JEE Mains (Dropper)

Sample Paper - II

DURATION : 180 Minutes | **M. MARKS : 300**

ANSWER KEY

PHYSICS

1.	(4)
2.	(4)
3.	(2)
4.	(3)
5.	(4)
6.	(4)
7.	(1)
8.	(1)
9.	(3)
10.	(3)
11.	(3)
12.	(3)
13.	(2)
14.	(3)
15.	(1)
16.	(4)
17.	(1)
18.	(2)
19.	(4)
20.	(2)
21.	(8)
22.	(3)
23.	(5)
24.	(10)
25.	(3)
26.	(6)
27.	(1)
28.	(2)
29.	(9)
30.	(3)

CHEMISTRY

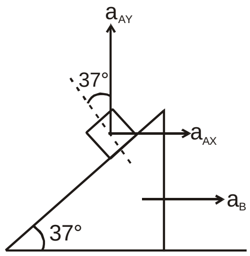
31.	(2)
32.	(3)
33.	(1)
34.	(3)
35.	(3)
36.	(3)
37.	(3)
38.	(4)
39.	(3)
40.	(4)
41.	(3)
42.	(1)
43.	(1)
44.	(3)
45.	(1)
46.	(1)
47.	(2)
48.	(2)
49.	(2)
50.	(2)
51.	(3)
52.	(152)
53.	(11)
54.	(5)
55.	(4)
56.	(4)
57.	(8)
58.	(4)
59.	(5)
60.	(3)

MATHEMATICS

61.	(2)
62.	(2)
63.	(3)
64.	(2)
65.	(3)
66.	(4)
67.	(4)
68.	(4)
69.	(3)
70.	(3)
71.	(4)
72.	(3)
73.	(2)
74.	(3)
75.	(1)
76.	(4)
77.	(4)
78.	(2)
79.	(2)
80.	(3)
81.	(8)
82.	(1)
83.	(3)
84.	(615)
85.	(8)
86	(23)
87.	(108)
88.	(49)
89.	(2021)
90.	(13)

PHYSICS

1. **(4)**



From wedge constraint

$(a_A)_\perp = (a_B)_\perp$

$a_{AX}\cos 53° - a_{AY}\cos 37°$

$= a_B \cos 53°$

$a_B = -5$ m/s

$\vec{a}_B = -5\hat{i}$

2. **(4)**

For spherical surface

Using $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

$\Rightarrow \quad \frac{n}{2R} - \frac{1}{\infty} = \frac{n-1}{R}$

$\Rightarrow \quad n = 2n - 2$

$\Rightarrow \quad n = 2$

3. **(2)**

Zero error (excess reading) = 0.3 mm.

observed thickness of block = 13.8 mm.

Actual thickness = 13.8 – 0.3 = 13.5 mm

4. **(3)**

Initial extension will be equal to 6 m.

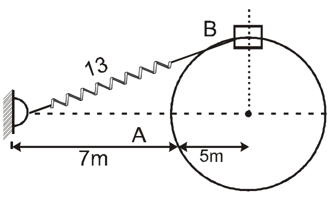
$\therefore$ Initial energy = $\frac{1}{2}$ (200) $(6)^2$ = 3600 J

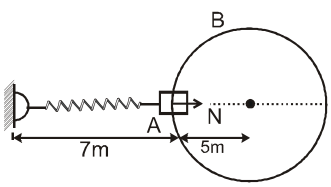
Reaching at A : $\frac{1}{2}$ MV^2 = 3600 J

$\Rightarrow$ mv^2 = 7200 J

From F.B.D. at A :

$N = \frac{mv^2}{R} = \frac{7200}{5}$ = 1440 N





5. **(4)**

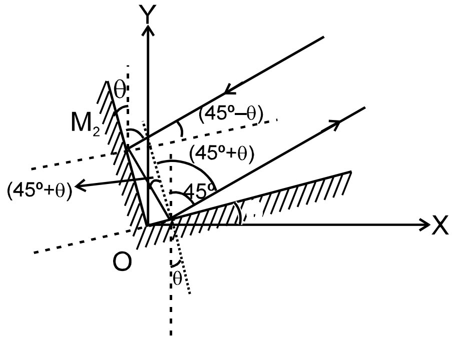
Since tension is the two rods will be same, hence

$A_1Y_1\,\alpha_1\,\Delta\theta = A_2Y_2\,\alpha_2\,\Delta\theta$

$\Rightarrow$ $A_1Y_1\,\alpha_1 = A_2Y_2\,\alpha_2$.

6. **(4)**

Figure shows the slope of the final reflected ray is tan 45° = 1



Hence the most appropriate option is (D)

7. **(1)**

Rate of heat transfer is same through all walls

$$\frac{K_1.A.(10)}{40\,cm} = \frac{K_2A(20)}{20\,cm} = \frac{K_2\,A(40)}{10\,cm}$$

$\Rightarrow \quad \frac{K_1}{4} = K_2 = 4K_3 \Rightarrow K_1 = 4K_2 = 16K_3.$

8. **(1)**

$\frac{hc}{\lambda} = 5\,eV_0 + \phi$

$\frac{hc}{3\lambda} = eV_0 + \phi \qquad \Rightarrow \frac{2hc}{3\lambda} = 4eV_0$

$\Rightarrow \phi = \frac{hc}{6\lambda}$

9. **(3)**

Here path difference will be :

$\Delta x = (\mu_2 - \mu_1)t \qquad \Rightarrow \delta = \frac{2\pi}{\lambda}(\mu_2 - \mu_1)t$

