Vernacular

Ultimate kcet crash course 2026
Maths
Integrals

DPP: 3

Q1 Evaluate : $\int \frac{1}{\sqrt{9+8x-x^2}}dx$

(A) $-\sin^{-1}\left(\frac{x-4}{5}\right)+c$

(B) $\sin^{-1}\left(\frac{x+4}{5}\right)+c$

(C) $\sin^{-1}\left(\frac{x-4}{5}\right)+c$

(D) None of these

Q2 Find $\int \frac{dx}{\sqrt{x^2-a^2}}$

(A) $\log\left|x+\sqrt{a^2-x^2}\right|+c$

(B) $\log\left|\sqrt{x^2-a^2}\right|+c$

(C) $\sin^{-1}\frac{x}{a}+c$

(D) $\log\left|x+\sqrt{x^2-a^2}\right|+c$

Q3 The value of $\int \frac{dx}{\sqrt{x^2+16}}$ is

(A) $\log\left|4+\sqrt{x^2+16}\right|+c$

(B) $\log\left|x+\sqrt{x^2+16}\right|+c$

(C) $\log\left|\frac{x}{4}+\sqrt{x^2+16}\right|+c$

(D) $\frac{1}{8}\tan^{-1}\left(\frac{x}{4}\right)+c$

Q4 The value of $\int \frac{dx}{5-2x+x^2}$ is

(A) $\log\left|2+\sqrt{x-1}\right|+c$

(B) $\frac{1}{2}\log\left|x+\sqrt{x-1}\right|+c$

(C) $\tan^{-1}\left(\frac{x-1}{2}\right)+c$

(D) $\frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}\right)+c$

Q5 Evaluate : $\int \frac{dx}{\sqrt{3+4x-4x^2}}$

(A) $\frac{1}{2}\cos^{-1}\left(\frac{2x-1}{2}\right)+c$

(B) $2\cos^{-1}\left(\frac{2x-1}{2}\right)+c$

(C) $\frac{1}{2}\sin^{-1}\left(\frac{2x-1}{2}\right)+c$

(D) $2\sin^{-1}\left(\frac{2x-1}{2}\right)+c$

Q6 Find $\int \frac{dx}{a^2-x^2}$

(A) $\frac{1}{2a}\log\left|\frac{x-a}{x+a}\right|+c$

(B) $\frac{1}{2a}\log\left|\frac{a+x}{a-x}\right|+c$

(C) $\frac{1}{2a}\log\left|\frac{a-x}{a+x}\right|+c$

(D) $\frac{1}{2a}\log\left|a-x\right|+c$

Q7 Evaluate : $\int \frac{1}{x\left\{6(\log x)^2+7\log x+2\right\}}dx$

(A) $\log\left|\frac{2\log x+1}{3\log x+2}\right|+c$

(B) $\frac{1}{2}\log\left|\frac{2\log x+1}{3\log x+2}\right|+c$

(C) $\log\left|\frac{3\log x+2}{2\log x+1}\right|+c$

(D) None of these

Q8 The value of $\int \frac{x^2-1}{x^4+1}dx$ is

(A) $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}x}\right)+c$

(B) $\frac{1}{2\sqrt{2}}\ln\left(\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right)+c$

