JEE Mains (Dropper)

Sample Paper - I

DURATION : 180 Minutes **M. MARKS : 300**

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(2)	31.	(1)	61.	(4)
2.	(3)	32.	(2)	62.	(3)
3.	(2)	33.	(3)	63.	(3)
4.	(2)	34.	(3)	64.	(4)
5.	(1)	35.	(4)	65.	(2)
6.	(3)	36.	(1)	66.	(3)
7.	(2)	37.	(3)	67.	(2)
8.	(4)	38.	(3)	68.	(3)
9.	(3)	39.	(3)	69.	(1)
10.	(4)	40.	(1)	70.	(2)
11.	(1)	41.	(3)	71.	(4)
12.	(1)	42.	(2)	72.	(3)
13.	(1)	43.	(3)	73.	(4)
14.	(3)	44.	(2)	74.	(2)
15.	(1)	45.	(4)	75.	(4)
16.	(2)	46.	(4)	76.	(3)
17.	(3)	47.	(4)	77.	(4)
18.	(3)	48.	(2)	78.	(2)
19.	(2)	49.	(2)	79.	(3)
20.	(1)	50.	(1)	80.	(2)
21.	(2)	51.	(144)	81.	(96)
22.	(5)	52.	(150)	82.	(0)
23.	(3)	53.	(5)	83.	(3)
24.	(7)	54.	(7)	84.	(780)
25.	(6)	55.	(5)	85.	(13)
26.	(6)	56.	(4)	86.	(27)
27.	(5)	57.	(2)	87.	(192)
28.	(2)	58.	(5)	88.	(219)
29.	(5)	59.	(8)	89.	(1)
30.	(5)	60.	(6)	90.	(3)

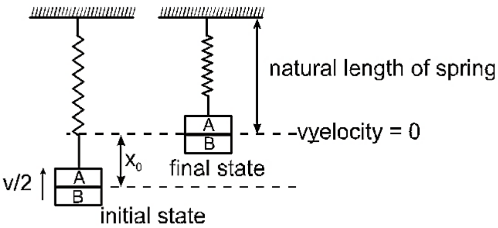
PHYSICS

1. **(2)**

The initial extension in spring is $x_0 = \frac{mg}{k}$

Just after collision of B with A the speed of combined mass is $\frac{v}{2}$.

For the spring to just attain natural length the combined mass must rise up by $x_0 = \frac{mg}{k}$ (sec fig.) and comes to rest.



Applying conservation of energy between initial and final states

$$\frac{1}{2}2m\left(\frac{v}{2}\right)^2 + \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = 2mg\left(\frac{mg}{k}\right)$$

Solving we get v = $\sqrt{\frac{6mg^2}{k}}$

Alternative solution by SHM

$$\frac{v}{2} = \sqrt{\frac{k}{2m}}\sqrt{\left(\frac{2mg}{k}\right)^2 - \left(\frac{mg}{k}\right)^2};$$

$$v = \sqrt{\frac{2k}{m}}\sqrt{\frac{3m^2g^2}{k^2}} \qquad = \sqrt{\frac{6mg^2}{k}}$$

2. **(3)**

$$\frac{1}{F_{lens}} = (1.5-1)\left[\frac{1}{-40} - \frac{1}{-20}\right] = \frac{1}{80}$$

$$F_l = 80\text{cm}$$

$$F_m = -\frac{20}{2} = -10\text{cm}$$

$$\frac{1}{F_{eq}} = \frac{1}{f_m} - \frac{2}{f_l} \quad = \frac{1}{-10} - \frac{2}{80}$$

f_{eq} = – 8 cm

Hence object should be placed at x = 16 cm, i.e. at the centre of curvature.

3. **(2)**

Averaging a number of readings makes the measurement more precise.

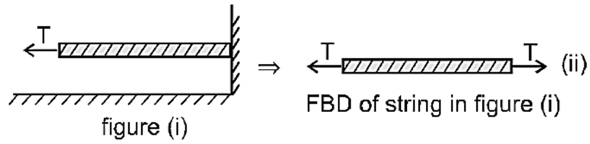
4. **(2)**

$x = x_1$ and $x = x_3$ are not equilibrium positions because $\frac{dU}{dx} \neq 0$ at these points.

$x = x_2$ is unstable, as U is maximum at this point.

5. **(1)**

Tension in both string shall be same which can be observed by making FBD of string in figure (1)



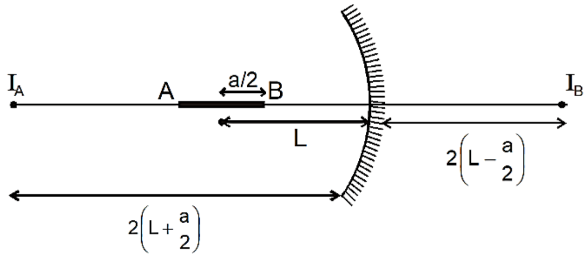
6. **(3)**

Let the length of rod be ‘a’. The magnitude of transverse magnification of ends A and B is 2 each. The image of B is virtual of A is real.

