MATHEMATICS

Maximum Marks: 80

Time Allowed: Three hours

*(Candidates are allowed **additional 15 minutes** for **only** reading the paper.*

*They must **NOT** start writing during this time).*

This Question Paper consists of three sections A, B and C.

*Candidates are required to attempt all questions from **Section A** and all questions **EITHER** from **Section B OR Section C.***

Section A**: Internal choice has been provided in **two questions of two marks each, two questions of four marks each and two questions of six marks each.

Section B:** Internal choice has been provided in **one question of two marks and one question of four marks.

Section C:** Internal choice has been provided in **one question of two marks and one question of four marks.

All working, including rough work, should be done on the same sheet as, and adjacent to the rest of the answer.

The intended marks for questions or parts of questions are given in brackets [ ].

Mathematical tables and graph papers are provided.

SECTION A - 65 MARKS

Question 1

In subparts (i) to (x) choose the correct options and in subparts (xi) to (xv), answer the questions as instructed.

(i) Which of the following is **NOT** an equivalence relation on **Z**? **[1]**

(a) $aRb \Leftrightarrow a + b \text{ is an even integer}$

(b) $aRb \Leftrightarrow a - b \text{ is an even integer}$

(c) $aRb \Leftrightarrow a < b$

(d) $aRb \Leftrightarrow a = b$

(ii) Let A be the set of all 50 cards numbered from 1 to 50. Let $f: A \to N$ be a function defined by $f(x)$ = card number of the card 'x'. Then the function 'f' is: **[1]**

(a) one to one but not onto.

(b) onto but not one to one.

(c) neither one to one nor onto.

(d) one to one and onto.

(iii) The value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to **[1]**

(a) $\frac{\pi}{3}$

(b) $\frac{2\pi}{3}$

(c) $\frac{-\pi}{3}$

(d) $\frac{\pi}{4}$

(iv) If $A = \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}$, then A^n $(n \in N)$ is equal to **[1]**

(a) $\begin{pmatrix} 1 & nb \\ 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & b^n \\ 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & n^b \\ 0 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & nb \\ 0 & 0 \end{pmatrix}$

(v) If A is a 3×3 matrix such that $A(adj\ A) = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, then the value of $|A^T|$ will be **[1]**

(a) 27

(b) 9

(c) 3

(d) -3

(vi) If $\begin{vmatrix} a & b & c \\ x+a & y+2a & z+3a \\ 1 & 2 & 3 \end{vmatrix} = \begin{vmatrix} a & b & c \\ x & y & z \\ 1 & 2 & 3 \end{vmatrix} + |B|$, then the value of $|B|$ is equal to **[1]**

(a) $a + 2b + 3c$

(b) $a - b + 2c$

(c) 0

(d) $3a$

(vii) If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then $\frac{dy}{dx}$ is equal to **[1]**

(a) $\frac{x}{y}$

(b) $-\frac{x}{y}$

(c) $\frac{y}{x}$

(d) $-\frac{y}{x}$

(viii) If $f(x) = \frac{4-x^2}{4x-x^3}$ then the function is: **[1]**

(a) discontinuous at only one point.

(b) discontinuous at exactly two points.

(c) discontinuous at exactly three points.

(d) discontinuous at exactly four points.

(ix) The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is: **[1]**

(a) 2

(b) 1

(c) 3

(d) not defined

(x) If A and B are two events such that $P(A) > 0$ and $P(B) \neq 1$, then $P\left(\bar{A}/\bar{B}\right)$ is **[1]**

(a) $1 - P\left(\bar{A}/B\right)$

(b) $1 - P\left(A/B\right)$

(c) $\dfrac{1 - P(A \cup B)}{P(\bar{B})}$

(d) $\dfrac{P(\bar{A})}{P(B)}$

(xi) Write the smallest equivalence relation on the set $A = \{a, b, c\}$ **[1]**

(xii) If $\begin{pmatrix} 3 & 5 \\ 7 & 9 \end{pmatrix} = X + Y$, where X is skew- symmetric matrix and Y is symmetric matrix. Find $|X|$. **[1]**

(xiii) If $\int \log 2x \, dx = x\log 2x - k + c$ where k is a function of x, then find k. **[1]**

(xiv) 50 tickets in a box are numbered 00, 01, 02 , ... , 49. One ticket is drawn randomly from the box. Find the probability of the ticket having the product of its digits 7, given that the sum of the digits is 8? **[1]**

(xv) A speaks truth in 60% of cases and B speaks truth in 90% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact? **[1]**

Question 2 **[2]**

(a) If $y = \sqrt{\sin x + y}$, then find $\frac{dy}{dx}$.

