ICSE Class 10 Maths Selina Solutions Chapter 21: Trigonometry is the name of the branch of mathematics that studies triangular measurements. Trigonometric identities are demonstrated by the use of trigonometrical ratios and their relations. Other topics discussed in this chapter include the usage of trigonometrical tables and the trigonometrical ratios of complementary angles.

Since this chapter provides the groundwork for more advanced mathematics, students should become well-versed in it. For this reason, We have developed the Selina Solutions for Class 10 Mathematics, which were made by knowledgeable faculty members with extensive backgrounds in academia. Additionally, this helps pupils develop their problem-solving abilities, which are critical from the perspective of exams.

ICSE Class 10 Maths Selina Solutions Chapter 21 Overview

ICSE Class 10 Maths Selina Solutions Chapter 21 covers Trigonometrical Identities. This chapter focuses on the fundamental identities of trigonometry, which are crucial for solving various trigonometric problems. Key identities include the Pythagorean identities.

The chapter also involves proving these identities and applying them to simplify trigonometric expressions and solve equations. Understanding these identities is essential for mastering more complex trigonometric concepts and problems.

ICSE Class 10 Maths Selina Solutions Chapter 21

Below we have provided ICSE Class 10 Maths Selina Solutions Chapter 21. Students are advised to thoroughly practice the questions of ICSE Class 10 Maths Selina Solutions Chapter 21.

1. sec A – 1/ sec A + 1 = 1 – cos A/ 1 + cos A

Solution:

$$\text{LHS} = \frac{\sec A - 1}{\sec A + 1} = \frac{\frac{1}{\cos A} - 1}{\frac{1}{\cos A} + 1}$$

$$= \frac{1 - \cos A}{1 + \cos A} = \text{RHS}$$

– Hence Proved

2. 1 + sin A/ 1 – sin A = cosec A + 1/ cosec A – 1

Solution:

$$\text{LHS} = \frac{1+\sin A}{1-\sin A}$$

$$\text{RHS} = \frac{\operatorname{cosec} A + 1}{\operatorname{cosec} A - 1} = \frac{\frac{1}{\sin A}+1}{\frac{1}{\sin A}-1}$$

$$= \frac{1+\sin A}{1-\sin A}$$

– Hence Proved

3. 1/ tan A + cot A = cos A sin A

Solution:

Taking L.H.S,

$$\frac{1}{\tan A + \cot A} = \sin A \cos A$$

$$\text{LHS} = \frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}} = \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cos A}}$$

$$= \frac{1}{\frac{1}{\sin A \cos A}} \left(\because \sin^2 A + \cos^2 A = 1\right)$$

$$= \sin A \cos A = \text{RHS}$$

– Hence Proved

4. tan A – cot A = 1 – 2 cos^2 A/ sin A cos A

Solution:

Taking LHS,

$$\tan A - \cot A = \frac{\sin A}{\cos A} - \frac{\cos A}{\sin A}$$
$$= \frac{\sin^2 A - \cos^2 A}{\sin A \cos A}$$
$$= \frac{1 - \cos^2 A - \cos^2 A}{\sin A \cos A} \left(\because \sin^2 A = 1 - \cos^2 A\right)$$
$$= \frac{1 - 2\cos^2 A}{\sin A \cos A}$$

– Hence Proved

5. $\sin^4 A – \cos^4 A = 2 \sin^2 A – 1$

Solution:

Taking L.H.S,

$\sin^4 A – \cos^4 A$

$= (\sin^2 A)^2 – (\cos^2 A)^2$

$= (\sin^2 A + \cos^2 A) (\sin^2 A – \cos^2 A)$

$= \sin^2 A – \cos^2 A$

$= \sin^2 A – (1 – \sin^2 A)$ [Since, $\cos^2 A = 1 – \sin^2 A$]

$= 2\sin^2 A – 1$

– Hence Proved

6. $(1 – \tan A)^2 + (1 + \tan A)^2 = 2 \sec^2 A$

Solution:

Taking L.H.S,

$(1 – \tan A)^2 + (1 + \tan A)^2$

$= (1 + \tan^2 A + 2 \tan A) + (1 + \tan^2 A – 2 \tan A)$

$= 2 (1 + \tan^2 A)$

$= 2 \sec^2 A$ [Since, $1 + \tan^2 A = \sec^2 A$]

– Hence Proved

7. $\text{cosec}^4 A - \text{cosec}^2 A = \cot^4 A + \cot^2 A$

Solution:

$\text{cosec}^4 A - \text{cosec}^2 A$

$= \text{cosec}^2 A(\text{cosec}^2 A - 1)$

$= (1 + \cot^2 A)(1 + \cot^2 A - 1)$

$= (1 + \cot^2 A) \cot^2 A$

$= \cot^4 A + \cot^2 A$ = R.H.S

– Hence Proved

8. sec A (1 – sin A) (sec A + tan A) = 1

Solution:

Taking L.H.S,

sec A (1 – sin A) (sec A + tan A)

$$= \frac{1}{\cos A}(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$$
$$= \frac{(1 - \sin A)}{\cos A}\left(\frac{1 + \sin A}{\cos A}\right) = \left(\frac{1 - \sin^2 A}{\cos^2 A}\right)$$
$$= \left(\frac{\cos^2 A}{\cos^2 A}\right) = 1 = \text{RHS}$$

– Hence Proved

9. cosec A (1 + cos A) (cosec A – cot A) = 1

Solution:

Taking L.H.S,

$$= \frac{1}{\sin A}(1+\cos A)\left(\frac{1}{\sin A} - \frac{\cos A}{\sin A}\right)$$
$$= \frac{(1+\cos A)}{\sin A}\left(\frac{1-\cos A}{\sin A}\right)$$
$$= \frac{1-\cos^2 A}{\sin^2 A} = \frac{\sin^2 A}{\sin^2 A} = 1 = \text{RHS}$$

– Hence Proved

10. $\sec^2 A + \text{cosec}^2 A = \sec^2 A \cdot \text{cosec}^2 A$

Solution:

Taking L.H.S,

$$= \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A} = \frac{\sin^2 A + \cos^2 A}{\cos^2 A . \sin^2 A}$$
$$= \frac{1}{\cos^2 A . \sin^2 A} = \sec^2 A \, \text{cosec}^2 A = \text{RHS}$$

– Hence Proved

11. $(1 + \tan^2 A) \cot A / \text{cosec}^2 A = \tan A$

Solution:

Taking L.H.S,

$$\frac{(1+\tan^2 A)\cot A}{\text{cosec}^2 A}$$
$$= \frac{\sec^2 A \cot A}{\text{cosec}^2 A} (\because \sec^2 A = 1 + \tan^2 A)$$
$$= \frac{\frac{1}{\cos^2 A} \times \frac{\cos A}{\sin A}}{\frac{1}{\sin^2 A}} = \frac{\frac{1}{\cos A \sin A}}{\frac{1}{\sin^2 A}}$$
$$= \frac{\sin A}{\cos A} = \tan A$$

= RHS

– Hence Proved

12. $\tan^2 A - \sin^2 A = \tan^2 A.\sin^2 A$

Solution:

Taking L.H.S,

$\tan^2 A - \sin^2 A$

$$= \frac{\sin^2 A}{\cos^2 A} - \sin^2 A = \frac{\sin^2 A(1-\cos^2 A)}{\cos^2 A}$$
$$= \frac{\sin^2 A}{\cos^2 A}.\sin^2 A = \tan^2 A.\sin^2 A = RHS$$

– Hence Proved

13. $\cot^2 A - \cos^2 A = \cos^2 A.\cot^2 A$

Solution:

Taking L.H.S,

$\cot^2 A - \cos^2 A$

$$= \frac{\cos^2 A}{\sin^2 A} - \cos^2 A = \frac{\cos^2 A(1-\sin^2 A)}{\sin^2 A}$$
$$= \cos^2 A\frac{\cos^2 A}{\sin^2 A} = \cos^2 A.\cot^2 A = RHS$$

– Hence Proved

14. $(\text{cosec } A + \sin A)(\text{cosec } A - \sin A) = \cot^2 A + \cos^2 A$

Solution:

Taking L.H.S,

$(\text{cosec } A + \sin A)(\text{cosec } A - \sin A)$

$= \text{cosec}^2 A - \sin^2 A$

$= (1 + \cot^2 A) - (1 - \cos^2 A)$

$= \cot^2 A + \cos^2 A$ = R.H.S

– Hence Proved

15. (sec A – cos A)(sec A + cos A) = $\sin^2$ A + $\tan^2$ A

Solution:

Taking L.H.S,

(sec A – cos A)(sec A + cos A)

= ($\sec^2$ A – $\cos^2$ A)

= (1 + $\tan^2$ A) – (1 – $\sin^2$ A)

= $\sin^2$ A + $\tan^2$ A = RHS

– Hence Proved

16. (cos A + sin A)^2 + (cosA – sin A)^2 = 2

Solution:

Taking L.H.S,

(cos A + sin A)^2 + (cosA – sin A)^2

= cos2 A + sin2 A + 2cos A sin A + cos2 A – 2cosA.sinA

= 2 ($\cos^2$ A + $\sin^2$ A) = 2 = R.H.S

– Hence Proved

17. (cosec A – sin A)(sec A – cos A)(tan A + cot A) = 1

Solution:

Taking LHS,

(cosec A – sin A)(sec A – cos A)(tan A + cot A)

$$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)\left(\tan A + \frac{1}{\tan A}\right)$$
$$= \left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$$
$$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)\left(\frac{\sin^2 A + \cos^2 A}{\sin A.\cos A}\right)$$
$$= 1$$

= RHS

– Hence Proved

18. 1/ sec A + tan A = sec A – tan A

Solution:

Taking LHS,

$$\frac{1}{\sec A + \tan A}$$
$$= \frac{1}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A}$$
$$= \frac{\sec A - \tan A}{\sec^2 A - \tan^2 A}$$
$$= \sec A - \tan A$$

= RHS

– Hence Proved

19. cosec A + cot A = 1/ cosec A – cot A

Solution:

Taking LHS,

cosec A + cot A

$$= \frac{\operatorname{cosec} A + \cot A}{1} \times \frac{\operatorname{cosec} A - \cot A}{\operatorname{cosec} A - \cot A}$$
$$= \frac{\operatorname{cosec}^2 A - \cot^2 A}{\operatorname{cosec} A - \cot A} = \frac{1 + \cot^2 A - \cot^2 A}{\operatorname{cosec} A - \cot A}$$
$$= \frac{1}{\operatorname{cosec} A - \cot A}$$

= RHS

– Hence Proved

20. sec A – tan A/ sec A + tan A = 1 – 2 secA tanA + 2 tan^2 A

Solution:

Taking LHS,

$$\frac{\sec A - \tan A}{\sec A + \tan A}$$

$$= \frac{\sec A - \tan A}{\sec A + \tan A} \times \frac{\sec A - \tan A}{\sec A - \tan A}$$

$$= \frac{(\sec A - \tan A)^2}{\sec^2 A - \tan^2 A}$$

$$= \frac{\sec^2 A + \tan^2 A - 2\sec A \tan A}{1}$$

= 1 + $\tan^2$ A + $\tan^2$ A – 2 sec A tan A

= 1 – 2 sec A tan A + 2 $\tan^2$ A = RHS

– Hence Proved

21. (sin A + cosec A)^2 + (cos A + sec A)^2 = 7 + $\tan^2$ A + $\cot^2$ A

Solution:

Taking LHS,

(sin A + cosec A)^2 + (cos A + sec A)^2

= $\sin^2$ A + cosec^2 A + 2 sin A cosec A + $\cos^2$ A + $\sec^2$ A + 2cos A sec A

= ($\sin^2$ A + $\cos^2$ A) + cosec^2 A + $\sec^2$ A + 2 + 2

= 1 + cosec^2 A + $\sec^2$ A + 4

= 5 + (1 + $\cot^2$ A) + (1 + $\tan^2$ A)

= 7 + $\tan^2$ A + $\cot^2$ A = RHS

– Hence Proved

22. $\sec^2$ A. cosec^2 A = $\tan^2$ A + $\cot^2$ A + 2

Solution:

Taking,

RHS = $\tan^2$ A + $\cot^2$ A + 2 = $\tan^2$ A + $\cot^2$ A + 2 tan A. cot A

= (tan A + cot A)^2 = (sin A/cos A + cos A/ sin A)^2

= (sin2 A + cos2 A/ sin A.cos A)^2 = 1/ $\cos^2$ A. $\sin^2$ A

$= \sec^2 A. \operatorname{cosec}^2 A$ = LHS

– Hence Proved

23. 1/ 1 + cos A + 1/ 1 – cos A = 2 cosec² A

Solution:

Taking LHS,

$$\frac{1}{1+\cos A}+\frac{1}{1-\cos A}$$
$$=\frac{1-\cos A+1+\cos A}{(1+\cos A)(1-\cos A)}$$
$$=\frac{2}{1-\cos^2 A}$$
$$=\frac{2}{\sin^2 A}$$
$$=2\operatorname{cosec}^2 A$$

= RHS

– Hence Proved

24. 1/ 1 – sin A + 1/ 1 + sin A = 2 sec² A

Solution:

Taking LHS,

$$\frac{1}{1-\sin A}+\frac{1}{1+\sin A}$$
$$=\frac{1+\sin A+1-\sin A}{(1-\sin A)(1+\sin A)}$$
$$=\frac{2}{1-\sin^2 A}$$
$$=\frac{2}{\cos^2 A}$$
$$=2\sec^2 A$$

= RHS

– Hence Proved

ICSE Class 10 Maths Selina Solutions Chapter 21 Exercise 21B

1. Prove that:

(i) $\dfrac{\cos A}{1-\tan A}+\dfrac{\sin A}{1-\cot A}=\sin A+\cos A$

(ii) $\dfrac{\cos^3 A+\sin^3 A}{\cos A+\sin A}+\dfrac{\cos^3 A-\sin^3 A}{\cos A-\sin A}=2$

(iii) $\dfrac{\tan A}{1-\cot A}+\dfrac{\cot A}{1-\tan A}=\sec A\operatorname{cosec} A+1$

(iv) $\left(\tan A+\dfrac{1}{\cos A}\right)^2+\left(\tan A-\dfrac{1}{\cos A}\right)^2=2\left(\dfrac{1+\sin^2 A}{1-\sin^2 A}\right)$