Applying mirror formula to B and A

$$\frac{1}{2\left(L-\frac{a}{2}\right)} - \frac{1}{\left(L-\frac{a}{2}\right)} = -\frac{1}{f} \qquad ...(1)$$

$$-\frac{1}{2\left(L+\frac{a}{2}\right)} - \frac{1}{\left(L+\frac{a}{2}\right)} = -\frac{1}{f} \qquad ...(2)$$



Solving we get $a = L$.

7. **(2)**

The slope of temperature variation is more in inner

$$\frac{dQ}{dt} = \frac{kA}{l}.\Delta T$$

$$\Delta T = \frac{l}{KA}.\frac{dQ}{dt}$$

Slope α $\frac{1}{K}$

Larger the conductivity, smaller is the slope.

8. **(4)**

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - \phi$$

$$\frac{1}{2}m'^2 = \frac{hc}{(3\lambda/4)} - \phi = \frac{4hc}{3\lambda} - \phi$$

Clearly $v' > \sqrt{\frac{4}{3}}v$

9. **(3)**

There can be three minima from central point to ∞ corresponding to $\frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$ path differences.

∴ total number of minima = $2n_{max}$ = 6

10. **(4)**

Law of equipartition of energy

$<KE_R> = 2 \cdot \frac{1}{2}$ kT = E

$\therefore <KE_T> = 3 \cdot \frac{1}{2}$ kT

$<KE_T> = \frac{3E}{2}$

11. **(1)**

$KE_{A/CM} = \frac{1}{2}.1.(v_{A/CM})^2 = 2$ Joules

$\Rightarrow$ $V_{A/CM} = 2$ m/s.

Let ; COM move towards +ve *x*-direction.

Then, $\vec{v}_{A/CM} = 2\hat{i}$

$\Rightarrow$ $v_{B/CM} = -\hat{i}$ (Use ; $\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$)

$\therefore$ $KE_{System} = \frac{1}{2}.1.\left(2\hat{i} + \vec{v}_{CM}\right)^2 + \frac{1}{2}.2\left(-\hat{i} + \vec{v}_{CM}\right)^2$

$= \frac{1}{2}\left[4 + v_{CM}^2 + 2.2\hat{i}.\vec{v}_{CM}\right] + \frac{1}{2}.2.\left[1 + v_{CM}^2 - 2.\hat{i}.\vec{v}_{CM}\right]$

$= (2 + 2 + 2\hat{i}.\vec{v}_{CM}) + (1 + 4 - 2\hat{i}.\vec{v}_{CM})$

= 9 J

12. **(1)**

Focal length is minimum in case I, therefore power is maximum.

13. **(1)**

Least count = $1 - \frac{8}{10}$ = 0.2 mm

Length = 4 + 5 × 0.2 = 5.0 mm

= 5.0×10^{-3} m.

14. **(3)**

$\vec{f} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j}$

$= -[6\hat{i}] + [8\hat{j}] = -6\hat{i} + 8\hat{j}$

$\therefore$ $\vec{a} = -3\hat{i} + 4\hat{j}$ has same direction as that of

$\vec{u} = \frac{-3\hat{i} + 4\hat{j}}{2} = \left(\frac{\vec{a}}{2}\right)$

$|\vec{a}| = 5$, $|\vec{u}| = 5/2$

Since $\vec{u}$ and $\vec{a}$ are in same direction, particle will move along a straight line

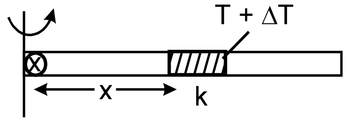
$\therefore S = \frac{5}{2} \times 2 + \frac{1}{2} \times 5 \times 2^2 = 5 + 10 = 15$ m.

15 m. Ans

15. **(1)**

$-\int_T^0 \Delta T = \int_0^\ell \frac{m}{\ell} dx\omega^2 x \Rightarrow T = \frac{m}{\ell}\omega^2 \frac{x^2}{2}$

$\Rightarrow Y = \frac{F\ell}{A\Delta\ell}$ $\Delta\ell = \frac{F\ell}{Ay}$



$\Delta\ell = \frac{\frac{m}{\ell}\frac{\omega^2 x^2}{2}dx}{AY}$

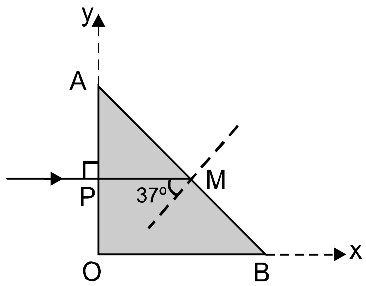
$\Delta\ell = \frac{m}{\ell}\frac{\omega^2\ell^3}{6AY}$

