LIVE

JEE MAIN 2024

ATTEMPT – 02 , 09th April 24, SHIFT - 01

PAPER DISCUSSION

Mathematics

$f(x) = 3ax^3 + bx^2 + cx + 41, f(1) = 41, f'(1) = 2, f''(1) = 4$. Find $a^2 + b^2 + c^2$.

$f'(x) = 9ax^2 + 2bx + c$

$f''(x) = 18a(x) + 2b$

$4 = 18a + 2b$

$9a + b = 2 \quad -1$

$2 = 9a + 2b + c \quad -2$

$41 = 3a + b + c + 41 \quad -3$

$0^2 + 2^2 + (-2)^2 = 8$

#CNF

$9a + b = 2$

$2 = 9a + 2b + c$

$\cancel{9a} + b = \cancel{9a} + 2b + c$

$b + c = 0$

$41 = 3a + 41$

$a = 0$

$b = 2$

$c = -2$

$\int \frac{2-\tan x}{3+\tan x} = \alpha x + \beta \ell n(3\cos x + \sin x) + \gamma$, where γ is constant integration. Find $\alpha + \beta$.

$$\int \frac{2-\left(\frac{\sin x}{\cos x}\right)}{3+\left(\frac{\sin x}{\cos x}\right)}$$

$$\int \left(\frac{2\cos x - \sin x}{3\cos x + \sin x}\right) dx$$

$$(2\cos x - \sin x) = \overset{\frac{1}{2}}{A}(3\cos x + \sin x) + \overset{\frac{1}{2}}{B}(-3\sin x + \cos x)$$

$$3A + B = 2 \quad \times 3$$

$$A - 3B = -1$$

$$10A = 5$$

$$A = \frac{1}{2}$$

$$\frac{1}{2} + 1 = 3B$$

$$B = \frac{1}{2}$$

$$\frac{1}{2}(x)+\frac{1}{2}\ln(3\cos x+\sin x)+C$$

If the domain of function $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $R - (\alpha, \beta)$. Then find α, β.

$12(-4)(-2/3) = 32$

$12\alpha\beta$?

$-1 \le \frac{x-1}{2x+3} \le 1$

$\frac{x-1}{2x+3} \ge -1$

$\frac{x-1}{2x+3} + 1 \ge 0$

$\frac{3x+2}{2x+3} \ge 0$

$+ \quad -\frac{3}{2} \quad - \quad -\frac{2}{3} \quad +$

$\frac{x-1}{2x+3} \le 1$

$\left(\frac{x-1}{2x+3}\right) - 1 \le 0$

$(x-1) - (2x+3)$

$\left(\frac{-x-4}{2x+3}\right) \le 0$

$\frac{x+4}{2x+3} \ge 0$

$+ \quad -4 \quad - \quad -\frac{3}{2} \quad +$

$-4 \quad -\frac{3}{2} \quad -\frac{2}{3}$

$(-\infty, -4] \cup [-2/3, \infty)$

$R - (-4, -2/3)$

$$\frac{1}{1+d}+\frac{1}{(1+d)(1+2d)}+\frac{1}{(1+2d)(1+3d)}+\ldots+\frac{1}{(1+9d)(1+10d)}=5.$$ Find the value of $50d$.

(Ans=5)

ANS = 5

$$\frac{1}{d}\left(\frac{(1+d-1)}{1\times(1+d)}+\frac{(1+2d)-(1+d)}{(1+d)(1+2d)}\cdots+\frac{(1+10d)-(1+9d)}{(1+9d)(1+10d)}\right)$$

$$\frac{1}{d}\left(1-\cancel{\frac{1}{1+d}}+\cancel{\frac{1}{1+d}}-\cancel{\frac{1}{1+2d}}\cdots+\cancel{\frac{1}{1+9d}}-\frac{1}{1+10d}\right)$$

$$\frac{1}{d}\left(1-\frac{1}{1+10d}\right)=5$$

$$\frac{1}{\cancel{d}}\left(\frac{\cancel{10}\,\cancel{d}}{1+10d}\right)=\cancel{5}\,2$$