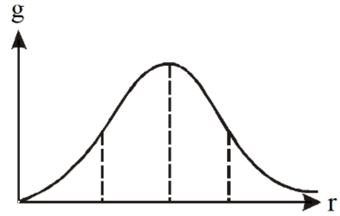
Hence (3)

10. **(3)**

$\frac{1}{2}mV^2 = \frac{3}{2}kT \qquad \frac{1}{2}I\omega^2 = \frac{2}{2}kT$

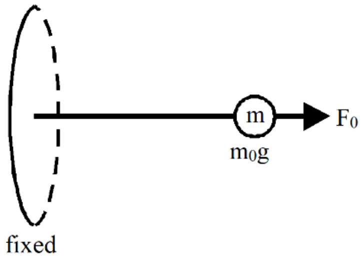
$\frac{V}{\omega} = \sqrt{\frac{3I}{2m}}$

11. **(3)**



$x < \frac{R}{\sqrt{2}} \qquad x = \frac{R}{\sqrt{2}} \qquad x > \frac{R}{\sqrt{2}}$

$x > \frac{R}{\sqrt{2}}$: If we displace the particle in forward direction, if $m_0g \downarrow$ so the particle will move away from equilibrium = unstable equilibrium.



$x < \frac{R}{\sqrt{2}}$: If we displace the particle in forward direction, g ↓ = m_0g↑. So, particle will come back to equilibrium. So, stable equilibrium.
So, stable equilibrium.

12. **(3)**
At t = 0

Displacement x = $x_1 + x_2 = 4\sin\frac{\pi}{3} = 2\sqrt{3}$ m

Resulting Amplitude

A = $\sqrt{2^2+4^2+2(2)(4)\cos\pi/3} = \sqrt{4+16+8}$

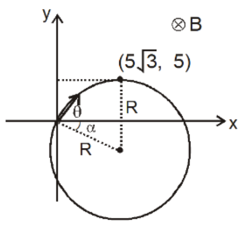
$= \sqrt{28} = 2\sqrt{7}$ m

Maximum speed = $A\omega = 20\sqrt{7}$ m/s

Maximum acceleration = $A\omega^2 = 200\sqrt{7}$ m/s^2

Energy of the motion = $\frac{1}{2}m\omega^2A^2$ = 28 J

13. **(2)**



$R^2 - (R-5)^2 = (5\sqrt{3})^2$

$R^2 - R^2(R-5)^2 = (5\sqrt{3})^2$

$R^2 - R^2 - 25 + 10R = 75$

R = 10 m

sin α = $\frac{1}{2}$, a = 30°, θ = 90

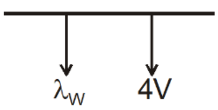
– α = 60°

$\frac{mv}{qB} = R \Rightarrow v = \frac{RqB}{m} = \frac{10\times10^{-6}\times10}{5\times10^{-5}}$ = 2 m/s

14. **(3)**
For observer O_1,

$\lambda_1 = \frac{V - V_s}{f} = \frac{V - V/5}{f} = \frac{4V}{5f}$

For O_2, there is change of medium hence at the surface of water, keeping frequency unchanged



$\frac{V}{\lambda_a} = \frac{4V}{\lambda_w}$

$\Rightarrow \lambda_w = 4\lambda_a = \frac{16V}{5f}$

$f'' = \frac{\text{velo. of wave relative to observer}}{\lambda_w}$

$= \frac{4V + \frac{V}{5}}{\lambda_w} = \frac{21V}{5}.\frac{5f}{16V} = \frac{21f}{16}$

15. **(1)**

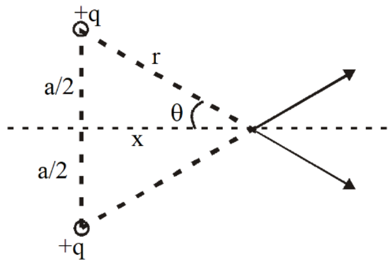
$I_{max} = \left(\sqrt{I_1}+\sqrt{I_2}\right)^2 = \left(\sqrt{I_1}+\sqrt{\frac{I}{2}}\right)^2 < 4I$

$I_{min} = \left(\sqrt{I_1}-\sqrt{\frac{I}{2}}\right)^2 > 0$

16. **(4)**
If unlike
⇒ dipole, E will monotonically decrease as we move away.
∴ Like charge (can be both + or both –)
∴ Direction of E at all points on the perpendicular bisector will be along the perpendicular bisector.
There is a point at which E is maximum as derived below:

$E = \frac{2Kq}{r^2}$. cos θ; $\quad$ $r = \frac{a}{2\sin\theta}$

∴ $E = \frac{2Kq}{a^2}$. 4 sin θ cos θ



E is maximum when $\sin^2\theta\cos\theta$ is maximum.
$\sin^2\theta\cos\theta = \cos\theta - \cos^3\theta$
Differentiate and equate to zero.
$-\sin\theta + 3\cos^2\theta\sin\theta = 0$
$\Rightarrow \sin\theta = 3\cos^2\theta\sin\theta$
$\Rightarrow \sin\theta = 0$ $\quad$ ⇒ at infinity
⇒ minimum or $1 = 3\cos^2\theta$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \sqrt{2}$

$x = \frac{a/2}{\tan\theta} = \frac{a}{2\sqrt{2}}$

∴ P should be closer than $\frac{a}{2\sqrt{2}}$

17. **(1)**
Initial state is same for all three processes (say initial internal energy = E_0)
In the final state, $V_A = V_B = V_C$
and $P_A > P_B > P_C$
⇒ $P_AV_A > P_BV_B > P_CV_C$
⇒ $E_A > E_B > E_C$
if $T_1 > T_2$
then $E_0 > E_f$ for all three processes
and hence $(E_0 - E_A) < (E_0 - E_B) < (E_0 - E_C)$
$\Rightarrow |\Delta E_A| < |\Delta E_B| < |\Delta E_C|$
If $T_1 < T_2$, then $E_0 < E_f$ for all three processes
and hence $(E_A - E_0) > (E_B - E_0) > (E_C - E_0)$
$\Rightarrow |\Delta E_A| > |\Delta E_B| > |\Delta E_C|$