$\Delta\ell = \frac{\rho\omega^2\ell^3}{6y}$

$\Delta\ell = \omega^2$

$\omega^2 = 2\omega_1$

16. **(2)**



Clearly, $PM = \frac{3}{2}$ cm

$37^\circ > \sin^{-1}\frac{1}{n_0 + a(3/2)}$

$\frac{3}{5} > \frac{1}{n_0 + \frac{3a}{2}}$

$3n_0 + \frac{9a}{2} > 5$

$\frac{9a}{2} > 1$

$a > \frac{2}{9}$

17. **(3)**

$P = Ae\sigma T^4$

$2 = 2 \times 10^{-6} \times 0.9 \times 5.6 \times 10^{-8} \times T^4$

$T^4 = \frac{10^{14}}{0.9x5.6}$

$T = 2110\ k$

18. **(3)**

As the accelerating potential difference is changed only the minimum wavelength changes.

It has no effect on wavelengths of characteristic *x*-rays (whether they are produced or not)

$\therefore$ (3) is the correct choice.

19. **(2)**

$\Delta = 2\mu t$

$= 2 \times \frac{4}{3} \times 1 = \frac{8}{3}$

There are two phase reversal of π. One at the air water surface 8 another at the water-ground surface.

$\therefore$ condition for maximum interference will be $\Delta = n\lambda$

$\frac{8}{3} = n\lambda$

$\lambda = \frac{8}{3n}$ $\qquad$ $\lambda_{max} = \frac{8}{3}$ m

20. **(1)**

In an adiabatic expansion, internal energy decreases and hence temperature decreases.

$\therefore$ from equation of state of ideal gas

$PV = nRT$

$\Rightarrow$ The product of P and V decreases.

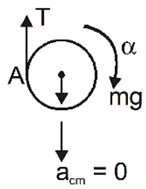
21. **(2)**

$a_A = a = \alpha \cdot R$ ...(i)

$T - mg = 0$ ...(ii)

$T.R = \frac{mR^2}{2}.\alpha$ ...(iii)

$\therefore$ $g = \frac{a}{2}$



22. **(5)**

Area perpendicular to the light = 1 × cos 60°

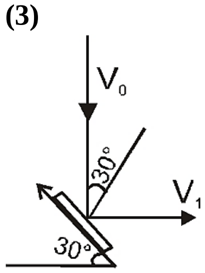
Energy falling on the surface = intesity × perp.area =

$3 \times 1 \times \cos 60° = \frac{3}{2}$ watt

Momentum carried by the light per sec

$= 3/(2c) = 5 \times 10^{-9}$

23. **(3)**



$V_0 \sin 30° = V_1 \cos 30°$ (i)

$eV_0 \cos 30° = V_1 \sin 30°$ (ii)

Dividing (i) & (ii)

$\frac{1}{e}\tan 30° = \frac{1}{\tan 30°}$ $\qquad$ $\frac{1}{e} = 3$

24. **(7)**

$W = \int \vec{F}.d\vec{r}$

$= \int (ydx + xydy)$ $\qquad$ $\because 2x = 3y$

$= \int_0^2 \left(\frac{3}{2}ydy + \frac{3}{2}y^2dy\right)$ $\qquad$ $\therefore dx = \frac{3}{2}dy$

$= \left[\frac{3}{4}y^2 + \frac{y^3}{2}\right]_0^2 = 7$ joule

25. **(6)**

$\frac{A_1}{A_0} = \left(\frac{1}{2}\right)^{t/\tau}$

$\Rightarrow \frac{100/20}{141/20} = \frac{1}{\sqrt{2}} = \left(\frac{1}{2}\right)^{t/\tau}$

$\Rightarrow \frac{t}{\tau} = \frac{1}{2}$

$\Rightarrow \tau = 2t = 2 \times 3$ days = 6 days

26. **(6)**

Minimum velocity required at $D \frac{mv^2}{R-r} = mg$

$\Rightarrow$ $v = \sqrt{g(R-r)}$

Energy conservation between *A* and *D*

$mg(h - 2R + r) = \frac{1}{2}mv2 + \frac{mr^2}{2} \times \frac{v^2}{r^2}$

$g(h - 2R + r) = \frac{3}{4}g(R - r)$

$R = \frac{4h + 7r}{11} = \frac{52 + 14}{11} = 6$ cm.