$$2=1+10d$$

$$10d=1$$

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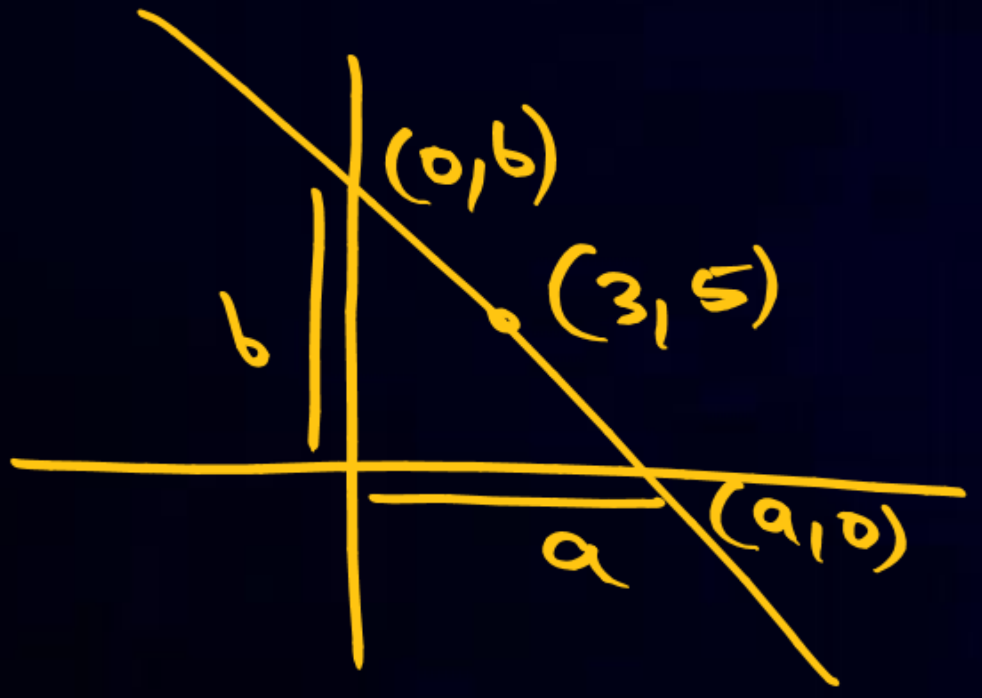
A variable line passing through (3, 5) cut x & y axis. Find minimum area made between axes and the given line.



$$\Delta = \frac{1}{2}(a)(b)$$

$$\Delta_{min} = 30$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{3}{a} + 5/b = 1$$

$$\frac{\frac{3}{a} + \frac{5}{b}}{2} \geq \sqrt{\frac{15}{ab}}$$

$$\frac{1}{4} \geq \frac{15}{ab}$$

$$ab \geq 60$$

The remainder when $(428)^{2024}$ is divided by 21 is

$$(420+8)^{2024}$$

$$\frac{8^{2024}}{21}$$

$$8^1 = 8$$
$$8^2 = 64$$

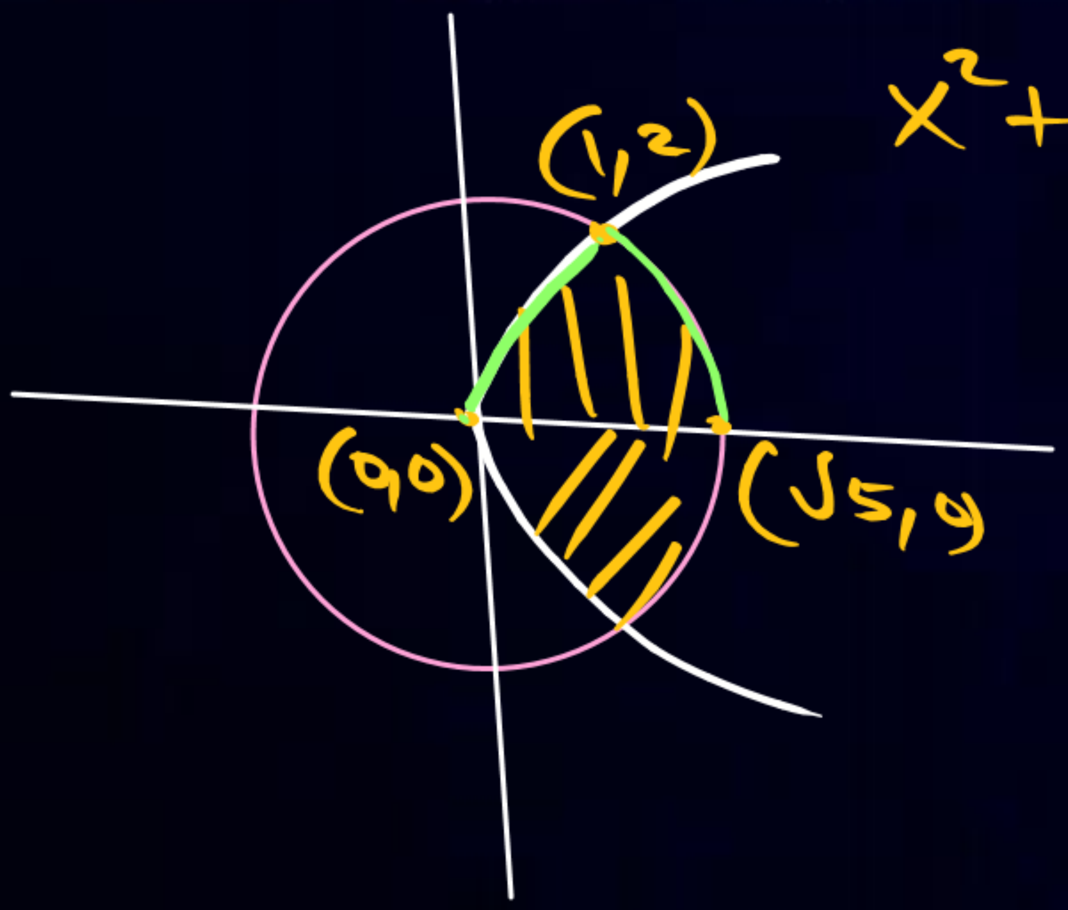
$$\frac{(64)^{1012}}{21} = \frac{(1)^{1012}}{21}$$

