
ULTIMATE KCET
CRASH COURSE 2026
Mathematics
Lecture - 03
vectors and 3D
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Topics *to be covered*

1. 3D — Class 11^{th} / Class 12^{th}
2.
3.
4.

2D
⇓
4 - Quadrants

	I	II	III	IV
x	+	−	−	+
y	+	+	−	−

3D
⇓
8 - Octants

	I	II	III	IV	V	VI	VII	VIII
x	+	−	−	+	+	−	−	+
y	+	+	−	−	+	+	−	−
z	+	+	+	+	−	−	−	−

upper Part

Lower Part

Vectors

line

Direction Angles :-

2 pairs

$\Downarrow$

1^{st} Pair $\Rightarrow$ α, β, γ

2^{nd} Pair $\Rightarrow$ $\pi-\alpha, \pi-\beta, \pi-\gamma$

Direction ratios

(a, b, c) or $(-a, -b, -c)$

$\downarrow$ 1^{st} Pair

$\downarrow$ 2^{nd} Pair.

Ex:

$(+2, -3, +4)$ or $(-2, +3, -4)$

Direction cosines

$$l = \pm\frac{a}{r}$$

$$m = \pm\frac{b}{r}$$

$$n = \frac{\pm c}{r}$$

$$r = \sqrt{a^2+b^2+c^2}$$

(*) $l^2+m^2+n^2=1$

$\Downarrow$

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

$\boxed{D.R's}$

+ve x-axis → $(k, 0, 0)$

+ve y-axis → $(0, k, 0)$

+ve z-axis → $(0, 0, +k)$

−ve x-axis → $(-k, 0, 0)$

−ve y-axis → $(0, -k, 0)$

−ve z-axis → $(0, 0, -k)$

$\boxed{\text{D.C's}}$

+ve x-axis $\rightarrow (1,0,0)$

+ve y-axis $\rightarrow (0,1,0)$

+ve z-axis $\rightarrow (0,0,1)$

-ve x-axis $\rightarrow (-1,0,0)$

-ve y-axis $\rightarrow (0,-1,0)$

-ve z-axis $\rightarrow (0,0,-1)$

2D

① eqⁿ of x-axis :-

$$\boxed{y=0}$$

② eqⁿ of y-axis :-

$$\boxed{x=0}$$

3D :-

① eqⁿ of x-axis :-

$y=0$ & $z=0$

② eqⁿ of y-axis

$x=0$ & $z=0$

③ eqⁿ of z-axis

$x=0$ & $y=0$

Co-ordinates – 3D

① on x-axis :-

$(x, 0, 0)$

② on y-axis :-

$(0, y, 0)$

③ on z-axis :-

$(0, 0, z)$

① In XY Plane

$(x, y, 0)$

② In YZ Plane

$(0, y, z)$

③ In XZ Plane

$(x, 0, z)$

Question

A vector $\vec{a}$ makes equal acute angles on the coordinate axis. Then the projection of vector $\vec{b} = 5\hat{i} + 7\hat{j} - \hat{k}$ on $\vec{a}$ is

(A) $\frac{11}{15}$

(B) $\frac{11}{\sqrt{3}}$

(C) $\frac{4}{5}$

(D) $\frac{3}{5\sqrt{3}}$

Question

If $\vec{\alpha} = \hat{i} - 3\hat{j}, \vec{\beta} = \hat{i} + 2\hat{j} - \hat{k}$ then, express $\vec{\beta}$ in the form $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$ then $\vec{\beta}_1$ is given by

(A) $\frac{-1}{2}(\hat{i} - 3\hat{j})$

(B) $\frac{\hat{i} - 3\hat{j}}{2}$

(C) $\frac{3}{2}(\hat{i} + 3\hat{j})$

(D) $\hat{i} + 3\hat{j}$

Question

If vector $\vec{b} = 3\hat{j} + 4\hat{k}$ is written as the sum of a vector $\vec{b}_1$, parallel to $\vec{a} = \hat{i} + \hat{j}$ and a vector $\vec{b}_2$, perpendicular to $\vec{a}$, then $\vec{b}_1 \times \vec{b}_2$ is equal to

A $3\hat{i} - 3\hat{j} + 9\hat{k}$

B $-3\hat{i} + 3\hat{j} - 9\hat{k}$

C $-6\hat{i} + 6\hat{j} - 9/2\hat{k}$

D $6\hat{i} - 6\hat{j} + 9/2\hat{k}$

Question

The component of $\hat{i}$ in the direction of vector $\hat{i} + \hat{j} + 2\hat{k}$ is