27. **(5)**

By moseley's law,

$\sqrt{\upsilon} = a(z - b)$

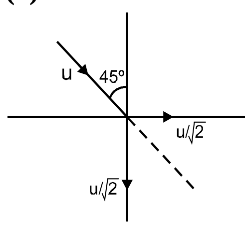
$\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{z_1 - 1}{z_2 - 1}$

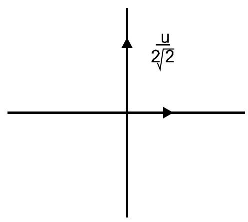
$2 = \frac{z_1 - 1}{z_2 - 1}$

$2 = \frac{36}{z_2 - 1}$ $\qquad$ $z_2 - 1 = 18$

$z_2 = 19 = 95/n$ $\qquad$ $\Rightarrow n = 5$

28. **(2)**





$$Ndt = \frac{mu}{2\sqrt{2}} + \frac{mu}{\sqrt{2}} \quad \text{.............(i)}$$

$$mNdt = m.\frac{u}{\sqrt{2}} \quad \text{.............(ii)}$$

(i) & (ii)

$$m = \frac{2}{3}$$

29. **(5)**

.$\varepsilon_i = \varepsilon_f$

$$mg\frac{L}{2}\sin\theta + 0 = m_1 g\frac{(L-x)}{2}\sin\theta - mg\frac{x}{2} + \frac{1}{2}mv^2$$

$$mg\frac{L}{2}\sin\theta = \frac{g}{L}(L^2 + x^2 - 2Lx)\sin\theta - \frac{x^2}{L} + v^2$$

$$gL\sin\theta = \frac{g}{L}(L^2 + x^2 - 2Lx)\sin\theta - \frac{x^2}{L}g + v^2$$

$$v = \sqrt{\frac{5}{8}g\ell}$$

30. **(5)**

P = F.V = constant

$$F = \frac{P}{V}$$

$$F \propto \frac{1}{V} \qquad \text{as } V\uparrow, F\downarrow$$

When

Net force on block becomes zero i.e. its maximum velocity

$$P = (\mu mg)V_{max};\ V_{max} = \frac{100}{1\times 2\times 10} = 5 \text{ m/s}$$

CHEMISTRY

31. **(1)**

32. **(2)**

33. **(3)**

34. **(3)**

35. **(4)**
Degree of adsorption decreases with temperature.

36. **(1)**
$3A(g) \rightarrow 2B(g) + 3C(g) + D(g)$
P_0
$P_0 - x \qquad \frac{2}{3}x \qquad x \qquad \frac{x}{3}$

$$P_t \Rightarrow P_0 - x + \frac{2}{3}x + x + \frac{x}{3} = P_0 + x$$

$x = (P_t - P_0)$

$$K = \frac{1}{t}\ln\frac{P_0}{P_0 - (P_t - P_0)}$$

$$K = \frac{1}{t}\ln\frac{P_0}{2P_0 - P_t}$$

37. **(3)**
N_2O is isoelectronic with CO_2. Both have 22 electrons.

38. **(3)**

39. **(3)**

40. **(1)**

$$Pb(NO_3)_2 \xrightarrow{\Delta} PbO + NO_2 + O_2$$

41. **(3)**

42. **(2)**

43. **(3)**

$$Hg_2Cl_2 + NH_4OH \rightarrow Hg + \underbrace{Hg(NH_2)Cl}_{\text{black}}$$

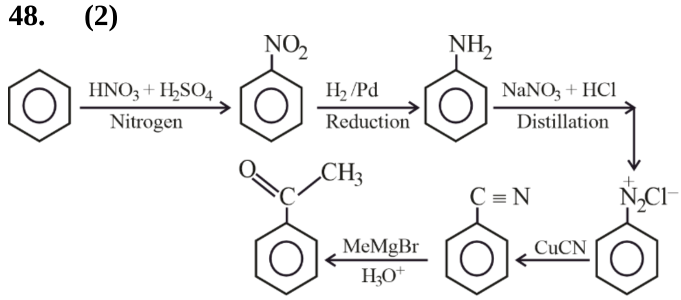
44. **(2)**

45. **(4)**

46. **(4)**

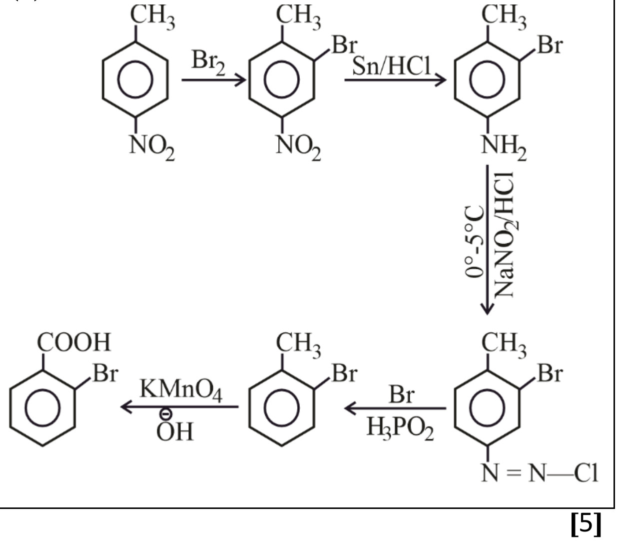
47. **(4)**

48. **(2)**



B.D.C.

