iii) Sect ion B : Question Numbers 21 to 26 comprises of 6 question of two marks each.

(iv) Section C : Question Numbers 27 to 34 comprises of 8 question of three marks each.

(v) Section D : Question Numbers 35 to 40 comprises of 6 question of four marks each.

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in 2 question of the mark, 2 question of one mark, 2 questions of two marks. 3 question of three marks and 3 question of four marks. You have to attempt only one of the choices in such questions.

(vii) In addition to this. Separate instructions are given with each section and question, wherever necessary.

(viii) Use of calculations is not permitted.

Section A

Question numbers 1 to 20 carry 1 mark each.
Question numbers 1 to 10 are multiple choice questions. Choose the correct option.

1. On dividing a polynomial p(x) by $x^2 - 4$, quotient and remainder are found to be x and 3 respectively. The polynomial p(x) is

(A) $3x^2 + x - 12$

(B) $x^3 - 4x + 3$

(C) $x^2 + 3x - 4$

(D) $x^2 - 4x - 3$

Answer:
Correct Answer: (B) $x^3 - 4x + 3$
Explanation:
P(x) = (divisor)×(quotient) + Remainder
$= (x^2 - 4)x + 3$
$= x^3 - 4x + 3$

2) In Figure-1, ABC is an isosceles triangle, right-angled at C. Therefore

(A) $AB^2 = 2AC^2$

(B) $BC^2 = 2AB^2$

(C) $AC^2 = 2AB^2$

(D) $AB^2 = 4AC^2$

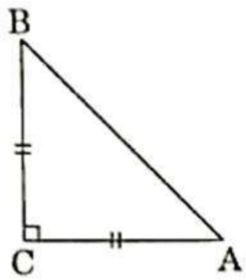


Figure-1

Answer:
Correct Answer: (A) $AB^2 = 2AC^2$
Explanation:

Given that ACB is an isosceles triangle right angled at C.

Therefore, AC = BC

Using Pythagoras theorem in the given triangle,

we have

$$AB^2 = AC^2 + BC^2$$
$$= AC^2 + AC^2$$
$$= 2AC^2$$

3) The point on the x-axis which is equidistant from (−4, 0) and 10, 0) is

(A) (7, 0)

(B) (5, 0)

(C) (0, 0)

(D) (3, 0)

OR

The centre of a circle whose end points of a diameter are (– 6, 3) and 6, 4) is

(A) (8, – 1)

(B) (4, 7)

(C) $\left(0, \frac{7}{2}\right)$

(D) $\left(4, \frac{7}{2}\right)$

Answer:

Correct Answer: (D) (3, 0)

Explanation:

The required point and the given points as well lie on the x-axis.

The required point (x, 0) is the mid-point of the line joining points (–4, 0) and (10, 0).

So, $x = (-4+10)/2$

$= 6/2$

$= 3$

Required point = (x, 0)

= (3, 0)

OR

Correct Answer: (C) (0, 7/2)
Explanation:
The centre of a circle is the mid-point of its diameter.
End points of the diameter are: (–6, 3) and (6, 4)
Coordinates of the centre = ((– 6+6)/2, (3+4)/2)
= (0, 7/2)

4) The value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots, is

(A) 4

(B) ±4

(C) −4

(D) 0

Answer:
Correct Answer: (B) ±4
Explanation:
The given equation is:
$2x^2 + kx + 2 = 0$
Discriminant = $b^2 - 4ac$
Here, b =k, a =2, and c =2
So, Discriminant = $k^2 - 4 \times 2 \times 2$
= $k^2 - 16$
A quadratic equation has equal roots if its discriminant is zero.
$k^2 - 16 = 0$

$k^2 = 16$

$k = \pm 4$

5) Which of the following is not an A.P.?

(A) $-1.2, 0.8, 2.8 \ldots$

(B) $3, 3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, \ldots$

(C) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$

(D) $\frac{-1}{5}, \frac{-2}{5}, \frac{-3}{5}, \ldots$

Answer:

Correct Answer: (C) $\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$

Explanation:

$\frac{4}{3}, \frac{7}{3}, \frac{9}{3}, \frac{12}{3}, \ldots$

$$\frac{7}{3} - \frac{4}{3} = \frac{7-4}{3}$$

$$= \frac{3}{3}$$

$$= 1$$

$$\frac{9}{3} - \frac{7}{3} = \frac{9-7}{3}$$

$$= \frac{2}{3}$$

$$\Rightarrow \quad \frac{3}{3} \neq \frac{2}{3}$$

Difference between consecutive terms is not same. So, this is not an A.P.

6) The pair of linear equations $\frac{3x}{2} + \frac{5y}{3} = 7$ and $9x + 10y = 14$ is

(A) consistent

(B) inconsistent

(C) consistent with one solution

(D) consistent with many solutions

Answer:

Correct Answer: (B) Inconsistent

Explanation:

$$\frac{3x}{2}+\frac{5y}{3}=7$$

$$\frac{9x+10y}{6}=7$$

$$9x+10y=42 \quad ...(1)$$

$$9x+10y=14 \quad ...(2)$$

Ratios of coefficients of x and that of y are

$$\frac{9}{9}=\frac{10}{10}=\frac{1}{1}$$

Ratio of constants$=\frac{42}{14}=\frac{3}{1}\neq\frac{1}{1}$

Ratios of coefficients of x and y are equal but they are not equal to the ratio of constants.

So, the given equations represent a pair of parallel lines and so they do not have a common solution.

7) In Figure-2 PQ is tangent to the circle with centre at O, at the point B. If $\angle AOB = 100°$, then $\angle ABP$ is equal to

(A) 50°

(B) 40°

(C) 60°

(D) 80°

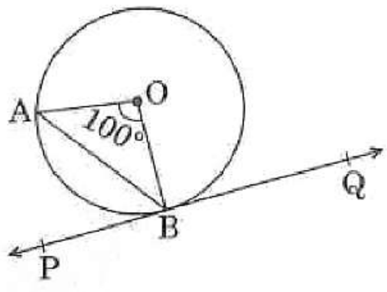


Figure-2

Answer:

Correct Answer: (A) 50°

Explanation:

OA = OB (radii)

So, ∠OAB = ∠OBA

= (180° –100°)/2

= 40°

Now, a radius of a circle meets a tangent at 90°.