$$= 1$$

$$\frac{16^{13}}{15} = \frac{16 \cdot 16 \cdot 16 \cdot 16}{15}$$

$$\frac{6^{99}}{5}$$

The parabola $\boxed{y^2 = 4x}$ divides the area of the circle $\boxed{x^2 + y^2 = 5}$ in 2 parts. The area of smaller part is equal to



$x^2 + 4x = 5$

$x = 1$

$$2\left[\int_0^1 \sqrt{4x}\,dx + \int_1^{\sqrt{5}} \sqrt{5-x^2}\,dx\right]$$

$$2\left(\frac{x^{3/2}}{3/2}\right)_0^1 + \frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\left(\frac{x}{\sqrt{5}}\right)$$

$$2\left[\frac{4}{3} + \left(\frac{5}{2}\left(\frac{\pi}{2}\right) - \left(\frac{1}{2}(2) + \frac{5}{2}\sin^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)\right)\right]$$

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$\cos\theta\cos(60-\theta)\cos(60+\theta) \le \frac{1}{8}$, $\theta \in [0, 2\pi]$. Find the sum of value of θ for which $\cos 3\theta$ is maximum. 6π

$\frac{1}{\cancel{4}}\cos 3\theta \le \frac{1}{\cancel{8}\,2}$

$\cos 3\theta \le \frac{1}{2}$

$\cos 3\theta = \frac{1}{2}$

$3\theta = 2n\pi \pm \pi/3$

$n=0$ $\quad n=1$

$2\pi - \pi/3,\ 2\pi + \pi/3$

$4\pi - \pi/3,\ 4\pi + \pi/3$

$6\pi - \pi/3$

$\theta = \pi/9,\ \frac{5\pi}{9},\ \frac{7\pi}{9},$

$\frac{11\pi}{9},\ \frac{13\pi}{9},\ \frac{17\pi}{9}$

$\frac{54\pi}{9} = 6\pi$

Given system as

$3x + 4y + \lambda z = 4$

$5x + 7y + 2z = 8$ 189

$97x + 197y + 83z = \mu$.

Find $\lambda + 3\mu$, if the system has infinite solutions.

$$\Delta = \begin{vmatrix} 3 & 4 & \lambda \\ 5 & 7 & 2 \\ 97 & 197 & 189 \end{vmatrix} = 0$$

$R_3 \to R_3 - 19R_2$

If the roots of equation $x^2 + 2\sqrt{2}x - 1 = 0$ are α and β. Find the equation whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$.