18. **(2)**

Magnetic field at centre (site of nucleus)

$$B = \frac{\mu_0 I}{2r} = \frac{\mu_0 qf}{2r} = \frac{\mu_0 qv}{2r \times 2\pi r}$$

$\Rightarrow B \alpha \frac{1}{r^2}$ and $B \alpha v$

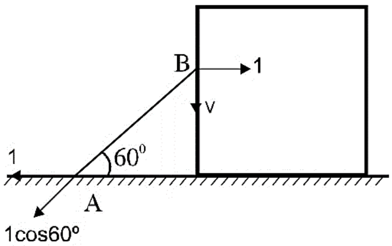
$\therefore B \alpha \frac{1}{n^5}$

$\frac{B_1}{B_2} = \frac{(2)^5}{(1)^5}$ (since, $n_1 = 1$ to $n_2 = 2$)

$\therefore B_1 = 32 B_2$

Also, $mvr = n.\frac{h}{2\pi}$ therefore angular momentum is changed by $\frac{h}{2\pi}$

19. **(4)**



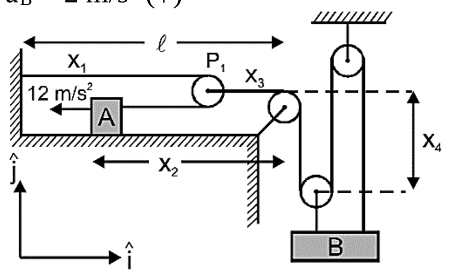
$1 \cos 60^\circ = v \cos 30^\circ - 1 \cos 60^\circ$

$v = \frac{2}{\sqrt{3}}$

$v_{net} = \sqrt{\frac{7}{3}}$

20. **(2)**

$a_B = 2$ m/s^2 ($\uparrow$)



$(l - x_3) + (x_2 - x_3) = k_1$

& $x_3 + 3x_4 = k_2$

$a_2 - 2a_3 = 0 \Rightarrow a_3 = \frac{a_2}{2} = \frac{12}{2} = 6$ m/s^2

[$a_3 \Rightarrow$ acceleration of P_1 pulley]

& $-a_3 + 3a_4 = 0$ $\Rightarrow \frac{a_3}{3} = \frac{6}{3} = 2$ m/s^2

[$a_4 \Rightarrow$ acceleration of block B]

21. **(8)**

When the structure becomes inverted, there is no decrease in the potential energy of the ring. therefore, Decrease in PE of the rod = Gain in rotational kinetic energy of the structure

$\Rightarrow$ M.g. 4R $= \frac{1}{2} I_{sy} \omega^2$

(As COM of the rod comes down by distance 4R)

Now I_{sy} =

$$\frac{MR^2}{2} + \left[\frac{M.4R^2}{12} + M.4R^2\right]$$

$$= \frac{MR^2}{2} + \frac{13MR^2}{3} = \frac{29MR^2}{6}$$

$$\omega = \sqrt{\frac{8M.gR}{\left(\frac{29}{6}\right)MR^2}} = \sqrt{\frac{48g}{29R}} = 8 \text{ rad/s}$$

22. **(3)**

Energy equivalent to III line of lyman series $= 13.6\left(\frac{1}{1^2} - \frac{1}{4^2}\right)$

So according to the question

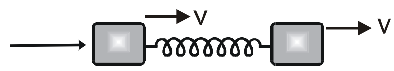
$$13.6\left(\frac{1}{1^2} - \frac{1}{4^2}\right) - \phi \geq 10.2$$

$\phi \leq 2.55$ or $\phi_{max} = 2.55$.

So the next integer is 3.

23. **($\alpha = 5$)**

$6 \times 2.2 - 3 \times 1.8 = (6 + 3)$ v



Momentum conservation

$v = \frac{7.8}{9} = \frac{13}{15}$

$$\frac{1}{2} \times 6 \times 2.2^2 + \frac{1}{2} \times 3 \times 1.8^2 = \frac{1}{2} 9 \times \left(\frac{13}{15}\right)^2 + \frac{1}{2} 512 x^2$$

$\Rightarrow x = 0.25$ $\Rightarrow \alpha = 5$

24. **(10)**

From given graphs:

$$a_x = \frac{3}{4}t \text{ and } a_y = -\left(\frac{3}{4}t + 1\right) \Rightarrow v_x = \frac{3}{8}t^2 + C$$

At $t = 0$: $v_x = -3 \Rightarrow C = -3$

$\therefore v_x = \frac{3}{8}t^2 - 3$

$\Rightarrow dx = \left(\frac{3}{8}t^2 - 3\right)dt$ …(1)

Similarly; $dy = \left(-\frac{3}{8}t^2 - t + 4\right)dt$ …(2)

As $dw = \vec{F}\cdot\vec{ds} = \vec{F}\cdot(dx\,\hat{i} + dy\,\hat{j})$

$$\therefore \int_0^W dw = \int_0^4 \left[\frac{3}{4}t\hat{i} - \left(\frac{3}{4}t+1\right)\hat{j}\right]\cdot\left[\left(\frac{3}{8}t^2-3\right)\hat{i}+\left(-\frac{3}{8}t^2-t+4\right)\hat{j}\right]dt$$

$\therefore$ **W = 10 J**

Alternate Solution:

Area of the graph;

$\int a_x\, dt = 6 = V_{(x)f} - (-3) \Rightarrow V_{(x)f} = 3.$

And $\int a_y\, dt = -10 = V_{(y)f} = (4) \Rightarrow V_{(y)f} = -6$

Now work done = Δ KE = 10 J

25. **(3)**

$$\frac{\lambda_1}{\lambda_2} = \frac{\lambda_1 N}{\lambda_2 N} = \frac{\text{decay rate of } \alpha \text{ decay}}{\text{decay rate of } \beta \text{ decay}} =$$

$$\frac{\text{probability of } \alpha \text{ decay}}{\text{probability of } \beta \text{ decay}} = \frac{\frac{75}{100}}{\frac{25}{100}} = 3$$

26. **(6)**

$$X_{CM} = \frac{\int dm.x}{\int dm} = \frac{2L}{3}$$

Mass of rod;

$$M = \int_0^L \lambda_0 x\, dx = \frac{10^3}{2}$$

Torque about 'O';

$$F_B \frac{L}{2}\cos 37° - Mg\frac{2L}{3}\cos 37° = TL\cos 37°$$

$$10^3\pi\left(\frac{1}{\pi}\right)\times 10\frac{1}{2} - \frac{10^4}{2}\times\frac{2}{3} = T$$

$$T = \frac{10^4}{6}\ \text{N}$$

27. **(1)**

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{q}{R} = \varepsilon$$

$$\frac{q}{2\pi\varepsilon_0 R} = \varepsilon$$

$\therefore$ K = 1

28. **(2)**

$$\alpha = \frac{d\omega}{dt} = 6t^2 - 2t$$

$$\int_0^{\omega} d\omega = \int_0^t \left(6t^2 - 2t\right)dt$$

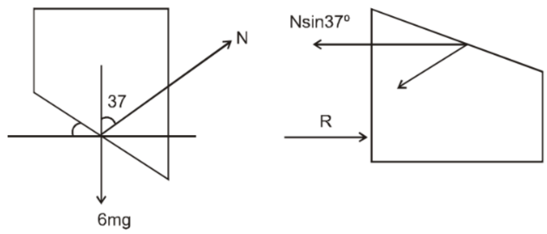
so $\omega = 2t^3 - t^2 + 10$

and $\frac{d\theta}{dt} = 2t^3 - t^2 + 10$

so $\int_4^{\theta} d\theta = \int_0^t \left(2t^3 - t^2 + 10\right)dt$

$$\theta = \frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$$

29. **(9)**



$\frac{N4}{5} = 6$ mg

$\Rightarrow 7.5$ mg

R = 7.5 mg $\frac{3}{5}$ = 4.5 mg $\frac{9}{2}$ mg

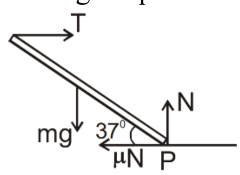
30. **(3)**

T the FBD of any one rod of

T = μN … (i)

mg = N … (ii)

Taking torque about point 'P'



mg $\frac{L}{2}$ cos 37° = TL sin 37°

$\frac{2mg}{3} = \mu mg \qquad \Rightarrow \mu = 2/3$

CHEMISTRY

31. **(2)**

(i) $S^- \rightarrow S^{-2}$

ΔH_{eg2} always positive.

(ii) $Na^+_{(g)} + Cl^-_{(g)} \rightarrow NaCl(s)$

(bond formation is an exothermic)

(iii) Addition of 1st electron in N atom is endothermic process

(iv) IP always endothermic process

32. **(3)**

33. **(1)**

34. **(3)**

$Br_2 + F^- \rightarrow Br^- + F_2$ reaction not possible

order of oxidising $\rightarrow F_2 > Cl_2 > Br_2 > I$

35. **(3)**
pH = 5
$\therefore$ [H+] = 10–5 M
Total $[H^+] = 10^{-8} + 10^{-7}$
$= 10^{-7}[10^{-1} + 1]$
$= 10^{-7}[1.1]$
$pH = -\log(1.1 \times 10^{-7})$
= 6.95

36. **(3)**

$$A \rightarrow \frac{1}{8} \times 8 = 1$$

$$B \rightarrow \frac{1}{2} \times 5 = \frac{5}{2}$$

A_2B_5

37. **(3)**

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$$

15 10 10
1R 10 – 7.5 = 2.5 mL O_2
= 2.5 mL O_2

38. **(4)**
Cation stability $\propto$ (+) I-effect.

$$\propto \frac{1}{(-)\text{I effect}}$$

[O and N shows (-) I effect in carbon chain.

39. **(3)**

40. **(4)**

41. **(3)**

42. **(1)**

43. **(1)**

44. **(3)**
In HCP, the number of octahedral void is equal to the number of atoms. Hence, the formula is $AC_{2/3}$ i.e. A_3C_2.

45. **(1)**
$k_1 = k_2$
$10^{15} e^{-25000/8.314\,T} = 10^{14} e^{-15000/8.314\,T}$

$$\frac{10^{15}}{10^{14}} = e^{10000/8.314\,T}$$

$$2.303\log_{10} 10 = \frac{10000}{8.314\,T}$$

$$T = \frac{10000}{8.314 \times 2.303} = 522\text{ K}$$

Hence the correct answer is (1).