49. **(2)**



50. **(1)**

51. **(144)**

52. **(150)**

$W = nC_{v,m}\,\Delta T$

$W = 1 \times 20 \times (T_2 - 300)$

$-3000J = 20\,(T_2 - 300)$

$-150 = T_2 - 300$

$T_2 = +150K$

53. **(5)**

54. **(7)**

Fe^{+2} : $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^6$

$3p^6 \rightarrow 4e^-$

$3d^6 \rightarrow$ maximum $3e^-$ having $|m| = 1$

55. **(5)**

CaO and K_2O do not evolve O gas upon heating.

56. **(4)**

57. **(2)**

$B_rF_5 = sp^3d^2$

$SF_6 = sp^3d^2$

$XeF_4 = sp^3d^2$

$PCl_5 = sp^3d$

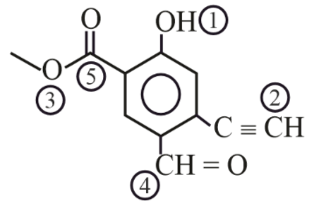
$ICl_3 = sp^3d$

$XeOF_4 = sp^3d^2$

$CCl_4 = sp^3$

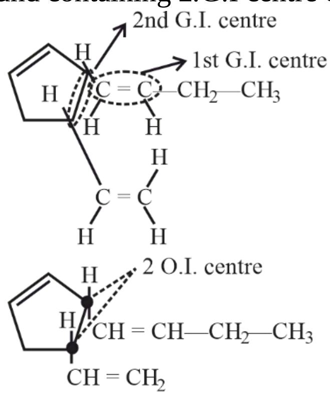
$IF_7 = sp^3d^3$

58. **(5)**



59. **(8)**

Compound containing 2.G.I centre & 2.O.I centre



Total stereoisomer

= O.I.+G.I.

$= 2^n + 2^n$

$= 2^2 + 2^2$

Total S.I. = 8

60. **(6)**

MATHEMATICS

61. **(4)**

$$t_{r+1} = {}^{50}C_r.(1)^{50-r}.\left(-2x^{1/2}\right)^r$$

$$= {}^{50}C_r \cdot 2^r \cdot x^{r/2}(-1)^r$$

$\Rightarrow$ $r =$ even integer.

$\Rightarrow$ Sum of coefficient

$$= \sum_{r=0}^{25} {}^{50}C_{2r} \cdot 2^{2r} = \frac{1}{2}\left((1+2)^{50}\right)$$

$$= \frac{1}{2}\left(3^{50}+1\right)$$

62. **(3)**

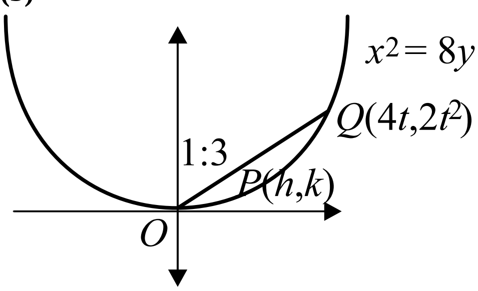
New sum $\sum y_i = (16\times16-16)+(3+4+5)$ = 252

Number of observation = 18

$\Rightarrow$ New mean

$\Rightarrow$ $\bar{y} = \dfrac{252}{18} = 14.$

63. **(3)**



$$h = \frac{4t}{4} = t$$

$$k = \frac{2t^2}{4} = \frac{t^2}{2}$$

$\Rightarrow$ $x^2 = 2y$

64. **(4)**

Required probability

$$= \left(\frac{{}^3C_1\,{}^{12}C_3\,2^9 - {}^3C_2\,{}^{12}C_3\,{}^9C_3}{3^{12}}\right)$$