A $6\sqrt{6}$

B $\sqrt{6}$

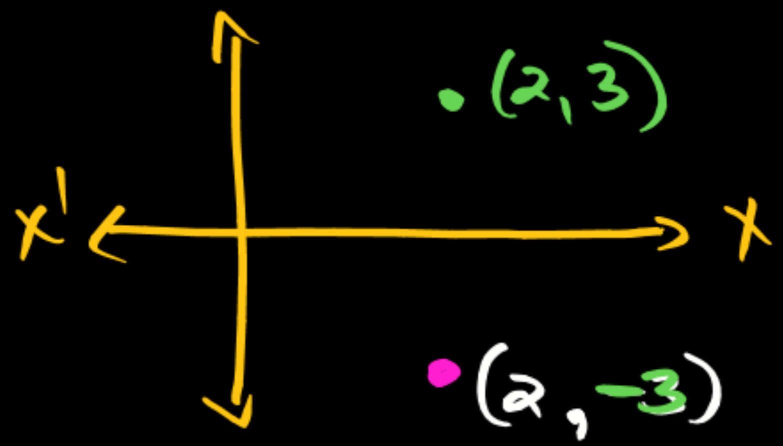
C $\frac{\sqrt{6}}{6}$

D 6

(*) image of $(2,3) \rightarrow$ 1st quad

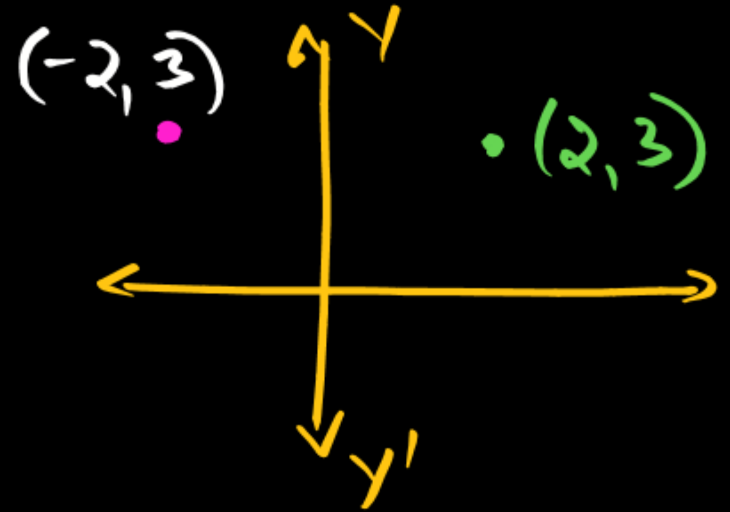
(1) along x-axis :-

$(2,-3)$



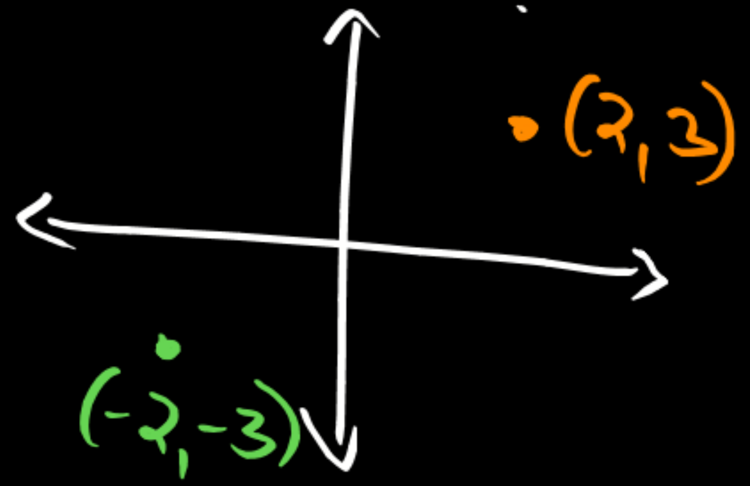
(2) along y-axis :-

$(-2,3)$



(3) along origin :-

$(-2,-3)$



(x, y, z)

xz Plane

↳ sign of 'y' changes

$(x, -y, z)$

(x, y, z)

yz Plane

↳ Sign of 'x' changes

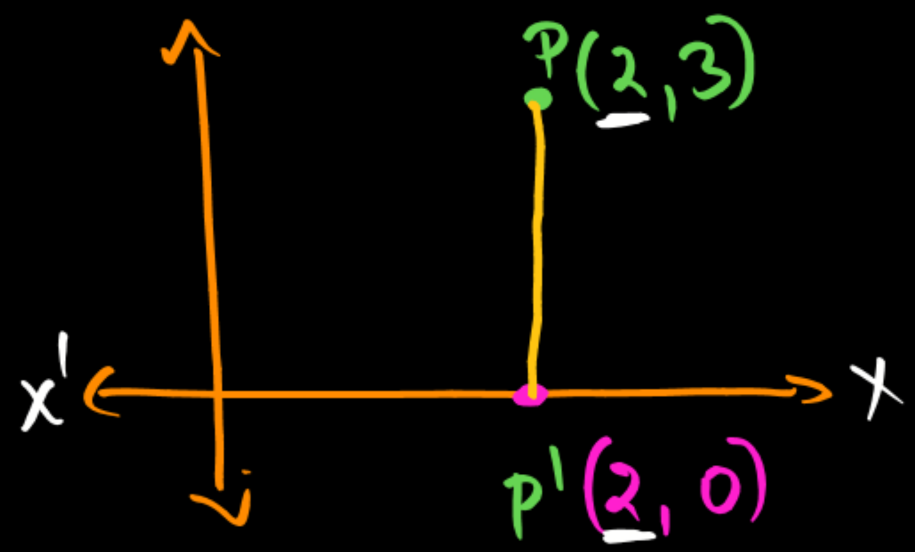
$(-x, y, z)$

(x, y, z)