(v) $2\sin^2 A+\cos^4 A=1+\sin^4 A$

(vi) $\dfrac{\sin A-\sin B}{\cos A+\cos B}+\dfrac{\cos A-\cos B}{\sin A+\sin B}=0$

(vii) $(\operatorname{cosec} A-\sin A)(\sec A-\cos A)=\dfrac{1}{\tan A+\cot A}$

(viii) $(1+\tan A\tan B)^2+(\tan A-\tan B)^2=\sec^2 A\sec^2 B$

(ix) $\dfrac{1}{\cos A+\sin A-1}+\dfrac{1}{\cos A+\sin A+1}=\operatorname{cosec} A+\sec A$

Solution:

$$\text{LHS}=\frac{\cos A}{1-\tan A}+\frac{\sin A}{1-\cot A}$$

$$=\frac{\cos A}{1-\frac{\sin A}{\cos A}}+\frac{\sin A}{1-\frac{\cos A}{\sin A}}=\frac{\cos A}{\frac{\cos A-\sin A}{\cos A}}+\frac{\sin A}{\frac{\sin A-\cos A}{\sin A}}$$

$$=\frac{\cos^2 A}{\cos A-\sin A}+\frac{\sin^2 A}{\sin A-\cos A}=\frac{\cos^2 A-\sin^2 A}{(\cos A-\sin A)}$$

$$=\sin A+\cos A=\text{RHS}$$ (i)

– Hence Proved

(ii) Taking LHS,

$$\frac{\cos^3 A+\sin^3 A}{\cos A+\sin A}+\frac{\cos^3 A-\sin^3 A}{\cos A-\sin A}$$

$$=\frac{\left(\cos^3 A+\sin^3 A\right)\left(\cos A-\sin A\right)+\left(\cos^3 A-\sin^3 A\right)\left(\cos A+\sin A\right)}{\cos^2 A-\sin^2 A}$$

$$=\frac{\cos^4 A-\cos^3 A\sin A+\sin^3 A\cos A-\sin^4 A+\cos^4 A+\cos^3 A\sin A-\sin^3 A\cos A-\sin^4 A}{\cos^2 A-\sin^2 A}$$

$$=\frac{2\left(\cos^4 A-\sin^4 A\right)}{\cos^2 A-\sin^2 A}$$

$$=\frac{2\left(\cos^2 A+\sin^2 A\right)\left(\cos^2 A-\sin^2 A\right)}{\left(\cos^2 A-\sin^2 A\right)}$$

$$=2\left(\cos^2 A+\sin^2 A\right)$$

$$=2\ \left(\because \cos^2 A+\sin^2 A=1\right)$$

– Hence Proved

$$\frac{\tan A}{1-\cot A}+\frac{\cot A}{1-\tan A}$$

$$=\frac{\tan A}{1-\frac{1}{\tan A}}+\frac{\frac{1}{\tan A}}{1-\tan A}$$

$$=\frac{\tan^2 A}{\tan A-1}+\frac{1}{\tan A(1-\tan A)}$$

$$=\frac{\tan^3 A-1}{\tan A(\tan A-1)}$$

$$=\frac{(\tan A-1)(\tan^2 A+1+\tan A)}{\tan A(\tan A-1)}$$

$$=\frac{\sec^2 A+\tan A}{\tan A}$$

$$=\frac{\frac{1}{\cos^2 A}}{\frac{\sin A}{\cos A}}+1$$

$$=\frac{1}{\sin A\cos A}+1$$

$$=\sec A\operatorname{cosec} A+1$$

(iii)

– Hence Proved

$$\left(\tan A + \frac{1}{\cos A}\right)^2 + \left(\tan A - \frac{1}{\cos A}\right)^2$$
$$= \left(\frac{\sin A + 1}{\cos A}\right)^2 + \left(\frac{\sin A - 1}{\cos A}\right)^2$$
$$= \frac{\sin^2 A + 1 + 2\sin A + \sin^2 A + 1 - 2\sin A}{\cos^2 A}$$
$$= \frac{2 + 2\sin^2 A}{\cos^2 A}$$
$$= 2\left(\frac{1 + \sin^2 A}{1 - \sin^2 A}\right)$$

(iv)

– Hence Proved

(v) Taking LHS,

$2 \sin^2 A + \cos^2 A$

$= 2 \sin^2 A + (1 – \sin^2 A)^2$

$= 2 \sin^2 A + 1 + \sin^4 A – 2 \sin^2 A$

$= 1 + \sin^4 A =$ RHS

– Hence Proved

$$\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B}$$
$$= \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$
$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)}$$
$$= \frac{1 - 1}{(\cos A + \cos B)(\sin A + \sin B)}$$
$$= 0$$

(vi)

– Hence Proved

LHS

$= (\text{cosec}A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$

$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$

$= \sin A \cos A$

$\text{RHS} = \frac{1}{\tan A + \cot A}$

$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$

$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$

$= \sin A \cos A$

LHS = RHS

(vii)

LHS

$= (\text{cosec}A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$

$= \left(\frac{\cos^2 A}{\sin A}\right)\left(\frac{\sin^2 A}{\cos A}\right)$

$= \sin A \cos A$

$\text{RHS} = \frac{1}{\tan A + \cot A}$

$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$

$= \frac{\sin A \cos A}{\sin^2 A + \cos^2 A}$

$= \sin A \cos A$

LHS = RHS

– Hence Proved

$(1 + \tan A \tan B)^2 + (\tan A - \tan B)^2$

$= 1 + \tan^2 A \tan^2 B + 2 \tan A \tan B + \tan^2 A + \tan^2 B - 2 \tan A \tan B$

$= 1 + \tan^2 A + \tan^2 B + \tan^2 A \tan^2 B$

$= \sec^2 A + \tan^2 B(1 + \tan^2 A)$

$= \sec^2 A + \tan^2 B \sec^2 A$

$= \sec^2 A(1 + \tan^2 B)$

$= \sec^2 A \sec^2 B$

(viii)

– Hence Proved

$$\frac{1}{(\cos A + \sin A) - 1} + \frac{1}{(\cos A + \sin A) + 1}$$

$$= \frac{\cos A + \sin A + 1 + \cos A + \sin A - 1}{(\cos A + \sin A)^2 - 1}$$

$$= \frac{2(\cos A + \sin A)}{\cos^2 A + \sin^2 A + 2\cos A \sin A - 1}$$

$$= \frac{2(\cos A + \sin A)}{1 + 2\cos A \sin A - 1} = \frac{\cos A + \sin A}{\cos A \sin A}$$

$$= \frac{\cos A}{\cos A \sin A} + \frac{\sin A}{\cos A \sin A}$$

$$= \frac{1}{\sin A} + \frac{1}{\cos A}$$

$$= \operatorname{cosec} A + \sec A$$

(ix)

– Hence Proved

2. If x cos A + y sin A = m and x sin A – y cos A = n, then prove that:

$x^2 + y^2 = m^2 + n^2$

Solution:

Taking RHS,

$m^2 + n^2$

$= (x \cos A + y \sin A)^2 + (x \sin A - y \cos A)^2$

$= x^2 \cos^2 A + y^2 \sin^2 A + 2xy \cos A \sin A + x^2 \sin^2 A + y^2 \cos^2 A - 2xy \sin A \cos A$