OR

(b) Find the point on the curve $y^2 = 4x + 8$ for which the abscissa and ordinate changes at the same rate.

Question 3 **[2]**

Let $f: R \rightarrow R$ defined as $f(x) = 2x - 3$. Find

(i) $f^{-1}(x)$

(ii) domain and range of $f^{-1}(x)$

Question 4 **[2]**

The function f is defined for all $x \in \mathrm{R}$. The line with equation $y = 6x - 1$ is the tangent to the graph of f at $x = 4$.

(i) Write down the value of $f'(4)$.

(ii) Find $f(4)$.

Question 5 **[2]**

(i) Evaluate: $\int [\sin(\log x) + \cos(\log x)]\, dx$

OR

(ii) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{a \sin x + b \cos x}{\sin x + \cos x}\, dx$

Question 6 **[2]**

Solve the differential equation: $(1 + y^2)(1 + \log x)dx + xdy = 0$

Question 7 **[4]**

If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$, prove that $\frac{x^2}{a^2} - \frac{2xy}{ab} cos\alpha + \frac{y^2}{b^2} = sin^2\alpha$

Question 8 **[4]**

If $y = \left(x + \sqrt{1 + x^2}\right)^n$, then prove that $(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = n^2 y$.

Question 9 **[4]**

(i) Evaluate : $\int_{-1}^{2} |x^3 - x|\, dx$

OR

(ii) Evaluate: $\int \sqrt{\operatorname{cosec} x - 1}\, dx$

Question 10 **[4]**

(i) A student answers a multiple choice question with 5 alternatives, of which exactly one is correct. The probability that he knows the correct answer is $\frac{1}{5}$. If he does not know the correct answer, he randomly ticks one answer. Given that he has answered the question correctly, find the probability that he did not tick the answer randomly.

OR

(ii) A candidate takes three tests in succession and the probability of passing the first test is $\frac{1}{2}$. The probability of passing each succeeding test is $\frac{1}{2}$ or $\frac{1}{4}$ depending on whether he passes or fails in the preceding one. The candidate is selected, if he passes at least two tests. Find the probability that the candidate is selected.

Question 11 **[6]**

Find A^{-1}, if $A = \begin{pmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{pmatrix}$. Using A^{-1}, solve the system of linear equations: $x - 2y = 10, 2x - y - z = 8, -2y + z = 7$

Question 12 **[6]**

(i) Find the particular solution of the differential equation given below $(1 + x^2)dy = (\tan^{-1} x - y)dx$, given that $y = 1\ when\ x = 0$

OR

(ii) Evaluate: $\int \frac{1}{\sin^4 x + \cos^4 x} dx$

Question 13 **[6]**

(i) Find the maximum volume of the cylinder which can be inscribed in a sphere of radius $3\sqrt{3}\ cm$. (find the answer in terms of π)

OR

(ii) Find the coordinates of a point on the curve $y = x^2 + 7x + 2$ which is closest to the straight line $y = 3x - 3$.

Question 14 **[6]**

A biased four-sided die with faces labelled 1, 2,3 and 4 is rolled and recorded. Let X be the result obtained when the die is rolled. The probability distribution for X is given in the following table where p and q are constants.

x	1	2	3	4
$P(X = x)$	p	0·3	q	0·1

For the probability distribution, it is known that $E(X) = 2$. Find p and q.

Also, find $P(X > 2)$.

Ajay plays a game with this four-sided die. In this game he is allowed a maximum of five rolls. His score is calculated by adding the results of each roll. He wins the game if his score is at least 10. After 3 rolls, Ajay has score of four points. Assuming that rolls of the die are independent, find the probability that Ajay wins the game.