$\alpha+\beta=-2\sqrt{2}$

$\alpha\beta=-1$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=8-2(-1)=10$

$\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2$

$\Rightarrow 10^2-2(1)=98$

$(\alpha^2+\beta^2)^3=\alpha^6+\beta^6+3\alpha^2\beta^2(\alpha^2+\beta^2)$

$10^3=\alpha^6+\beta^6+3(10)$

$\alpha^6+\beta^6=10\times 97$

$\frac{\alpha^6+\beta^6}{10}=97$

$x^2-(97+98)x+(97)(98)=0$

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A ray of light passing through $P(1, 2)$ after reflecting on x-axis at point Q it passes through $R(3, 4)$. If $S(h, k)$ is such that $PQRS$ is a parallelogram, then find the value of hk^2.

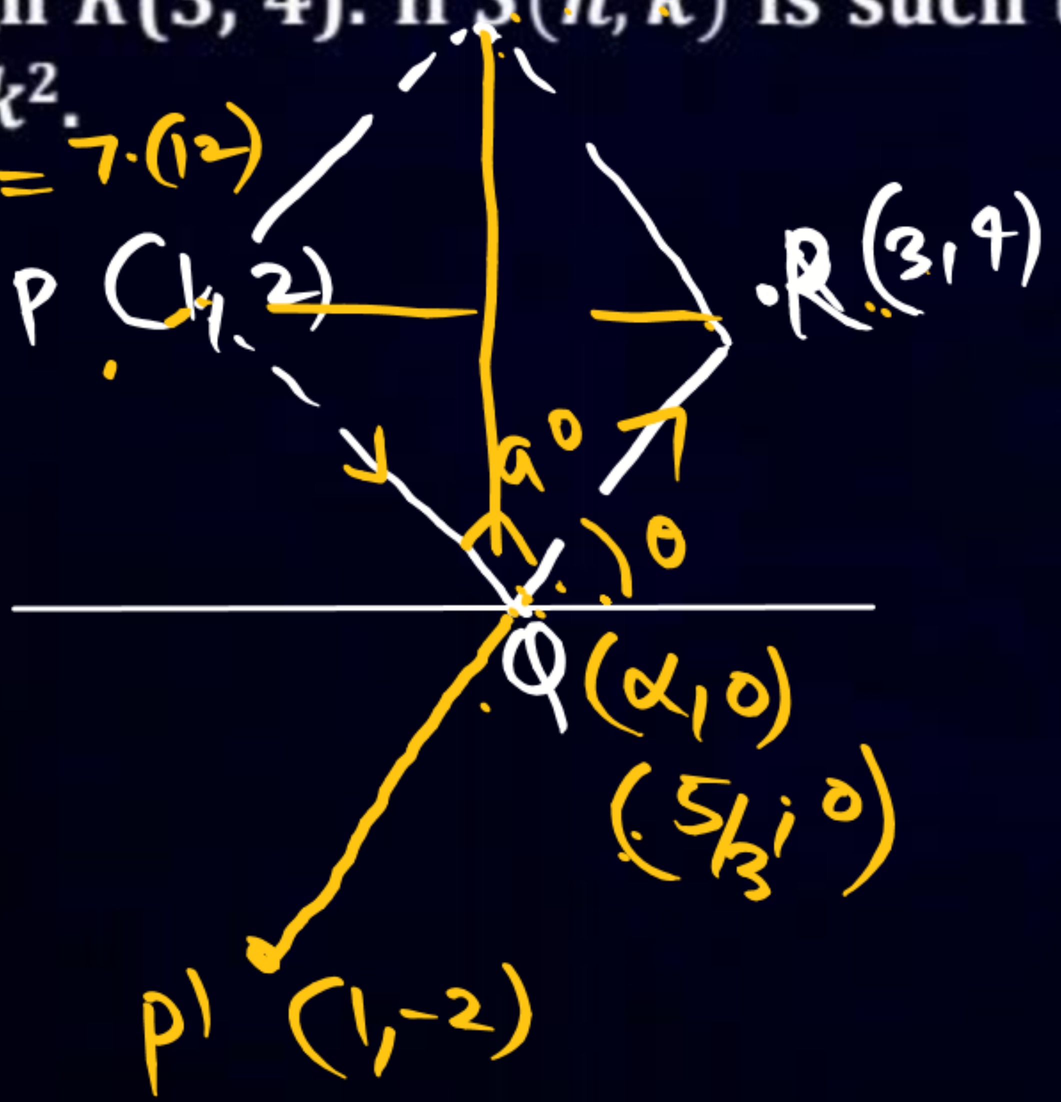
$\frac{7}{3} \cdot 6^2 = 7 \cdot (12)$

(A) 90

(B) 84 ✓

(C) 96

(D) 108



$m_{QR} = \frac{4-0}{3-\alpha}$, $m_{P'R} = \frac{6}{2} = 3$

$\frac{4}{3-\alpha} = 3$

$4/3 = 3 - \alpha$

$\alpha = 3 - 4/3$

$\alpha = 5/3$

$K + 0 = 4 + 2$

$K = 6$

$h + 5/3 = 4$

$h = 4 - 5/3 = 7/3$

Ans=8

Given $f(x) = x^2 + 9$ and $g(x) = \frac{x}{x-9}$. Given a curve $\frac{x^2}{a} + \frac{y^2}{b} = 1$, where $a = \text{fog}(10)$, b = gof(3). Then find $8e^2 + l^2$, (where e = eccentricity, l = latus rectum length)

$g(10) = \frac{10}{1}$

$f(10) = 10^2 + 9 = 109 = a$

$f(3) = 18$

$g(18) = \frac{18}{9} = 2 = b$