So, ∠ABP = ∠OBP – ∠OBA

= 90° –40° = 50°

8) The radius of a sphere (in cm) whose volume is 12π cm^3, is

(A) 3

(B) $3\sqrt{3}$

(C) $3^{2/3}$

(D) $3^{1/3}$

Answer:

Correct Answer: (C)$3^{2/3}$

Explanation:

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$12\pi = \frac{4}{3}\pi r^3$$

$$r^3 = 3^2$$

$$r = 3^{2/3}$$

9) The distance between the points (m,−n) and (−m, n) is

(A) $\sqrt{m^2+n^2}$

(B) **m + n**

(C) $2\sqrt{m^2+n^2}$

(D) $\sqrt{2m^2+2n^2}$

Answer:

Correct Answer: (C) $2\sqrt{m^2+n^2}$

Explanation:

$$\text{Distance} = \sqrt{\,m-(-m)\,^{2}+(-n-n)^2}$$

$$= \sqrt{(m+m)^2+(-2n)^2}$$

$$= 2\sqrt{m^2+n^2}$$

10) In Figure-3. From an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If ∠QPR = 90°, then length of PQ is

(A) 3cm

(B) 4cm

(C) 2cm

(D) $2\sqrt{2}$ cm

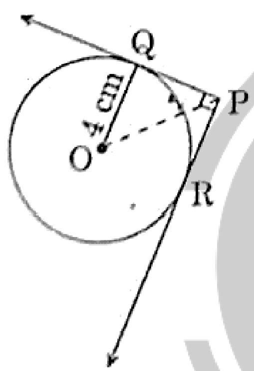


Figure-3

Answer:
Correct Answer: (B) 4 cm
Explanation:
Tangents are drawn from an external point P.

So, line joining centre O and point P bisects ∠PQR.

OP bisects ∠QPR = 90°.

In Δ OQP,

∠Q = 90° (radius meets tangent at 90°)

∠QPO = 45° = ∠QOP

Thus, OQ = PQ = 4 cm

Fill in the blanks in question number 11 to 15

11) The probability of an event that is sure to happen is __.

Answer: 1

12) Simplest form of $\frac{1+\tan^2 A}{1+\cot^2 A}$ is _______.

Answer:

$\cot^2 A$

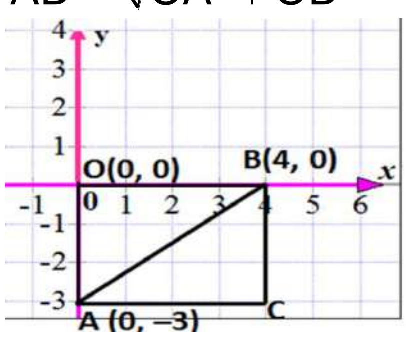
$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\operatorname{cosec}^2 A} = \frac{\sin^2 A}{\cos^2 A} = \cot^2 A$$

13) AOBC is a rectangle whose three vertices are A(0, −3), O(0, 0) and B (4, 0). The length of its diagonal is ____.

Answer:

In right-angled triangle AOB,

$$AB = \sqrt{OA^2 + OB^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$



14) In the formula $\bar{x} = a + \left(\frac{\sum f_i u_i}{\sum f_i}\right) \times h$, u_i = ______.

Answer:

$$\frac{x_i - a}{h}$$

15) All concentric circles are _______ to each other.

Answer: similar

Answer the following question numbers 16 to 20.

16) Find the sum of the first 100 natural numbers.

Answer:

$1 + 2 + 3 + ...100$ is an A. P.

Here first term $a = 1$

Common difference d =1

Sum of n terms of an A.P. $= \frac{n}{2}\left[2a + (n-1)d\right]$

The sum of first 100 natural numbers

$$= \frac{100}{2}\left[2 \times 1 + (100-1) \times 1\right]$$

$$= \frac{100\ (101)}{2}$$

$$= 50 \times 101$$

$$= 5050$$

17) In Figure-4 the angle of elevation of the top of a tower from a point C on the ground, which is

30 m away from the foot of the tower, is 30°. Find the height of the tower.

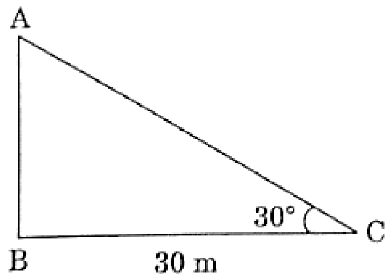


Figure-4

Answer:

$$\tan 30^\circ = \frac{AB}{30}$$

$$\frac{1}{\sqrt{3}} = \frac{AB}{30}$$

$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3}$$

So, the height of the tower is $10\sqrt{3}$ m.

18) The LCM of two numbers is 182 and their HCF is 13. If one of the numbers is 26, Find the other.

Answer:

$LCM \times HCF =$ Product of the two numbers

$182 \times 13 = 26 \times x$

$$x = \frac{182 \times 13}{26} = 91$$

So, the other number is 91.

19) Form a quadratic polynomial, the sum and product of whose zeroes are (-3) and 2 respectively.

OR

Can (x^2 – 1) be a remainder while dividing

$x^4 - 3x^2 + 5x - 9$ by (x^2 +3)? Justify your answer with reasons.

Answer:

x^2 – (sum of zeroes)x + product of zeroes

= $x^2 - (-3)x+2$

= $x^2 + 3x+2$

So, the required polynomial is $x^2 +3x+2$.

OR

When a polynomial p(x) is divided by another polynomial g(x), then the degree of remainder r(x) < degree of g(x)

Therefore, for the given question $x^2 - 1$ cannot be a remainder while dividing $x^4 - 3x^2 + 5x - 9$ by $x^2 + 3$ because deg ($x^2 - 1$) = deg ($x^2 + 3$).

20) Evaluate:

$$\frac{2 \tan 45^0 \times \cos 60^0}{\sin 30^0}$$

Answer:

$$\frac{2\tan 45° \times \cos 60°}{\sin 30°}$$

$$= \frac{2 \times 1 \times \frac{1}{2}}{\frac{1}{2}}$$

$$= 2$$

SECTION B

Question number 21 to 26 carry 2 marks each.

21) In the given Figure-5, DE ||AC and DF||AE.

Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

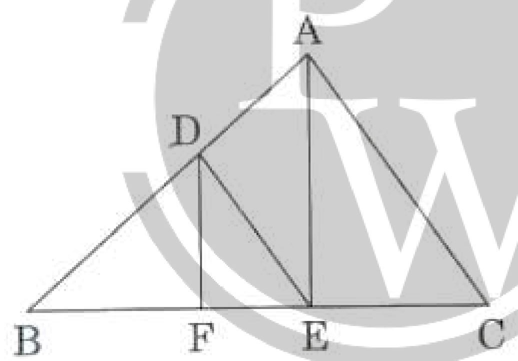


Figure-5

Answer:

In ΔABC, $DE \parallel AC$

So, using basic proportionality theorem, we get

$$\frac{BD}{DA} = \frac{BE}{EC} \qquad ...(1)$$

In ΔBAE, $DF \parallel AE$

So, using basic proportionality theorem, we get

$$\frac{BD}{DA} = \frac{BF}{FE} \qquad ...(2)$$

From (1) and (2), we get

$$\frac{BE}{EC} = \frac{BF}{FE}$$

22) Show that $5+2\sqrt{7}$ is an irrational number, where $\sqrt{7}$ is given to be an irrational number.