xy Plane

↳ Sign of 'z'-changes

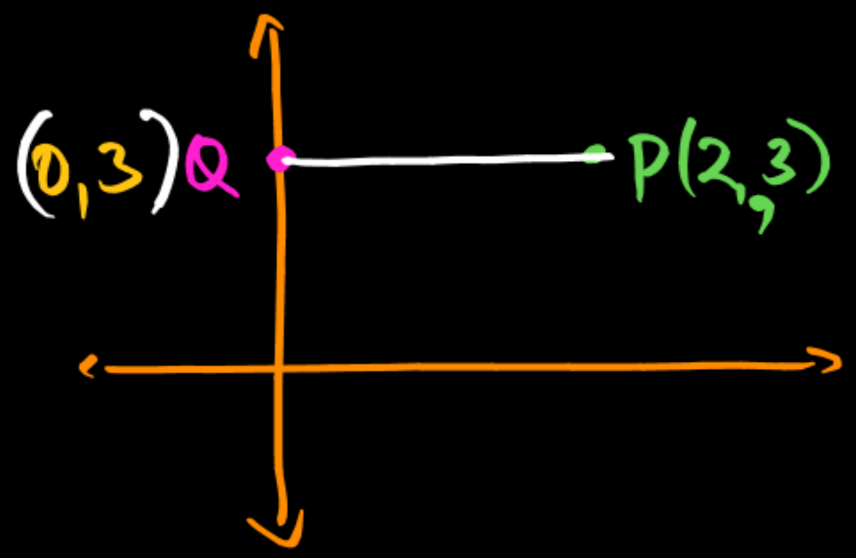
$(x, y, -z)$



$PP' = 3$

Foot of P (2, 0)