a>b

$\frac{x^2}{109} + \frac{y^2}{2} = 1$

$l = \frac{2(2)}{\sqrt{109}}$

$\frac{2}{109} = 1 - e^2$

$e^2 = 1 - 2/109$

$e = \sqrt{\frac{107}{109}}$

8×2

$\frac{8 \times 107}{109} + \frac{16}{109}$

$\frac{8(\cancel{109})}{\cancel{109}} = 8$

$\vec{a}$, $\vec{b}$ and $\vec{c}$ are the unit vectors. If $\vec{a} = 2\vec{b} + \vec{c}$, angle between $\vec{a}$ & $\vec{b} = \cos^{-1}\left(\frac{5}{7}\right)$. Find angle between $\vec{a}$ & $\vec{c}$.

SKIP

Not Used

$$\vec{a} - \vec{c} = 2\vec{b}$$

$$|a|^2 + |c|^2 - 2\vec{a}\cdot\vec{c} = 4|b|^2$$

$$1 + 1 - 2\vec{a}\cdot\vec{c} = 4$$

$$-2 = 2\vec{a}\cdot\vec{c}$$

$$\vec{a}\cdot\vec{c} = -1$$

$$|a||c|\cos\theta = -1$$

$$\cos\theta = -1$$

A triangle ABC is made of three vectors $\vec{a}, \vec{b}$ and $\vec{c}$. $\vec{a}, \vec{b}$ and $\vec{c}$ are $(\alpha\hat{\imath} + 5\hat{\jmath} + 4\hat{k})$, $(3\hat{\imath} + 5\hat{\jmath} + 4\hat{k})$ and $\vec{a} - \vec{b}$ respectively. Area of ΔABC is given as $5\sqrt{6}$. Find $|\vec{c}|^2$.

$\vec{c} = \vec{a} - \vec{b}$

$\vec{c} = (\alpha - 3)\hat{i}$

$|c| = |\alpha - 3| = \frac{10\sqrt{6}}{\sqrt{41}}$

$|c|^2 = \frac{100 \times 6}{41}$

$$\frac{1}{2}\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 5 & 4 \\ 3 & 5 & 4 \end{vmatrix}\right| = 5\sqrt{6}$$