OR

Check whether 12^n can end with the digit 0 for any natural number n.

Answer:

Let us assume, to the contrary, that $5+2\sqrt{7}$ is rational.

That is, we can find coprime a and b ($b \neq 0$) such that

$$5+2\sqrt{7} = \frac{a}{b}$$

$$\therefore\ 2\sqrt{7} = \frac{a}{b} - 5$$

Rearranging this equation, we get $\sqrt{7} = \frac{1}{2}\left(\frac{a}{b} - 5\right) = \frac{a-5b}{2b}$

Since, a and b are integers, we get $\frac{a-5b}{2b}$ is rational, and so $\sqrt{7}$ is a rational.

But this contradicts the fact that $\sqrt{7}$ is irrational.

This contradiction has arisen because of our incorrect assumption that $5+2\sqrt{7}$ is rational.

So, we conclude that $5+2\sqrt{7}$ is irrational.

OR

If the number 12^n, for any n, were to end with the digit zero, then it would be divisible by 5.

That is, the prime factorisation of 12^n would contain the prime 5. This is not possible

$\because$ $12^n = (2 \times 2 \times 3)^n$

So, the prime numbers in the factorisation of 12^n are 2 and 3.

So, the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of 12^n.

So, there is no natural number n for which 12^n ends with the digit zero.

23) If A, B and C are interior angles of a $\angle$ ABC, then show that

$$\cos\left(\frac{B+C}{2}\right)=\sin\frac{A}{2}.$$

Answer:

Given that A, B and C are interior angles of a triangle ABC.

$\therefore \quad A + B + C = 180^\circ$

or $\quad A = 180^\circ - B - C$

Now,

$$\cos\left(\frac{B+C}{2}\right) = \sin\left(90^\circ - \frac{B+C}{2}\right)$$

$$= \sin\left(\frac{180^\circ - B - C}{2}\right)$$

$$= \sin\left(\frac{A}{2}\right)$$

24) In Figure 6, a quadrilateral ABCD is drawn to circumscribe a circle.

Prove that

AB + CD = BC +AD.

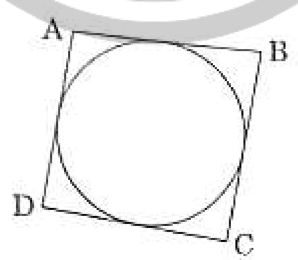


Figure-6

OR

In Figure-7, find the perimeter of ∠ ABC, if AP = 12 cm.

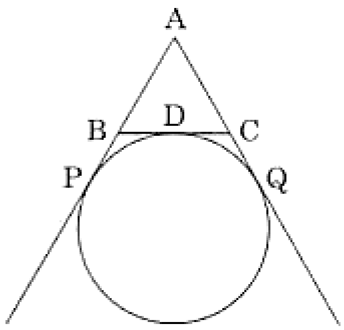
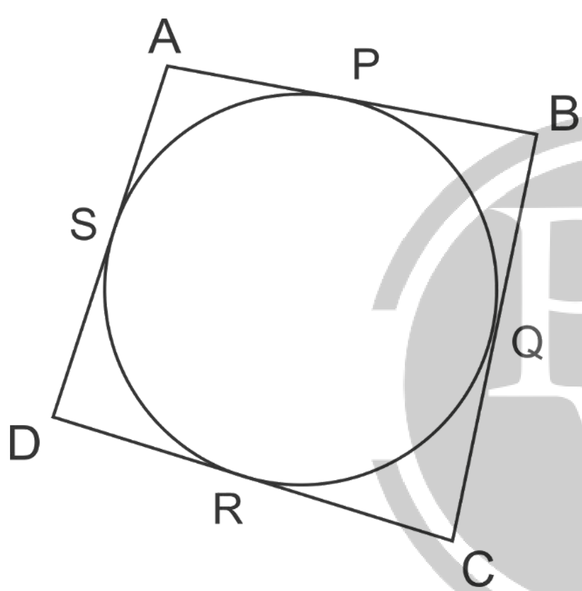


Figure-7

Answer:



We have to prove that

$AB + CD = BC + AD$

We know that lengths of tangents drawn from a point to a circle are equal.

Therefore, from figure, we have

$DR = DS,\ CR = CQ,\ AS = AP,\ BP = BQ$

Now,

$$
\begin{aligned}
LHS = AB + CD &= (AP + BP) + (CR + DR) \\
&= (AS + BQ) + (CQ + DS) \\
&= BQ + CQ + AS + DS \\
&= BC + AD \\
&= RHS
\end{aligned}
$$

OR

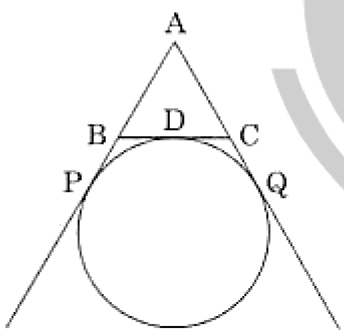


Figure-7

From the given figure, we have AP = 12 cm

Since AQ and AB are the tangent to the circle from a common point A, hence AP = AQ = 12

Similarly, PB = BD and CD =CQ

Also, AP =AB + PB and AQ = AC + CQ

Perimeter of ABC = AB + BD + CD + AC

= AB + PB + CQ + AC

(since PB = BM and CM = CQ)

= (AB + PB) + (CQ + AC)

= AP + AQ

= 12 +12

= 24 cm

Therefore, the perimeter of triangle ABC =24 cm

25) Find the mode of the following distribution:

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of Students	4	6	7	12	5	6

Answer:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of Students	4	6	7	12	5	6

From the given data, we have

$l = 30,\ f_1 = 12,\ f_0 = 7,\ \ f_2 = 5,\ h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{12 - 7}{2 \times 12 - 7 - 5}\right) \times 10$$

$$= 34.1667$$

$\therefore$ Mode of the given data is 34.1667.

26) 2 cubes, each of volume 125 cm³, are joined end to end. Find the surface area of the resulting cuboid.

Answer:

Let the side of the old cube = a

The volume of the old cube = 125 cm^3 (Given)

The volume of the cube = a^3

$a^3 = 125\ cm^3$

$a^3 = 5^3$

a = 5 cm

The dimensions of the resulting cuboid are:

Length, l = 10 cm

Breadth, b = 5 cm

Height, h = 5 cm

Total surface area of the resulting cuboid:

= 2(lb+bh+hl)

= 2[10(5)+5(5)+5(10)]

= 2[50+25+50]

= 2[125]

= 250 cm^2

Section C

27) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

OR

The present age of a father is three years more than three times the age of his son. Three years

hence the father's age will be 10 years more than twice the age of the son. Determine their present ages.