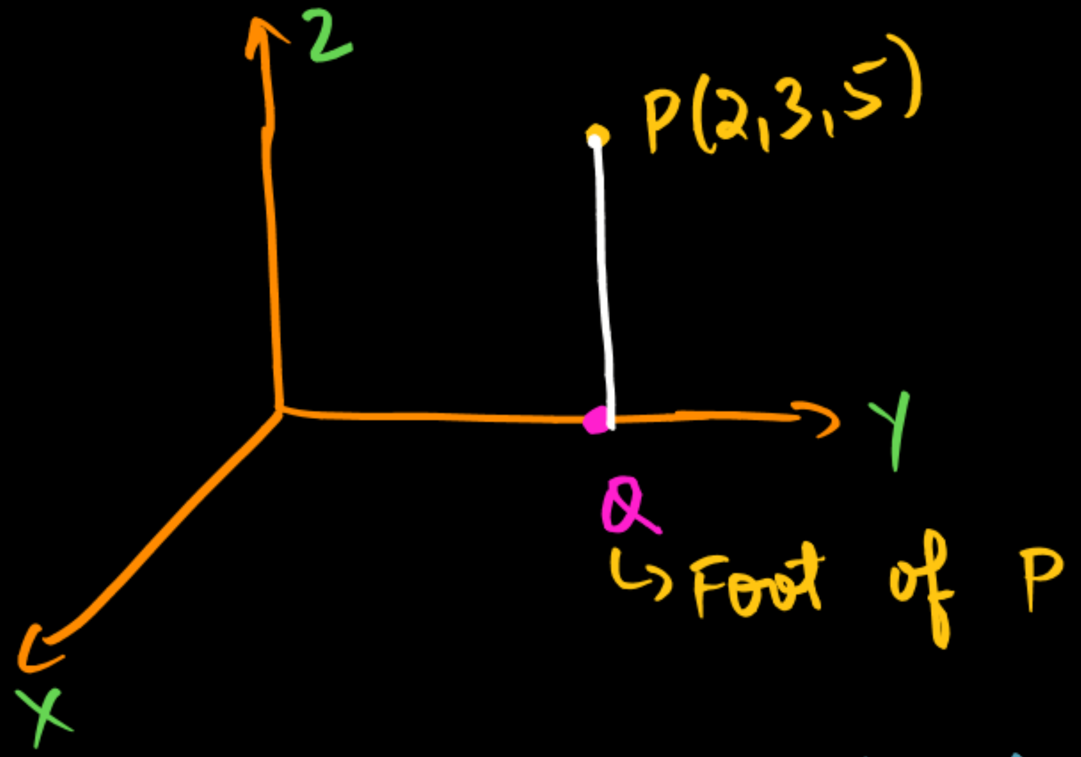
coordinates



$$QQ' = \sqrt{(2-0)^2 + (3-3)^2}$$

$$= \sqrt{2^2}$$

$$QQ' = 2$$



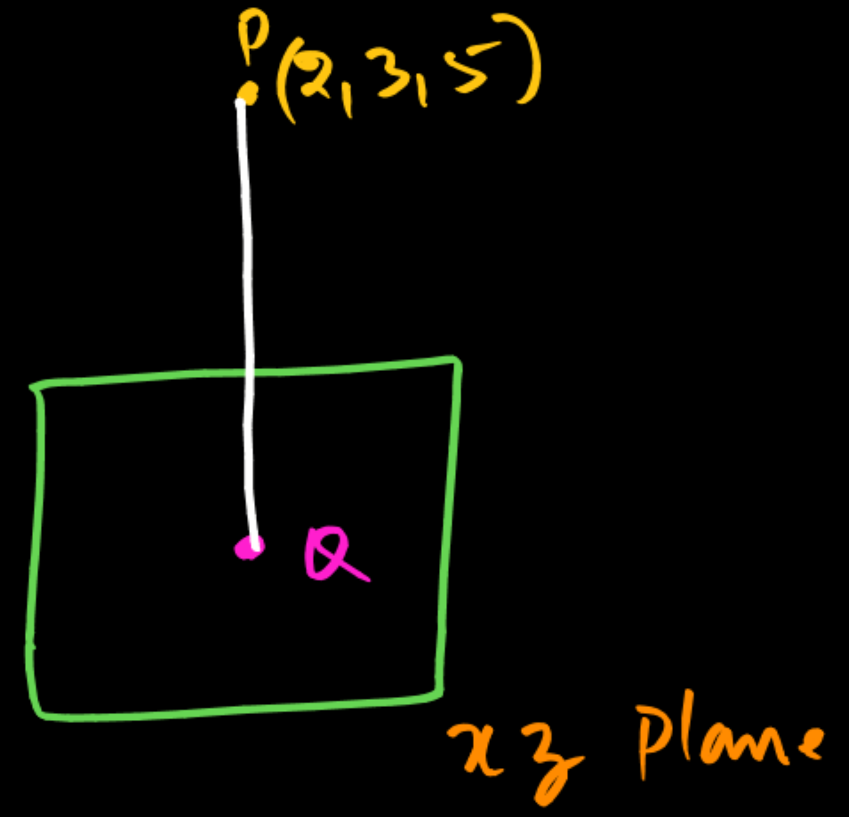
on y-axis

$\Downarrow$

$x = 0$ & $z = 0$

Coordinates of Q = $(0,3,0)$

$$PQ = \sqrt{2^2+0^2+5^2} = \sqrt{4+25} = \sqrt{29}$$



in xz plane

$\Downarrow$

$y=0$

Along xz-Plane

↳ sign of y changes

Coordinates of $Q = (2,0,5)$

$PQ = \sqrt{0^2+3^2+0^2} = \sqrt{9} = 3$

image of $P = \underline{(2,-3,5)}$

Question

The image of the point $(-2,0,0)$ in the xz-plane is

$\Downarrow$

$(-2,-0,0)$

$\Downarrow$

$(-2,0,0)$

$\hookrightarrow$ sign of y changes

(A) $(0,-2,0)$

(B) $(0,0,-2)$

(C) $(-2,0,0)$

(D) $(0,0,2)$

Question

image

The coordinates of the reflection of the point $P(4,2,3)$ in the xy-plane is

$\Downarrow$

$P(4,2,-3)$

A $(4, -2, 3)$

B $(4, 2, 3)$

C $(4, 2, -3)$

D $(4, -2, -3)$

Question

M is the foot of the perpendicular drawn from the point $A(6,7,8)$ on the yz-plane. What are the coordinates of point M ?

$\Downarrow$

$x = 0$

$\downarrow$

$M(0, 7, 8)$

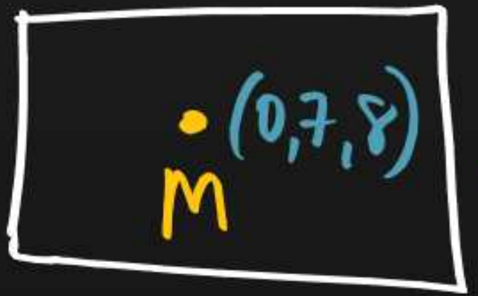
(A) (6,0,0)

(B) (6,7,0)

(C) (6,0,8)

(D) (0,7,8)

$\cdot\ A(6,7,8)$



yz plane

$\Downarrow$

$x = 0$

Question

The octant in which the point $(2,6,-2)$ lie is

$\underbrace{+,+}_{\downarrow \, 5^{th}},-$ ↳ lower

(A) IV

(B) V

(C) VI

(D) None of these

Question

Name the octant in which the point $(\underbrace{-3,-1,6}_{3^{rd}})$ lies.

