NCERT Solutions for Class 10 Maths Chapter 5 Exercise 5.4: NCERT Solutions for Class 10 Maths Chapter 5 Exercise 5.4 focus on practical applications of Arithmetic Progressions (AP). This exercise emphasizes solving real-life problems using AP concepts such as finding missing terms, the nth term, and the sum of n terms. It covers various scenarios, including profit calculations, ticket pricing, and patterns in sequences, making the concepts relatable and easy to understand.

Designed to enhance problem-solving skills, the solutions are step-by-step and aligned with the NCERT guidelines, ensuring conceptual clarity. This helps students strengthen their understanding of arithmetic progressions and apply them effectively in exams and daily scenarios.

NCERT Solutions for Class 10 Maths Chapter 5 Exercise 5.4 Overview

NCERT Solutions for Class 10 Maths Chapter 5 Exercise 5.4 on Arithmetic Progressions are vital for building a strong mathematical foundation. This exercise deals with real-life applications of arithmetic sequences, such as calculating the nth term or the sum of terms in a sequence, which are crucial for competitive exams and higher studies. Understanding APs helps develop logical thinking and problem-solving skills, as they appear frequently in physics, economics, and everyday scenarios like budgeting or planning.

These solutions provide clear, step-by-step explanations, ensuring students grasp concepts thoroughly and improve their accuracy and confidence in solving AP-related problems effectively.

NCERT Solutions for Class 10 Maths Chapter 5 Exercise 5.4 Arithmetic Progressions

Below is the NCERT Solutions for Class 10 Maths Chapter 5 Exercise 5.4 Arithmetic Progressions -

1. Which term of the AP: 121, 117, 113, . . ., is its first negative term? [Hint: Find n for $a_n < 0$]

Solution:

Given, the AP series is 121, 117, 113, . . .,

Thus, the first term, a = 121

The common difference, d = 117-121= -4

By the nth term formula,

$a_n = a+(n-1)d$

Therefore,

$a_n = 121+(n-1)(-4)$

= 121-4n+4

=125-4n

To find the first negative term of the series, $a_n < 0$

Therefore,

125-4n < 0

125 < 4n

n>125/4

n>31.25

Therefore, the first negative term of the series is the 32^{nd} term.

2. The sum of the third and the seventh terms of an AP is 6, and their product is 8. Find the sum of the first sixteen terms of the AP.

Solution:

From the given statements, we can write

$a_3 + a_7 = 6$ ……………………………(i)

And

$a_3 \times a_7 = 8$ ……………………………..(ii)

By the nth term formula,

$a_n = a+(n-1)d$

Third term, $a_3 = a+(3-1)d$

$a_3 = a + 2d$………………………………(iii)

And Seventh term, $a7= a+(7-1)d$

$a_7 = a + 6d$ ………………………………..(iv)

From equations (iii) and (iv), putting in equation(i), we get

$a+2d +a+6d = 6$

$2a+8d = 6$

$a+4d=3$

or

$a = 3–4d$ …………………………………(v)

Again, putting the eq.(iii) and (iv) in eq. (ii), we get

$(a+2d)×(a+6d) = 8$

Putting the value of a from equation (v), we get

$(3–4d +2d)×(3–4d+6d) = 8$

$(3 –2d)×(3+2d) = 8$

$3^2 – 2d^2 = 8$

$9 – 4d^2 = 8$

$4d^2 = 1$

$d = 1/2$ or $-1/2$

Now, by putting both the values of d, we get

$a = 3 – 4d = 3 – 4(1/2) = 3 – 2 = 1$, when $d = 1/2$

$a = 3 – 4d = 3 – 4(-1/2) = 3+2 = 5$, when $d = -1/2$

We know that the sum of the nth term of AP is

$S_n = n/2\ [2a +(n – 1)d]$

So, when a = 1 and d=1/2

Then, the sum of the first 16 terms is

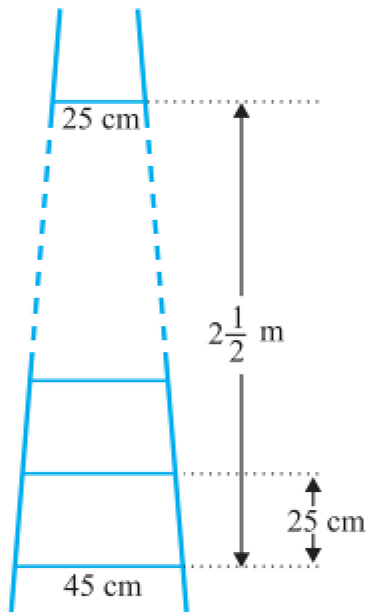
S_{16} = 16/2 [2 +(16-1)1/2] = 8(2+15/2) = 76

And when a = 5 and d= -1/2

Then, the sum of the first 16 terms is

S_{16} = 16/2 [2(5)+(16-1)(-1/2)] = 8(5/2)=20

3. A ladder has rungs 25 cm apart (see Fig. 5.7). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and the bottom rungs are $2\frac{1}{2}$m apart, what is the length of the wood required for the rungs? [Hint: Number of rungs = -250/25].