Answer:

Let the numerator of the fraction be x and denominator be y.

Therefore, the fraction is $\frac{x}{y}$.

According to question,

$$\frac{x-1}{y} = \frac{1}{3}$$

$$3(x-1) = y$$

$$3x - 3 = y \quad ...(1)$$

and $\frac{x}{y+8} = \frac{1}{4}$

$$4x = y + 8$$

$$4x - 8 = y \quad ...(2)$$

From equations (1) and (2), we get

$$3x - 3 = 4x - 8$$

$$4x - 3x = 8 - 3$$

$$x = 5$$

Putting $x = 5$ in equation (1),

$$3 \times 5 - 3 = y$$

$$y = 12$$

So, the required fraction $= \frac{5}{12}$.

OR

Let the son's present age be x.

So, father's present age = $3x+3$

3 years later:

Son's age = $x+3$

Father's age = $3x+3+3=3x+6$

But, according to the given condition,

3 years later father's age= $2(x+3)+10$

$$=2x+6+10$$

$$=2x+16$$

So, we can write

$$3x+6=2x+16$$

$$3x-2x=16-6$$

$$x=10$$

So, son's present age $=10$ years

and father's present age $=10\times3+3$

$$=33 \text{ years}$$

28) Use Euclid Division Lemma to show that the square of any positive integer is either of the form 3q or 3q + 1 for some integer q.

Answer:

Let a be a positive integer and $b = 3$.

By Euclid's Algorithm,

$a = 3m + r$ for some integer $m \geq 0$ and $0 \leq r < 3$.

The possible remainders are 0, 1 and 2. Therefore,

a can be $3m$ or $3m + 1$ or $3m + 2$.

Thus,

$$\begin{aligned} a^2 &= 9m^2 \text{ or } (3m+1)^2 \text{ or } (3m+2)^2 \\ &= 9m^2 \text{ or } (9m^2 + 6m + 1) \text{ or } (9m^2 + 12m + 4) \\ &= 3 \times (3m^2) \text{ or } 3(3m^2 + 2m) + 1 \text{ or } 3(3m^2 + 4m + 1) + 1 \\ &= 3k_1 \text{ or } 3k_2 + 1 \text{ or } 3k_3 + 1 \end{aligned}$$

where k_1, k_2 and k_3 are some positive integers.

Hence, square of any positive integer is either of the form $3q$ or $3q + 1$ for some integer q.

29) Find the ratio in which y-axis divides the line segment joining the points (6, - 4) and (-2, -7). Also find the point of intersection.

OR

Show that the points (7, 100, (-2, 5) and (3, -4) are vertices of an isosceles right triangle.

Answer:

Let the ratio in which the line segment joining A$(6, \ -4)$ and B$(-2, \ -7)$ is divided by the y-axis be $k:1$.

Let the coordinate of point on y-axis be $(0, y)$.

Therefore,

$$0 = \frac{-2k+6}{k+1} \quad \text{and} \quad y = \frac{-7k-4}{k+1}$$

Now,

$$0 = \frac{-2k+6}{k+1}$$

or $0 = -2k + 6$

or $k = 3$

Therefore, the required ratio is 3:1.

Also,

$$y = \frac{-7k-4}{k+1}$$

$$= \frac{-7 \times 3 - 4}{3+1}$$

$$= \frac{-25}{4}$$

Therefore, the given line segment is divided by the point $\left(0, \ \frac{-25}{4}\right)$ in the ratio 3:1.

OR

Let the given points are P(7, 10), Q(-2, 5) and R(3, -4).
Now, using distance formula we find distance between these points i.e., PQ, QR and PR.
Distance between points P(7, 10) and Q(-2, 5),

$$PQ = \sqrt{(-2-7)^2 + (5-10)^2}$$
$$= \sqrt{81+25}$$
$$= \sqrt{106}$$

Distance between points Q(-2, 5) and R(3, -4),

$$QR = \sqrt{(3+2)^2 + (-4-5)^2}$$
$$= \sqrt{25+81}$$
$$= \sqrt{106}$$

Distance between points P(7, 10) and R(3, -4),

$$PR = \sqrt{(3-7)^2 + (-4-10)^2}$$
$$= \sqrt{16+196}$$
$$= \sqrt{212}$$

Now,

$$PQ^2 + QR^2 = 106 + 106$$
$$= 212 = PR^2$$

i.e., $PQ^2 + QR^2 = PR^2$

Therefore, points P(5, -2), Q(6, 4) and R(7, -2) form an isosceles right triangle because sides PQ and QR are equal.

30) Prove that:

$$\sqrt{\frac{\mathbf{1+\sin A}}{\mathbf{1-\sin A}}} = \mathbf{\sec\ A + \tan\ A}$$

Answer:

$$\begin{aligned}
\text{LHS} &= \sqrt{\frac{1+\sin A}{1-\sin A}} \\
&= \sqrt{\frac{1+\sin A}{1-\sin A}} \times \sqrt{\frac{1+\sin A}{1+\sin A}} \\
&= (1+\sin A)\sqrt{\frac{1}{1-\sin^2 A}} \\
&= \frac{1+\sin A}{\sqrt{\cos^2 A}} \\
&= \frac{1+\sin A}{\cos A} \\
&= \frac{\sin A}{\cos A} + \frac{1}{\cos A} \\
&= \tan A + \sec A = \text{RHS}
\end{aligned}$$

31) For an A.P., it is given that the first term (a) = 5, common difference (d) = 3, and the n^{th} term (a_n) = 50. Find n and sum of first n terms (S_n) of the A.P.

Answer:

Here, $a = 5$, $d = 3$, $a_n = 50$

We need to find S_n.

Firstly, we will find the value of n.