A I

B III

C V

D VIII

Question

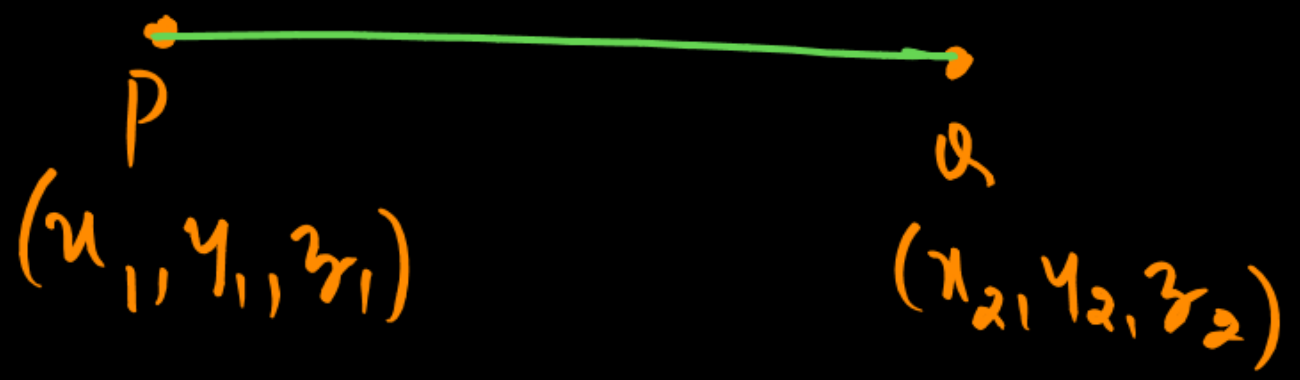
The point $(-2, -3, -4)$ lies in the

(A) First octant

(B) Seventh octant

(C) Second octant

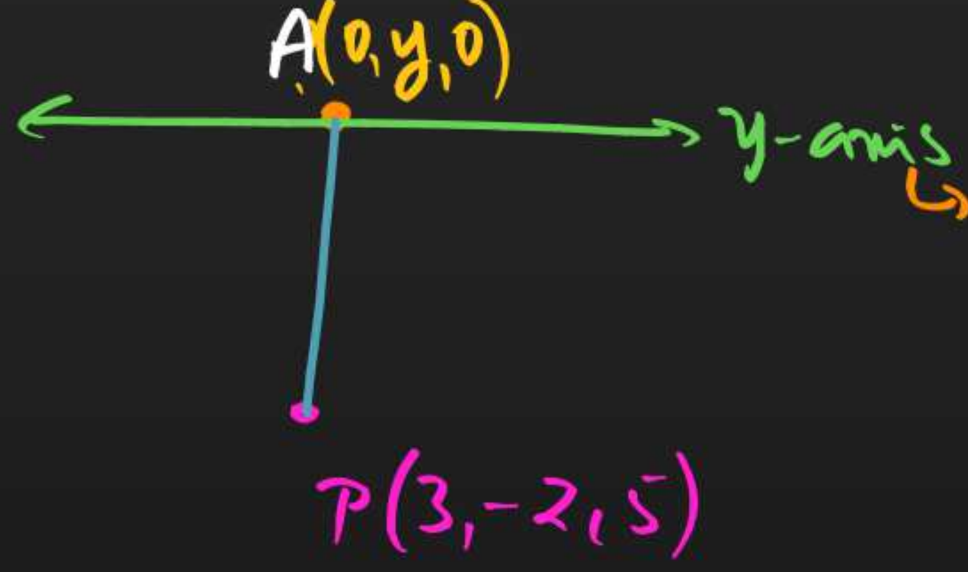
(D) Eighth octant



$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Question

Find the coordinates of a point on y-axis which is at a distance of $5\sqrt{2}$ from the point $P(3,-2,5)$.



↳ on y-axis

$x = 0$ & $z = 0$

(A) $(0,-6,0)$

(B) $(0,2,0)$

(C) Both (A) and (B)