Solution:

Given,

The distance between the rungs of the ladder is 25cm.

Distance between the top rung and bottom rung of the ladder is =

$2\frac{1}{2}\text{m} = 2\frac{1}{2} \times 100 \text{ cm}$ = 5/2 ×100cm

= 250cm

Therefore, total number of rungs = 250/25 + 1 = 11

As we can see from the figure, the ladder has rungs in decreasing order from top to bottom. Thus, we can conclude that the rungs are decreasing in the order of AP.

And the length of the wood required for the rungs will be equal to the sum of the terms of the AP series formed.

So,

The first term, a = 45

The last term, l = 25

Number of terms, n = 11

Now, as we know, the sum of nth terms is equal to

S_n= n/2(a+ l)

S_n= 11/2(45+25) = 11/2(70) = 385 cm

Hence, the length of the wood required for the rungs is 385cm.

4. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x, such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find this value of x. [Hint :Sx – 1 = S49 – Sx]

Solution:

Given,

Row houses are numbers from 1,2,3,4,5…….49.

Thus, we can see the houses numbered in a row are in the form of AP.

So,

The first term, a = 1

The common difference, d=1

Let us say the number of the houses can be represented as

Sum of nth term of AP = n/2[2a+(n-1)d]

Sum of number of houses beyond x house = S_{x-1}

= (x-1)/2[2(1)+(x-1-1)1]

= (x-1)/2 [2+x-2]

= x(x-1)/2 ………………………………………(i)

By the given condition, we can write

$S_{49} – S_x$ = {49/2[2(1)+(49-1)1]}–{x/2[2(1)+(x-1)1]}

= 25(49) – x(x + 1)/2 ………………………………….(ii)

As per the given condition, eq.(i) and eq(ii) are equal to each other.

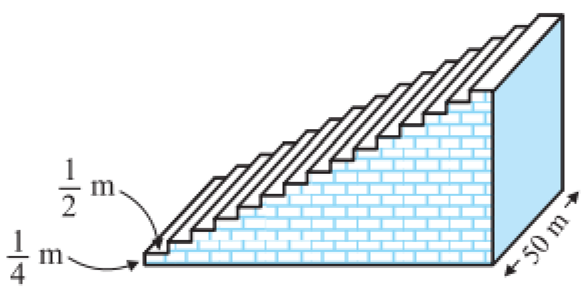
Therefore,

x(x-1)/2 = 25(49) – x(x+1)/2

x = ±35

As we know, the number of houses cannot be a negative number. Hence, the value of x is 35.

5. A small terrace at a football ground comprises of 15 steps, each of which is 50 m long and built of solid concrete. Each step has a rise of 1 4 m and a tread of 1 2 m (see Fig. 5.8). Calculate the total volume of concrete required to build the terrace. [Hint: Volume of concrete required to build the first step = ¼ ×1/2 ×50 m^3.]



Solution:

As we can see from the given figure, the first step is ½ m wide, 2nd step is 1m wide and 3rd step is 3/2m wide. Thus, we can understand that the width of the step by ½ m each time when the height is ¼ m. And also, given the length of the steps is 50m the time. So, the width of steps forms a series AP in such a way that

½ , 1, 3/2, 2, ……..

The volume of steps = Volume of Cuboid

= Length × Breadth Height

Now,

The volume of concrete required to build the first step = ¼ ×1/2 ×50 = 25/4

The volume of concrete required to build the second step =¼ ×1×50 = 25/2

The volume of concrete required to build the second step = ¼ ×3/2 ×50 = 75/4

Now, we can see the volumes of concrete required to build the steps are in the AP series.

25/4 , 25/2 , 75/4 …..

Thus, applying the AP series concept,

The first term, a = 25/4

The common difference, d = 25/2 – 25/4 = 25/4

As we know, the sum of n terms is

S_n = n/2[2a+(n-1)d] = 15/2(2×(25/4)+(15/2 -1)25/4)

Upon solving, we get

$S_{n=}$ 15/2 (100)

S_n=750

Hence, the total volume of concrete required to build the terrace is 750 m^3.

Benefits of Solving NCERT Solutions for Class 10 Maths Chapter 5 Exercise 5.4

Comprehensive Understanding: Provides step-by-step explanations, ensuring conceptual clarity in arithmetic progressions.

Exam-Ready Preparation: Solutions are aligned with NCERT guidelines, helping students excel in board exams.

Real-Life Applications: Explains AP concepts with practical examples, enhancing problem-solving skills.

Saves Time: Offers accurate, well-structured answers to difficult problems, aiding efficient study.

Boosts Confidence: Strengthens fundamentals, helping students tackle related competitive exams.

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