We know that

$$a_n = a + (n-1)d$$

So, $\quad 50 = 5 + (n-1)3$

or $\quad 50 - 5 = (n-1)3$

or $\quad \frac{45}{3} + 1 = n$

or $\quad n = 16$

We know that sum of first n terms of an AP is given by

$$S_n = \frac{n}{2}(a + a_n)$$

So, $\quad S_{16} = \frac{16}{2}(5 + 50)$

$$= 8 \times 55$$

or $\quad S_{16} = 440$

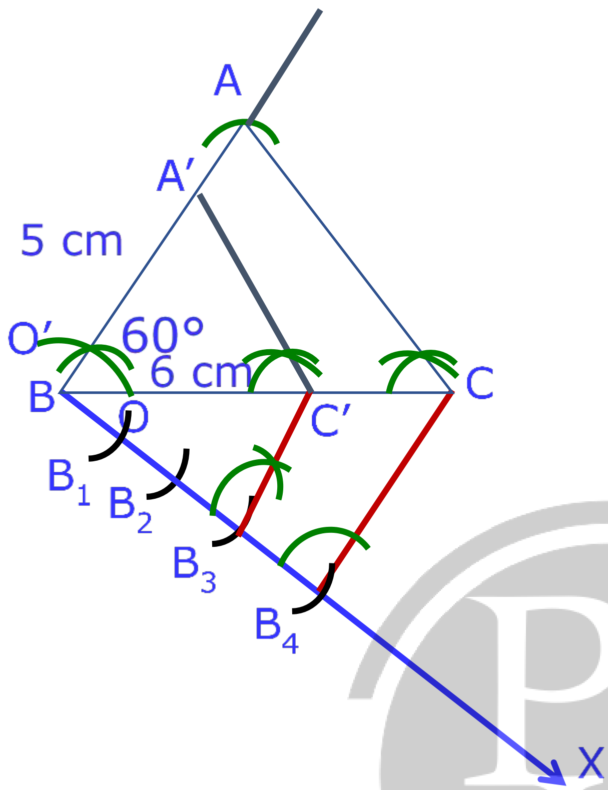
32) Construct a ΔABC with sides BC = 6 cm, AB = 5 cm and ∠ABC = 60°. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

OR

Draw a circle of radius 3.5 cm. Take a point P outside the circle at a distance of 7 cm from the centre of the circle and construct a pair of tangents

to the circle from that point.

Answer:



Steps of Construction :

Step 1: Draw a $\triangle ABC$ with sides $AB = 5$ cm, $BC = 6$ cm and $\angle ABC = 60°$.

Step 2: Draw a ray BX making an acute angle with line BC on the opposite side of vertex A.

Step 3: Locate 4 points B_1, B_2, B_3, B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$.

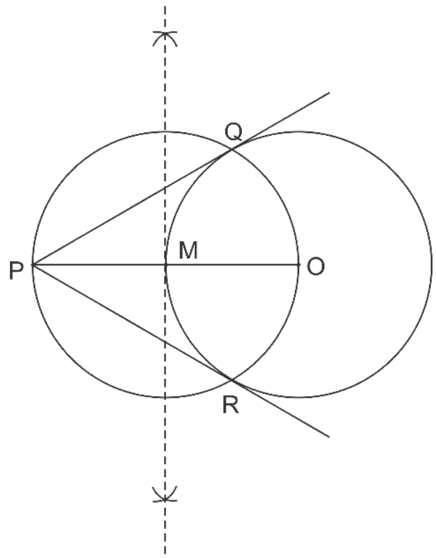
Step 4: Join the points C and B_4.

Step 5: Through the point B_3, draw a line parallel to CB_4 intersecting line segment BC at point C'.

Step 6: Draw a line through C' parallel to the line AC to intersect line segment AB at A'.

The required triangle is $\triangle A'BC'$.

OR



Steps of Construction :

Step 1: Draw a circle of radius 3.5 cm with centre at point O. Locate a point P, at a distance of 7 cm from O, and join O and P.

Step 2: Bisect OP. Let M be the mid-point of OP.

Step 3: Draw a circle with centre at M and MO as radius. Q and R are points of intersections of this circle with the circle having centre at O.

Step 4: Join PQ and PR.

PQ and PR are the required tangents.

33) Read the following passage and answer the question given at the end:

Diwali Fair.

A game in a booth at a Diwali Fair involves using a spinner first. Then, if the spinner stops on an

even number, the player is allowed to pick a marble from a bag. The spinner and the marbles in the bag are represented in Figure – 8.

Prizes are given when a black marble is picked. Shweta plays the game once.

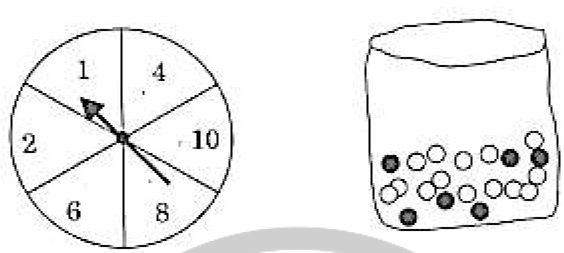


Figure-8

(i) What is the probability that she will be allowed to pick a marble from the bag?

(ii) (ii) Suppose she is allowed to pick a marble from the bag, what is the probability of getting a prize, when it is given that the bag contains 20 balls out of which 6 are black?

Answer:

Numbers on spinner $= 1,\ 2,\ 4,\ 6,\ 8, 10$

Even numbers on spinner $= 2,\ 4,\ 6,\ 8, 10$

Shweta will pick black marble, if spinner stops on even number.

Therefore,

$$n(\text{Even number}) = 5$$

$$n(\text{Possible number}) = 6$$

$(i)\, P(\text{Shweta allowed to pick a marble})$

$= P(\text{Even number})$

$= \dfrac{n(\text{Even number})}{n(\text{Possible number})}$

$= \dfrac{5}{6}$

Therefore, the probability of allowing Shweta to pick a marble is $\dfrac{5}{6}$.

(ii) Since, prizes are given, when a black marble is picked.

Number of black marbles $= 6$

Total number of marbles $= 20$

Therefore, $P(\text{getting a prize}) = P(\text{a black marble})$

$$= \frac{n(\text{Black marbles})}{n(\text{Total marbles})}$$

$$= \frac{6}{20}$$

$$= \frac{3}{10}$$

Therefore, the probabiltiy of geting prize is $\frac{3}{10}$.

34. In figure – 9, a square OPQR is inscribed in a quadrant OAQB of a circle. If the radius of circle is $6\sqrt{2}$ cm, find the area of the shaded region.

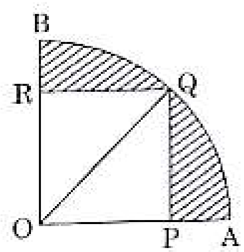


Figure-9

Answer:

Given that, $OQ = 6\sqrt{2}$ cm

OPQR is a square.

Let the side of square $= a$

The diagonal of square $= a\sqrt{2}$

Here, OQ is a diagonal of square.

$\Rightarrow \quad a\sqrt{2} = 6\sqrt{2}$

$\Rightarrow \quad a = 6$ cm

Area of square OPQR $= 6^2$

$= 36 \text{ cm}^2$

Radius of the quadrant OAQB = Diagonal of the square OPQR

$= 6\sqrt{2}$ cm

Area of the quadrant OAQB $= \dfrac{90^\circ}{360^\circ} \times \dfrac{22}{7} \times \left(6\sqrt{2}\right)^2$

$= \dfrac{396}{7} \text{ cm}^2$

Area of shaded region = Area of the quadrant OAQB − Area of square OPQR

$= \dfrac{396}{7} - 36$

$= \dfrac{144}{7}$

$= 20.6 \text{ cm}^2$

SECTION D

35) Obtain other zeroes of the polynomial $p(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$ if two of its zeroes are $\sqrt{5}$ and $-\sqrt{5}$.

