ULTIMATE KCET

CRASH COURSE 2026

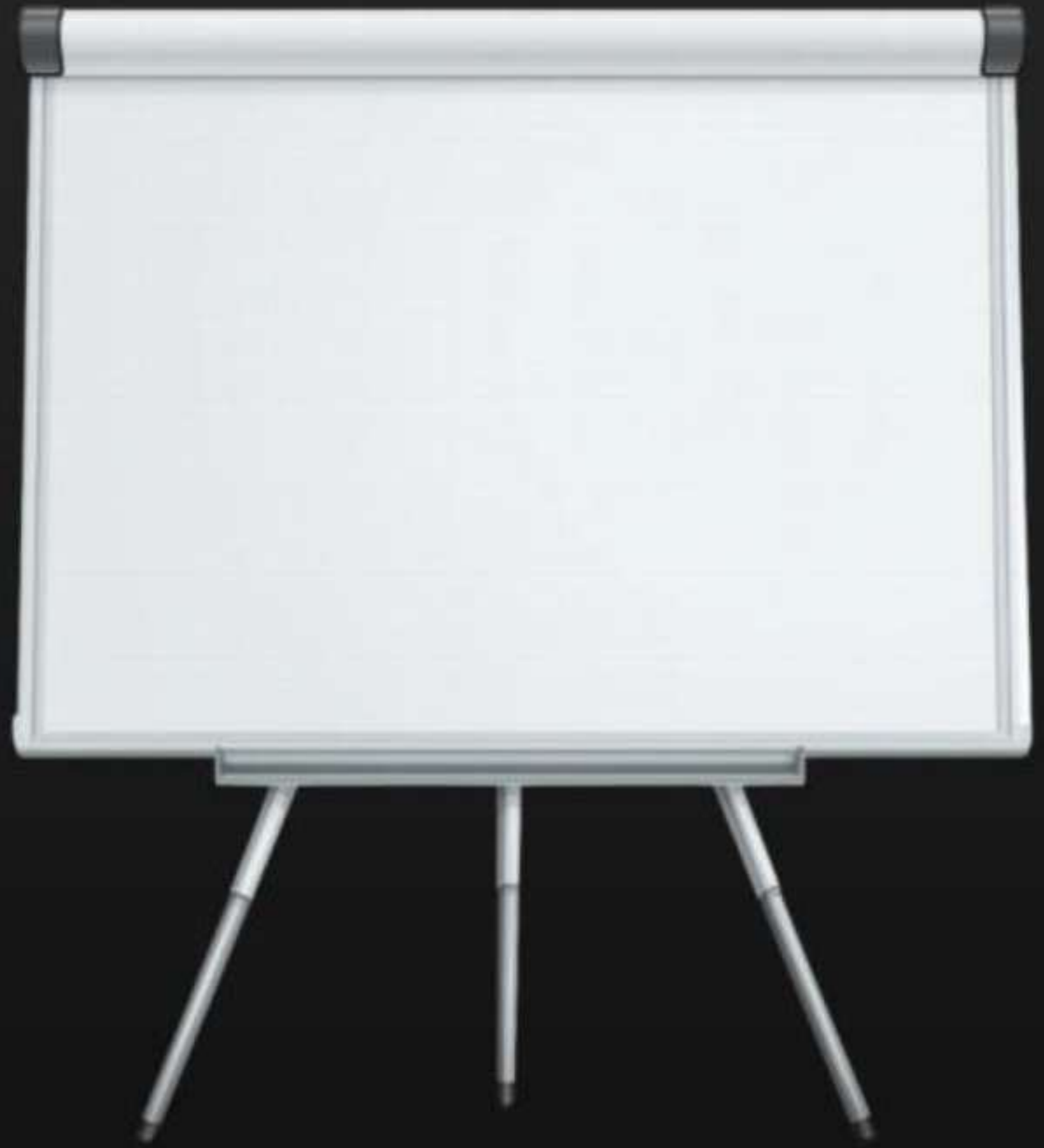
Mathematics | Lecture - 01

Relations and Functions

By – Guru sir

Recap *of previous lecture*

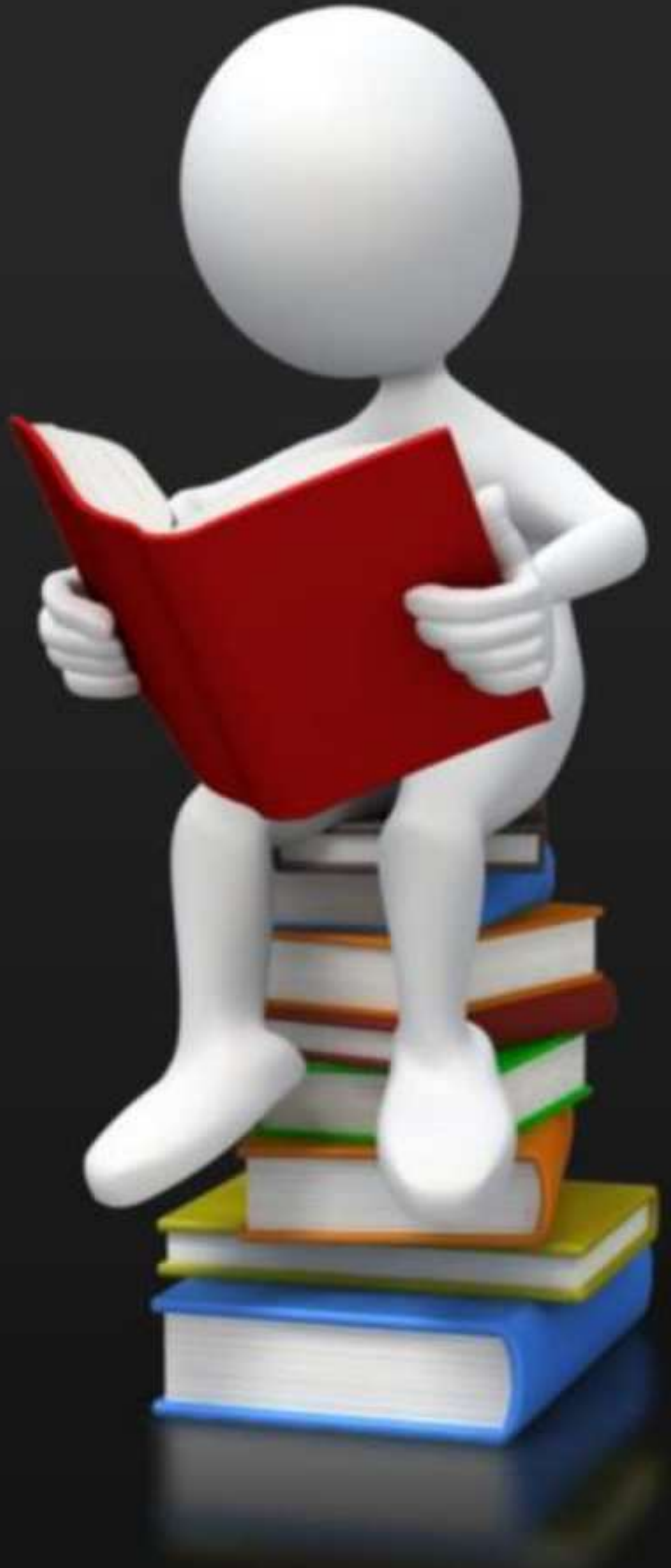
1. ITF
2.
3.
4.



Topics *to be covered*

1. Types of Relations
2. Types of functions
3. No. of functions
4.

3 to 4 marks



Easy → Counting of functions

No of one-one functions :-

if $A \to 1^{st}$ set

$B \to 2^{nd}$ set $\Rightarrow f : A \to B$

no of one-one func from A to B exists only if $n(A) \leq n(B)$

$\Downarrow$

no of elements in the 1^{st} set should be lesser than or equal to that of 2^{nd} set if $n(A)=n$ & $n(B)=m$

Then no. of one-one func. from A to B $= \begin{cases} {}^mP_n & \text{if } n(A) \le n(B) \\ 0 & \text{if } n(A) > n(B) \end{cases}$

$${}^mP_n = \frac{m!}{(m-n)!}$$

Ex: if $n(A) = 3$ & $n(B) = 5$

(1) no of one-one funct from A to B $= {}^5P_3 = \frac{5!}{2!} = 60$

(2) —"— —"— — from B to A $= 0$

B ↓ 1st set, A ↓ 2nd set

(3) —"— —"— — from A to A $= {}^3P_3 = 3! = 6$

(4) —"— —"— — from B to B $= {}^5P_5 = 5! = 120$

$${}^nC_n = 1 \quad \Big| \quad {}^nP_n = n!$$

if $n(A) = n$ & $n(B) = m$

Then no of func from A to B $= [n(B)]^{n(A)}$

$$= \left[\begin{matrix}\text{No of elements} \\ \text{in 2nd set}\end{matrix}\right]^{\text{no of elements in 1st set}}$$

