ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture - 02

vectors and 3D

By – Guru sir

Recap of previous lecture

1. Vector Algebra
2.
3.
4.

Topics *to be covered*

1. Vector Algebra - continue
2. 3D
3.
4.

⊛ Section formula:-

① internal division:-

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}$$

② external division:-

$$\vec{r} = \frac{m\vec{b} - n\vec{a}}{m-n}$$

③ Midpoint :-

$$\vec{r} = \frac{\vec{a} + \vec{b}}{2}$$

Question

The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a $\triangle ABC$. The length of the median through A is

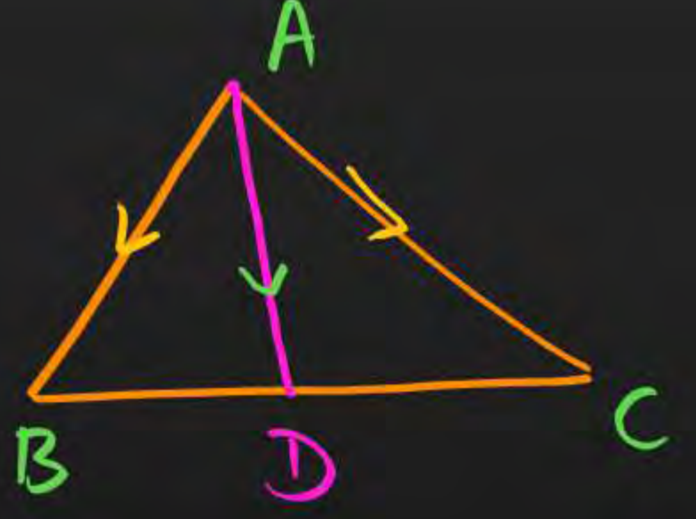
(A) $\sqrt{18}$

(B) $\sqrt{72}$

(C) $\sqrt{33}$

(D) $\sqrt{288}$

'D' is the midpoint of B & C



$|\overrightarrow{AD}| = ?$

$$\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

$$= \frac{8\hat{i} - 2\hat{j} + 8\hat{k}}{2}$$

$$\overrightarrow{AD} = 4i - j + 4\hat{k}$$

$$|\overrightarrow{AD}| = \sqrt{16+1+16} = \sqrt{33}$$

Question

The two vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}+5\hat{k}$ represents the two sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of a $\triangle ABC$. The length of the median though A is

(A) 14

(B) 7

(C) $\sqrt{14}$

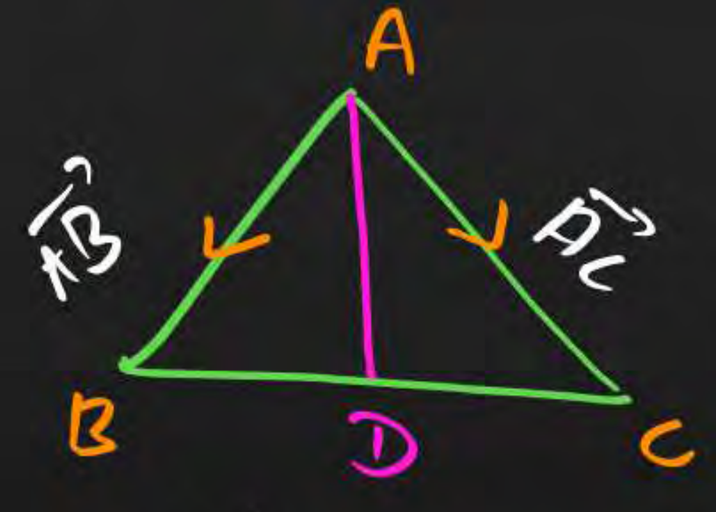
(D) $\frac{\sqrt{14}}{2}$

$$\overrightarrow{AD} = \frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$$

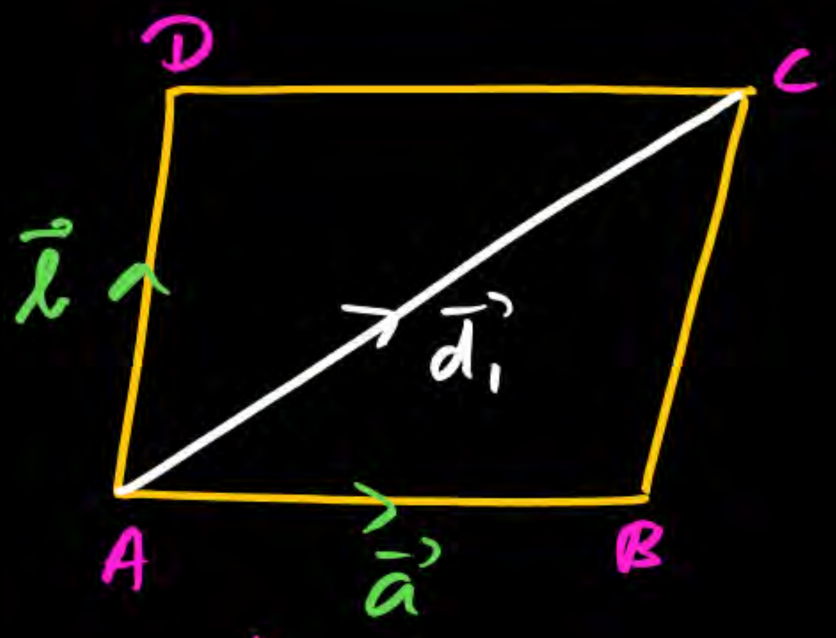
$$= \frac{2\hat{i}+4\hat{j}+6\hat{k}}{2}$$

$$\overrightarrow{AD} = \hat{i}+2\hat{j}+3\hat{k}$$

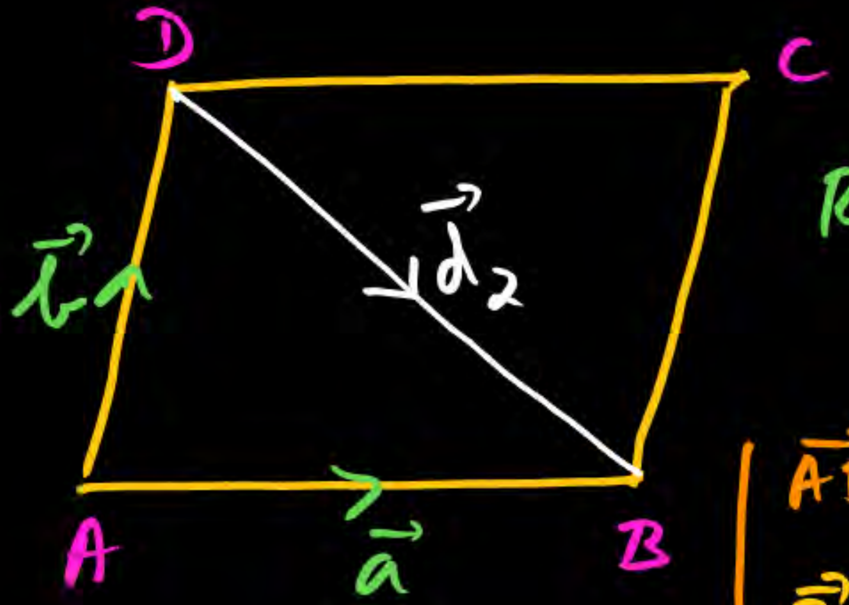
$$|\overrightarrow{AD}| = \sqrt{1+4+9} = \sqrt{14}$$