OR

What minimum must be added to $2x^3 - 3x^2 + 6x + 7$ so that resulting polynomial will be divisible by $x^2 - 4x + 8$?

Answer:

The given polynomial is $p(x) = 2x^4 - x^3 - 11x^2 + 5x + 5$.

The two zeroes of p(x) are $\sqrt{5}$ and $-\sqrt{5}$.

Therefore, $\left(x - \sqrt{5}\right)$ and $\left(x + \sqrt{5}\right)$ are factors of p(x).

Also, $\left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right) = x^2 - 5$

and so $x^2 - 5$ is a factor of p(x).

Now,

$$
\begin{array}{r|l}
 & 2x^2 - x - 1 \\
x^2 - 5 & 2x^4 - x^3 - 11x^2 + 5x + 5 \\
 & 2x^4 \qquad - 10x^2 \\
 & - \qquad\quad + \\
\hline
 & -x^3 - x^2 + 5x + 5 \\
 & -x^3 \qquad + 5x \\
 & + \qquad\quad - \\
\hline
 & -x^2 \qquad + 5 \\
 & -x^2 \qquad + 5 \\
 & + \qquad\quad - \\
\hline
 & 0
\end{array}
$$

$$2x^4 - x^3 - 11x^2 + 5x + 5 = (x^2 - 5)(2x^2 - x - 1)$$

$$= (x^2 - 5)(2x^2 - 2x + x - 1)$$

$$= (x^2 - 5)(2x + 1)(x - 1)$$

Equating $(x^2 - 5)(2x + 1)(x - 1)$ to zero, we get the zeroes of the given polynomial.

Hence, the zeroes of the given polynomial are :

$\sqrt{5}, -\sqrt{5}, -\frac{1}{2}$ and 1.

OR

The given polynomial is $2x^3 - 3x^2 + 6x + 7$.

Here, divisor is $x^2 - 4x + 8$.

Divide $2x^3 - 3x^2 + 6x + 7$ by $x^2 - 4x + 8$ and find the remainder.

$$
\begin{array}{r|l}
 & 2x + 5 \\
\hline
x^2 - 4x + 8 & 2x^3 - 3x^2 + 6x + 7 \\
 & 2x^3 - 8x^2 + 16x \\
 & - \quad + \quad - \\
\hline
 & 5x^2 - 10x + 7 \\
 & 5x^2 - 20x + 40 \\
 & - \quad + \quad - \\
\hline
 & 10x - 33
\end{array}
$$

$\text{Remainder} = 10x - 33$

Therefore, we should add $-(10x - 33)$ to make it exactly divisible by $x^2 - 4x + 8$.

Thus, we should add $-10x + 33$ to $2x^3 - 3x^2 + 6x + 7$.

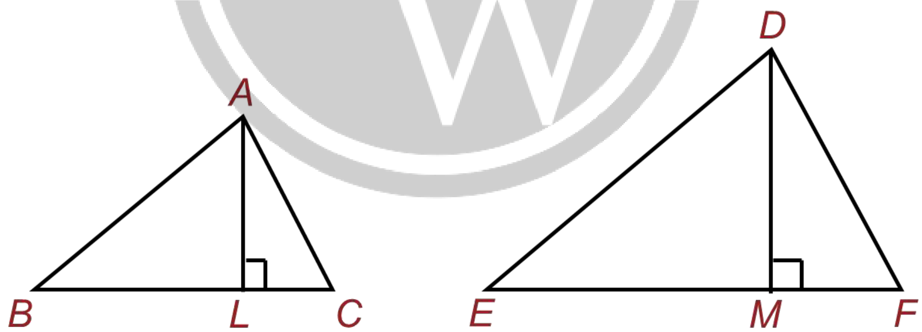
36) Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Answer:

Given : $\Delta ABC \sim \Delta DEF$

To prove : $\dfrac{\text{Area } \Delta ABC}{\text{Area } \Delta DEF} = \left(\dfrac{AB}{DE}\right)^2 = \left(\dfrac{BC}{EF}\right)^2 = \left(\dfrac{AC}{DE}\right)^2$

Construction: Draw $AL \perp BC$ and $DM \perp EF$



Proof: Here $\frac{\text{Area } \Delta ABC}{\text{Area } \Delta DEF} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times EF \times DM} = \frac{BC \times AL}{EF \times DM} \quad \cdots 1$

In ΔALB and ΔDME

$\angle ALB = \angle DME$ Each 90°

and $\angle B = \angle E$ Since $\Delta ABC \sim \Delta DEF$

So, $\Delta ALB \sim \Delta DME$ AA similarity criterion

$\Rightarrow \frac{AL}{DM} = \frac{AB}{DE}$

But $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ Since $\Delta ABC \sim \Delta DEF$

Therefore, $\frac{AL}{DM} = \frac{BC}{EF} \quad \cdots (2)$

From (1) and (2), we have

$$\frac{\text{Area } (\Delta ABC)}{\text{Area } (\Delta DEF)} = \frac{BC}{EF} \times \frac{AL}{DM} = \frac{BC}{EF} \times \frac{BC}{EF} = \left(\frac{BC}{EF}\right)^2$$

But $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ Since $\Delta ABC \sim \Delta DEF$

This implies that,

$$\frac{\text{Area } \Delta ABC}{\text{Area } \Delta DEF} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DE}\right)^2$$

37) Sum of the areas of two squares is $544m^2$. If the difference of their perimeters is 32 m, find the sides of the two squares.

OR

A motorboat whose speed is 18km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

Answer:

Let the sides of first and second square be x any y. Then,

Area of first square = x^2
And,
Area of second square = y^2

According to the question,

$x^2 + y^2 = 544 \quad \ldots(1)$

Now,

Perimeter of first square = 4x
And,
Perimeter of second square = 4y

According to the question,

$4x - 4y = 32 \qquad ...(2)$

From equation (2), we get

$$4(x - y) = 32$$

or, $x - y = \frac{32}{4}$

or, $x - y = 8$

or, $x = 8 + y \qquad ...(3)$

Subsituting this value of x in equation(1), we get

$$x^2 + y^2 = 544$$

or, $(8 + y)^2 + y^2 = 544$

or, $64 + y^2 + 16y + y^2 = 544$

or, $2y^2 + 16y + 64 = 544$

or, $2y^2 + 16y + 64 - 544 = 0$

or, $2y^2 + 16y - 480 = 0$

or, $2(y^2 + 8y - 240) = 0$

or, $y^2 + 8y - 240 = 0$

or, $y^2 + 20y - 12y - 240 = 0$

or, $y(y + 20) - 12(y + 20) = 0$

or, $(y + 20)(y - 12) = 0$

$\Rightarrow$ $y + 20 = 0$ or $y - 12 = 0$

$\Rightarrow$ $y = -20$ or $y = 12$

Since side of a square cannot be negative, therefore $y = 12$.