$= x^2 (\cos^2 A + \sin^2 A) + y^2 (\sin^2 A + \cos^2 A)$

$= x^2 + y^2$ [Since, $\cos^2 A + \sin^2 A = 1$]

= RHS

3. If m = a sec A + b tan A and n = a tan A + b sec A, prove that $m^2 – n^2 = a^2 – b^2$

Solution:

Taking LHS,

$m^2 – n^2$

= $(a \sec A + b \tan A)^2 – (a \tan A + b \sec A)^2$

= $a^2 \sec^2 A + b^2 \tan^2 A + 2 ab \sec A \tan A – a^2 \tan^2 A – b^2 \sec^2 A – 2ab \tan A \sec A$

= $a^2 (\sec^2 A – \tan^2 A) + b^2 (\tan^2 A – \sec^2 A)$

= $a^2 (1) + b^2 (-1)$ [Since, $\sec^2 A – \tan^2 A = 1$]

= $a^2 – b^2$

= RHS

ICSE Class 10 Maths Selina Solutions Chapter 21 Exercise 21C

1. Show that:

(i) tan 10° tan 15° tan 75° tan 80° = 1

Solution:

Taking, tan 10° tan 15° tan 75° tan 80°

= tan (90° – 80°) tan (90° – 75°) tan 75° tan 80°

= cot 80° cot 75° tan 75° tan 80°

= 1 [Since, tan θ x cot θ = 1]

(ii) sin 42° sec 48° + cos 42° cosec 48° = 2

Solution:

Taking, sin 42° sec 48° + cos 42° cosec 48°

= sin 42° sec (90° – 42°) + cos 42° cosec (90° – 42°)

= sin 42° cosec 42° + cos 42° sec 42°

= 1 + 1 [Since, sin θ x cosec θ = 1 and cos θ x sec θ = 1]

= 2

(iii) sin 26°/ sec 64° + cos 26°/ cosec 64° = 1

Solution:

Taking,

$$\frac{\sin 26°}{\sec 64°} + \frac{\cos 26°}{\operatorname{cosec} 64°}$$
$$= \frac{\sin 26°}{\sec(90° - 26°)} + \frac{\cos 26°}{\operatorname{cosec}(90° - 26°)}$$
$$= \frac{\sin 26°}{\operatorname{cosec} 26°} + \frac{\cos 26°}{\sec 26°}$$
$$= \sin^2 26° + \cos^2 26°$$
$$= 1$$

2. Express each of the following in terms of angles between 0°and 45°:

(i) sin 59°+ tan 63°

(ii) cosec 68°+ cot 72°

(iii) cos 74°+ sec 67°

Solution:

(i) sin 59°+ tan 63°

= sin (90 – 31)°+ tan (90 – 27)°

= cos 31°+ cot 27°

(ii) cosec 68°+ cot 72°

= cosec (90 – 22)°+ cot (90 – 18)°

= sec 22°+ tan 18°

(iii) cos 74°+ sec 67°

= cos (90 – 16)°+ sec (90 – 23)°

= sin 16°+ cosec 23°

3. Show that:

(i) $\frac{\sin A}{\sin(90^\circ - A)} + \frac{\cos A}{\cos(90^\circ - A)} = \sec A \operatorname{cosec} A$

(ii) $\sin A \cos A - \frac{\sin A \cos(90^\circ - A) \cos A}{\sec(90^\circ - A)} - \frac{\cos A \sin(90^\circ - A) \sin A}{\operatorname{cosec}(90^\circ - A)} = 0$

Solution:

(i) $\frac{\sin A}{\sin(90^\circ - A)} + \frac{\cos A}{\cos(90^\circ - A)}$

$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} = \frac{\sin^2 A + \cos^2 A}{\cos A \sin A}$

$= \frac{1}{\cos A \sin A}$

$= \sec A \operatorname{cosec} A$

(ii) $\sin A \cos A - \frac{\sin A \cos(90^\circ - A) \cos A}{\sec(90^\circ - A)} - \frac{\cos A \sin(90^\circ - A) \sin A}{\operatorname{cosec}(90^\circ - A)}$

$= \sin A \cos A - \frac{\sin A \sin A \cos A}{\operatorname{cosec} A} - \frac{\cos A \cos A \sin A}{\sec A}$

= sin A cos A – $\sin^3$ A cos A – $\cos^3$ A sin A

= sin A cos A – sin A cos A ($\sin^2$ A + $\cos^2$ A)

= sin A cos A – sin A cos A (1) [Since, $\sin^2$ A + $\cos^2$ A = 1]

= 0

4. For triangle ABC, show that:

(i) sin (A + B)/ 2 = cos C/2

(ii) tan (B + C)/ 2 = cot A/2

Solution:

We know that, in triangle ABC

∠A + ∠B + ∠C = 180° [Angle sum property of a triangle]

(i) Now,

$(\angle A + \angle B)/2 = 90^\circ - \angle C/2$

So,

$\sin((A + B)/2) = \sin(90^\circ - C/2)$

$= \cos C/2$

(ii) And,

$(\angle C + \angle B)/2 = 90^\circ - \angle A/2$

So,

$\tan((B + C)/2) = \tan(90^\circ - A/2)$

$= \cot A/2$

5. Evaluate:

(i) $3\dfrac{\sin 72^\circ}{\cos 18^\circ} - \dfrac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

(ii) $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ$

(iii) $\dfrac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$

(iv) $\tan(55^\circ - A) - \cot(35^\circ + A)$

(v) $\operatorname{cosec}(65^\circ + A) - \sec(25^\circ - A)$

(vi) $2\dfrac{\tan 57^\circ}{\cot 33^\circ} - \dfrac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2}\cos 45^\circ$

(vii) $\dfrac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2\dfrac{\sin^2 75^\circ}{\cos^2 15^\circ}$

(viii) $\dfrac{\cos 70^\circ}{\sin 20^\circ} + \dfrac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$

(ix) $14\sin 30^\circ + 6\cos 60^\circ - 5\tan 45^\circ$

Solution:

$$3\frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$
$$= 3\frac{\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ}$$
$$= 3\frac{\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ} = 3 - 1 = 2$$

(i)

(ii) $3 \cos 80^o \operatorname{cosec} 10^o + 2 \cos 59^o \operatorname{cosec} 31^o$

$= 3 \cos (90 – 10)^o \operatorname{cosec} 10^o + 2 \cos (90 – 31)^o \operatorname{cosec} 31^o$

$= 3 \sin 10^o \operatorname{cosec} 10^o + 2 \sin 31^o \operatorname{cosec} 31^o$

$= 3 + 2 = 5$

(iii) $\sin 80^o / \cos 10^o + \sin 59^o \sec 31^o$

$= \sin (90 – 10)^o / \cos 10^o + \sin (90 – 31)^o \sec 31^o$

$= \cos 10^o / \cos 10^o + \cos 31^o \sec 31^o$

$= 1 + 1 = 2$

(iv) $\tan (55^o – A) – \cot (35^o + A)$

$= \tan [90^o – (35^o + A)] – \cot (35^o + A)$

$= \cot (35^o + A)] – \cot (35^o + A)$

$= 0$

(v) $\operatorname{cosec} (65^o + A) – \sec (25^o – A)$

$= \operatorname{cosec} [90^o – (25^o – A)] – \sec (25^o – A)$

$= \sec (25^o – A) – \sec (25^o – A)$

$= 0$

$$2\frac{\tan 57^o}{\cot 33^o} - \frac{\cot 70^o}{\tan 20^o} - \sqrt{2}\cos 45^o$$
$$= 2\frac{\tan(90^o - 33^o)}{\cot 33^o} - \frac{\cot(90^o - 20^o)}{\tan 20^o} - \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$$
$$= 2\frac{\cot 33^o}{\cot 33^o} - \frac{\tan 20^o}{\tan 20^o} - 1$$
$$= 2 - 1 - 1$$
$$= 0$$

