CBSE Class 11 Maths Notes Chapter 13: In CBSE Class 11 Maths, Chapter 13 is all about Limits and Derivatives. These are important ideas in calculus, a branch of math used in many fields. In this chapter, students learn about limits, which show how values get closer to each other.

They also study derivatives, which help find rates of change and slopes of curves. By understanding these concepts, students can solve problems in math, science, and engineering. The notes for this chapter explain these ideas clearly, with examples to help students learn and practice.

Mastering limits and derivatives in this chapter sets a strong foundation for future math studies and real-world applications.

CBSE Class 11 Maths Notes Chapter 13 PDF

You can access the CBSE Class 11 Maths Notes Chapter 13 on Limits and Derivatives in PDF format using the provided link. These notes cover important concepts such as limits and derivatives, which are fundamental in calculus. Understanding limits helps in approaching values or points, while derivatives are crucial for understanding rates of change and slopes of curves.

By studying these notes, students can strengthen their understanding of calculus and prepare themselves for higher-level math studies.

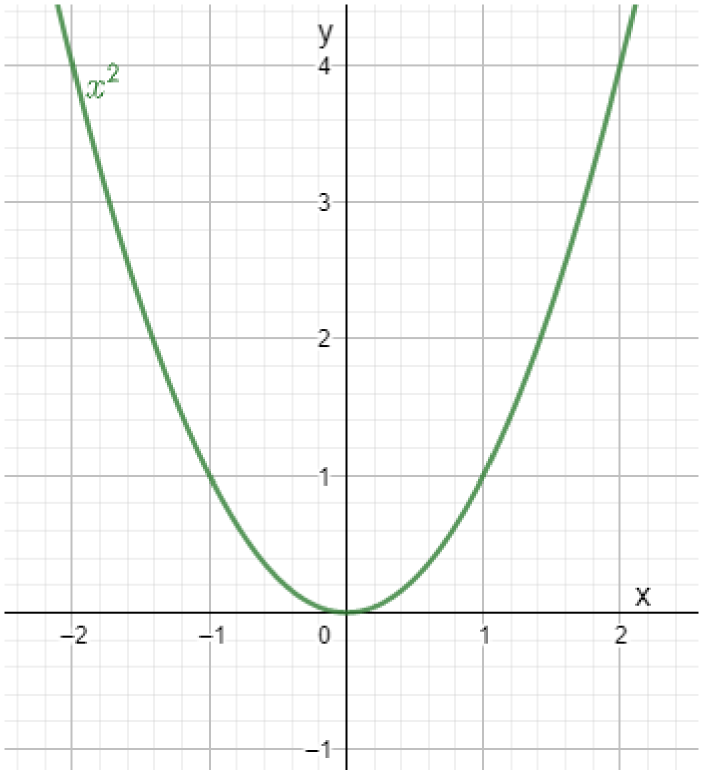
CBSE Class 11 Maths Notes Chapter 13 PDF

CBSE Class 11 Maths Notes Chapter 13 Limits and Derivatives

The solutions for CBSE Class 11 Maths Notes Chapter 13 on Limits and Derivatives are provided below, provide a detailed guide to understanding these fundamental concepts in calculus. Limits help us understand how values approach each other, while derivatives enable us to find rates of change and slopes of curves.

With clear explanations and examples, these notes facilitate a deeper understanding of calculus principles, preparing students for further studies in mathematics and related fields. By mastering the concepts covered in this chapter, students can build a strong foundation for tackling more advanced topics in calculus and applying mathematical principles to real-world problems.

Limits



Consider the function $f(x)=x^2$. When plotted, we see that as the value of x approaches 0, the value of $f(x)$ also moves towards 0.

In general, when x approaches a certain value a, and $f(x)$ approaches a specific value l, then l is termed as the limit of the function $f(x)$, symbolized as $\lim_{x \to a} f(x) = l$.

Regardless of the limits, a function should assume a particular value at a given point $x=a$.

There are two ways in which x can approach a number: from the left or from the right. This implies that all x values near a could be either less than a or greater than a.

The right-hand limit represents the value of $f(x)$ determined by $f(x)$ values when x tends towards a from the right, denoted as $\lim_{x \to a^+} f(x)$.

Similarly, the left-hand limit signifies the value of $f(x)$ dictated by $f(x)$ values when x approaches a from the left, expressed as $\lim_{x \to a^-} f(x)$.

In our example, the right and left-hand limits differ. Hence, the limit of $f(x)$ as x approaches zero does not exist, even though the function is defined at $x=0$.

If the right and left-hand limits converge to the same value, then that common value represents the limit and is denoted by $\lim_{x \to a} f(x)$.

Algebra of limits

Theorem 1 states various properties of limits for two functions f and g:

If both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then:

- The limit of the sum of two functions is the sum of their limits: $\lim_{x \to a}[f(x)+g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$.
- The limit of the difference of two functions is the difference of their limits: $\lim_{x \to a}[f(x)-g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$.
- The limit of the product of two functions is the product of their limits: $\lim_{x \to a}[f(x)\cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$.
- The limit of the quotient of two functions is the quotient of their limits (provided the denominator is non-zero): $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$.