(C) $\frac{1}{\sqrt{2}}\ln\left(\frac{x^2-1}{\sqrt{2}x}\right)+c$

(D) $\frac{1}{2\sqrt{2}}\ln\left(\frac{x^2-1}{x^2+1}\right)+c$

Q9 Evaluate : $\int \frac{1}{\sqrt{(2-x)^2-1}}dx$

(A) $\log\left|x-2+\sqrt{(2-x)^2-1}\right|+c$

(B) $-\log\left|2-x+\sqrt{(2-x)^2-1}\right|+c$

(C) $\log\left|x+2+\sqrt{(2+x)^2-1}\right|+c$

(D) $-\log\left|2+x+\sqrt{(2+x)^2+1}\right|+c$

Q10

$\int \frac{x^2}{x^6+2x^3+2}\,dx =$

(A) $\tan^{-1}(x^3+1)+c$

(B) $\frac{1}{3}\tan^{-1}\left(x^3+1\right)+c$

(C) $-\frac{1}{3}\tan^{-1}\left(x^3+1\right)+c$

(D) $\frac{1}{3}\cot^{-1}\left(x^3+1\right)+c$

Q11 $\int \frac{dx}{\sin x(3+2\cos x)} =$

(A) $\frac{1}{10}\log(1+\cos x)+\frac{1}{2}\log|1+\cos x| + \frac{2}{5}\log|2+3\cos x|+c$

(B) $\frac{1}{10}\log(1-\cos x)-\frac{1}{2}\log|1+\cos x| + \frac{2}{5}\log|3+2\cos x|+c$

(C) $\frac{1}{10}\log(1-\cos x)-\frac{1}{2}\log|1+\cos x|+c$

(D) None of these

Q12 $\int \frac{e^x}{(1+e^x)(2+e^x)}dx =$

(A) $\log\left[(1+e^x)(2+e^x)\right]+c$

(B) $\log\left[\frac{1+e^x}{2+e^x}\right]+c$

(C) $\log\left[(1+e^x)\sqrt{2+e^x}\right]+c$

(D) None of these

Q13 $\int \frac{1}{x-x^3}dx =$

(A) $log\ \frac{x}{\sqrt{1+x^2}}+c$

(B) $-\log\frac{x}{\sqrt{1-x^2}}+c$

(C) $\log\frac{x}{\sqrt{1-x^2}}+c$

(D) $\log\left(\frac{-x}{\sqrt{1-x^2}}\right)+c$

Q14 $\int \frac{1}{(2x-3)(3x-2)}dx =$

(A) $\frac{-1}{5}\log\left|\frac{2x-3}{3x-2}\right|+c$

(B) $\frac{1}{5}\log\left|\frac{2x-3}{3x-2}\right|+c$

(C) $\frac{1}{3}\log\left|\frac{2x-3}{3x-2}\right|+c$

(D) $\frac{-1}{3}\log\left|\frac{3x-2}{2x-3}\right|+c$

Q15 $\int \frac{1}{(3x^2+13x-10)}dx =$

(A) $\log\left|\frac{x+5}{3x-2}\right|+c$

(B) $\frac{1}{15}\log\left|\frac{x+5}{3x-2}\right|+c$

(C) $\frac{1}{17}\log\left|\frac{3x-2}{x+5}\right|+c$

(D) $\log\left|\frac{3x-5}{x+2}\right|+c$

Q16 $\int \frac{x^2+x+1}{x^2(x+2)}dx =$

(A) $\frac{1}{4}\log|x|-\frac{1}{2x}+\frac{3}{4}\log|x+2|+c$

(B) $\frac{3}{4}\log x-\frac{1}{2x}-\frac{3}{4}\log|x+2|+c$

(C) $\frac{1}{4}\log(x)+\frac{1}{2x}-\frac{3}{4}\log|x+2|+c$

(D) None of these

Q17 $\int \frac{x^2}{(x^2+2)(x^2+3)}dx =$

(A) $-\sqrt{2}\tan^{-1}x+\sqrt{3}\tan^{-1}x+c$

(B) $-\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}+\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}+c$