(vi)

$$\frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2\frac{\sin^2 75^\circ}{\cos^2 15^\circ}$$
$$= \frac{[\cot(90^\circ - 49^\circ)]^2}{\tan^2 49^\circ} - 2\frac{[\sin(90^\circ - 15^\circ)]^2}{\cos^2 15^\circ}$$
$$= \frac{\tan^2 49^\circ}{\tan^2 49^\circ} - 2\frac{\cos^2 15^\circ}{\cos^2 15^\circ}$$

(vii)

= 1 – 2 = -1

$$\frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8\sin^2 30^\circ$$
$$= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} - 8\left(\frac{1}{2}\right)^2$$
$$= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 2$$
$$= 1 + 1 - 2 = 0$$

(viii)

(ix) 14 sin 30° + 6 cos 60° – 5 tan 45°

= 14 (1/2) + 6 (1/2) – 5(1)

= 7 + 3 – 5

= 5

6. A triangle ABC is right angled at B; find the value of (sec A. cosec C – tan A. cot C)/ sin B

Solution:

As, ABC is a right angled triangle right angled at B

So, A + C = 90°

(sec A. cosec C – tan A. cot C)/ sin B

= (sec (90° – C). cosec C – tan (90° – C). cot C)/ sin 90°

= (cosec C. cosec C – cot C. cot C)/ 1 = cosec^2 C – $\cot^2$ C

= 1 [Since, cosec^2 C – $\cot^2$ C = 1]

ICSE Class 10 Maths Selina Solutions Chapter 21 Exercise 21D

1. Use tables to find sine of:

(i) 21°

(ii) 34° 42′

(iii) 47° 32′

(iv) 62° 57′

(v) 10° 20′ + 20° 45′

Solution:

(i) sin 21° = 0.3584

(ii) sin 34° 42’= 0.5693

(iii) sin 47° 32’= sin (47° 30′ + 2′) =0.7373 + 0.0004 = 0.7377

(iv) sin 62° 57′ = sin (62° 54′ + 3′) = 0.8902 + 0.0004 = 0.8906

(v) sin (10° 20′ + 20° 45′) = sin 30°65′ = sin 31°5′ = 0.5150 + 0.0012 = 0.5162

2. Use tables to find cosine of:

(i) 2° 4’

(ii) 8° 12’

(iii) 26° 32’

(iv) 65° 41’

(v) 9° 23’ + 15° 54’

Solution:

(i) cos 2° 4’ = 0.9994 – 0.0001 = 0.9993

(ii) cos 8° 12’ = cos 0.9898

(iii) cos 26° 32’ = cos (26° 30’ + 2’) = 0.8949 – 0.0003 = 0.8946

(iv) cos 65° 41' = cos (65° 36' + 5') = 0.4131 -0.0013 = 0.4118

(v) cos (9° 23' + 15° 54') = cos 24° 77' = cos 25° 17' = cos (25° 12' + 5') = 0.9048 – 0.0006 = 0.9042

3. Use trigonometrical tables to find tangent of:

(i) 37°

(ii) 42° 18′

(iii) 17° 27′

Solution:

(i) tan 37° = 0.7536

(ii) tan 42° 18′ = 0.9099

(iii) tan 17° 27′ = tan (17° 24′ + 3′) = 0.3134 + 0.0010 = 0.3144

ICSE Class 10 Maths Selina Solutions Chapter 21 exercise 21E

1. Prove the following identities:

(i) $\frac{1}{\cos A + \sin A} + \frac{1}{\cos A - \sin A} = \frac{2\cos A}{2\cos^2 A - 1}$

(ii) $\operatorname{cosec} A - \cot A = \frac{\sin A}{1 + \cos A}$

(iii) $1 - \frac{\sin^2 A}{1 + \cos A} = \cos A$

(iv) $\frac{1 - \cos A}{\sin A} + \frac{\sin A}{1 - \cos A} = 2\operatorname{cosec} A$

(v) $\frac{\cot A}{1 - \tan A} + \frac{\tan A}{1 - \cot A} = 1 + \tan A + \cot A$

(vi) $\frac{\cos A}{1 + \sin A} + \tan A = \sec A$

(vii) $\frac{\sin A}{1 - \cos A} - \cot A = \operatorname{cosec} A$

(viii) $\frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \frac{\cos A}{1 - \sin A}$

(ix) $\sqrt{\frac{1 + \sin A}{1 - \sin A}} = \frac{\cos A}{1 - \sin A}$

(x) $\sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{\sin A}{1 + \cos A}$

(xi) $\frac{1 + (\sec A - \tan A)^2}{\operatorname{cosec} A(\sec A - \tan A)} = 2\tan A$

(xii) $\frac{(\operatorname{cosec} A - \cot A)^2 + 1}{\sec A(\operatorname{cosec} A - \cot A)} = 2\cot A$

(xiii) $\cot^2 A\left(\frac{\sec A - 1}{1 + \sin A}\right) + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right) = 0$

(xiv) $\frac{(1 - 2\sin^2 A)^2}{\cos^4 A - \sin^4 A} = 2\cos^2 A - 1$

(xv) $\sec^4 A(1 - \sin^4 A) - 2\tan^2 A = 1$

(xvi) $\operatorname{cosec}^4 A(1 - \cos^4 A) - 2\cot^2 A = 1$

(xvii) $(1 + \tan A + \sec A)(1 + \cot A - \operatorname{cosec} A) = 2$

Solution:

(i) Taking LHS,

1/ (cos A + sin A) + 1/ (cos A – sin A)

$$= \frac{\cos A + \sin A + \cos A - \sin A}{(\cos A + \sin A)(\cos A - \sin A)}$$
$$= \frac{2\cos A}{\cos^2 A - \sin^2 A}$$
$$= \frac{2\cos A}{\cos^2 A - (1 - \cos^2 A)}$$
$$= \frac{2\cos A}{2\cos^2 A - 1}$$

= RHS

– Hence Proved

(ii) Taking LHS, cosec A – cot A

$$= \frac{1}{\sin A} - \frac{\cos A}{\sin A}$$
$$= \frac{1 - \cos A}{\sin A}$$
$$= \frac{1 - \cos A}{\sin A} \times \frac{1 + \cos A}{1 + \cos A}$$
$$= \frac{1 - \cos^2 A}{\sin A(1 + \cos A)}$$
$$= \frac{\sin^2 A}{\sin A(1 + \cos A)}$$
$$= \frac{\sin A}{1 + \cos A}$$

= RHS

– Hence Proved

(iii) Taking LHS, 1 – $\sin^2 A$/ (1 + cos A)

$$= \frac{1 + \cos A - \sin^2 A}{1 + \cos A}$$
$$= \frac{\cos A + \cos^2 A}{1 + \cos A}$$
$$= \frac{\cos A(1 + \cos A)}{1 + \cos A}$$
$$= \cos A$$