(D) None of these

$AP = 5\sqrt{2}$

$AP^2 = 50$

$3^2 + (y+2)^2 + 5^2 = 50$

$34 + y^2 + 4 + 4y = 50$

$y^2 + 4y = 12$

$y^2 + 4y - 12 = 0$

$(y+6)(y-2) = 0$

-12

$+6 \quad -2$

$y = -6$ & $y = 2$

$\Downarrow \qquad \Downarrow$

$(0,-6,0) \qquad (0,2,0)$

Question

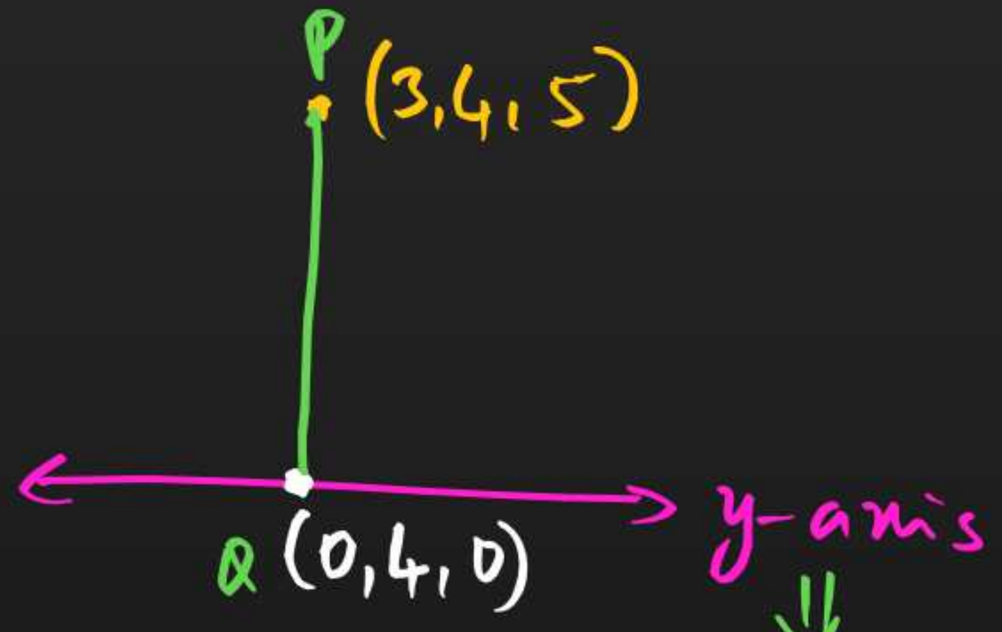
The length of the foot of perpendicular drawn from the point $P(3,4,5)$ on y-axis is

(A) 10 units

(B) $\sqrt{34}$ units

(C) $\sqrt{113}$ units

(D) $5\sqrt{2}$ units



$\Downarrow$

$x = 0$ & $z = 0$

$$PQ = \sqrt{3^2 + 0 + 5^2}$$

$$= \sqrt{9+25} = \sqrt{34}$$

Question

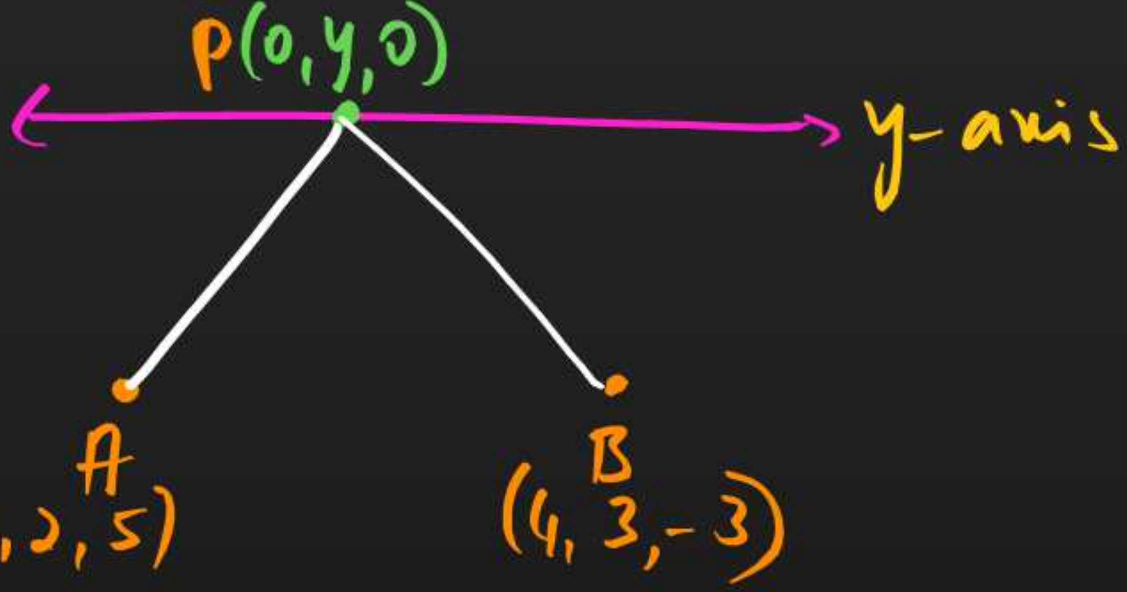
Find the point on the y-axis which is equidistant from the points $\overset{A}{(0,2,5)}$ and $\underset{B}{(4,3,-3)}$.

A $\left(0, \frac{1}{2}, 0\right)$

B $\left(0, \frac{3}{2}, 0\right)$

C $\left(0, \frac{7}{2}, 0\right)$

D $\left(0, \frac{5}{2}, 0\right)$ ✓



$AP = BP$

$AP^2 = BP^2$

$0 + (2-y)^2 + 5^2 = 4^2 + (3-y)^2 + (-3)^2$

$4 + y^2 - 4y + \cancel{25} = \cancel{16} + \cancel{9} + y^2 - 6y + 9$

$\cancel{y^2} - 4y + 4 = \cancel{y^2} - 6y + 9$

$2y = 5$

$y = \frac{5}{2}$

$\Downarrow$

$\left(0, \frac{5}{2}, 0\right)$

Question

Find the point on y-axis which is at a distance of $\sqrt{10}$ units from the point $\overset{A}{(1,2,3)}$.