In a special case when g is a constant function such that $g(x)=\lambda$ for some real number λ:

- The limit of a constant multiple of a function is equal to the constant multiplied by the limit of the function: $\lim_{x \to a}[(\lambda \cdot f)(x)] = \lambda \cdot \lim_{x \to a} f(x)$.

Limits of polynomials and rational functions

A polynomial function f is one where $f(x)$ is either a zero function or takes the form $f(x)=a_0+a_1x+a_2x^2+...+a_nx^n$, where a_i are real numbers and $a_n \neq 0$ for some natural number n.

We know that

$$\lim_{x \to a} x = a$$
$$\lim_{x \to a} x^2 = \lim_{x \to a}(x.x) = \lim_{x \to a} x . \lim_{x \to a} x = a.a = a^2$$

Hence,

$$\lim_{x \to a} x^n = a^n$$

Let

$$f(x)=a_0+a_1x+a_2x^2+...+a_nx^n$$

be a polynomial function

$$\lim_{x \to a} f(x) = \lim_{x \to a}[a_0+a_1x+a_2x^2+...+a_nx^n]$$

$$= \lim_{x\to a} a_0 + \lim_{x\to a} a_1 x + \lim_{x\to a} a_2 x^2 + ... + \lim_{x\to a} a_n x^n$$

$$= a_0 + a_1 \lim_{x\to a} x + a_2 \lim_{x\to a} x^2 + ... + a_n \lim_{x\to a} x^n$$

$$= a_0 + a_1 a + a_2 a^2 + ... + a_n a^n$$

$$= f(a)$$

A rational function f is one where $f(x) = \frac{g(x)}{h(x)}$, and $g(x)$ and $h(x)$ are polynomials such that $h(x) \neq 0$.

Then, $\lim_{x\to a} f(x) = \lim_{x\to a} \frac{g(x)}{h(x)} = \frac{\lim_{x\to a} g(x)}{\lim_{x\to a} h(x)} = \frac{g(a)}{h(a)}$

However, if $h(a) = 0$, there are two scenarios:

- If $g(a) \neq 0$, the limit does not exist.
- If $g(a) = 0$, we have $g(x) = (x-a)^k g_1(x)$ and $h(x) = (x-a)^l h_1(x)$, where k is the maximum power of $(x-a)$ in $g(x)$ and l is the maximum power of $(x-a)$ in $h(x)$.
  - If $k \geq l$, then the limit is 0.
  - If $k < l$, the limit is not defined.

Theorem 2

For any positive integer n, $\lim_{x\to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$.

The proof is shown below.

Dividing $(x^n - a^n)$ by $(x - a)$,

$$\lim_{x\to a} \frac{x^n - a^n}{x - a} = \lim_{x\to a} (x^{n-1} + x^{n-2}a + x^{n-3}a^2 + ... + x a^{n-2} + a^{n-1})$$

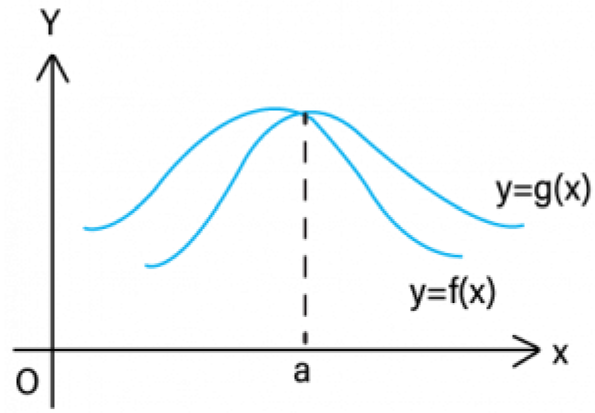
$$= a^{n-1} + a a^{n-2} + ... + a^{n-2}(a) + a^{n-1}$$