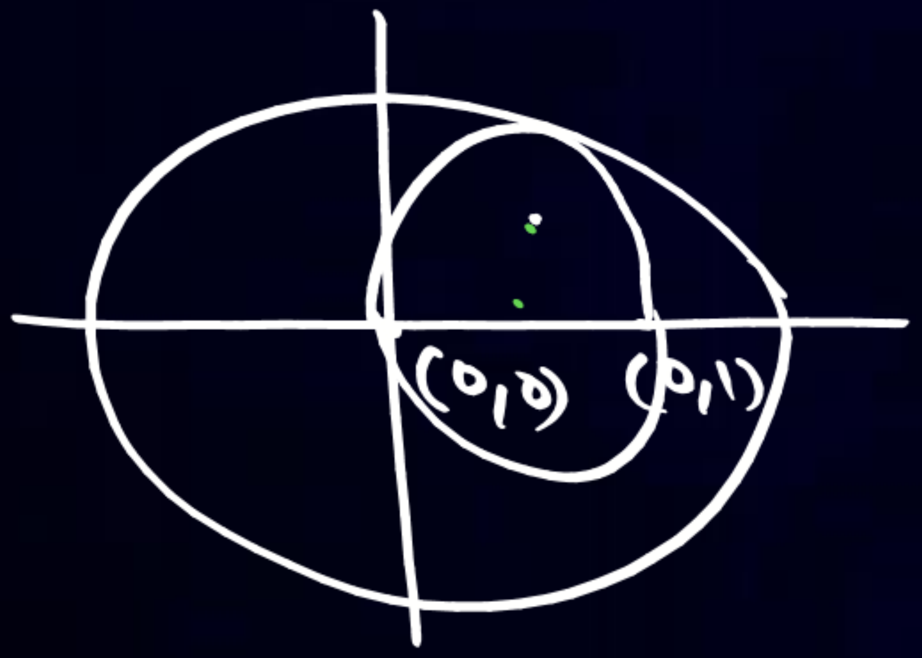
$\frac{1}{2}\left|(0) + \hat{j}(12 - 4\alpha) + \hat{k}(5\alpha - 15)\right|$

$\sqrt{(12 - 4\alpha)^2 + (5\alpha - 15)^2} = 10\sqrt{6}$

$|\alpha - 3|\sqrt{16 + 25} = 10\sqrt{6}$

$|\alpha - 3| = \frac{10\sqrt{6}}{\sqrt{41}}$

A circle with centre (α, β) passes through point $(0, 0)$ and $(0, 1)$ and touches the circle $x^2 + y^2 = 9$ for all possible values of (α, β). Find value of $4(\alpha^4 + \beta^4)$?



$4\left(9/4\right) = 9$

$|C_1C_2| = r_1 - r_2$

$C_1 = (0,0),\ r_1 = 3$

$C_2 = (\alpha, \beta),\ r_2 = \sqrt{\alpha^2+\beta^2}$

$\sqrt{\alpha^2+\beta^2} = 3 - \sqrt{\alpha^2+\beta^2}$

$4(\alpha^2+\beta^2) = 9$

$\alpha^2+\beta^2 = 9/4$

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SKIP

$m = 7/2, n = 7/2$

If A is 3×3 matrix, $\det(3\text{adj}(2\text{adj}A)) = 2^{-13} \cdot 3^{-10}$ and $\det(3\text{adj}(2A)) = 2^{-m} \cdot 3^{-n}$, then $2m + 2n$ is equal to $2\left(\frac{7}{2}+\frac{7}{2}\right) = 14$

$$|\text{Adj}A| = |A|^{n-1}$$

$$\text{Adj}(kA) = k^{n}(\text{Adj}(A))$$

$$|3\,\text{adj}(\text{adj}A)| = 2^{-13}\,3^{-10}$$

$$3^3\,|\text{adj}(2\,\text{adj}A)| = 2^{-13}\,3^{-10}$$

$$3^3\,|2\,\text{Adj}A|^2 = 2^{-13}\,3^{-10}$$

$$3^3\,2^6\,|\text{Adj}A|^2 = 2^{-13}\,3^{-10}$$

$$3^3\,2^6\,|A|^4 = 2^{-13}\,3^{-10}$$

$$|A|^4 = 2^{-19}\,3^{-13} = 2^{-\frac{19}{2}}\,3^{-13/2}$$

$$3^3\,|\text{Adj}(2A)| = 3^3\,(|2A|)^2$$

$$= 3^3\,2^6\,|A|^2$$

$$= 3^3\,2^6\,(2^{-19/2})(3^{-13/2})$$

$$\Rightarrow 2^{-7/2}\,3^{-7/2}$$

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If a function f satisfies $\boxed{f(m+n) = f(m) + f(n)}$ for all $m, n \in N$ and $\boxed{f(1) = 1}$, then find the largest natural number λ such that $\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2$.

$f(x+y) = f(x) + f(y)$

$f(x) = k(x)$

$f(1) = k(1)$

$k = 1$

$f(x) = x$

$(\lambda+1) + (\lambda+2) \ldots + (\lambda+2022)$

$(2022)\lambda + \frac{(2022)(2023)}{2} \leq (2022)^2$

$\lambda + \frac{2023}{2} \leq 2022$

$\lambda \leq \frac{4044-2023}{2}$

$\lambda \leq \frac{2021}{2}$

$\lambda \leq 1010.5$

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THANK YOU