'D' is the midpoint



$\vec{d_1} = \vec{a} + \vec{b}$



Resultant $= \vec{AB}$

$\vec{d_2} = \vec{a} - \vec{b}$

$\vec{AB} = \vec{AD} + \vec{DB}$

$\vec{a} = \vec{b} + \vec{d_2}$

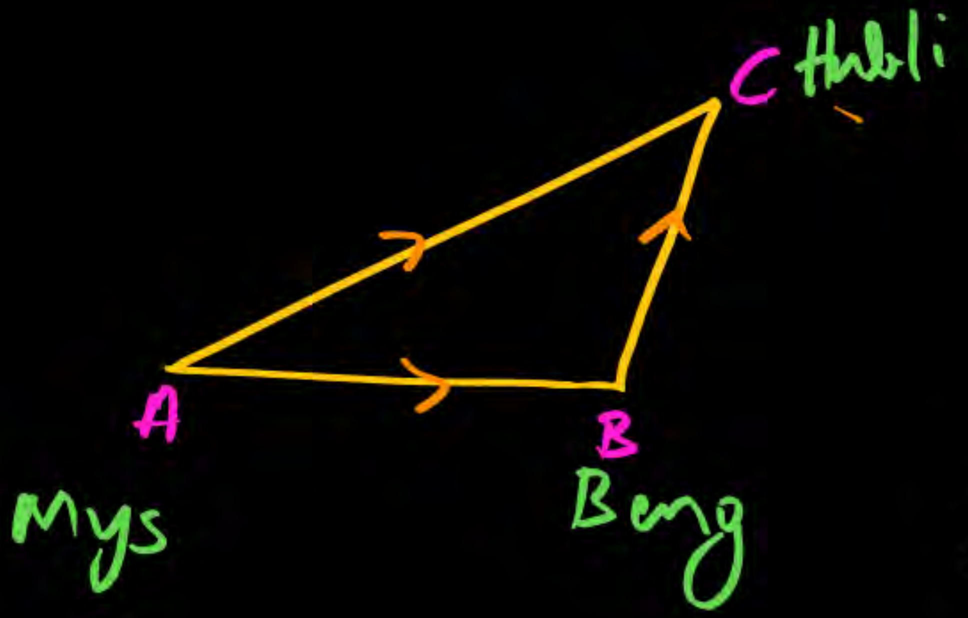
$\vec{d_2} = \vec{a} - \vec{b}$

$\therefore$ if $\vec{a}$ & $\vec{b}$ represents adjacent sides of a parallelogram

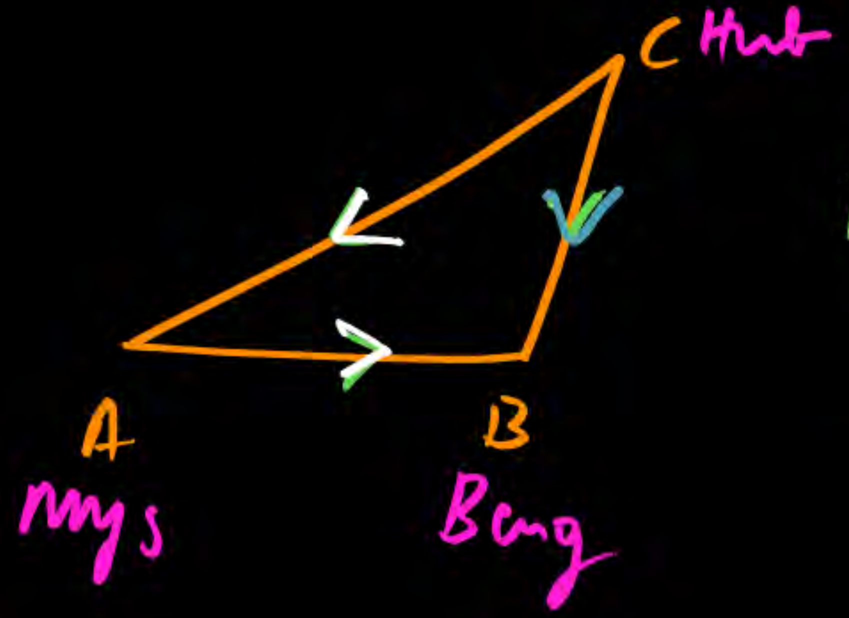
Then diagonals

$\vec{d_1} = \vec{a} + \vec{b}$

& $\vec{d_2} = \vec{a} - \vec{b}$



$\vec{AB} + \vec{BC} = \vec{AC} \rightarrow$ resultant



Resultant $= \vec{CB}$

$\vec{CB} = \vec{CA} + \vec{AB}$

Question

If $\hat{i}+\hat{j}-\hat{k}$ and $2\hat{i}-3\hat{j}+\hat{k}$ are adjacent sides of a parallelogram, then the lengths of its diagonals are

$\vec{a}$ $\quad$ $\vec{b}$

A. $\sqrt{21}, \sqrt{13}$

B. $\sqrt{3}, \sqrt{14}$

C. $\sqrt{13}, \sqrt{14}$

D. $\sqrt{21}, \sqrt{3}$

$\vec{d_1} = \vec{a}+\vec{b} = 3\hat{i}-2\hat{j}+0\hat{k}$

$|\vec{d_1}| = \sqrt{9+4} = \sqrt{13}$

$\vec{d_2} = \vec{a}-\vec{b} = -\hat{i}+4\hat{j}-2\hat{k}$

$|\vec{d_2}| = \sqrt{1+16+4} = \sqrt{21}$

Question

If direction cosines of a vector of magnitude 3 are $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$ and $a > 0$, then vector is

A $2\hat{i} - \hat{j} + 2\hat{k}$

B $\hat{i} + 2\hat{j} + 2\hat{k}$

C $2\hat{i} + \hat{j} + 2\hat{k}$

D $\hat{i} - 2\hat{j} + 2\hat{k}$

Question

$|\vec{a}|=3$ $|\vec{b}|=4$ $|\vec{c}|=5$

If $a=3, b=4, c=5$ each one of $\vec{a}, \vec{b} \& \vec{c}$ is perpendicular to the sum of the remaining then $|\vec{a}+\vec{b}+\vec{c}|$ is equal to