46. **(1)**
Total volume = 500 + 1500 = 2000 ml = 2L
Moles of $[Na^+]$ ions = 0.2 × 0.5 = 0.1

Concentration of $[Na^+]$ ions $= \frac{0.1}{2} = 0.05$ M

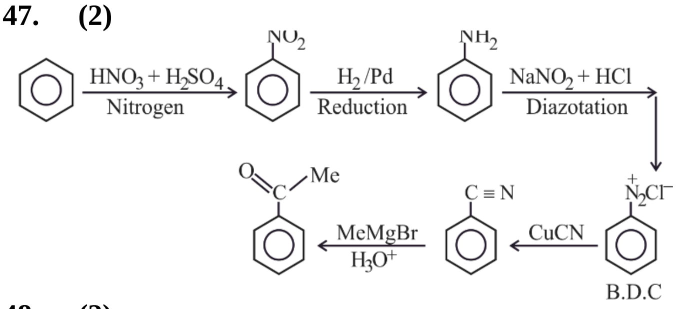
Moles of $[Mg^{2+}]$ ions = 0.4 × 1.5 = 0.6
Concentration of $[Mg^{2+}]$ ions

$$= \frac{0.6}{2} = 0.3\text{ M} = 7.2\text{ gm/L}$$

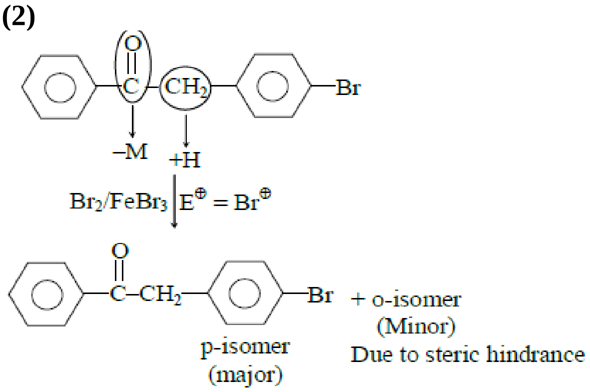
Moles of $[Cl^-]$ ions
= 0.2 × 0.5 + 2 × 0.4 × 1.5 = 1.3

Concentration of $[Cl^-]$ ions $= \frac{1.3}{2} = 0.65$ M

47. **(2)**



48. **(2)**



49. **(2)**
Morphine.

50. **(2)**

51. **(3)**

52. **(152)**

$$\frac{200-190}{200} = n_B = \frac{n_B}{n_b + \frac{624}{78}}$$

$$\frac{1}{20} = \frac{n_B}{n_B + B}$$

$$19n_B = 8 \Rightarrow n_B = \frac{8}{19}$$

Mol. wt. = 152 g

53. **(11.00)**
$CH_3NHg + HCl \longrightarrow CH_3N^{\oplus}H_3Cl^-$
0.08 0.02 0.02
0.06 –

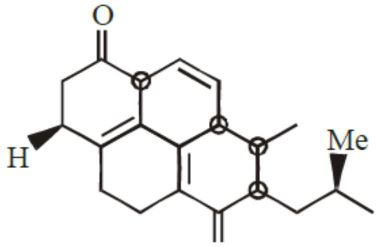
$$pOH = pK_b + \frac{\log 0.02}{0.06}$$

$$pOH = 3.48 + \log\frac{1}{3}$$

pOH = 3
pH = 14 – 3.11 = 11

54. **(5)**

55. **(4)**

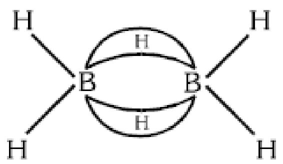


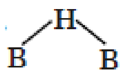
Number of chiral centre = 4

56. **(4)**

Polar ($\mu \neq 0$)	Non-polar ($\mu = 0$)
XeF_6	XeF_2, XeF_4, IF_7
ClF_3	SO_3, CO_2, CCl_4
SO_2	
$CHCl_3$	

57. **(8)**

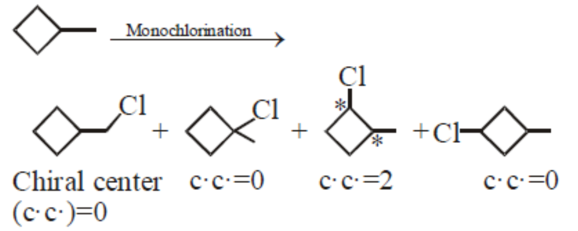


The structure of diborane B_2H_6, 

$\Rightarrow$ 3C – 2e $\Rightarrow$ 2 bonds

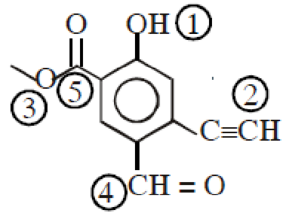
x = 2

58. **(4)**

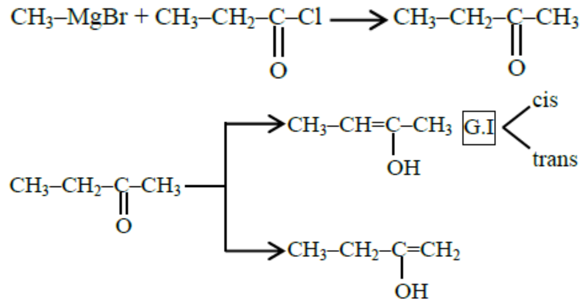


$\therefore$ optically active isomer = 4.