Substituting y = 12 in equation (3), we get

$x = 8 + y = 8 + 12 = 20$

Therefore,

Side of first square = x = 20 cm

And,

Side of second square = y = 12 cm

OR

Let the speed of the stream be x km/h.

Therefore, speed of the boat upstream = (18 – x) km/h
and the speed of the boat downstream = (18 + x) km/h.

The time taken to go upstream = $\frac{\text{distance}}{\text{speed}}$

$$= \frac{24}{18-x} \text{ hours}$$

Similarly, the time taken to go downstream = $\frac{24}{18+x}$ hours

According to the question,

$$\frac{24}{18-x}-\frac{24}{18+x}=1$$

or, $$\frac{24(18+x)-24(18-x)}{(18+x)(18-x)}=1$$

or, $$24(18+x)-24(18-x)=(18+x)(18-x)$$

or, $$432+24x-432+24x=324-x^2$$

or, $$x^2+48x-324=0$$

Using the quadratic formula, we get

$$x=\frac{-48\pm\sqrt{48^2-4(1)(-324)}}{2}$$

$$=\frac{-48\pm\sqrt{2304+1296}}{2}$$

$$=\frac{-48\pm\sqrt{3600}}{2}$$

$$=\frac{-48\pm 60}{2}$$

Therefore, $x=\frac{-48+60}{2}$ or $x=\frac{-48-60}{2}$

$\Rightarrow$ $x=\frac{12}{2}$ or $x=\frac{-108}{2}$

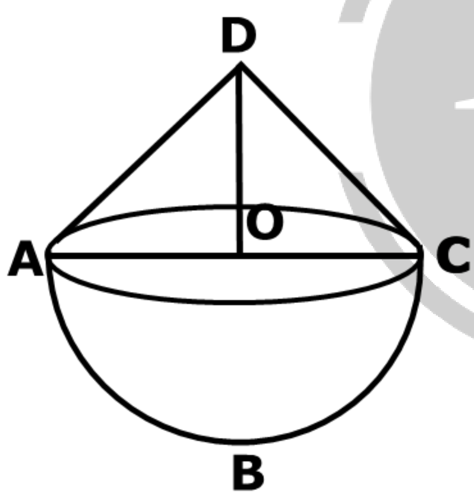
$\Rightarrow$ $x=6$ or $x=-54$

Since x is the speed of the stream, it cannot be negative. So, we ignore the root x = –54. Therefore, x = 6 gives the speed of the stream as 6 km/h.

38. A solid toy is in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10 cm and the radius of the base is 7 cm. Determine the volume of the toy. Also find the area of the coloured sheet required to cover the toy.

(Use $\pi = \frac{22}{7}$ and $\sqrt{149} = 12.2$)

Answer:



Let ABC be the hemisphere and ADC be the cone standing on the base of the hemisphere.

Height of the cone (h_1) = 10 cm (Given)

Radius of the cone (r_1) = 7 cm (Given)

Since the hemisphere is surmounted by the right circular cone of same radius, therefore

Radius of the hemisphere (r_2) = 7 cm

So,

Volume of the toy

= Volume of the cone + Volume of the hemisphere

$$= \frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_2^3$$

$$= \left[\left(\frac{1}{3}\times\frac{22}{7}\times 7\times 7\times 10\right)+\left(\frac{2}{3}\times\frac{22}{7}\times 7\times 7\times 7\right)\right]cm^3$$

$$= \left[\frac{1540}{3}+\frac{2156}{3}\right]cm^3$$

$$= \frac{3696}{3}cm^3$$

$$= 1232 \text{ cm}^3$$

Area of the coloured sheet required to cover the toy

= CSA of hemisphere + CSA of cone

$$= 2\pi r_2^2 + \pi r\ell$$

Where ℓ is the slant height of the cone

$$\ell = \sqrt{r_1^2 + h_1^2}$$

$$= \sqrt{7^2 + 10^2}$$

$$= \sqrt{49 + 100}$$

$$= \sqrt{149}$$

$$= 12.2 \text{ cm}$$

So,

Area of the coloured sheet required to cover the toy

$$= \left[\left(2 \times \frac{22}{7} \times 7 \times 7\right) + \left(\frac{22}{7} \times 7 \times 12.2\right)\right] cm^2$$

$$= (308 + 268.4) cm^2$$

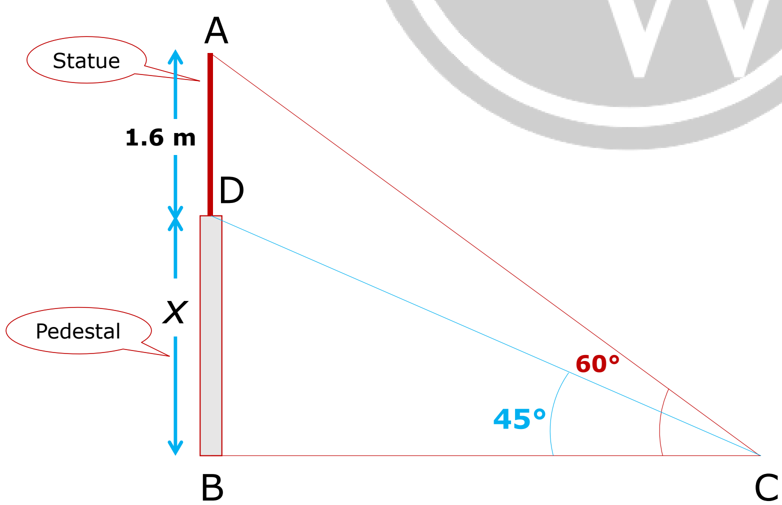
$$= 576.4 \text{ cm}^2$$

39. A statue 1.6 m tall, stands on the top of a pedestal.

From a point on the ground, the angle of elevation of the top of the statue is 60^0 and from the same point the angle of elevation of the top of the pedestal is 45^0. Find the height of the pedestal.

(Use $\sqrt{3} = 1.73$)

Answer:



Let BD be a pedestal of height x m and AD be a statue of height 1.6 m. The angle of elevation of the top of pedestal from a point C is 45° and that of point statue from C is 60°.