(C) $\sqrt{2}\tan^{-1}\frac{x}{\sqrt{2}}+\sqrt{3}\tan^{-1}\frac{x}{\sqrt{3}}+c$

(D) none of these

Q18 $\int \frac{2e^t}{e^{3t}-6e^{2t}+11e^t-6}dt =$

(A) $\log\left|\left\{\frac{(e^t+1)(e^t-3)}{(e^t-2)^2}\right\}\right|+c$

(B) $\log\left|\left\{\frac{(e^t+1)(e^t+3)}{(e^t-2)^2}\right\}\right|+c$

(C) $\log\left|\left\{\frac{(e^t-1)(e^t+3)}{(e^t-2)^2}\right\}\right|+c$

(D) $\log\left|\left\{\frac{(e^t-1)(e^t-3)}{(e^t-2)^2}\right\}\right|+c$

Q19 If $\int \frac{3x+4}{x^3-2x-4}dx = \log|x-2|+k\log f(x)+c,$ then

(A) $f(x)=|x^2-2x+2|$

(B) $f(x)=x^2-2x-2$

(C) $k=-\frac{1}{2}$

(D) $k=\frac{1}{2}$

Q20 $\int \frac{2x+1}{(x+1)(x-2)}dx =$

(A) 1/3 log |x + 1| + 5/3 log |x – 2| + c

(B) 1/3 log |x + 1| - 5/3 log |x – 2| + c

(C) 1/3 log |x + 1| - 5/3 log |x + 2| + c

(D) 1/3 log |x + 1| + 5/3 log |x + 2| + c

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Q21 $\int \tan^{-1}\sqrt{x}\,dx =$