= RHS

– Hence Proved

(iv) Taking LHS,

(1 – cos A)/ sin A + sin A/ (1 – cos A)

$$= \frac{(1-\cos A)^2 + \sin^2 A}{\sin A(1-\cos A)}$$
$$= \frac{1+\cos^2 A - 2\cos A + \sin^2 A}{\sin A(1-\cos A)}$$
$$= \frac{2-2\cos A}{\sin A(1-\cos A)}$$
$$= \frac{2(1-\cos A)}{\sin A(1-\cos A)}$$
$$= 2\operatorname{cosec} A$$

= RHS

– Hence Proved

(v) Taking LHS, cot A/ (1 – tan A) + tan A/ (1 – cot A)

$$= \frac{\frac{1}{\tan A}}{1-\tan A} + \frac{\tan A}{1-\frac{1}{\tan A}}$$
$$= \frac{1}{\tan A(1-\tan A)} + \frac{\tan^2 A}{\tan A - 1}$$
$$= \frac{1-\tan^3 A}{\tan A(1-\tan A)}$$
$$= \frac{(1-\tan A)(1+\tan A+\tan^2 A)}{\tan A(1-\tan A)}$$
$$= \frac{1+\tan A+\tan^2 A}{\tan A}$$
$$= \cot A + 1 + \tan A$$

= RHS

– Hence Proved

(vi) Taking LHS, cos A/ (1 + sin A) + tan A

$$= \frac{\cos A}{1+\sin A} + \frac{\sin A}{\cos A}$$
$$= \frac{\cos^2 A + \sin A + \sin^2 A}{(1+\sin A)\cos A}$$
$$= \frac{1+\sin A}{(1+\sin A)\cos A}$$
$$= \frac{1}{\cos A}$$
$$= \sec A$$

= RHS

– Hence Proved

(vii) Consider LHS,

= (sin A/(1 – cos A)) – cot A

We know that, cot A = cos A/sin A

So,
= ($\sin^2$ A – cos A + $\cos^2$ A)/(1 – cos A) sin A

= (1 – cos A)/(1 – cos A) sin A

= 1/sin A

= cosec A

(viii) Taking LHS, (sin A – cos A + 1)/ (sin A + cos A – 1)

$$= \frac{\sin A - \cos A + 1}{\sin A + \cos A - 1} \times \frac{\sin A - (\cos A - 1)}{\sin A - (\cos A - 1)}$$

$$= \frac{(\sin A - \cos A + 1)^2}{\sin^2 A - (\cos A - 1)^2}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 - 2\sin A\cos A - 2\cos A + 2\sin A}{\sin^2 A - \cos^2 A - 1 + 2\cos A}$$

$$= \frac{1 + 1 - 2\sin A\cos A - 2\cos A + 2\sin A}{-\cos^2 A - \cos^2 A + 2\cos A}$$

$$= \frac{2(1 - \cos A) + 2\sin A(1 - \cos A)}{2\cos A(1 - \cos A)}$$

$$= \frac{1 + \sin A}{\cos A}$$

$$= \frac{1 + \sin A}{\cos A} \times \frac{1 - \sin A}{1 - \sin A}$$

$$= \frac{\cos^2 A}{\cos A(1 - \sin A)}$$

$$= \frac{\cos A}{1 - \sin A}$$

= RHS

– Hence Proved

(ix) Taking LHS,

$$\sqrt{\frac{1 + \sin A}{1 - \sin A}}$$

$$= \sqrt{\frac{1 + \sin A}{1 - \sin A} \times \frac{1 - \sin A}{1 - \sin A}}$$

$$= \sqrt{\frac{1 - \sin^2 A}{(1 - \sin A)^2}}$$

$$= \sqrt{\frac{\cos^2 A}{(1 - \sin A)^2}}$$

$$= \frac{\cos A}{1 - \sin A}$$

= RHS

– Hence Proved

(x) Taking LHS,

$$\sqrt{\frac{1-\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{1-\cos A}{1+\cos A} \times \frac{1+\cos A}{1+\cos A}}$$

$$= \sqrt{\frac{1-\cos^2 A}{(1+\cos A)^2}}$$

$$= \sqrt{\frac{\sin^2 A}{(1+\cos A)^2}}$$

$$= \frac{\sin A}{1+\cos A}$$

= RHS

– Hence Proved

(xi) Taking LHS,

$$\frac{1+(\sec A-\tan A)^2}{\operatorname{cosec} A(\sec A-\tan A)}$$

$$= \frac{(\sec^2 A-\tan^2 A)+(\sec A-\tan A)^2}{\operatorname{cosec} A(\sec A-\tan A)}$$

$$= \frac{(\sec A-\tan A)(\sec A+\tan A)+(\sec A-\tan A)^2}{\operatorname{cosec} A(\sec A-\tan A)}$$

$$= \frac{(\sec A+\tan A)+(\sec A-\tan A)}{\operatorname{cosec} A}$$

$$= \frac{2\sec A}{\operatorname{cosec} A}$$

$$= 2\frac{\frac{1}{\cos A}}{\frac{1}{\sin A}}$$

$$= 2\tan A$$

= RHS

– Hence Proved

(xii) Taking LHS,

$$\frac{(\operatorname{cosec} A - \cot A)^2 + 1}{\sec A(\operatorname{cosec} A - \cot A)}$$
$$= \frac{(\operatorname{cosec} A - \cot A)^2 + (\operatorname{cosec}^2 A - \cot^2 A)}{\sec A(\operatorname{cosec} A - \cot A)}$$
$$= \frac{(\operatorname{cosec} A - \cot A)^2 + (\operatorname{cosec} A - \cot A)(\operatorname{cosec} A + \cot A)}{\sec A(\operatorname{cosec} A - \cot A)}$$
$$= \frac{(\operatorname{cosec} A - \cot A) + (\operatorname{cosec} A + \cot A)}{\sec A}$$
$$= \frac{2\operatorname{cosec} A}{\sec A}$$
$$= 2\cot A$$

= RHS

– Hence Proved

(xiii) Taking LHS,

$$\cot^2 A\left(\frac{\sec A - 1}{1 + \sin A}\right) + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right)$$
$$= \cot^2 A\left(\frac{\sec A - 1}{1 + \sin A} \times \frac{\sec A + 1}{\sec A + 1}\right) + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right)$$
$$= \cot^2 A\left[\frac{\sec^2 A - 1}{(1 + \sin A)(\sec A + 1)}\right] + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right)$$
$$= \cot^2 A\left[\frac{\tan^2 A}{(1 + \sin A)(\sec A + 1)}\right] + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right)$$
$$= \frac{1}{(1 + \sin A)(\sec A + 1)} + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right)$$
$$= \frac{1 + \sec^2 A(\sin A - 1)(1 + \sin A)}{(1 + \sin A)(\sec A + 1)}$$
$$= \frac{1 + \sec^2 A(\sin^2 A - 1)}{(1 + \sin A)(\sec A + 1)}$$
$$= \frac{1 + \sec^2 A(-\cos^2 A)}{(1 + \sin A)(\sec A + 1)}$$
$$= \frac{1 - 1}{(1 + \sin A)(\sec A + 1)}$$
$$= 0$$