SECTION B - 15 MARKS

Question 15 **[5]**

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{c} = 3\hat{\imath} + \hat{\jmath}$, find t such that $\vec{a} + t\vec{b}$ is perpendicular to $\vec{c}$ is

(a) 0

(b) 5

(c) 4

(d) 2

(ii) The planes $2x – y + 4z = 5$ and $5x – 2·5y + 10z = 6$ are

(a) parallel

(b) intersect on y axis

(c) perpendicular

(d) pass through $(0, 0, \frac{5}{4})$

(iii) Find a vector of magnitude of 10 units and parallel to the vector $2\hat{\imath} + 3\hat{\jmath} - \hat{k}$.

(iv) Find the position vector of a point R which divides the line joining the two-points P and Q with position vectors $2\hat{\imath}+\hat{\jmath}$ and $\hat{\imath} - 2\hat{\jmath}$ respectively in the ratio of 2:1 externally.

(v) Find the equation of the plane with intercept 3 on the $y-$ axis and parallel to xz $-$plane.

Question 16 **[2]**

(i) Find the area of the triangle whose adjacent sides are $\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and $\hat{\imath} + \hat{\jmath} + 2\hat{k}$.

OR

(ii) For *any two* non-zero vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$, then find the angle between them.

Question 17 **[4]**

(i) Show that $\frac{4-x}{-1} = \frac{y+3}{-4} = \frac{z+1}{7}$ and $\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ intersect each other. Also find out the point of intersection.

OR

(ii) Find the equation of the plane passing through the points $(0,4,-3)$ and $(6,-4,3)$ if the sum of their intercepts on three axes is 0.

Question 18 **[4]**

If the area is bounded by the parabola $y^2 = 16x$ and the line $y = 4mx$ is $\frac{1}{12}$ sq units, then using integration, find the value of m. $(m > 0)$.

SECTION C - 15 MARKS

Question 19 **[5]**

In subparts (i) and (ii) choose the correct options and in subparts (iii) to (v), answer the questions as instructed.

(i) Which relation is correct for breakeven point where $R(X)$ is revenue function and $C(x)$ is cost function?

(a) $R(X) > C(X)$

(b) $R(X) < C(X)$

(c) $R(X) = C(X)$

(d) $R(X) = 2C(X)$

(ii) If correlation coefficient $r = 0$, then regression lines are

(a) parallel to each other.

(b) not mutually perpendicular.

(c) parallel to coordinate axis.

(d) overlapping lines.

(iii) Average revenue of a commodity is given by $AR(x) = a + \frac{b}{x}$. Find the demand function when marginal revenue is zero.

(iv) If $R(X) = 36x + 3x^2 + 5$ then find actual revenue from selling 50^{th} item.

(v) For a given bi-variant distribution, the mean of variable $x = 4$ and the mean of variable $y = 6$. Find the point of intersection of two regression lines.

Question 20 **[2]**

(i) The total revenue function $R(x) = x + 2x^3 - 3{\cdot}5x^2$. Find the point where marginal revenue curve cuts the co-ordinate axis.

OR

(ii) For the revenue function $R(x) = \frac{bx}{a+x}$, show that the marginal revenue function is increasing for all $b < 0$ and $a > 0$.

Question 21 **[4]**

The line of regression of marks in Maths (X) and marks in English (Y) for a class of 50 students is $3Y - 5X + 180 = 0$. The average score in English is 44 and variance of marks in Maths is $\frac{9}{16} th$ of the variance of marks in English. Find the average score in Maths. Also, find out the coefficient of correlation between marks in Maths and English.

Question 22 **[4]**

(i) Solve the following linear programming problem graphically and interpret your result of $Z = 2x - 5y$ subject to the constraints $x + y \geq 2,\ x - y \geq 0,\ x \leq 1,\ x \geq 0,\ y \geq 0$.

OR

(ii) The standard weight of a special purpose brick is 5 kg and it must contain two basic ingredients B_1 and B_2 . B_1 costs ₹ 5 per kg and B_2 costs ₹ 8 per kg. Strength considerations dictate that the brick should not contain more than 4 kg of B_1 and minimum 2 kg of B_2 .Since the demand for the product is likely to be related to the price of the bricks, find the minimum cost of the brick satisfying the above conditions. Formulate this situation as an L.P.P. and solve it graphically.

(NOTE: The total weightage of each Unit covered in the question paper shall be as specified in the syllabus, covering all the chapters of each Unit. The weightage of each question in the Question Paper shall be as indicated in this Specimen Paper. However, the number of MCQs given in Question Nos. 1, 15 and 19 may vary from year to year.)