59. **(5)**



60. **(3)**



MATHEMATICS

61. **(1)**

$$2^{\frac{1}{4}+\frac{2}{16}+\frac{3}{48}+......\infty} = 2^{\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+......\infty} = \sqrt{2}$$

62. **(2)**

Mean $(x_1 - 5) = \dfrac{\sum(x_i - 5)}{10} = 1$

$\therefore$ λ = {mean(x_i – 5)} + 2 = 3

μ = var(x_i – 5) = $\dfrac{\sum(x_i-5)^2}{10} - \dfrac{\sum(x_i-5)}{10} = 3$

63. **(1)**

$e_1 = \sqrt{1-\dfrac{4}{18}} = \sqrt{\dfrac{7}{9}} = \dfrac{\sqrt{7}}{3}$

$e_2 = \sqrt{1+\dfrac{4}{9}} = \sqrt{\dfrac{13}{9}} = \dfrac{\sqrt{13}}{3}$

$15e_1^2 + 3e_2^2 = k \Rightarrow k = 15\left(\dfrac{7}{9}\right) + 3\left(\dfrac{13}{9}\right)$

$\therefore$ k = 16

64. **(2)**

$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix}$ = ((9 + 4) –1(3 – 4) +

2(–1 – 3)) = 13 + 1 – 8 = 6

$|adjB| = |adjadjA| = |A|^{(n-1)^2} = |A|^4 = (36)^2$

$|C| = |BA| = 3^3 \times 6$

$\dfrac{|adj\,B|}{|C|} = \dfrac{36 \times 36}{3^3 \times 6} = 8$

65. **(3)**

LHL = $a + 3$

$f(0) = b$

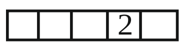
RHL = $\lim\limits_{h\to 0}\left(\dfrac{(1+3h)^{\frac{1}{3}}-1}{h}\right) = 1$

$\therefore$ $a = -2$, $b = 1$

$\therefore$ $a + 2b = 0$

66. **(4)**

Number of numbers



= 8 × 8 × 7 × 6 = 2688 = 336k $\Rightarrow$ k = 8

67. **(4)**

$AA + ABA + BAA + ABBA + BBAA + BABA$

$= \frac{1}{4}+\frac{1}{8}+\frac{1}{8}+\frac{1}{16}+\frac{1}{16}+\frac{1}{16}=\frac{11}{16}$

68. **(4)**

Let $e^x = t \in (0, \infty)$

Given equation $t^4 + t^3 - 4t^2 + t + 1 = 0$

$t^2 + t - 4 + \frac{1}{t}+\frac{1}{t^2}=0$

$\left(t^2+\frac{1}{t^2}\right)+\left(t+\frac{1}{t}\right)-4=0$

Let $t+\frac{1}{t}=\alpha$

$\left(\alpha^2-2\right)+\alpha-4=0$

$\alpha^2 + \alpha - 6 = 0$

$\alpha = -3, 2$

$\Rightarrow \alpha = 2$

$\Rightarrow e^x + e^{-x} = 2$

$x = 0$ only solution

69. **(3)**

Let thickness = x cm

Total volume $v = \frac{4}{3}\pi(10+x)^3$

$\frac{dv}{dt}=4\pi(10+x)^2\frac{dx}{dt}$ ....(i)

Given, $\frac{dv}{dt}$ = 50 cm^3/min

At $x = 5$ cm

$50 = 4\pi(10+5)^2\frac{dx}{dt}$

$\frac{dx}{dt}=\frac{1}{18\pi}$ cm/min

70. **(3)**

$I=\int_0^{\pi}\left\{\frac{x\sin^8 x}{\sin^8 x+\cos^8 x}+\frac{(2\pi-x)\sin_x^8}{\sin^8 x+\cos^8 x}\right\}dx$

$= \int_0^{\pi}\frac{2\pi\sin^8 x}{\sin^8 x+\cos^8 x}dx$

$= 2\pi\int_0^{\pi/2}\left\{\frac{\sin^8 x}{\sin^8 x+\cos^8 x}+\frac{\cos^8 x}{\sin^8 x+\cos^8 x}\right\}$

$= 2\pi\int_0^{\pi/2}1\,dx = 2\pi\times\frac{\pi}{2}=\pi^2$

71. **(4)**

$x^2 + (y-1)^2 = x^2 + (y+2)^2$

$-2y + 1 = 4y + 4$

$6y = -3 \Rightarrow y = -\frac{1}{2}$

$x^2 + y^2 = \frac{24}{4} \Rightarrow x^2 = \frac{24}{4}=6$

$\Rightarrow z = \pm\sqrt{6}-\frac{i}{2}$

$|z + 3i| = \sqrt{6+\frac{24}{4}}=\sqrt{\frac{49}{4}}$

$|z + 3i| = \frac{7}{2}$

72. **(3)**

$\sqrt{5}$ is not an integer and 5 is not an irrational number $\sim(p \vee q) = \sim p \wedge \sim q$

73. **(2)**

$D(2, 2)$

Point of intersection $P\left(-\frac{1}{5},\frac{2}{5}\right)$ equation of line DP

$\Rightarrow 8x - 11y + 6 = 0$

74. **(3)**

$\Delta = 0 \Rightarrow \begin{vmatrix}1 & 4 & -2\\ 1 & 7 & -5\\ 1 & 5 & \alpha\end{vmatrix}=0$

$(7\alpha + 25) - (4\alpha + 10) + (-20 + 14) = 0$

$3\alpha + 9 = 0 \Rightarrow \alpha = -3$

Also $D_z = 0 \Rightarrow \begin{vmatrix}1 & 4 & 1\\ 1 & 7 & \beta\\ 1 & 5 & 5\end{vmatrix}=0$

$1(35 - 5\beta) - (15) + 1(4\beta - 7) = 0$

$\beta = 13$

75. **(1)**

Let use LMVT for $x \in [a, c]$

$\frac{f(c)-f(a)}{c-a} = f'(\alpha),\alpha\in(a,c)$

Also use LMVT for $x \in [c, b]$

$\frac{f(b)-f(c)}{b-c}=f'(\beta),b\in(c,b)$

$\because f''(x)<0 \Rightarrow f'(x)$ is decreasing

$f'(\alpha)>f'(\beta)$

$\frac{f(c)-f(a)}{c-a}>\frac{f(b)-f(c)}{b-c}$

$\frac{f(c)-f(a)}{f(b)-f(c)}>\frac{c-a}{b-c}$ ($\because$ $f(x)$ is increasing)