65. **(2)**

$$\left|\frac{z_1-2z_2}{2-z_1\bar{z}_2}\right|=1$$

$$(z_1-2z_2)(\bar{z}_1-2\bar{z}_2)$$

$$=(2-z_1\bar{z}_2)(2-\bar{z}_1z_2)$$

$$\Rightarrow\quad |z_1|^2-2z_2\bar{z}_1-2\bar{z}_2z_1+4|z_2|^2$$

$$=4-2z_1\bar{z}_2-2\bar{z}_1z_2+|z_1|^2|z_2|^2$$

$$\Rightarrow\quad |z_1|^2+4|z_2|^2-4-|z_1|^2|z_2|^2=0$$

$$|z_1|^2\left(1-|z_2|^2\right)-4\left(1-|z_2|^2\right)=0$$

$$\Rightarrow\quad |z_1|=2\left(\text{as } |z_2|\neq 1\right)$$

66. **(3)**

$$\int\frac{dx}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}$$

$$1+\frac{1}{x^4}=t$$

$$\frac{-4}{x^5}dx=dt$$

$$\Rightarrow\quad -\frac{1}{4}\int\frac{1}{t^{3.4}}dt$$

$$=-\frac{1}{4}\times 4t^{1/4}+c=-\left(1+\frac{1}{x^4}\right)^{1/4}+c.$$

67. **(2)**

Let equation of plane is

$$(2x-5y+z-3)+\lambda(x+y+4z-5)=0$$

$$\frac{2+\lambda}{1}=\frac{\lambda-5}{3}=\frac{1+4\lambda}{6}$$

$$\Rightarrow\quad 6+3\lambda=\lambda-5$$

$$11=-2\lambda$$

$$\lambda=-\frac{11}{2}$$

Also, $6\lambda-30=3+12\lambda=-6\lambda=33$

$$\lambda=-\frac{11}{2}.$$

So the equation of required plane is

$$(4x-10y+2z-6)-11(x+y+4z-5)=0$$

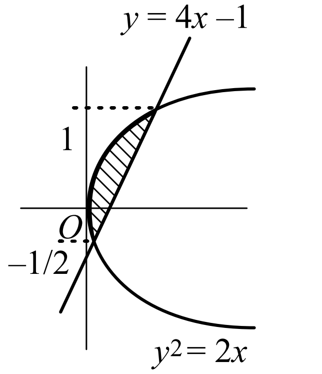
$$\Rightarrow\quad -7x-21y-42z+49=0$$

$$\Rightarrow\quad x+3y+6z-7=0.$$

68. **(3)**

The required region

$$=\int_{-\frac{1}{2}}^{1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dy$$



$$=\frac{1}{4}\left(\frac{y^2}{2}+y\right)_{-\frac{1}{2}}^{1}-\frac{1}{2}\left(\frac{y^3}{3}\right)_{-\frac{1}{2}}^{1}$$

$$=\frac{1}{4}\left(\frac{1}{2}+1-\left(\frac{1}{8}-\frac{1}{2}\right)\right)-\frac{1}{2}\frac{1}{3}\left(1+\frac{1}{8}\right)$$

$$=\frac{1}{4}\times\frac{15}{8}-\frac{3}{16}=\frac{9}{32}.$$

69. **(1)**

Given

$l+n=2m$ …(i)

l, G_1, G_2, G_3, n are in G.P.

$\Rightarrow\quad G_1, = lr$ (let r be the common ratio)

$G_2 = lr^2$

$G_3 = lr^3$

$n = lr^4$

$$r=\left(\frac{n}{l}\right)^{1/4}$$

$$\Rightarrow G_1^4+2G_2^4+G_3^4=(lr)^4+2\left(lr^2\right)^4+\left(lr^3\right)^4$$

$$=l^4\times r^4\left[l+2r^4+r^8\right]$$

$$=l^4\times r^4\left[r^4+l\right]^2$$

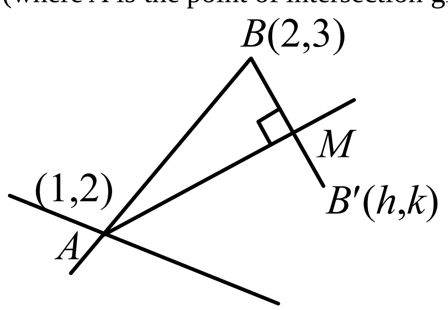
$$=l^4\times\frac{n}{l}\left[\frac{n+l}{l}\right]^2$$

$= l\,n \times 4m^2$

$= 4l\,nm^2$.

70. **(2)**

Let M is mid point of BB' and AM is ⊥ bisector of BB' (where A is the point of intersection given lines)



$$(x-2)(x-1)+(y-2)(y-3)=0$$

$\Rightarrow \quad \left(\frac{h+2}{2}-2\right)\left(\frac{h+2}{2}-1\right)+$

$\left(\frac{k+3}{2}-2\right)\left(\frac{k+3}{2}-3\right)=0$

$\Rightarrow \quad (h-2)(h)+(k-1)(k-3)=0$

$\Rightarrow \quad x^2-2x+y^2-4y+3=0$

$\Rightarrow \quad (x-1)^2+(y-2)^2=2.$

71. (4)

$\tan^{-1}y=\tan^{-1}x+\tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$=\tan^{-1}\left(\frac{x+\frac{2x}{1-x^2}}{1-x\left(\frac{2x}{1-x^2}\right)}\right)$