(A) $x\tan^{-1}x - \frac{1}{2}\log\left(1+x^2\right)+c$

(B) $x\tan^{-1}\sqrt{x} - \frac{1}{2}\log\left(1+x^2\right)+c$

(C) $x\tan^{-1}\sqrt{x} - \sqrt{x} + \log(1+x)+c$

(D) $(x+1)\tan^{-1}\sqrt{x} - \sqrt{x}+c$

Q22 Find the value of $\int e^x\left(\frac{2+\sin 2x}{1+\cos 2x}\right)dx$

(A) $e^x \tan x + c$ (B) $e^x \tan^2 x + c$

(C) $e^x \sec^2 x + c$ (D) $e^x \sec x + c$

Q23 $\int x\tan^{-1}x\,dx =$

(A) $\frac{1}{2}\left(x^2+1\right)\tan^{-1}x - \frac{1}{2}x + c$

(B) $\frac{1}{2}\left(x^2-1\right)\tan^{-1}x - \frac{1}{2}x + c$

(C) $\frac{1}{2}\left(x^2+1\right)\tan^{-1}x + \frac{1}{2}x + c$

(D) $\frac{1}{2}\left(x^2+1\right)\tan^{-1}x - x + c$

Q24 $\int \frac{\log x dx}{x^3} =$

(A) $\frac{1}{4x^2}(2\log x - 1)+c$

(B) $-\frac{1}{4x^2}(2\log x + 1)+c$

(C) $\frac{1}{4x^2}(2\log x + 1)+c$

(D) $\frac{1}{4x^2}(1 - 2\log x)+c$

Q25 $\int 32x^3(\log x)^2 dx$ is equal to

(A) $x^4\{8(\log x)^2 - 4(\log x) + 1\} + c$

(B) $x^3\{(\log x)^2 + 2\log x\} + c$

(C) $x^4\{8(\log x)^2 - 4\log x\} + c$

(D) $8x^4(\log x)^2 + c$

Q26 $\int e^x \frac{(1-x)^2}{(1+x^2)^2}dx =$

(A) $\frac{e^x}{(1-x)^2}+c$

(B) $\frac{e^x}{1+x^2}+c$

(C) $e^x\left(\frac{1-x^2}{1+x^2}\right)+c$

(D) $\frac{e^x}{(1+x^2)^2}+c$

Q27 If $\int \frac{e^x(1+\sin x)dx}{1+\cos x} = e^x f(x) + c$, then $f(x)$ =

(A) sin x/2 (B) cos x/2

(C) tan x/2 (D) log x/2

Q28 $\int\left[\log(\log x) + \frac{1}{(\log x)^2}\right]dx =$

(A) $x\log(\log x) + \frac{x}{\log x}+c$

(B) $x\log(\log x) - \frac{x}{\log x}+c$

(C) $x\log(\log x) + \frac{\log x}{x}+c$

(D) $x\log(\log x) - \frac{\log x}{x}+c$

Q29 $\int\left(f(x)g''(x) - f''(x)g(x)\right)dx =$

(A) $f(x)g'(x) - f'(x)g(x)+c$

(B) $f(x)g'(x) + f'(x)g(x)+c$

(C) $f(x)g(x) - f(x)g'(x)+c$

(D) $\frac{f(x)}{g'(x)}+c$

Q30 $\int x^3 e^{x^2}dx =$

(A) $\frac{1}{2}\left(x^2+1\right)e^{x^2}+c$

(B) $\left(x^2+1\right)e^{x^2}+c$

(C) $\frac{1}{2}\left(x^2-1\right)e^{x^2}+c$

(D) $\left(x^2-1\right)e^{x^2}+c$

Answer Key

Q1	(C)	Q16	(A)
Q2	(D)	Q17	(B)
Q3	(B)	Q18	(D)
Q4	(D)	Q19	(C)
Q5	(C)	Q20	(A)
Q6	(B)	Q21	(D)
Q7	(A)	Q22	(A)
Q8	(B)	Q23	(A)
Q9	(B)	Q24	(B)
Q10	(B)	Q25	(A)
Q11	(B)	Q26	(B)
Q12	(B)	Q27	(C)
Q13	(C)	Q28	(B)
Q14	(B)	Q29	(A)
Q15	(C)	Q30	(C)

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

$$\text{We have, } \int \frac{1}{\sqrt{9+8x-x^2}}dx$$

$$= \int \frac{1}{\sqrt{-\{x^2-8x-9\}}}dx$$

$$= \int \frac{1}{\sqrt{-\left\{(x-4)^2-5^2\right\}}}dx = \int \frac{1}{\sqrt{5^2-(x-4)^2}}dx$$

$$= \sin^{-1}\left(\frac{x-4}{5}\right)+c$$

Video Solution:

Q2 Text Solution:

$$\text{Put } x = a\sec\theta$$

$$dx = a\sec\theta\tan\theta d\ \theta$$

$$\Rightarrow I = \int \frac{a\sec\theta\tan\theta d\theta}{\sqrt{a^2\sec^2\theta - a^2}}$$

$$= \int \frac{a\sec\theta\tan\theta d\theta}{a\tan\theta}$$

$$= \int \sec\theta d\theta$$

$$= \log|\sec\theta+\tan\theta| + c_1$$

$$= \log\left|\frac{x}{a}+\sqrt{\frac{x^2}{a}-1}\right| + c_1$$

$$= \log\left|x+\sqrt{x^2-a^2}\right| - \log|a| + c_1$$

$$= \log\left|x+\sqrt{x^2-a^2}\right| + c_1.$$

Video Solution:

Q3 Text Solution:

$$I = \int \frac{dx}{\sqrt{x^2+4^2}}$$

$$= \log\left|x+\sqrt{x^2+16}\right| + c$$

Video Solution:

Q4 Text Solution:

$$5-2x+x^2 = t\left(x^2-2x+1-1+5\right)$$

$$= (x-1)^2+4$$

$$I = \int \frac{dx}{(x-1)^2+2^2}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{x-1}{2}\right)+c$$

Video Solution:

Q5 Text Solution:

$$\text{Let } I = \int \frac{dx}{\sqrt{3+4x-4x^2}} = \frac{1}{2}\int \frac{1}{\sqrt{\frac{3}{4}+x-x^2}}dx$$

$$= \frac{1}{2}\int \frac{1}{\sqrt{1-\left(x^2-x+\frac{1}{4}\right)}}dx = \frac{1}{2}$$

$$\int \frac{1}{\sqrt{(1)^2-\left(x-\frac{1}{2}\right)^2}}dx$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{2x-1}{2}\right)+c$$

Video Solution:

Q6 **Text Solution:**

$I = \int \frac{dx}{a^2-x^2}$

$= \frac{1}{2a}\left[\int \frac{dx}{a-x} + \int \frac{dx}{a+x}\right]$

$= \frac{1}{2a}\left[-\log|a-x| + \log|a+x|\right] + c.$

$= \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c.$

Video Solution:

Q7 **Text Solution:**

Let $I = \int \frac{1}{x\left\{6(\log x)^2+7\log x+2\right\}}dx$

Putting $\log x = t \Rightarrow \frac{1}{x}dx = dt$

$I = \int \frac{1}{6t^2+7t+2}dt = \frac{1}{6}\int \frac{1}{\left(t+\frac{7}{12}\right)^2-\left(\frac{1}{12}\right)^2}dt$

$= \frac{1}{6}\times\frac{1}{2\left(\frac{1}{12}\right)}\log\left|\frac{t+\frac{7}{12}-\frac{1}{12}}{t+\frac{7}{12}+\frac{1}{12}}\right| + K$

$= \log\left|\frac{2t+1}{3t+2}\right| + c \quad \left[\text{where, } c = K + \log\frac{3}{2}\right]$

$= \log\left|\frac{2\log x+1}{3\log x+2}\right| + c$

Video Solution:

Q8 **Text Solution:**

$I = \int \frac{x^2\left[1-\frac{1}{x^2}\right]}{x^2\left[x^2+\frac{1}{x^2}\right]}dx$

$= \int \frac{1-\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2-2}dx$

Put $x + \frac{1}{x} = t$

$\left(1-\frac{1}{x^2}\right)dx = dt$

$I = \int \frac{dt}{t^2-2}$

$= \frac{1}{2\sqrt{2}}\log\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + c$

$= \frac{1}{2\sqrt{2}}\log\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right| + c$

Video Solution:

Q9 **Text Solution:**

$I = \int \frac{1}{\sqrt{(2-x)^2-1}}dx$

Put $2 - x = t$

$dx = -dt$

$I = -\int \frac{dt}{\sqrt{t^2-1}}$

$I = -\log\left|t+\sqrt{t^2-1}\right| + c$

$I = -\log\left|2-x+\sqrt{(2-x)^2-1}\right| + c$

Video Solution:

Q10 **Text Solution:**

$\int \frac{x^2}{x^6+2x^3+2}\,dx = \frac{1}{3}\tan^{-1}\left(x^3+1\right)+c$

Video Solution:

Q11 **Text Solution:**

$l = \int \frac{\sin x dx}{\sin^2 x(3+2\cos x)}$ Let $\cos x = t \Rightarrow$

$-\sin x dx = dt$

$\therefore l = -\int \frac{dt}{(1-t^2)(3+2t)} = \int \frac{-dt}{(1-t)(1+t)(3+2t)}$

Let $\frac{-1}{(1-t)(1+t)(3+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{3+2t}$

$A = \frac{-1}{(1+t)(3+2t)}$, put $t = 1$ and B

$= \frac{-1}{(1-t)(3+2t)}$,

put $t = -1$ and $C = \frac{-1}{(1-t)(1+t)}$, put $t = \frac{-3}{2}$

$= \frac{-1}{1-t} + \frac{\frac{-1}{2}}{1+t} + \frac{\frac{4}{5}}{3+2t}$

$l = \frac{1}{10}\log|1-t| - \frac{1}{2}\log|1+t| + \frac{4}{5}$

$\times \frac{1}{2}\log|3+2t| + c = \frac{1}{10}\log|1-\cos x|$

$-\frac{1}{2}\log|1+\cos x| + \frac{2}{5}\log|3+2\cos x| + c$

Video Solution:

Q12 **Text Solution:**

$\int \frac{e^x}{(1+e^x)(2+e^x)}dx = \int\left\{\frac{e^x}{1+e^x} - \frac{e^x}{2+e^x}\right\}dx$

Now put

$1+e^x = t$ *and* $2+e^x = t$ *then the required integral*

$= \log(1+e^x) - \log(2+e^x) = \log\left(\frac{1+e^x}{2+e^x}\right)$

$+ c$

Video Solution:

Q13 **Text Solution:**

$\int \frac{dx}{x-x^3} = \int \frac{dx}{x(1-x^2)} = \int \frac{dx}{x(1+x)(1-x)}$

Let $\frac{1}{x(1+x)(1-x)} = \frac{A}{x} + \frac{B}{1+x} + \frac{C}{1-x} = \frac{1}{x}$