76. **(4)**

$$\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8}-3\cos\frac{\pi}{8}\right]+\sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8}-4\sin^3\frac{\pi}{8}\right]$$

$$= 4\cos^6\frac{\pi}{8}-4\sin^6\frac{\pi}{8}-3\cos^4\frac{\pi}{8}+3\sin^4\frac{\pi}{8}$$

$$= 4\left[\left(\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}\right)\right]\left[\left(\sin^4\frac{\pi}{8}+\cos^4\frac{\pi}{8}+\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)\right]$$

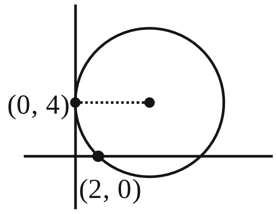
$$-3\left[\left(\cos^2\frac{\pi}{8}-\sin^2\frac{\pi}{8}\right)\right]$$

$$= \cos\frac{\pi}{4}\left[4\left(1-\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)-3\right]$$

$$= \frac{1}{\sqrt{2}}\left[1-\frac{1}{2}\right]=\frac{1}{2\sqrt{2}}$$

77. **(4)**

Equation of family of circle



$(x-0)^2 + (y-4)^2 + \lambda x = 0$

$\Rightarrow$ Passes (2, 0)

$4 + 16 + 2\lambda = 0 \Rightarrow \lambda = -10$

$x^2 + y^2 - 10x - 8y + 16 = 0$

centre (5, 4). and $R = \sqrt{25+16-16} = 5$

Check the options,

Option (D)

$$\left|\frac{4\times5+3\times4-8}{5}\right|=\frac{24}{5}\neq5$$

78. **(2)**

$$\int_0^1\left(a+bx+cx^2\right)dx = ax+\frac{bx^2}{2}+\frac{cx^3}{3}\Big|_0^1$$

$$= a+\frac{b}{2}+\frac{c}{3}$$

$f(1) = a + b + c \quad \Rightarrow \quad f(0) = a$

$$f\left(\frac{1}{2}\right)=a+\frac{b}{2}+\frac{c}{4}$$

Now $\frac{1}{6}\left(f(1)+f(0)+4f\left(\frac{1}{2}\right)\right)$

$$=\frac{1}{6}\left(a+b+c+a+4\left(a+\frac{b}{2}+\frac{c}{4}\right)\right)$$

$$=\frac{1}{6}(6a+3b+2c)=a+\frac{b}{2}+\frac{c}{3}$$

79. **(2)**

$$f'(x)=\tan^{-1}(\sec x+\tan x)=\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}+x\right)}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right) = \frac{\pi}{4}+\frac{x}{2}$$

$$\int\left(f'(x)\right)dx=\int\left\{\frac{\pi}{4}+\frac{x}{2}dx\right\}$$

Integrating both side

$$f(x)=\frac{\pi}{4}x+\frac{x^2}{4}+c$$

$f(0) = c = 0 \Rightarrow f(x) = \frac{\pi}{4}x+\frac{x^2}{4}$

So, $f(1) = \frac{\pi+1}{4}$

80. **(3)**

$$\int\left(\frac{x-3}{x+4}\right)^{\frac{-6}{7}}\frac{1}{(x+4)^2}dx$$

Let $\frac{x-3}{x+4}=t^7\Rightarrow\frac{7}{(x+4)^2}dx=7t^6dt$

$$\int t^{-6}t^6dt=t+c$$

81. **(8)**

Let $A(1, -1, 3)$, $B(2, -4, 11)$

$\overrightarrow{AB}=\hat{i}-3j+8k$

$C(-1, 2, 3)$, $D(3, -2, 10)$

$\overrightarrow{CD}=4\hat{i}-4j+7k$

Projection of $\overrightarrow{AB}$ on $\overrightarrow{CD}$ = $\frac{\overrightarrow{AB}.\overrightarrow{CD}}{\left|\overrightarrow{CD}\right|}$

$$=\left(\frac{4+12+56}{\sqrt{16+16+49}}\right)=\frac{72}{9}=8$$

82. **(1)**

$$\begin{vmatrix}a+1 & a & a\\ a & a+1 & a\\ a & a & a+1\end{vmatrix}=0$$

$\Rightarrow a + 1 + a + a = 0 \Rightarrow a=-\frac{1}{3}$

$\vec{P}=\frac{2}{3}\hat{i}-\frac{1}{3}j-\frac{1}{3}k$

$\vec{Q}=\frac{1}{3}\left(-\hat{i}+2j-k\right)$

$\vec{R}=\frac{1}{3}\left(-\hat{i}-j+2k\right)$

$\vec{P}.\vec{Q}=\frac{1}{9}\begin{vmatrix} i & j & k \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{vmatrix}$

$=\frac{1}{9}\left(i(4-1)-j(-2-1)+k(1+2)\right)$

$=\frac{1}{9}(3i+3j+3k)=\frac{i+j+k}{3}$

$\left|\vec{R}\times\vec{Q}\right|=\frac{1}{3}\sqrt{3}\Rightarrow \left|\vec{R}\times\vec{Q}\right|^2=\frac{1}{3}$

$3\left(\vec{P}\times\vec{Q}\right)^2-\lambda\left|\vec{R}\times\vec{Q}\right|^2=0$

$3.\frac{1}{9}-\lambda.\frac{1}{3}=0 \Rightarrow \lambda = 1$

83. (3)