$=\tan^{-1}\left(\frac{x-x^3+2x}{1-x^2-2x^2}\right)$

$\tan^{-1}y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$y=\frac{3x-x^3}{1-3x^2}.$

72. (3)

$\sim(\sim s\vee(\sim r\wedge s))$

$\equiv s\wedge(r\vee\sim s)\equiv(s\wedge r)\vee(s\wedge\sim s)$

$\equiv(s\wedge r)\vee F$

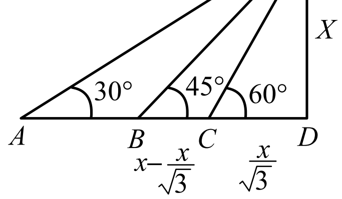
$\equiv s\wedge r.$

73. (4)

$Ab=\sqrt{3}x-x$

$BC=x-\frac{x}{\sqrt{3}}$

$\frac{AB}{BC}=\frac{\sqrt{3}x-x}{x-\frac{x}{\sqrt{3}}}=\frac{\sqrt{3}}{1}.$



74. (2)

$\lim_{x\to 0}\frac{2\sin^2 x\times(3+\cos x)}{x\times\left(\frac{\tan 4x}{4x}\right)\times 4x}$

75. (4)

$(\vec{a}.\vec{c})\vec{b}-(\vec{b}.\vec{c})\vec{a}=\frac{1}{3}|\vec{b}||\vec{c}|\vec{a}$

$\Rightarrow \quad \vec{b}.\vec{c}=\frac{1}{3}|\vec{b}||\vec{c}|$

$\Rightarrow \quad -|\vec{b}||\vec{c}|\cos\theta=\frac{1}{3}|\vec{b}||\vec{c}|$

$\Rightarrow \quad \cos\theta=-\frac{1}{3}$

$\Rightarrow \quad \sin\theta=\frac{2\sqrt{2}}{3}.$

76. (3)

$A=\begin{bmatrix}1&2&2\\2&1&-2\\a&2&b\end{bmatrix}, A^T=\begin{bmatrix}1&2&a\\2&1&2\\2&-2&b\end{bmatrix}$

$AA^T=[b_{ij}]_{3\times3}$

$b_{13}=0\Rightarrow 0=a+4+2b$

$b_{23}=0\Rightarrow 0=2a+2+2b$

$\Rightarrow \quad 3a+6=0$

$\Rightarrow \quad a=-2,\ b=-1.$

77. (4)

for $f(x)$ to be continuous

$2k=3m+2$

$2k-3m=2$

for $f(x)$ to be differentiable…(i)

$\frac{k}{4}=m$

$K=4m.$

from (i), $8m-3m=2$

$5m=2$

$m=\frac{2}{5}$

$k=4\times\frac{2}{5}=\frac{8}{5}$

$k+m=\frac{2}{5}+\frac{8}{5}=\frac{10}{5}=2.$

78. (2)

$(2-\lambda)x_1-2x_2+x_3=0$

$2x_1-(3+\lambda)x_2+2x_3=0$

$-x_1+2x_2-\lambda x_3=0$

$\Rightarrow \quad \begin{vmatrix}2-\lambda&-2&1\\2&-3-\lambda&2\\-1&2&-\lambda\end{vmatrix}=0$

$(2-\lambda)(3\lambda+\lambda^2-4)+2(-2\lambda+2)$

$+1(4-3-\lambda)=0$

$(2-\lambda)(\lambda^2+3\lambda-4)+4(1-\lambda)+(1-\lambda)=0$

$(2-\lambda)((\lambda+4)(\lambda-1))+5(1-\lambda)=0$

$(1-\lambda)((\lambda+4)(\lambda-2)+5)=0$

$\Rightarrow\quad \lambda = 1, 1, -3.$

79. **(3)**

$x^2+3xy-xy-3y^2=0$

$x(x+3y)-y(x+3y)=0$

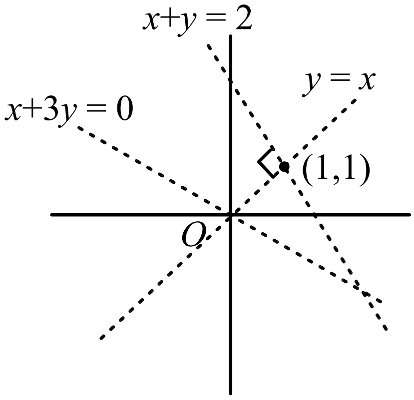
$(x+3y)(x-y)=0$

Equation of normal is $(y-1)=-1\,(x-1)$

$\Rightarrow\quad x+y=2$

It intersects $x+3y=0$ at $(3,-1)$

And hence meets the curve again in the 4th quadrant.