A. $\frac{5}{\sqrt{2}}$

B. $\frac{2}{\sqrt{5}}$

C. $5\sqrt{2}$

D. $\sqrt{5}$

$\vec{a} \perp (\vec{b}+\vec{c})$

$\vec{a}\cdot(\vec{b}+\vec{c})=0$

$\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0 \rightarrow (1)$

$\vec{b} \perp (\vec{a}+\vec{c})$

$\vec{b}\cdot(\vec{a}+\vec{c})=0$

$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}=0 \rightarrow (2)$

$\vec{c} \perp (\vec{a}+\vec{b})$

$\vec{c}\cdot(\vec{a}+\vec{b})=0$

$\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}=0 \rightarrow (3)$

$(1)+(2)+(3)$

$2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{a}\cdot\vec{c})=0$

WKT

$$|\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2\underbrace{(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})}_{0}$$

$$= 9+16+25$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 50$$

$$|\vec{a}+\vec{b}+\vec{c}| = \sqrt{50} = 5\sqrt{2}$$

Question

The value of x if $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector is

A $\pm\frac{1}{\sqrt{3}}$

B $\pm\sqrt{3}$

C ± 3

D $\pm\frac{1}{3}$

Question

If $\cos\alpha, \cos\beta, \cos\gamma$ are the direction cosines of a vector $\vec{a}$, then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is equal to

A 2

B 3

C -1

D 0

Question

A vector $\vec{a}$ makes equal acute angles on the coordinate axis. Then the projection of vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ on $\vec{a}$ is

A $\frac{11}{15}$

B $\frac{11}{\sqrt{3}}$

C $\frac{4}{5}$

D $\frac{3}{5\sqrt{3}}$

Question

The diagonals of a parallelogram are the vectors $3\hat{i}+6\hat{j}-2\hat{k}$ and $-\hat{i}-2\hat{j}-8\hat{k}$, then the length of the shorter side of parallelogram is

(A) $\sqrt{29}$ ✓

(B) $\sqrt{30}$

(C) $\sqrt{26}$

(D) $\sqrt{22}$

$\vec{d_1} = \vec{a}+\vec{b} = 3i+6j-2\hat{k} \rightarrow (1)$

$\vec{d_2} = \vec{a}-\vec{b} = -i-2j-8\hat{k} \rightarrow (2)$

$(1)+(2)$

$2\vec{a} = 2i+4j-10\hat{k}$

$\vec{a} = i+2j-5\hat{k}$

$|\vec{a}| = \sqrt{1+4+25} = \sqrt{30}$

$(1)-(2)$

$2\vec{b} = 4i+8j+6\hat{k}$

$\vec{b} = 2i+4j+3\hat{k}$

$|\vec{b}| = \sqrt{4+16+9} = \sqrt{29}$

Question

$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + |\vec{b}|^2$

If $\vec{a}\cdot\vec{b} = 0$ and $\underbrace{\overbrace{(\vec{a}+\vec{b})}^{\vec{c}} \text{ makes an angle } 60^\circ \text{ with } \vec{a}}$ then

$\cos\theta = \dfrac{\vec{a}\cdot\vec{c}}{|\vec{a}||\vec{c}|}$

(A) $|\vec{a}| = 2|\vec{b}|$

(B) $2|\vec{a}| = |\vec{b}|$

(C) $|\vec{a}| = \sqrt{3}|\vec{b}|$

(D) $\sqrt{3}|\vec{a}| = |\vec{b}|$

$\cos\theta = \dfrac{\vec{a}\cdot(\vec{a}+\vec{b})}{|\vec{a}|\,|\vec{a}+\vec{b}|}$

$\dfrac{1}{2} = \dfrac{|\vec{a}|^2 + \cancel{\vec{a}\cdot\vec{b}} \to 0}{|\vec{a}|\,|\vec{a}+\vec{b}|}$

$\dfrac{1}{2} = \dfrac{|\vec{a}|^2}{|\vec{a}||\vec{a}+\vec{b}|}$

$\dfrac{1}{2} = \dfrac{|\vec{a}|}{|\vec{a}+\vec{b}|}$

$|\vec{a}+\vec{b}| = 2|\vec{a}|$

$|\vec{a}+\vec{b}|^2 = 4|\vec{a}|^2$

$|\vec{a}|^2 + |\vec{b}|^2 = 4|\vec{a}|^2$

$|\vec{b}|^2 = 3|\vec{a}|^2$

$|\vec{b}| = \sqrt{3}|\vec{a}|$

Question

If vector $\vec{b} = 3\hat{j} + 4\hat{k}$ is written as the sum of a vector $\vec{b}_1$, parallel to $\vec{a} = \hat{i} + \hat{j}$ and a vector $\vec{b}_2$, perpendicular to $\vec{a}$, then $\vec{b}_1 \times \vec{b}_2$ is equal to

A $3\hat{i} - 3\hat{j} + 9\hat{k}$

B $-3\hat{i} + 3\hat{j} - 9\hat{k}$

C $-6\hat{i} + 6\hat{j} - 9/2\hat{k}$

D $6\hat{i} - 6\hat{j} + 9/2\hat{k}$

Question

The component of $\hat{i}$ in the direction of vector $\hat{i} + \hat{j} + 2\hat{k}$ is

A $6\sqrt{6}$

B $\sqrt{6}$

C $\frac{\sqrt{6}}{6}$

D 6

Question

If $\hat{i}, \hat{j}, \hat{k}$ are unit vectors along the positive direction of X, Y and Z - axes, then a FALSE statement in the following is [2010]

$\hat{i} = \hat{i} \times \hat{i} = \vec{0}$

(A) $\sum \hat{i} \times (\hat{j} \times \hat{k}) = \vec{0}$ True

(B) $\sum \hat{i} \times (\hat{j} + \hat{k}) = \vec{0}$ True

$i \cdot j + i \cdot k = 0$

(C) $\sum \hat{i} \cdot (\hat{j} + \hat{k}) = 0$ True

(D) $\sum \hat{i} \cdot (\hat{j} \times \hat{k}) = \vec{0}$ False

$\hat{i} \cdot \hat{i} + j \cdot j + k \cdot \hat{k} = 3$

$i \times j = \hat{k}$

$j \times k = \hat{i}$

$\hat{k} \times i = j$

$j \times \hat{i} = -\hat{k}$

$\hat{k} \times j = -i$

$i \times \hat{k} = -j$

$$\sum \hat{i}\times(\hat{j}+\hat{k})$$

$$=\hat{i}\times(\hat{j}+\hat{k})+\hat{j}\times(\hat{k}+\hat{i})+\hat{k}\times(\hat{i}+\hat{j})$$

$$=(\hat{i}\times\hat{j})+(\hat{i}\times\hat{k})+(\hat{j}\times\hat{k})+(\hat{j}\times\hat{i})+(\hat{k}\times\hat{i})+(\hat{k}\times\hat{j})$$

$$=\hat{k}-\hat{j}+\hat{i}-\hat{k}+\hat{j}-\hat{i}$$

$$=\vec{0}$$

Question

HW

If $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0}$ and $(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) = \lambda(\vec{b} \times \vec{c})$ then the value of λ is equal to [2023]

A 4

B 2

C 6

D 3

Question

If $\vec{p} = \hat{i} - 2\hat{j} + \hat{k}, \vec{q} = \hat{i} + 4\hat{j} - 2\hat{k}$ are the position vectors of points P, Q respectively and point $R(\vec{r})$ divides the line PQ internally in the ratio $2:1$, then coordinates of R are