In the triangle ABC:

$$\frac{AB}{BC} = \tan 60^\circ$$

$$\frac{1.6 + x}{BC} = \sqrt{3}$$

Or, $BC = \frac{1.6 + x}{\sqrt{3}}$ ... 1

In the triangle DBC:

$$\frac{DB}{BC} = \tan 45^\circ$$

Or, $\frac{x}{BC} = 1$

Or, $x = BC$ ... 2

By equations 1 and 2, we get

$$x = \frac{1.6 + x}{\sqrt{3}}$$

Or, $\sqrt{3}x = 1.6 + x$

$(\sqrt{3} - 1)x = 1.6$

Or, $x = \frac{1.6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$$= \frac{1.6\ 1.73+1}{3-1}$$

$$= \frac{1.6 \times 2.73}{2}$$

$$= 2.184\,\text{m}$$

Therefore, the height of the pedestal is $2.184\,\text{m}$.

40) For the following data, draw a 'less than' ogive and hence find the median of the distribution.

Age(in years):	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**
Number of persons:	**5**	**15**	**20**	**25**	**15**	**11**	**9**

OR

The distribution given below show the number of wickets taken by bowlers in one-day cricket matches. Find the mean and the median of the number of wickets taken.

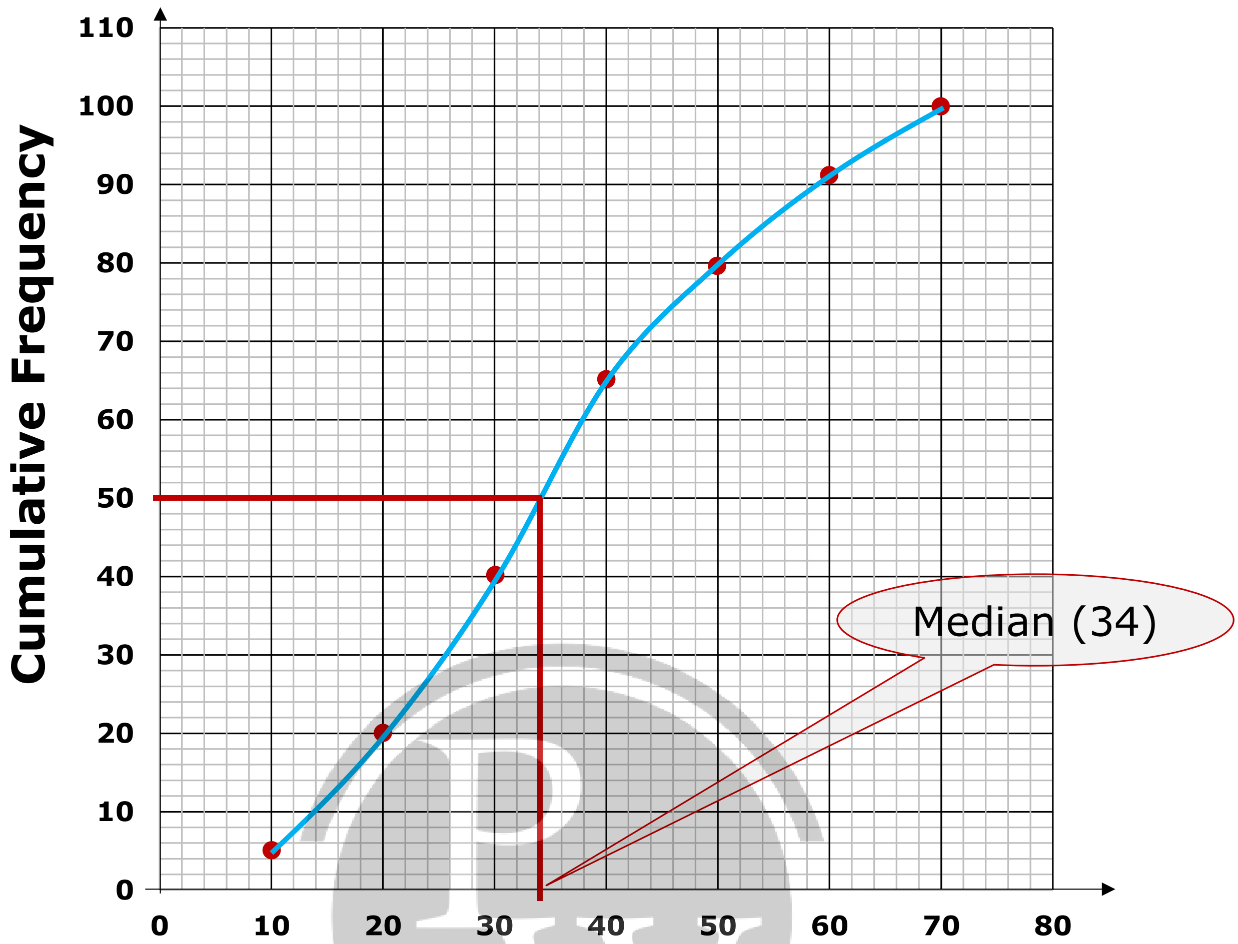
Number of wickets	**20-60**	**60-100**	**100-140**	**140-180**	**180-220**	**220-260**
Number of bowlers:	**7**	**5**	**16**	**12**	**2**	**3**

Answer:

Age	Number of Persons (Cumulative frequency)
Less than 10	5
Less than 20	5 + 15 = 20
Less than 30	20 + 20 = 40
Less than 40	40 + 25 = 65
Less than 50	65 + 15 = 80
Less than 60	80 + 11 = 91
Less than 70	91 + 9 = 100

Age	No. of Persons (f)	Cumulative frequency (cf)
0 – 10	5	5
10 – 20	15	20
20 – 30	20	40
30 – 40	25	65
40 – 50	15	80
50 – 60	11	91
60 – 70	9	100

Plot the points (10, 5), (20, 20), …, (70, 100) on a graph paper.


Cumulative Frequency
110
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
Median (34)
Upper Limits

OR

Class interval	No. of bowlers f_i	Class mark x_i	$f_i x_i$
20 – 60	7	40	280
60 – 100	5	80	400
100 – 140	16	120	1920
140 – 180	12	160	1920
180 – 220	2	200	400
220 – 260	3	240	720
Total	$\sum f_i = 45$		$\sum f_i x_i = 5640$

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{5640}{45} = 125.33$$

Number of wickets	Number of bowlers	Cumulative Frequency
20 – 60	7	7
60 – 100	5	12
100 – 140	16	28
140 – 180	12	40
180 – 220	2	42
220 – 260	3	45

$n = 45$

$$\Rightarrow \frac{n}{2} = \frac{45}{2} = 22.5$$

Median class $= 100 - 140$

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f} \times h$$

$$l = 100,\ \frac{n}{2} = 22.5,\ cf = 12,\ f = 16,\ h = 40$$

$$\text{Median} = 100 + \frac{22.5 - 12}{16} \times 40$$

$$= 100 + 26.25$$

$$= 126.25$$