$$= a^{n-1} + a^{n-1} + ... + a^{n-1} + a^{n-1} \ (n \text{ terms})$$

$$= n a^{n-1}$$

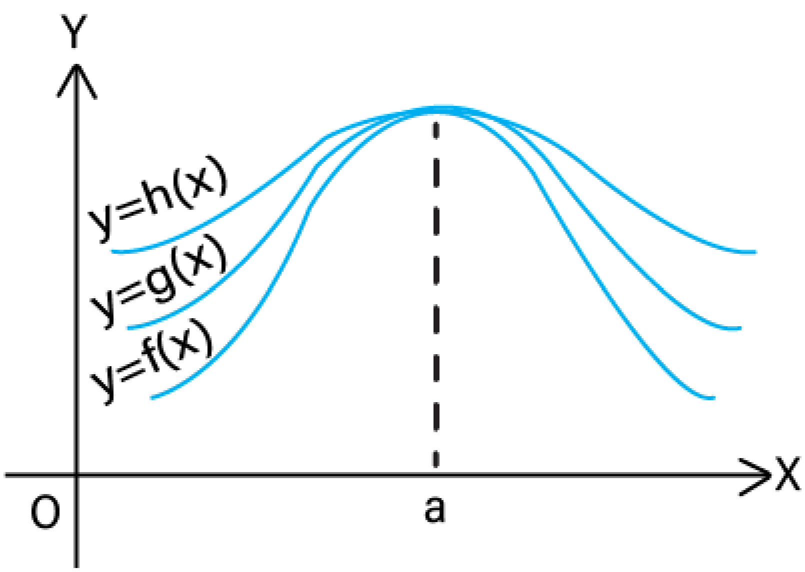
Limits of Trigonometric Functions

Theorem 3



Theorem 3 states that if f and g are two real-valued functions with the same domain, and $f(x) \leq g(x)$ for all x in their domain, then if both $\lim_{x \to a} f(x)$ and $\lim_{x \to a} g(x)$ exist, then $\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$.

Theorem 4



The Sandwich Theorem, or Theorem 4, asserts that if three real functions $f(x)$, $g(x)$, and $h(x)$ satisfy $f(x) \leq g(x) \leq h(x)$ for all x in their common domain, and if $\lim_{x \to a} f(x) = l = \lim_{x \to a} h(x)$, then $\lim_{x \to a} g(x) = l$.

To prove that $\cos(x) < \frac{\sin(x)}{x} < 1$ for $0 < |x| < \frac{\pi}{2}$, it is observed that $\sin(x)$ lies between $\cos(x)$ and $\tan(x)$. Since $0 < x < \frac{\pi}{2}$, $\sin(x)$ is positive. Thus, dividing throughout by $\sin(x)$, $1 < \frac{x}{\sin(x)} < \frac{1}{\cos(x)}$, leading to $\cos(x) < \frac{\sin(x)}{x} < 1$.

Two important limits are given:

- $\lim_{x\to 0}\frac{\sin(x)}{x}=1$
- $\lim_{x\to 0}\frac{1-\cos(x)}{x}=0$

Derivatives

The derivative of a function at a given point within its domain of definition is a fundamental concept in calculus.

Definition 1 states that if f is a real-valued function and a is a point in its domain, the derivative of f at a is defined as $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$, provided this limit exists. It is denoted as $f'(a)$.

Definition 2 defines the derivative of f at x as $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ where the limit exists. This definition is also known as the first principle of derivative.

The derivative of a function $f(x)$ with respect to x can be denoted as $f'(x)$, represented as $\frac{d}{dx}(f(x))$, or if $y=f(x)$, it is represented as $\frac{dy}{dx}$. Another notation used is $D(f(x))$.

Further, derivative of f at $x=a$ is also denoted by

Moreover, the derivative of f at $x=a$ can be denoted as $\frac{d}{dx}f(x)\big|_{x=a}$, $\frac{df}{dx}\big|_{x=a}$, or $\left(\frac{df}{dx}\right)_{x=a}$.

Theorem 5

Theorem 5 provides fundamental rules for finding derivatives of functions.

For two functions f and g with defined derivatives in a common domain:

- The derivative of their sum is the sum of their derivatives: $\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$.
- The derivative of their difference is the difference of their derivatives: $\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$.
- The product rule states that the derivative of the product of two functions is the first function's derivative times the second function plus the first function times the second function's derivative: $\frac{d}{dx}[f(x)\cdot g(x)]=\frac{d}{dx}f(x)\cdot g(x)+f(x)\cdot\frac{d}{dx}g(x)$.
- The quotient rule states that the derivative of the quotient of two functions is the derivative of the numerator times the denominator minus the numerator times the derivative of the denominator, all divided by the square of the denominator: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{\frac{d}{dx}f(x)\cdot g(x)-f(x)\cdot\frac{d}{dx}g(x)}{(g(x))^2}$.

Furthermore, the derivative of the function $f(x)=x$ is a constant.

Theorem 6

Theorem 6 states that the derivative of a function $f(x)=x^n$ is nx^{n-1} for any positive integer n.

Proof:

By the definition of the derivative function, we have:

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^n-x^n}{h}=\lim_{h\to 0}\frac{h(nx^{n-1}+\ldots+h^{n-1})}{h}$$

$$=\lim_{h\to 0}(nx^{n-1}+\ldots+h^{n-1})=nx^{n-1}$$

This can also be proved alternatively:

$\frac{d}{dx}(x^n)=\frac{d}{dx}(x\cdot x^{n-1})$ $=\frac{d}{dx}(x)\cdot(x^{n-1})+x\cdot\frac{d}{dx}(x^{n-1})$ (By the product rule) $=1\cdot x^{n-1}+x\cdot((n-1)x^{n-2})$ (By induction hypothesis) $=x^{n-1}+(n-1)x^{n-1}=nx^{n-1}$

Theorem 7

Theorem 7 states that for a polynomial function $f(x)=a_nx^n+a_{n-1}x^{n-1}+\ldots+a_1x+a_0$, where a_is are real numbers and $a_n\neq 0$, the derivative function is given by:

$$\frac{df(x)}{dx}=na_nx^{n-1}+(n-1)a_{n-1}x^{n-2}+\ldots+2a_2x+a_1$$