(A) $(1, 2, 1)$

(B) $(1, -2, 1)$

(C) $(1, -2, -1)$

(D) $(1, 2, -1)$

$$\vec{r} = \frac{m\vec{q} + n\vec{p}}{m+n}$$

$$= \frac{(2\hat{i} + 8\hat{j} - 4\hat{k}) + (\hat{i} - 2\hat{j} + \hat{k})}{3}$$

$$= \hat{i} + 2\hat{j} - \hat{k}$$

$$(1, 2, -1)$$

Question

$\vec{r} = -8\vec{a} + 13\vec{b}$ | $\vec{q} = -2\vec{a} + 3\vec{b} + 2\vec{c}$

The position vectors of points P, Q, R are given by $\vec{p} = \vec{a} - 2\vec{b} + 3\vec{c}, \vec{q} = -2\vec{a} + 3\vec{b} + 2\vec{c}, \vec{r} = -8\vec{a} + 13\vec{b}$ If the points P, Q, R are collinear, then the ratio in which point P divides the line segment RQ is

(A) 3:1 externally

(B) 3:1 internally

(C) 1:3 externally

(D) 1:3 internally

Assume the ratio $m : n = K : 1$

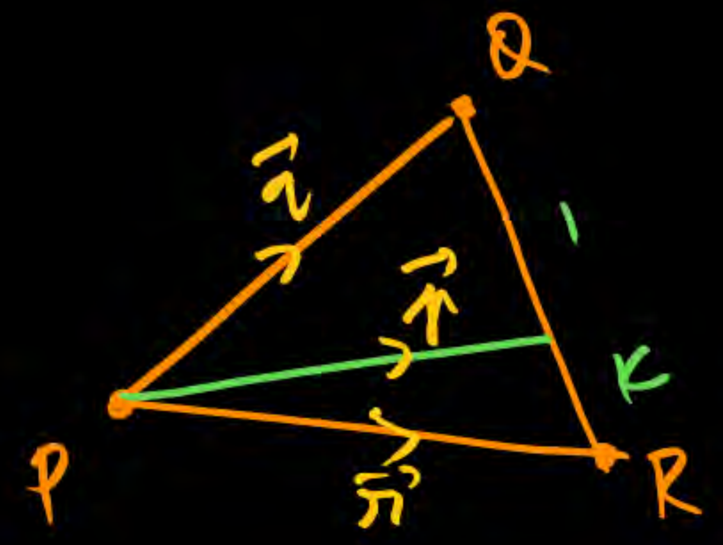
$$\vec{p} = \frac{K\vec{q} + (1)\vec{r}}{K+1}$$

$$\overset{\vec{a}}{1}, \overset{\vec{b}}{-2}, \overset{\vec{c}}{3} = \frac{(-2K\vec{a} + 3K\vec{b} + 2K\vec{c}) + (-8\vec{a} + 13\vec{b})}{K+1}$$

$$1 = \frac{-2K-8}{K+1}$$ | $K+1 = -2K-8$, $3K = -9$ | $K = -3$, $\frac{K}{1} = \frac{-3}{1}$

$K = +ve$ ↓ internal division

$K = -ve$ ↓ external division



Assume internal division

in the ratio $K:1$

$\downarrow \quad \downarrow$

$m \quad n$

$$\vec{r} = \frac{K\vec{q} + (1)\vec{\pi}}{K+1}$$

Question

If $\vec{a} = (1, p, 1), \vec{b} = (q, 2,2), \vec{a} \cdot \vec{b} = r$ and $\vec{a} \times \vec{b} = (0, -3,3)$, then find the value of p, q, r respectively.

(A) 1, 5, 9

(B) 9, 5, 1

(C) 5, 1, 9

(D) None of these

$\vec{a} \cdot \vec{b} = r$

$q + 2p + 2 = r$

$-1 + 2 + 2 = r$

$r = 3$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & p & 1 \\ q & 2 & 2 \end{vmatrix} = 0\hat{i} - 3\hat{j} + 3\hat{k}$$

$\hat{i}(2p-2) - \hat{j}(2-q) + \hat{k}(2-pq) = 0\hat{i} - 3\hat{j} + 3\hat{k}$

$2p - 2 = 0$

$p = 1$

$-(2-q) = -3$

$2 - q = 3$

$2 - 3 = q$

$q = -1$

$(p, q, r) = (1, -1, 3)$

Question

If $\vec{a} + 2\vec{b} - \vec{c} = \vec{0}$ and $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \lambda(\vec{a} \times \vec{b})$ then the value of λ is equal to

$\Downarrow$

$\vec{c} = \vec{a} + 2\vec{b}$

$(\vec{a} \times \vec{b}) + \vec{b} \times (\vec{a} + 2\vec{b}) + (\vec{a} + 2\vec{b}) \times \vec{a}$

$(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{a}) + 2(0) + (0) + 2(\vec{b} \times \vec{a})$

$(\vec{a} \times \vec{b}) - (\vec{a} \times \vec{b}) - 2(\vec{a} \times \vec{b})$

$= (-2)(\vec{a} \times \vec{b})$

$\Downarrow$

$\lambda = -2$

A 5

B 4

C 2

D -2

Question

If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 676$ and $|\vec{b}| = 2$, then $|\vec{a}|$ is equal to

A 13

B 26

C 39

D None of these

Question

$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to [2024]

A 12

B 3

C 8

D 4

Question

If $|\vec{a}| = |\vec{b}| = 1$ and $|\vec{a}+\vec{b}| = \sqrt{3}$, then the value of $(3\vec{a}-4\vec{b})\cdot(2\vec{a}+5\vec{b})$ is **[2024]**

$\Downarrow$

$|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = 3$

$1 + 1 + 2\vec{a}\cdot\vec{b} = 3$

$2\vec{a}\cdot\vec{b} = 3 - 2 = 1$

$\vec{a}\cdot\vec{b} = \frac{1}{2}$

$6|\vec{a}|^2 + 15\vec{a}\cdot\vec{b} - 8\vec{b}\cdot\vec{a} - 20|\vec{b}|^2$

$6 + 7\vec{a}\cdot\vec{b} - 20$

$-14 + 7\left(\frac{1}{2}\right)$

$\frac{-28+7}{2} = -\frac{21}{2}$

(A) -21

(B) $-21/2$

(C) 21

(D) $21/2$

Question

Given, $\vec{p} = 3\hat{i} + 2\hat{j} + 4\hat{k}, \vec{a} = \hat{i} + \hat{j}, b = \hat{j} + \hat{k}, \vec{c} = \hat{i} + \hat{k}$ and $\vec{p} = x\vec{a} + y\vec{b} + z\vec{c}$, then x, y, z are respectively [2022]

(A) $\frac{3}{2}, \frac{1}{2}, \frac{5}{2}$

(B) $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ ✓

(C) $\frac{5}{2}, \frac{3}{2}, \frac{1}{2}$

(D) $\frac{1}{2}, \frac{5}{2}, \frac{3}{2}$

$p = x\vec{a} + y\vec{b} + z\vec{c}$

$3i + 2j + 4\hat{k} = x(i + j) + y(\hat{j} + \hat{k}) + z(i + \hat{k})$

$3i + 2j + 4\hat{k} = i(x + 0 \cdot y + z) + j(x + y + 0 \cdot z) + \hat{k}(0 \cdot x + y + z)$

$x + z = 3$, $z = 3 - x$ | $x + y = 2 \rightarrow (1)$ | $y + z = 4$, $z = 4 - y$