= RHS

– Hence Proved

(xiv) Taking LHS,

$$\frac{(1-2\sin^2 A)^2}{\cos^4 A - \sin^4 A}$$

$$= \frac{(1-2\sin^2 A)^2}{(\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)}$$

$$= \frac{(1-2\sin^2 A)^2}{1 - \sin^2 A - \sin^2 A}$$

$$= \frac{(1-2\sin^2 A)^2}{1-2\sin^2 A}$$

$$= 1 - 2\sin^2 A$$

$$= 1 - 2(1 - \cos^2 A)$$

$$= 2\cos^2 A - 1$$

= RHS

– Hence Proved

(xv) Taking LHS,

$\sec^4 A (1 - \sin^4 A) - 2 \tan^2 A$

$= \sec^4 A(1 - \sin^2 A) (1 + \sin^2 A) - 2 \tan^2 A$

$= \sec^4 A(\cos^2 A) (1 + \sin^2 A) - 2 \tan^2 A$

$= \sec^2 A + \sin^2 A/ \cos^2 A - 2 \tan^2 A$

$= \sec^2 A - \tan^2 A$

= 1 = RHS

– Hence Proved

(xvi) $\operatorname{cosec}^4 A(1 - \cos^4 A) - 2 \cot^2 A$

$= \operatorname{cosec}^4 A (1 - \cos^2 A) (1 + \cos^2 A) - 2 \cot^2 A$

$= \operatorname{cosec}^4 A (\sin^2 A) (1 + \cos^2 A) - 2 \cot^2 A$

$= \operatorname{cosec}^2 A (1 + \cos^2 A) - 2 \cot^2 A$

$= \operatorname{cosec}^2 A + \cos^2 A/\sin^2 A - 2 \cot^2 A$

= cosec^2 A + $\cot^2$ A – 2 $\cot^2$ A

= cosec^2 A – $\cot^2$ A

= 1 = RHS

– Hence Proved

(xvii) (1 + tan A + sec A) (1 + cot A – cosec A)

= 1 + cot A – cosec A + tan A + 1 – sec A + sec A + cosec A – cosec A sec A

= 2 + cos A/sin A+ sin A/cos A – 1/(sin A cos A)

= 2 + ($\cos^2$ A + $\sin^2$ A)/ sin A cos A – 1/(sin A cos A)

= 2 + 1/(sin A cos A) – 1/(sin A cos A)

= 2 = RHS

– Hence Proved

2. If sin A + cos A = p

and sec A + cosec A = q, then prove that: q(p^2 – 1) = 2p

Solution:

Taking the LHS, we have

q(p^2 – 1) = (sec A + cosec A) [(sin A + cos A)^2 – 1]

= (sec A + cosec A) [$\sin^2$ A + $\cos^2$ A + 2 sin A cos A – 1]

= (sec A + cosec A) [1 + 2 sin A cos A – 1]

= (sec A + cosec A) [2 sin A cos A]

= 2sin A + 2 cos A

= 2p

3. If x = a cos θ and y = b cot θ, show that:

a^2/ x^2 – b^2/ y^2 = 1

Solution:

Taking LHS,

$a^2/ x^2 – b^2/ y^2$

$$= \frac{a^2}{a^2 \cos^2 \theta} - \frac{b^2}{b^2 \cot^2 \theta}$$
$$= \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta}$$
$$= \frac{1 - \sin^2 \theta}{\cos^2 \theta}$$
$$= \frac{\cos^2 \theta}{\cos^2 \theta}$$
$$= 1$$

4. If sec A + tan A = p, show that:

$\sin A = (p^2 – 1)/ (p^2 + 1)$

Solution:

Taking RHS, $(p^2 – 1)/ (p^2 + 1)$

$$= \frac{(\sec A + \tan A)^2 - 1}{(\sec A + \tan A)^2 + 1}$$
$$= \frac{\sec^2 A + \tan^2 A + 2 \tan A \sec A - 1}{\sec^2 A + \tan^2 A + 2 \tan A \sec A + 1}$$
$$= \frac{\tan^2 A + \tan^2 A + 2 \tan A \sec A}{\sec^2 A + \sec^2 A + 2 \tan A \sec A}$$
$$= \frac{2 \tan^2 A + 2 \tan A \sec A}{2 \sec^2 A + 2 \tan A \sec A}$$
$$= \frac{2 \tan A (\tan A + \sec A)}{2 \sec A (\tan A + \sec A)}$$
$$= \sin A$$

5. If tan A = n tan B and sin A = m sin B, prove that:

$\cos^2 A = m^2 – 1/ n^2 – 1$

Solution:

Given,

tan A = n tan B

n = tan A/ tan B

And, sin A = m sin B

m = sin A/ sin B

Now, taking RHS and substitute for m and n

$m^2 – 1/ n^2 – 1$

$$= \frac{\left(\frac{\sin A}{\sin B}\right)^2 - 1}{\left(\frac{\tan A}{\tan B}\right)^2 - 1}$$

$$= \frac{\tan^2 B(\sin^2 A - \sin^2 B)}{\sin^2 B(\tan^2 A - \tan^2 B)}$$

$$= \frac{\sin^2 A - \sin^2 B}{\cos^2 B\left(\frac{\sin^2 A}{\cos^2 A} - \frac{\sin^2 B}{\cos^2 B}\right)}$$

$$= \frac{\cos^2 A(\sin^2 A - \sin^2 B)}{\sin^2 A\cos^2 B - (1 - \cos^2 B)\cos^2 A}$$

$$= \frac{\cos^2 A(1 - \cos^2 A - 1 + \cos^2 B)}{\cos^2 B(\sin^2 A + \cos^2 A) - \cos^2 A}$$

$$= \frac{\cos^2 A(\cos^2 B - \cos^2 A)}{\cos^2 B - \cos^2 A}$$

$$= \cos^2 A$$

6. (i) If 2 sin A – 1 = 0, show that:

sin 3A = 3 sin A – 4 $\sin^3$ A

(ii) If 4 $\cos^2$ A – 3 = 0, show that:

cos 3A = 4 $\cos^2$ A – 3 cos A

Solution:

(i) Given, 2 sin A – 1 = 0

So, sin A = ½

We know, $\sin 30^o = 1/2$

Hence, $A = 30^o$

Now, taking LHS

$\sin 3A = \sin 3(30^o) = \sin 30^o = 1$

RHS = $3 \sin 30^o – 4 \sin^3 30^o = 3 (1/2) – 4 (1/2)^3 = 3 – 4(1/8) = 3/2 – ½ = 1$

Therefore, LHS = RHS

(ii) Given, $4 \cos^2 A – 3 = 0$

$4 \cos^2 A = 3$

$\cos^2 A = 3/4$

$\cos A = \sqrt{3}/2$

We know, $\cos 30^o = \sqrt{3}/2$

Hence, $A = 30^o$

Now, taking

LHS = $\cos 3A = \cos 3(30^o) = \cos 90^o = 0$

RHS = $4 \cos^3 A – 3 \cos A = 4 \cos^3 30^o – 3 \cos 30^o = 4 (\sqrt{3}/2)^3 – 3 (\sqrt{3}/2)$