$\frac{dy}{dx}=(1+x)+\left(\frac{y-3}{1+x}\right)$

$\frac{dy}{dx}-\frac{1}{(1+x)}y=(1+x)-\frac{3}{(1+x)}$

I.F. $= e^{-\int\frac{1}{(1+x)}dx}=\frac{1}{(1+x)}$

$\therefore \frac{d}{dx}\left(\frac{y}{1+x}\right)=1-\frac{3}{(1+x)^2}$

$\frac{y}{1+x}=x+3(1+x)^{-1}+c$

$y=(1+x)\left[x+\frac{3}{(1+x)}+c\right]$

$\therefore$ at $x = 2$, $y = 0$

$\therefore$ $0 = 3(2 + 1 + c) \Rightarrow c = -3$

$\therefore$ at $x = 3$, $y = 3$

84. (615)

General term $\frac{10!}{\alpha!\beta!\gamma!}x^{\beta+2\gamma}$

For coefficient of $x^4 \Rightarrow \beta + 2\gamma = 4$

$\gamma = 0, \beta = 4, \alpha = 6 \Rightarrow \frac{10!}{6!4!0!}=210$

$\gamma = 1, \beta = 2, \alpha = 7 \Rightarrow \frac{10!}{7!2!1!}=360$

$\gamma = 2, \beta = 0, \alpha = 8 \Rightarrow \frac{10!}{8!0!2!}=45$

Total = 615

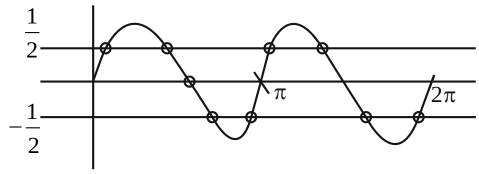
85. (8)

$\log_{1/2}|\sin x| = 2-\log_{1/2}|\cos x|$

$\log_{1/2}|\sin x\cos x| = 2$

$|\sin x\cos x| = \frac{1}{4}$

$\sin 2x = \pm\frac{1}{2}$



Number of solution = 8.

86. (23)

$\sin\theta + \cos\theta = \frac{1}{2}$

$\therefore \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{1}{4}$

Now, $\cos 4\theta = 1-2\sin^2 2\theta = 1-2\left(-\frac{3}{4}\right)^2=-\frac{1}{8}$

Also, $\sin 6\theta = 3\sin 2\theta - 4\sin^2 2\theta$

$= 3\left(-\frac{3}{4}\right)-4\left(-\frac{3}{4}\right)^3=-\frac{9}{16}$

$\therefore 16[\sin 2\theta + \cos 4\theta + \sin 6\theta] =$

$16\left(-\frac{3}{4}-\frac{1}{8}-\frac{9}{16}\right)=-23$

87. (108)

$A=\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$\therefore |A| = A$

$\left|3adj\left(2A^{-1}\right)\right| = 3^3\left|adj\left(2A^{-1}\right)\right|$

$= 3^3\left|2A^{-1}\right|^2 = 3^3\left(2^3\left|A^{-1}\right|\right)^2$

$= \frac{3^3 2^6}{|A|^2} = \frac{3^3 2^6}{4^2} = 108$

88. (49)

$A_k = \sum_{i=0}^{9}{}^9C_i\,{}^{12}C_{k-i}+\sum_{i=0}^{8}{}^8C_i\,{}^{13}C_{k-i}$

$= {}^{12}C_k + {}^{21}C_k = 2.{}^{21}C_k$

$\therefore A_4 - A_3 = 2\left({}^{21}C_4 - {}^{21}C_3\right)$

$= 2(5985 - 1330) = 190\,p$

$\Rightarrow p = 49$

89. **(2021)**

$$\therefore \sum_{k=1}^{10} c_k = \sum_{k=1}^{10}(a_k + b_k) = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$$

$$= \frac{10}{2}(2\times 11 + 9\times(-3)) + \frac{2\left(2^{10}-1\right)}{2-1}$$

$$= 5(22-27) + 2(1023)$$

$$= 2046 - 25 = 2021$$

90. **(13)**

$z = \frac{1-i\sqrt{3}}{2} = -\omega$, where ω is imaginary cube root of unity

$$\left(z^r + \frac{1}{z^r}\right)^3 = z^{3r} + \frac{1}{z^{3r}} + 3z^r + \frac{3}{z^r}$$

$$= (-\omega)^{3r} + \frac{1}{(-\omega)^{3r}} + 3(-\omega)^r + \frac{3}{(-\omega)^r}$$

$$= (-1)^r + \frac{1}{(-1)^r} + 3(-\omega)^r + \frac{3}{(-\omega)^r}$$

$$\therefore\ 21 + \sum_{r=1}^{21}\left(z^r + \frac{1}{z^r}\right)^3$$

$$= 21 + \sum_{r=1}^{21}\left[(-1)^r + \frac{1}{(-1)^r} + 3(-\omega)^r + \frac{3}{(-\omega)^r}\right]$$

$$= 21 + \left[-1-1+3\frac{(\omega)\left(1-(-\omega)^{21}\right)}{1+\omega} + 3\frac{\left(-\frac{1}{\omega}\right)\left(1-\left(-\frac{1}{\omega}\right)^{21}\right)}{1+\frac{1}{\omega}}\right]$$

$$= 21 + \left[-2 + 3\frac{(-\omega)(1+1)}{-\omega^2} - 3\frac{(1+1)}{-\omega^2}\right]$$

$$= 21 + \left[-2 + 6\left(\frac{1}{\omega} + \frac{1}{\omega^2}\right)\right] = 21 + [-2 + 6(-1)] = 13$$