$3 - x = 4 - y$

$x - y = -1 \rightarrow (2)$

(1) + (2): $2x = 1$, $x = 1/2$ | (1) − (2): $2y = 3$, $y = \frac{3}{2}$

$z = 4 - y = 4 - \frac{3}{2}$

$z = \frac{5}{2}$

Question

If $\vec{a} = 2\hat{i} - \hat{j} - m\hat{k}$ and $\vec{b} = \frac{4}{7}\hat{i} - \frac{2}{7}\hat{j} + 2\hat{k}$ are collinear, then the value of m is equal to

[2023]

(A) -7

(B) -1

(C) 2

(D) 7

Question

The vectors $3\hat{i}+5\hat{j}+2\hat{k}, 2\hat{i}-3\hat{j}-5\hat{k}$ and $5\hat{i}+2\hat{j}-3\hat{k}$ form the sides of [2020]

A. isosceles triangle

B. right triangle

C. scalene triangle

D. equilateral triangle

Question

$\vec{p} = \vec{OP} = \vec{a} + 2\vec{b} + 5\vec{c}$ | $\vec{q} = \vec{OQ} = 3\vec{a} + 2\vec{b} + \vec{c}$ | $\vec{r} = \vec{OR} = 2\vec{a} + 2\vec{b} + 3c$

If $\vec{p}, \vec{q}, \vec{r}$ are position vectors of the points P, Q, R respectively, where $\vec{p} = \vec{a} + 2\vec{b} + 5\vec{c}$, $\vec{q} = 3\vec{a} + 2\vec{b} + \vec{c}$, $\vec{r} = 2\vec{a} + 2\vec{b} + 3\vec{c}$, then the points P, Q, R are [2021]

(A) collinear

(B) non-collinear

(C) Form an acute angle triangle

(D) Both (B) and (C)

$\vec{PQ} = \vec{OQ} - \vec{OP} = 2\vec{a} + 0\vec{b} - 4\vec{c}$

$\vec{QR} = \vec{OR} - \vec{OQ} = -\vec{a} + 0\vec{b} + 2\vec{c}$

$\vec{PR} = \vec{OR} - \vec{OP} = \vec{a} + 0\vec{b} - 2\vec{c}$

$\vec{QR} = \underset{\lambda}{(-2)}\vec{PQ}$

$\Rightarrow \vec{PQ}$ & $\vec{QR}$ are parallel

$\vec{PR} = \underset{\lambda}{(-1)}\vec{QR}$

$\vec{QR}$ & $\vec{PR}$ are parallel

Here 'Q' is the common point

P, Q & R are collinear

Question

If $|\vec{a}| = 5, |\vec{b}| = 13$ and $|\vec{a} \times \vec{b}| = 25$, then find $\vec{a} \cdot \vec{b}$. [2018]

A ± 10

B ± 40

C ± 60

D ± 25

Question

If $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}$, then find $(\vec{a} + 3\vec{b}) \cdot (2\vec{a} - \vec{b})$. [2017]

$|\vec{a}| = \sqrt{1+1+4}$
$= \sqrt{6}$

$|\vec{b}| = \sqrt{9+4+1} = \sqrt{14}$

$2|\vec{a}|^2 - \vec{a}\cdot\vec{b} + 6\vec{b}\cdot\vec{a} - 3|\vec{b}|^2$

$= 2(6) + 5(3) - 3(14)$

$= 12 + 15 - 42$

$= 27 - 42$

$= \underline{-15}$

$\vec{a}\cdot\vec{b} = 3 + 2 - 2$
$= 3$

(A) -15

(B) 12

(C) 13

(D) -10

Question

If $|\vec{a}| = 15, |\vec{b}| = 12$ and $|\vec{a} + \vec{b}| = 20$, then $|\vec{a} - \vec{b}| =$ [2016]

(A) $\sqrt{338}$

(B) 338

(C) 769

(D) $\sqrt{769}$

Question

If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is [2016]

$|\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$

$\cos\theta = \sin\theta$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}$

A. $\pi/4$ only

B. $\pi/2$ only

C. $3\pi/4$ only

D. $\pi/4$ or $3\pi/4$

$\tan\frac{3\pi}{4} = -1$

Question

If $\vec{a}, \vec{b}, \vec{c}$ are three non-zero vectors such that each one of them are perpendicular to the sum of the other two vectors, then the value of $|\vec{a}+\vec{b}+\vec{c}|^2$ is [2015]

(A) $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2$

(B) $|\vec{a}| + |\vec{b}| + |\vec{c}|$

(C) $2(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)$

(D) $1/2(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2)$

Question

Find $\sin\theta$, if θ is the angle between the vectors $3\hat{i}+\hat{j}+2\hat{k}$ and $\hat{i}+\hat{j}+2\hat{k}$. [2014]

(A) $\sqrt{\frac{5}{21}}$

(B) $\frac{5}{\sqrt{21}}$

(C) $\frac{4}{\sqrt{21}}$

(D) $\sqrt{\frac{3}{21}}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}(4) + \hat{k}(2)$$

$$|\vec{a}\times\vec{b}| = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$|\vec{a}| = \sqrt{9+1+4} = \sqrt{14}$$

$$|\vec{b}| = \sqrt{1+1+4} = \sqrt{6}$$

$$|\vec{a}||\vec{b}| = \sqrt{6} \times \sqrt{14} = \sqrt{84}$$

$$= \sqrt{21\times 4} = 2\sqrt{21}$$

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|} = \frac{2\sqrt{5}}{2\sqrt{21}}$$

$$= \sqrt{\frac{5}{21}}$$

Question

$\rightarrow |\vec{a}| = |\vec{b}| = 2$

$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$

$|\vec{a}+\vec{b}|^2 = 4 + 4 + 2(2) = 12$

$|\vec{a}+\vec{b}| = \sqrt{12}$

$= 2\sqrt{3}$

If $\vec{a}$ and $\vec{b}$ are two vectors of magnitude 2 , each inclined at an angle 60°, then angle between $\vec{a}$ and $\vec{a}+\vec{b}$ is [2014]

A. 30°

B. 45°

C. 60°

D. 90°

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\cos 60 = \dfrac{\vec{a}\cdot\vec{b}}{2(2)}$

$\frac{1}{2}(4) = \vec{a}\cdot\vec{b}$

$\vec{a}\cdot\vec{b} = 2$

To find angle b/w $\vec{a}$ & $(\vec{a}+\vec{b})$

$\cos x = \dfrac{\vec{a}\cdot(\vec{a}+\vec{b})}{|\vec{a}|\,|\vec{a}+\vec{b}|}$

$= \dfrac{|\vec{a}|^2 + \vec{a}\cdot\vec{b}}{(2)\,|\vec{a}+\vec{b}|}$

$= \dfrac{4+(2)}{2(2\sqrt{3})}$

$\cos x = \dfrac{\cancel{6}^{\,3}}{2(\cancel{2})\sqrt{3}} = \dfrac{3}{2\sqrt{3}}$

$\cos x = \dfrac{\sqrt{3}}{2}$

$x = 30^\circ$

Question

If $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$, then the angle between $\vec{a}$ and $\vec{b}$ is [2012]