$P(0, y, 0)$

$PA = \sqrt{10}$

$PA^2 = 10$

$1^2 + (y-2)^2 + 3^2 = 10$

$(y-2)^2 = 0$

$y - 2 = 0$

$y = 2$

A (0,4,0)

B (0,3,0)

C (0,2,0)

D (0, −1,0)

Question

What is the perpendicular distance of the point $A(1,0,7)$ from the yz-plane?

$\Downarrow$
$x=0$

$P(0,0,7)$

(A) 1 unit

(B) 2 units

(C) 3 units

(D) 4 units

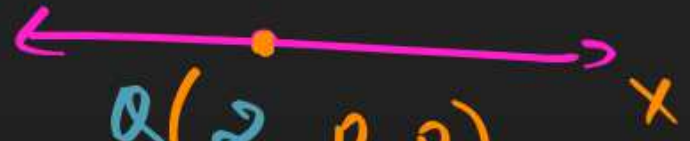
$AP = \sqrt{1^2+0^2+0^2} = 1$

Question

The distance of the point $A(2,3,2)$ from the x-axis is

$\Downarrow$

$y=0, z=0$



$Q(2,0,0)$

(A) 5 units

(B) $\sqrt{13}$ units

$PQ=\sqrt{0+3^2+2^2}=\sqrt{13}$

(C) $2\sqrt{5}$ units

(D) $5\sqrt{2}$ units

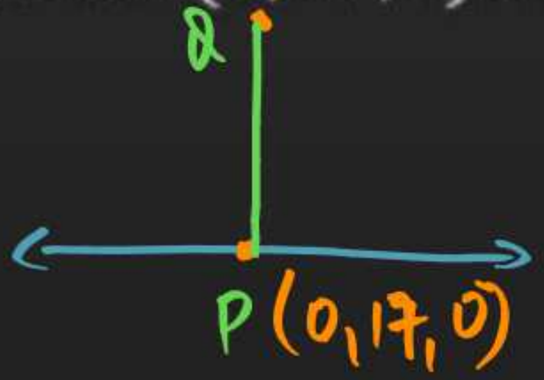
Question

The perpendicular distance of the point (6,17,8) from y-axis is

$\Downarrow$

$x = 0$

$z = 0$



(A) 5 units

(B) 6 units

(C) 8 units

(D) 10 units

$PQ = \sqrt{6^2 + 0 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$

Question

What is the perpendicular distance of the point $P(2,3,4)$ from xy-plane?

$z = 0$

$Q(2,3,0)$

$PQ = 4$

(A) 4 units

(B) 7 units

(C) 6 units

(D) 5 units

Question

The distance of the point $P(a, b, c)$ from the x-axis is [2014]

$y=0$

$z=0$

$Q(a, 0, 0)$

$PQ = \sqrt{b^2 + c^2}$

(A) $\sqrt{a^2 + b^2}$

(B) $\sqrt{b^2 + c^2}$

(C) a

(D) $\sqrt{a^2 + c^2}$

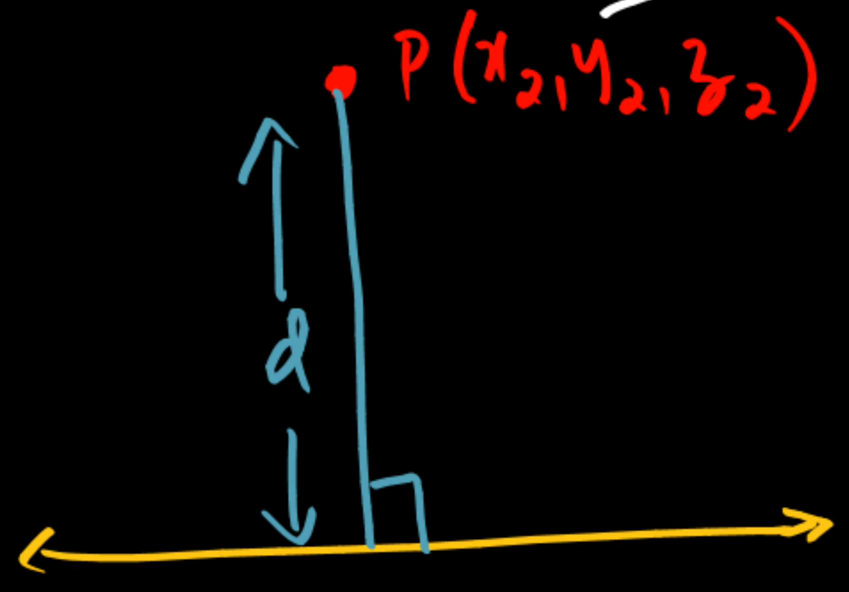
* Distance b/w 2 Parallel lines:-

$l_1 : \quad \vec{r} = \vec{a}_1 + \lambda \vec{b}$

$l_2 : \quad \vec{r} = \vec{a}_2 + \mu \vec{b}$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$$