$+ \frac{\frac{-1}{2}}{1+x} \quad + \frac{\frac{1}{2}}{1-x}$

$A = \frac{1}{(1+x)(1-x)}$ pu $tx = 0$ $B = \frac{1}{x(x-1)}$ put$x$

$= -1$ $C = \frac{1}{x(1+x)}$ put$x = 1$

$\int \frac{1}{x(1+x)(1-x)}dx = \log|x| - \frac{1}{2}\log|1+x|$

$-\frac{1}{2}\log|1-x| + c$

$= \log|x| - \frac{1}{2}\log|1-x^2| + c$

$= \log\left|\frac{x}{\sqrt{1-x^2}}\right| + c$

Video Solution:

Q14 **Text Solution:**

$\frac{1}{(2x-3)(3x-2)} = \frac{A}{(2x-3)} + \frac{B}{(3x-2)}$

$A = \frac{1}{3x-2}$ put $x = \frac{3}{2}$

$B = \frac{1}{2x-}$ put $x = \frac{2}{3}$

$\frac{1}{(2x-3)(3x-2)} = \frac{2}{5}\left(\frac{1}{2x-3}\right) - \frac{3}{5}\left(\frac{1}{3x-2}\right)$

$= \int \frac{1}{(2x-3)(3x-2)}\, dx = \frac{2}{5}\int \frac{1}{2x-3}dx - \frac{3}{5}$

$\int \frac{1}{3x-2}dx$

$= \frac{2}{5}\ \frac{1}{2}\log|2x-3| - \frac{3}{5} \times \frac{1}{3}\log|3x-2| + c$

$= \frac{1}{2}\left[\log|2x-3| - \log|3x-2|\right] + c$

$= \frac{1}{5}\left[\log|2x-3| - \log|3x-2|\right] + c$

$= \frac{1}{5}\log\left|\frac{2x-3}{3x-2}\right| + c$

Video Solution:

Q15 **Text Solution:**

$\int \frac{1}{(3x^2+15x-2x-10)}dx = \int \frac{1}{3x(x+5)-2(x+5)}dx =$

$\int \frac{1}{(3x-2)(x+5)}dx$

Let $\frac{1}{(3x-2)(x+5)} = \frac{A}{3x-2} + \frac{B}{x+5}$

$A = \frac{1}{x+5}$ put $x = \frac{2}{3}$

$B = \frac{1}{3x-2}$ put $x = -5$

$\therefore I = \frac{3}{17}\int \frac{dx}{3x-2} - \frac{1}{17}\int \frac{dx}{x+5} = \frac{3}{17}$

$\times \frac{1}{3}\log\left|3x-2\right| - \frac{1}{17}\log\left|x+5\right| + c$

$= \frac{1}{17}\log\left|\frac{3x-2}{x+5}\right| + c$

Video Solution:

Q16 **Text Solution:**

$\frac{x^2+x+1}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$

$\Rightarrow x^2 + x + 1 = Ax(x+2) + B(x+2)$

$+ C\left(x^2\right) \quad \ldots(i)$

Comparing both sides,

$1 = A + C,\ B = \frac{1}{2} \Rightarrow 2A + B = 1 \Rightarrow 2A$

$+ \frac{1}{2} = 1 \Rightarrow A = \frac{1}{4}$

and $C = 1 - A = \frac{3}{4}$

$I = 2A + B \Rightarrow B = \frac{1}{2}$

$\therefore I = \frac{1}{4}\int \frac{dx}{x} + \frac{1}{2}\int \frac{dx}{x^2} + \frac{3}{4}\int \frac{dx}{x+2}$

$= \frac{1}{4}\log\left|x\right| - \frac{1}{2x} + \frac{3}{4}\log\left|x+2\right| + c$

Video Solution:

Q17 **Text Solution:**

$\int \frac{x^2}{(x^2+2)(x^2+3)}dx = \int \left[\frac{3}{x^2+3} - \frac{2}{x^2+2}\right]dx$