$|\vec{a} \times \vec{b}| = \sqrt{9+4+36} = \sqrt{49} = 7$

A) $\pi/3$

B) $\pi/6$

C) $\pi/2$

D) $\pi/4$

$\sin\theta = \dfrac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \dfrac{7}{2(7)} = \dfrac{1}{2}$

$\underline{\theta} = 30^\circ = \dfrac{\pi}{6}$

Question

$\vec{b} \times \vec{a} = -(\vec{a} \times \vec{b})$

The area of the parallelogram with $\vec{a}$ and $\vec{b}$ as adjacent sides is 20 sq. units. Then the area of the parallelogram having $7\vec{a} + 5\vec{b}$ and $8\vec{a} + 11\vec{b}$ as adjacent sides, is

[2011]

(A) 2960 sq. units

(B) 740 sq. units

(C) 1340 sq. units

(D) 3400 sq. units

Area $= 20$

$|\vec{a} \times \vec{b}| = 20$

Consider

$|(7\vec{a} + 5\vec{b}) \times (8\vec{a} + 11\vec{b})|$

$= |56(0) + 77(\vec{a} \times \vec{b}) + 40(\vec{b} \times \vec{a}) + 55(0)|$

$= |(77 - 40)(\underline{\vec{a} \times \vec{b}})|$

$= |37(\vec{a} \times \vec{b})|$

$= 37(20)$

$= \underline{740}$

Question

If $|\vec{a}| = 4, |\vec{b}| = 2, |\vec{c}| = 6$ and each of the angles between the vectors is 60°, then $|\vec{a} + \vec{b} + \vec{c}| =$ [2011]

(A) 10

(B) $\sqrt{56}$

(C) $\sqrt{44}$

(D) 5

$|\vec{a}+\vec{b}+\vec{c}|^2 = 16+4+36+2(4+6+12)$

$= 56+2(22)$

$= 56+44$

$|\vec{a}+\vec{b}+\vec{c}|^2 = 100$

$|\vec{a}+\vec{b}+\vec{c}| = 10$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 60$

$= 4(2)\left(\frac{1}{2}\right) = 4$

$\vec{b}\cdot\vec{c} = 2(6)\frac{1}{2} = 6$

$\vec{a}\cdot\vec{c} = 4(6)\frac{1}{2} = 12$

Question

The position vector of the point which divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio $3:1$ is

A $\frac{3\vec{a}-2\vec{b}}{2}$

B $\frac{7\vec{a}-8\vec{b}}{4}$

C $\frac{3\vec{a}}{4}$

D $\frac{5\vec{a}}{4}$

Question

Let $\vec{a}-2\vec{b}$ and $2\vec{a}-\vec{b}$ be position vectors of the points A and B respectively. The position vector of the point which divides AB in the ratio $3:2$ externally is

A $\frac{8}{5}\vec{a}-\frac{7}{5}\vec{b}$

B $4\vec{a}+\vec{b}$

C $\vec{a}+4\vec{b}$

D $\frac{7}{5}\vec{b}-\frac{8}{5}\vec{a}$

Question

$\vec{a}$ | $|\vec{a}| = \sqrt{15+25+9} = \sqrt{49} = 7$

The vector in the direction of the vector $\sqrt{15}\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$ that has magnitude 14 is

$\vec{b} = 14\ \hat{a}$ | $\hat{a} = \dfrac{\sqrt{15}\hat{i} - 5\hat{j} + 3\hat{k}}{7}$

$\vec{b} = 14\left(\dfrac{\sqrt{15}i - 5j + 3\hat{k}}{7}\right)$

$= 2(\sqrt{15}i - 5j + 3\hat{k})$

(A) $\sqrt{15}\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

(B) $\frac{\sqrt{15}\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{14}$

(C) $2(\sqrt{15}\hat{\imath} - 5\hat{\jmath} + 3\hat{k})$

(D) $7(\sqrt{15}\hat{\imath} - 5\hat{\jmath} + 3\hat{k})$

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Question

Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\vec{c} = \hat{a} + 2\hat{b}$ and $\vec{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then the angle between $\hat{a}$ and $\hat{b}$ is

(A) $\pi/3$

(B) $\pi/4$

(C) $\pi/6$

(D) $\pi/2$

$\vec{c} \cdot \vec{d} = 0$

$(\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 4\vec{b}) = 0$

$5|\vec{a}|^2 - 4\vec{a}\cdot\vec{b} + 10\vec{b}\cdot\vec{a} - 8|\vec{b}|^2 = 0$

$5(1) + 6\vec{a}\cdot\vec{b} - 8(1) = 0$

$\vec{a}\cdot\vec{b} = \frac{3}{6} = \frac{1}{2}$

$\cos\theta = \frac{1/2}{(1)(1)}$

$\theta = \frac{\pi}{3}$

Question

If $\vec{a},\vec{b},\vec{c}$ are vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $|\vec{a}|=7, |\vec{b}|=5, |\vec{c}|=3$ then angle between vector $\vec{b}$ and $\vec{c}$ is

(A) 60°

(B) 30°

(C) 45°

(D) 90°

$\vec{a}+\vec{b}+\vec{c}=0$

$\vec{b}+\vec{c}=-\vec{a}$

$|\vec{b}+\vec{c}|^2=|-\vec{a}|^2$

$|\vec{b}|^2+|\vec{c}|^2+2|\vec{b}||\vec{c}|\cos\theta=49$

$25+9+2(5)(3)\cos\theta=49$

$30\cos\theta=49-34$

$\cos\theta=\frac{15}{30}=\frac{1}{2}$

$\theta=\frac{\pi}{3}$

Question

If $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are perpendicular and $\vec{b}=3\hat{i}-4\hat{j}+2\hat{k}$, then $|\vec{a}|$ is equal to

A $\sqrt{41}$

B $\sqrt{39}$

C $\sqrt{19}$

D $\sqrt{29}$

Question

The angle subtended at the point $(1,2,3)$ by the points $P(2,4,5)$ and $Q(3,3,1)$, is

A 90°

B 60°

C 30°

D 0°

Question

For non-zero vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}+\vec{b}| < |\vec{a}-\vec{b}|$, then $\vec{a}$ and $\vec{b}$ are

(A) collinear

(B) perpendicular to each other

(C) inclined at an acute angle

(D) inclined at an obtuse angle

$$|\vec{a}+\vec{b}|^2 < |\vec{a}-\vec{b}|^2$$

$$\cancel{|\vec{a}|^2}+\cancel{|\vec{b}|^2}+2\vec{a}\cdot\vec{b} < \cancel{|\vec{a}|^2}+\cancel{|\vec{b}|^2}-2\vec{a}\cdot\vec{b}$$

$$2\vec{a}\cdot\vec{b} < -2\vec{a}\cdot\vec{b}$$

$$4\vec{a}\cdot\vec{b} < 0$$

$$\vec{a}\cdot\vec{b} < 0$$

$$\cos\theta < 0$$

$$\cos\theta = -ve$$

$\theta \in 2^{nd}$ Quad

θ = obtuse

Question

$|\vec{a}| = \sqrt{1+1+2} = \sqrt{4}$

$\vec{a}\cdot\vec{b} = b_1 + b_2 + 2$

Let $\vec{a} = \hat{i} + \hat{j} + \sqrt{2}\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + \sqrt{2}\hat{k}$ and $\vec{c} = 5\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ be three vectors such that the projection of $\vec{b}$ on $\vec{a}$ is $|\vec{a}|$. If $\vec{a} + \vec{b}$ is perpendicular to $\vec{c}$, then $|\vec{b}|$ is equal to