→ The same formula is used to find the distance b/w a $\boxed{\text{point}}$ & a $\boxed{\text{line}}$



The point given is considered as

$$\vec{a}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$$\vec{a}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$$

Question

HW

The perpendicular distance of the point $P(6,7,8)$ from XY-plane is [2017]

(A) 7

(B) 6

(C) 8

(D) 5

Question

HW

Reflection of the point (α, β, γ) in XY plane is [2017]

A $(0, 0, \gamma)$

B $(\alpha, \beta, -\gamma)$

C $(-\alpha, -\beta, \gamma)$

D $(\alpha, \beta, 0)$

Question

The distance of the point $(-2,4,-5)$ from the line $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$ is [2017]

$\vec{a}_2 = -2\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{a}_2 - \vec{a}_1 = +1\hat{i} + 0\hat{j} + 3\hat{k}$

$\frac{x-(-3)}{4} = \frac{y-4}{5} = \frac{z-(-8)}{6}$

$\vec{a}_1 = -3\hat{i} + 4\hat{j} - 8\hat{k}$

$\vec{b} = 3\hat{i} + 5\hat{j} + 6\hat{k}$

$|\vec{b}| = \sqrt{9+25+36} = \sqrt{70}$

(A) $\sqrt{37}/10$

(B) $37/\sqrt{10}$

(C) $\sqrt{37/10}$

(D) $37/10$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{b} = \begin{vmatrix} i & j & k \\ +1 & 0 & +3 \\ 3 & 5 & 6 \end{vmatrix}$$

$= i(-15) - j(6-9) + k(+5)$

$= -15\hat{i} + 3\hat{j} + 5\hat{k}$

$|(\vec{a}_2 - \vec{a}_1) \times \vec{b}| = \sqrt{225+9+25} = \sqrt{259}$

$d = \sqrt{\frac{259}{70}} = \sqrt{\frac{37}{10}}$

Question

The distance of the point (1, 2, 1) from the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$ is [2019]

(1, 2, 3)

$|\vec{v}| = \sqrt{4+1+4} = \sqrt{9} = 3$

$\vec{a}_2 - \vec{a}_1 = (0, 0, 2)$

(A) $2\sqrt{3}/5$

(B) $\sqrt{5}/3$

(C) $2\sqrt{5}/3$

(D) $20/5$

$$(\vec{a}_2 - \vec{a}_1) \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= \hat{i}(-2) - \hat{j}(-4) + \hat{k}(0)$$

$$|(\vec{a}_2 - \vec{a}_1) \times \vec{v}| = \sqrt{4+16} = \sqrt{20}$$

$$= \sqrt{4 \times 5}$$

$$= 2\sqrt{5}$$

$$d = \frac{2\sqrt{5}}{3}$$

Question

$(3,3,-5)$

The distance of the point $(1,2,-4)$ from the line $\frac{x-3}{2}=\frac{y-3}{3}=\frac{z+5}{6}$ is [2020]

$|\vec{v}|=\sqrt{4+9+36}=7$

A. $\frac{\sqrt{293}}{7}$

B. $\frac{293}{49}$

C. $\frac{\sqrt{293}}{49}$

D. $\frac{293}{7}$

$$(a_2-a_1)\times\vec{v}=\begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix}$$

$$=i(9)-j(14)+\hat{k}(4)$$

$$|(\vec{a}_2-\vec{a}_1)\times\vec{v}|=\sqrt{81+196+16}$$

$$=\sqrt{293}$$

$$d=\frac{\sqrt{293}}{7}$$

Question

$(36)^2$
$= 9\ \underline{36}\ \underline{36}$
1296

→ Distance b/w a point & a line

The length of perpendicular drawn from the point $(3, -1, 11)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ is [2023]

$(0, 2, 3)$

(A) $\sqrt{33}$

(B) $\sqrt{66}$

(C) $\sqrt{53}$

(D) $\sqrt{29}$

$$(\vec{a_2} - \vec{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -3 & 8 \\ 2 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(-12-24) - \hat{j}(12-16) + \hat{k}(9+6)$$

$$= -36\hat{i} + 4\hat{j} + 15\hat{k}$$

$$|(\vec{a_2} - \vec{a_1}) \times \vec{b}| = \sqrt{1296+16+225}$$

$$= \sqrt{1537}$$

$$|\vec{b}| = \sqrt{4+9+16} = \sqrt{29}$$

$$d = \sqrt{\frac{1537}{29}}$$

$$= \underline{\sqrt{53}}$$

23-03

⇓

23 Inequalities

24 Domain & Range

25 Domain & Range

Next Sunday

⇓

Morning

⇓

2hr 30min

3D

Left

1. Point of intersection of 2 lines

2. Foot of perpendicular

one step more

3. Image of a point over a line

Thank

You