$= 4 (3\sqrt{3}/8) – 3\sqrt{3}/2$

$= 3\sqrt{3}/2 – 3\sqrt{3}/2$

$= 0$

Therefore, LHS = RHS

7. Evaluate:

(i) $2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right)$

(ii) $\sec 26^\circ \sin 64^\circ + \frac{\operatorname{coesec} 33^\circ}{\sec 57^\circ}$

(iii) $\frac{5\sin 66^\circ}{\cos 24^\circ} - \frac{2\cot 85^\circ}{\tan 5^\circ}$

(iv) $\cos 40^\circ \operatorname{cosec} 50^\circ + \sin 50^\circ \sec 40^\circ$

(v) $\sin 27^\circ \sin 63^\circ - \cos 63^\circ \cos 27^\circ$

(vi) $\frac{3\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$

(vii) $3\cos 80^\circ \operatorname{cosec} 10^\circ + 2\cos 59^\circ \operatorname{cosec} 31^\circ$

(viii) $\frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$

Solution:

$$2\left(\frac{\tan 35^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\cot 55^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec 40^\circ}{\operatorname{cosec} 50^\circ}\right)$$
$$= 2\left(\frac{\tan(90^\circ - 55^\circ)}{\cot 55^\circ}\right)^2 + \left(\frac{\cot(90^\circ - 35^\circ)}{\tan 35^\circ}\right)^2 - 3\left(\frac{\sec(90^\circ - 50^\circ)}{\operatorname{cosec} 50^\circ}\right)$$
$$= 2\left(\frac{\cot 55^\circ}{\cot 55^\circ}\right)^2 + \left(\frac{\tan 35^\circ}{\tan 35^\circ}\right)^2 - 3\left(\frac{\operatorname{cosec} 50^\circ}{\operatorname{cosec} 50^\circ}\right)$$
$$= 2(1)^2 + 1^2 + -3$$
$$= 2 + 1 - 3$$
$$= 0$$

(i)

$= 2\ (1)^2 + 1^2 – 3$

= 2 + 1 – 3 = 0

$$\sec 26^\circ \sin 64^\circ + \frac{\operatorname{coesec} 33^\circ}{\sec 57^\circ}$$
$$= \sec(90^\circ - 64^\circ)\sin 64^\circ + \frac{\operatorname{coesec}(90^\circ - 57^\circ)}{\sec 57^\circ}$$
$$= \operatorname{cosec} 64^\circ \sin 64^\circ + \frac{\sec 57^\circ}{\sec 57^\circ}$$

(ii)

= 1 + 1 = 2

$$\frac{5\sin 66^\circ}{\cos 24^\circ} - \frac{2\cot 85^\circ}{\tan 5^\circ}$$
$$= \frac{5\sin(90^\circ - 24^\circ)}{\cos 24^\circ} - \frac{2\cot(90^\circ - 5^\circ)}{\tan 5^\circ}$$
$$= \frac{5\cos 24^\circ}{\cos 24^\circ} - \frac{2\tan 5^\circ}{\tan 5^\circ}$$
$$= 5 - 2 = 3$$

(iii)

(iv) cos 40° cosec 50° + sin 50° sec 40°

= cos (90 – 50)° cosec 50° + sin (90 – 50)° sec 40°

= sin 50° cosec 50° + cos 40° sec 40°

= 1 + 1 = 2

(v) sin 27° sin 63° – cos 63° cos 27°

= sin (90 – 63)° sin 63° – cos 63° cos (90 – 63)°

= cos 63° sin 63° – cos 63° sin 63°

= 0

(vi)

$$\frac{3\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$
$$= \frac{3\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ}$$
$$= \frac{3\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ}$$
$$= 3 - 1 = 2$$

(vii) 3 cos 80° cosec 10° + 2 cos 59° cosec 31°

= 3 cos (90 – 10)° cosec 10° + 2 cos (90 – 31)° cosec 31°

= 3 sin 10° cosec 10° + 2 sin 31° cosec 31°

= 3 + 2 = 5

(viii)

$$\frac{\cos 75^\circ}{\sin 15^\circ} + \frac{\sin 12^\circ}{\cos 78^\circ} - \frac{\cos 18^\circ}{\sin 72^\circ}$$
$$= \frac{\cos(90^\circ - 15^\circ)}{\sin 15^\circ} + \frac{\sin(90^\circ - 78^\circ)}{\cos 78^\circ} - \frac{\cos(90^\circ - 72^\circ)}{\sin 72^\circ}$$
$$= \frac{\sin 15^\circ}{\sin 15^\circ} + \frac{\cos 78^\circ}{\cos 78^\circ} - \frac{\sin 72^\circ}{\sin 72^\circ}$$
$$= 1 + 1 - 1 = 1$$

8. Prove that:

(i) tan (55° + x) = cot (35° – x)

(ii) sec (70° – θ) = cosec (20° + θ)

(iii) sin (28° + A) = cos (62° – A)

(iv) 1/ (1 + cos (90° – A)) + 1/(1 – cos (90° – A)) = 2 cosec^2 (90° – A)

(v) 1/ (1 + sin (90° – A)) + 1/(1 – sin (90° – A)) = 2 sec^2 (90° – A)

Solution:

(i) tan (55° + x) = tan [90° – (35° – x)] = cot (35° – x)

(ii) sec (70° – **θ**) = sec [90° – (20° + **θ**)] = cosec (20° + **θ**)

(iii) sin (28° + A) = sin [90° – (62° – A)] = cos (62° – A)

(iv)

$$\frac{1}{1 + \cos(90^\circ - A)} + \frac{1}{1 - \cos(90^\circ - A)}$$
$$= \frac{1}{1 + \sin A} + \frac{1}{1 - \sin A}$$
$$= \frac{1 - \sin A + 1 + \sin A}{(1 + \sin A)(1 - \sin A)}$$
$$= \frac{2}{1 - \sin^2 A}$$
$$= \frac{2}{\cos^2 A}$$
$$= 2\sec^2 A$$
$$= 2\operatorname{cosec}^2(90^\circ - A)$$

(v)

$$\frac{1}{1+\sin(90^\circ - A)} + \frac{1}{1-\sin(90^\circ - A)}$$

$$= \frac{1}{1+\cos A} + \frac{1}{1-\cos A}$$

$$= \frac{1-\cos A + 1 + \cos A}{(1+\cos A)(1-\cos A)}$$

$$= \frac{2}{1-\cos^2 A}$$

$$= 2\operatorname{cosec}^2 A$$

$$= 2\sec^2(90^\circ - A)$$

Benefits of ICSE Class 10 Maths Selina Solutions Chapter 21

The ICSE Class 10 Maths Selina Solutions for Chapter 21 on Trigonometrical Identities offer several benefits for students:

Concept Clarity: The solutions provide detailed explanations and step-by-step methods to solve trigonometric problems, helping students understand the fundamental concepts and identities thoroughly.

Problem-Solving Skills: By practicing various types of problems, students can enhance their analytical and problem-solving skills, which are essential for tackling trigonometric equations and identities.

Exam Preparation: The solutions are aligned with the ICSE syllabus and exam pattern, offering students a comprehensive resource for effective preparation and improving their performance in exams.

Confidence Building: Regular practice with these solutions can boost students' confidence by making them familiar with different problem-solving techniques and reducing their anxiety towards trigonometry.

Error Minimization: The step-by-step approach helps students identify and learn from their mistakes, ensuring a better understanding and minimizing errors in solving trigonometric problems.