(A) $\sqrt{32}$

(B) $\sqrt{22}$

(C) 4

(D) 6

$\Downarrow$

$\dfrac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = |\vec{a}|$

$\vec{a}\cdot\vec{b} = |\vec{a}|^2$

$b_1 + b_2 + 2 = 4$

$b_1 + b_2 = 2$

$\Downarrow$

$b_2 = 2 - b_1$

$\Downarrow$

$(\vec{a} + \vec{b})\cdot\vec{c} = 0$

$\vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c} = 0$

$(5+1+2) + (5b_1 + b_2 + 2) = 0$

$10 + 5b_1 + b_2 = 0$

$10 + 5b_1 + 2 - b_1 = 0$

$12 + 4b_1 = 0$

$b_1 = -3$

$\therefore b_2 = 2-(-3) = 5$

$\therefore \vec{b} = -3\hat{i} + 5\hat{j} + \sqrt{2}\hat{k}$

$\Downarrow$

$|\vec{b}| = \sqrt{9+25+2}$

$= \sqrt{36} = \underline{6}$

Question

If the projection of $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ on $\vec{b} = 2\hat{i} + \lambda\hat{k}$ is zero, then the value of λ is

Projection $= 0$

(A) 0

$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = 0$

(B) 1

$\vec{a} \cdot \vec{b} = 0$

(C) $-2/3$

$2 + -2(0) + 3\lambda = 0$

$2 + 3\lambda = 0$

(D) $-3/2$

$\lambda = -\frac{2}{3}$

Question

Let $\vec{u}, \vec{v}, \vec{w}$ be such that $|\vec{u}| = 1, |\vec{v}| = 2, |\vec{w}| = 3$. If the projection of $\vec{v}$ along $\vec{u}$ is equal to that of $\vec{w}$ along $\vec{u}$ and $\boxed{\vec{v}, \vec{w} \text{ are perpendicular}}$ to each other, then $|\vec{u} - \vec{v} + \vec{w}|$ equals

(A) $\sqrt{14}$

(B) $\sqrt{7}$

(C) 2

(D) 14

Projection of $\vec{v}$ on $\vec{u}$ = Projection of $\vec{w}$ on $\vec{u}$

$$\frac{\vec{v}\cdot\vec{u}}{|\vec{u}|} = \frac{\vec{w}\cdot\vec{u}}{|\vec{u}|}$$

$$\vec{u}\cdot\vec{v} = \vec{u}\cdot\vec{w}$$

$$\vec{v}\cdot\vec{w} = 0$$

consider

$$|\vec{u}-\vec{v}+\vec{w}|^2$$

$$= |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(-\vec{u}\cdot\vec{v} - \vec{v}\cdot\vec{w} + \vec{u}\cdot\vec{w})$$

$$= 1 + 4 + 9 + 2(\cancel{-\vec{u}\cdot\vec{v}} - 0 + \cancel{\vec{u}\cdot\vec{w}})$$

$$|\vec{u}-\vec{v}+\vec{w}|^2 = 14$$

$$|\vec{u}-\vec{v}+\vec{w}| = \sqrt{14}$$

Question

$\rightarrow \cos\left(\frac{5\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

The angle between $\vec{a}$ and $\vec{b}$ is $\frac{5\pi}{6}$ and the projection of $\vec{a}$ on $\vec{b}$ is $\frac{-9}{\sqrt{3}}$, then $|\vec{a}|$ is equal to

A) 12

B) 8

C) 10

D) 6

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = -\frac{9}{\sqrt{3}}$$

$$\frac{|\vec{a}|\cancel{|\vec{b}|}\cos\theta}{\cancel{|\vec{b}|}} = \frac{-9}{\sqrt{3}}$$

$$|\vec{a}|\left(-\frac{\sqrt{3}}{2}\right) = -\frac{9}{\sqrt{3}}$$

$$|\vec{a}| = \frac{9(2)}{3} = 6$$

Question

If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 3$, then the range of $|\lambda\vec{a}|$ is

$\times^{ly}$ by $4 = |\vec{a}|$

(A) [0, 8]

$-12 \leq \lambda|\vec{a}| \leq 12$

(B) [−12, 8]

Take mod

(C) [0, 12]

$0 \leq |\lambda| |\vec{a}| \leq 12$

(D) [8, 12]

$0 \leq |\lambda\vec{a}| \leq 12$

Question

If $|\vec{a}| = 4$ and $-3 \leq \lambda \leq 1$, then the range of $|\lambda\vec{a}|$ is

$\Downarrow$

x^{ly} by $|\vec{a}| = 4$

A [0, 8]

$-12 \leq \lambda|\vec{a}| \leq 4$

B [−12, 8]

Take mod

C [0, 12]

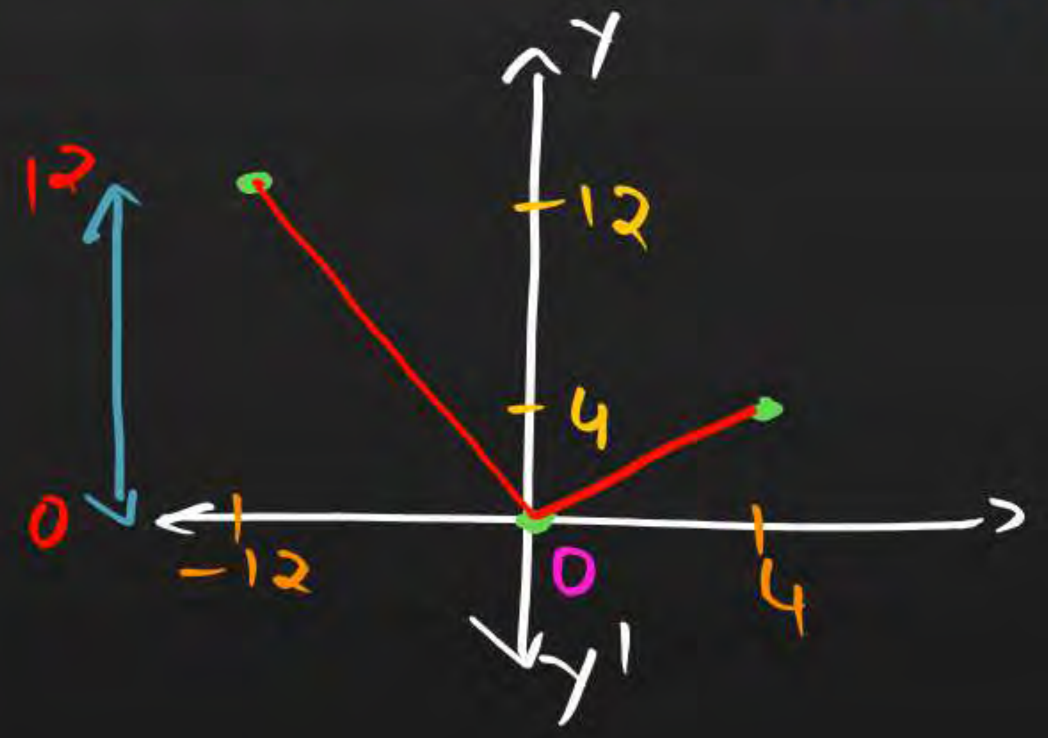
$0 \leq |\lambda||\vec{a}| \leq 12$

D [8, 12]