$= \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) - \sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$

Video Solution:

Q18 **Text Solution:**

Let $e^t = x \cdot \quad \Rightarrow e^t dt = dx$

$\therefore I = \int \frac{2dx}{x^3-6x^2+11x-6}$

$x^3 - 6x^2 + 11x - 6 = x^2(x-1)$

$-5x(x-1) + 6(x-1)$

$= (x-1)\left(x^2 - 5x + 6\right)$

$= (x-1)(x-2)(x-3)$

$\therefore I = 2\int \frac{dx}{(x-1)(x-2)(x-3)}$

Now, $\frac{1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$

Multiplying both sides by LCM

(x - 1) (x - 2) (x - 3)

1 = A(x - 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x - 2)

Putting x = 1, 2, 3 respectively, we get

$A = \frac{1}{2}, B = -1, C = \frac{1}{2}$

$I = \frac{2}{2}\int \frac{dx}{x-1} - 2\int \frac{dx}{x-2} + \frac{2}{2}\int \frac{dx}{x-3}$

$= \int \frac{dx}{x-1} - 2\int \frac{dx}{x-2} + \int \frac{dx}{x-3}$

$= \log|x-1| - \log|x-2|^2 + \log|x-3|$

$= \log\left|\left\{\frac{(x-1)(x-3)}{(x-2)^2}\right\}\right| + c$

Hence $\int \frac{2e^t}{e^{3t}-6e^{3t}+11e^t-6}dt$

$= \log\left|\left\{\frac{(e^t-1)(e^t-3)}{(e^t-2)^2}\right\}\right| + c$

Video Solution:

Q19 **Text Solution:**

$\frac{3x+4}{x^3-2x-4} = \frac{3x+4}{(x-2)(x^2+2x+2)}$

$= \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+2}$

$\Rightarrow 3x + 4 = A\left(x^2 + 2x + 2\right) + (Bx + C)(x - 2)$

$\therefore A + B = 0$

$2A - 2B + C = 3\} \Rightarrow A = 1, B = C = -1$

$2A - 2C = 4$

$\therefore \int \frac{3x+4}{x^3-2x-4}dx = \int \frac{dx}{x-2} - \frac{1}{2}\int \frac{2x+2}{x^2+2x+2}dx$

$= \log|x-2| - \frac{1}{2}\log\left|x^2+2x+2\right| + c$

$\Rightarrow k = -\frac{1}{2}$ and $f(x) = \left|x^2+2x+2\right|$

Video Solution:

Q20 **Text Solution:**

Let $\frac{2x+1}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{x-2}$

$A = \frac{2x+1}{x-2}$ put $x = -1$

$B = \frac{2x+1}{x+1}$ put $x = 2$

$\frac{2x+1}{(x+1)(x-2)} = \frac{\frac{1}{3}}{x+1} + \frac{\frac{5}{3}}{x-2}$

$\int \frac{2x+1}{(x+1)(x+2)3}dx = \frac{1}{3}\log\left|x+1\right| + \frac{5}{3}\log\left|x - 2\right| + c$

Video Solution:

Q21 **Text Solution:**

$$\int \tan^{-1}\sqrt{x}\cdot 1\,dx = \left(\tan^{-1}\sqrt{x}\right)\cdot x - \int \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}\cdot x dx$$

$$= x\tan^{-1}\sqrt{x} - \frac{1}{2}\int \frac{\sqrt{x}}{1+x}\,dx$$

Putting $\sqrt{x}=$ t in second term, we get

$$x\tan^{-1}\sqrt{x} - \int \frac{t^2}{1+t^2}\,dt$$

$$= x\tan^{-1}\sqrt{x} - \int\left(1-\frac{1}{1+t^2}\right)dt$$

$$= (x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + c$$

Video Solution:

Q22 **Text Solution:**

$$I = \int e^x\left(\frac{2+\sin 2x}{2\cos^2 x}\right)dx$$

$$= \int e^x\left(\frac{2}{2\cos^2 x} + \frac{2\sin x\cos x}{2\cos^2 x}\right)dx$$

$$= \int e^x\left(\sec^2 x + \tan x\right)dx$$

$$= \int e^x\left(\tan x + \sec^2 x\right)dx$$

$$= e^x\tan x + c$$

Video Solution:

Q23 **Text Solution:**

$$\int x\cdot\tan^{-1}x\,dx = \frac{x^2}{2}\tan^{-1}x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}\,dx$$

$$= \frac{1}{2}\tan^{-1}x\cdot\left(x^2+1\right) - \frac{1}{2}x + c$$

Video Solution:

Q24 **Text Solution:**

$$\int \frac{\log x}{x^3}dx = \int x^{-3}\log x\,dx$$

$$= -\frac{\log x}{2x^2} + \int \frac{1}{x}\cdot\frac{1}{2x^2}dx + c = -\frac{\log x}{2x^2} + \frac{1}{2}\cdot\frac{x^{-2}}{-2} + c$$

$$= -\frac{\log x}{2x^2} - \frac{1}{4x^2} + c = -\frac{1}{4x^2}(2\log x + 1) + c$$

Video Solution:

Q25 **Text Solution:**

$$\text{Let I} = \int 32x^3(\log x)^2 dx = 32\int x^3(\log x)^2 dx$$

$$= 32\left[(\log x)^2\int x^3 dx - \int\left(\frac{d}{dx}(\log x)^2\int x^3 dx\right)dx\right]$$

$$= 32\left[(\log x)^2\cdot\frac{x^4}{4} - \int 2\log x\cdot\frac{1}{x}\cdot\frac{x^4}{4}dx\right]$$

$$= 8x^4\left[(\log x)^2 - \frac{\log x}{2} + \frac{1}{8}\right] + c$$

$$= x^4\left[8(\log x)^2 - 4\log x + 1\right] + c$$

Video Solution:

Q26 **Text Solution:**

$I = \int e^x \left[\frac{1+x^2-2x}{(1+x^2)^2}\right] dx =$

$\int e^x \left[\frac{1}{1+x^2} + \frac{-2x}{(1+x^2)^2}\right] dx = \frac{e^x}{1+x^2} + c$

Video Solution:

Q27 **Text Solution:**

$I = \int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$

$= \int e^x \left[\frac{1+2\sin(x/2)\cos(x/2)}{2\cos^2(x/2)}\right] dx$

$I = \int e^x \left[\frac{1}{2}\sec^2(x/2) + \tan(x/2)\right] dx = e^x \cdot \tan(x/2) + c$

$\{\because \int e^x [f(x) + f'(x)] dx = e^x \cdot f(x) + c\}$

Video Solution:

Q28 **Text Solution:**

$\int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$

$= x\log(\log x) - \frac{x}{\log x} - \int \frac{1}{(\log x)^2} dx + \int \frac{1}{(\log x)^2} dx$

$= x\log(\log x) - \frac{x}{\log x} + c$

Video Solution:

Q29 **Text Solution:**

: Let $\mathrm{I} = \int \left(f(x)g''(x) - f''(x)g(x)\right) dx$

$= \int_{\mathrm{I}} f\left(x\right) g''_{\mathrm{II}}\left(x\right) dx - \int^{g}_{\mathrm{II}}(x)\, f''_{\mathrm{II}}\left(x\right) dx$

$= f(x)g'(x) - \int f'(x)g'(x)dx - g(x)f'(x) + \int g'(x)f'(x)dx$

$= f(x)g'(x) - g(x)f'(x) + c$

Video Solution:

Q30 **Text Solution:**

Put $x^2 = t \Rightarrow 2x\, dx = dt$, then

$\int x^e e^{x^2} dx = \frac{1}{2}\int te^t dt$

$= \frac{1}{2}\left[te^t - e^t\right] + c = \frac{1}{2}e^{x^2}\left(x^2 - 1\right) + c$

Video Solution:

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