80. **(2)**

$\frac{dy}{dx}+\frac{y}{x\ln x}=\frac{2x\ln x}{x\ln x}$

I.F. $= e^{\int\frac{1}{x\ln x}dx}=e^{\ln(\ln x)}=\ln x$

$y\ln x=\int 2\ln x\,dx$

$y\ln x=2(x\ln x-x)+c$

For $x=1$, $c=2$

$y\ln x=2(x\ln x-x+1)$

Put $x=e \Rightarrow y(e)=2.$

81. **(96)**

$t_r=\frac{\sum r^3}{\sum(2r-1)}=\frac{r^2(r+1)^2}{4r^2}=\frac{1}{4}(r+1)^2$

$S_9=\frac{1}{4}\sum_{r=1}^{9}(r+1)^2$, let $t=r+1$

$=\frac{1}{4}\left(\sum_{t=1}^{10}t^2-1\right)=96.$

82. **(0)**

$\lim_{x\to 0}\left(\frac{x^2+f(x)}{x^2}\right)=3,$ since limit exits hence

$x^2+f(x)=ax^4+bx^3+3x^2$

$\Rightarrow\quad f(x)=ax^4+bx^3+2x^2$

$\Rightarrow\quad f'(x)=4ax^3+3bx^2+4x$

also $f'(x)=0$ at $x=1,2$

$\Rightarrow\quad a=\frac{1}{2}, b=-2$

$\Rightarrow\quad f(x)=\frac{x^4}{2}-2x^3+2x^2$

$\Rightarrow\quad f(x)=8-16+8=0.$

83. **(3)**

$x^2=6x+2\Rightarrow\alpha^2=6\alpha+2$

$\Rightarrow\quad \alpha^{10}=6\alpha^9+2\alpha^8$ …(i)

and $\beta^{10}=6\beta^9+2\beta^8$ …(ii)

$\Rightarrow$ Subtract (ii) from (i)

$a_{10}=6a_9+2a_8$

$\Rightarrow\quad \frac{a_{10}-2a_8}{2a_9}=3.$

84. **(780)**

$x+y<41, x>0, y>0$ is bounded region.

Number of positive integral solutions of the equation $x+y+k=41$ will be number of integral co-ordinates in the bounded region.

$\Rightarrow\quad {}^{41-1}C_{3-1}={}^{40}C_2=780.$

85. **(13)**

Let the point of intersection be

$(2+3\lambda, 4\lambda-1, 12\lambda+2)$

$(2+3\lambda)-(4\lambda-1)+12\lambda+2=16$

$11\lambda = 11$

$\lambda = 1$

$\Rightarrow$ point of intersection is (5, 3, 14)

$\Rightarrow$ distance $=\sqrt{(5-1)^2+9+12^2}$

$=\sqrt{16+9+144}=13.$

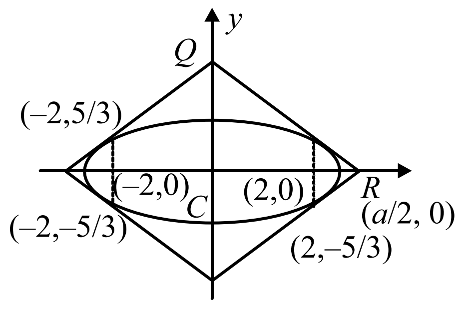
86. **(27)**

$a=3, b=\sqrt{5}$

$e=\sqrt{1-\frac{5}{9}}=\frac{2}{3}$

foci = $(\pm 2, 0)$

tangent at $P \Rightarrow \frac{2x}{9}+\frac{5y}{3.5}=1$



$$\frac{2x}{9}+\frac{y}{3}=1$$

$2x + 3y = 9$

Area of quadrilateral

$= 4 \times$ (area of triangle QCR)

$$=\left(\frac{1}{2}\times\frac{9}{2}\times 3\right)\times 4 = 27.$$

87. (192)

Four digit numbers will start from 6, 7, 8

$3 \times 4 \times 3 \times 2 = 72$

Five digit numbers = 5! = 120

Total number of integers = 192.

88. (219)

$n(A) = 4$, $n(B) = 2$

$n(A \times B) = 8$

number of subsets having atleast 3 elements

$$2^8 - \left(1 + {}^8C_1 + {}^8C_2\right) = 219$$

89. (1)

Apply the property

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

And then add

$$2I = \int_2^4 1dx$$

$2I = 2$

$I = 1.$

90. (3)

$C_1(2,3)$; $r_1 = \sqrt{4+9+12} = 5$

and $C_2(-3,-9)$; $r_2 = \sqrt{9+81-26} = 8$

$C_1C_2 = \sqrt{25+144} = 13$

$C_1C_2 = r_1 + r_2$ touching externally.

$\Rightarrow$ 3 common tangents