$0 \leq |\lambda\vec{a}| \leq 12$

if $x \in [-12, 4]$ blw -12 & 4

0 is also the input



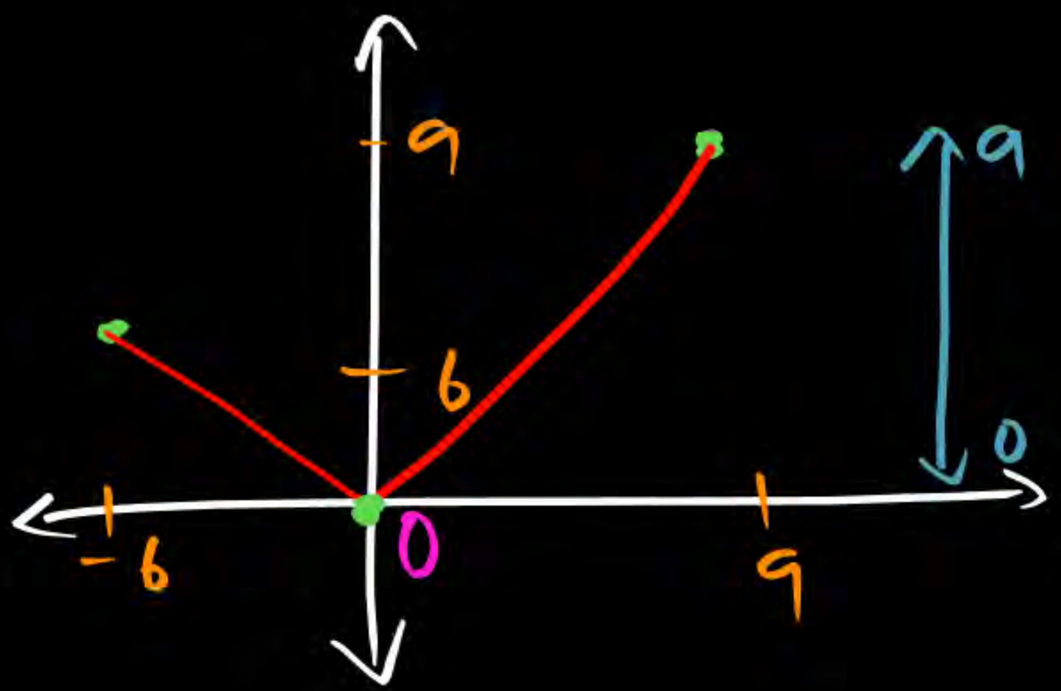
② if $x \in [-6, 9]$ → b/w -6 & 9

'0' is one of the input

Find $|x|$

Ans:

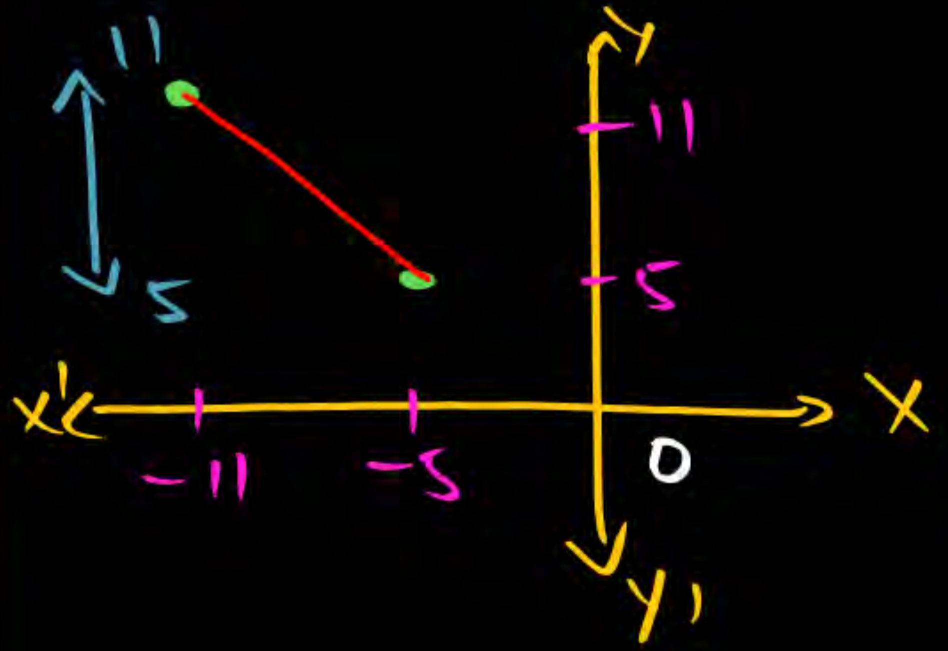
$[0, 9]$



③ if $x \in [-11, -5]$

'0' is not the input

Find $|x|$



$|x| \in [5, 11]$

Question

The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is

(A) One

(B) Two

(C) Three

(D) Infinite

$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

$\hat{n}_1 = \frac{-(\vec{b} \times \vec{a})}{|\vec{a} \times \vec{b}|}$

Question

If $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0}$, then $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} =$

$\Downarrow$

$\vec{a} = -2\vec{b} - 3\vec{c}$

$(-2\vec{b} - 3\vec{c}) \times \vec{b} + (\vec{b} \times \vec{c}) + \vec{c} \times (-2\vec{b} - 3\vec{c})$

$= -2(\vec{b} \times \vec{b}) - 3(\vec{c} \times \vec{b}) + (\vec{b} \times \vec{c}) - 2(\vec{c} \times \vec{b}) - 3(\vec{c} \times \vec{c})$

$= -2\vec{0} + 3(\vec{b} \times \vec{c}) + (\vec{b} \times \vec{c}) + 2(\vec{b} \times \vec{c}) - 2(\vec{0})$

$= \underline{6(\vec{b} \times \vec{c})}$

(A) $\vec{0}$

(B) $6(\vec{b} \times \vec{c})$

(C) $2(\vec{b} \times \vec{c})$

(D) $5(\vec{c} \times \vec{a})$

Question

If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a} \cdot \vec{b} \geq 0$ only when

$|\vec{a}|\,|\vec{b}| \cos\theta \geq 0$

$\cos\theta \geq 0$

$\theta \in \left[0, \frac{\pi}{2}\right]$

(A) $0 < \theta < \frac{\pi}{2}$

(B) $0 \leq \theta \leq \frac{\pi}{2}$

(C) $0 < \theta < \pi$

(D) $0 \leq \theta \leq \pi$

Thank

You