if $n(C) = 3$ & $n(B) = 5$

Then ① no of func from A to B $= 5^3 = 125$

② —"——"— B to A $= 3^5 = 243$

③ —"——"— A to A $= 3^3 = 27$

④ —"——"— B to B $= 5^5 = 3125$

many one func = A func which is not one-one is called as many one func

if $n(A) = 3$ & $n(B) = 4$

1. No of many one func from A to B $= 4^3 - {}^4P_3 = 64 - 24 = 40$
2. — " — " — " — B to A $= 3^4 - 0 = 81$
3. — " — " — " — A to A $= 3^3 - {}^3P_3 = 27 - 6 = 21$
4. — " — " — " — " — B to B $= 4^4 - {}^4P_4 = 256 - 24 = \underline{232}$

if $n(A) = p$ & $n(B) = q$ $\Rightarrow n(A \times B) = pq$

(1) Then no of relations from A to B $= 2^{pq}$

(2) $n(A \times B) = pq$

(3) $n(B \times A) = pq$

(4) $n(A \times A) = p^2$

(4) $n(B \times B) = q^2$

Relations are the subsets of the cartesian Product if $n(A) = p$ & $n(B) = q$

& if $n(A \times B) = 11$

Then $p^2 + q^2 =$

Soln:

$n(A \times B) = 11$

11×1 | 1×11

$p = 11$ & $q = 1$

$\Downarrow$

$p^2 + q^2 = 11^2 + 1^2$

$= 122$

$p = 1$ & $q = 11$

$\Downarrow$

$p^2 + q^2 = 1^2 + 11^2$

$= 122$

if $n(A) = 3 \Rightarrow n(A \times A) = 3 \times 3 = 9$

no of relations from A to A $= 2^{3\times 3} = 2^{9}$

No of func from A to A $= [n(A)]^{n(A)} = 3^{3} = 27$

if $n(A)=3$ & $n(B)=4$

① $n(A\times B)=3\times4=12$

② $n(B\times A)=4\times3=12$

③ $n(A\times A)=3\times3=9$

④ $n(B\times B)=4\times4=16$

① No of relations from A to B $=2^{12}$

② — " — " — from B to A $=2^{12}$

③ — " — " — from A to A $=2^{9}$

④ — " — " — from B to B $=2^{16}$

if $n(A)=3$ & $n(B)=2$

(1) No of relations from A to B which are not func $= 2^6 - 2^3 = 64-8 = 56$

(2) —" — —" – B to A —" —" — $= 2^6 - 3^2 = 64-9 = 55$

(3) —" —" — A to A —" —" — $= 2^9 - 3^3 = 512-27$

$= 485$

(4) —" —" —" — B to B —" —" — $= 2^4 - 2^2$

$= 16-4$

$= 12$

if $n(A) = 4$ $n(B) = 5$

① No of relations from A to B which are not func $= 2^{20} - 5^{4}$

② —"— —"— B to A —"— —"— $= 2^{20} - 4^{5}$

③ —"— —"— A to A —"— —"— $= 2^{16} - 4^{4}$

④ —"— —"— —"— B to B —"— —"— $= 2^{25} - 5^{5}$

No of onto function:-

if $A \to$ 1st set
$B \to$ 2nd set $\Rightarrow f: A \to B$

Then no of onto functions exists only if

$$n(B) \leq n(A)$$

ie, no of elements in 2nd set should be lesser than or equal to that of 1st set

onto function :-

special case :-

if $n(A) = p$ where $p \geq 2$

& $n(B) = 2 \rightarrow$ Fixed

$\therefore$ no of onto func from A to B $= 2^p - 2$

Ex:- if $n(A) = 6$ & $n(B) = 2$

$\therefore$ no of onto func from A to B $= 2^6 - 2 = 64 - 2 = 62$

if $n(A)=n$ & $n(B)=m$

Then no of onto func from A to B $= \begin{cases} \displaystyle\sum_{r=0}^{m-1} {}^{m}C_r (-1)^r (m-r)^n & \text{if } n(B) \le n(A) \\ 0 & \text{if } n(B) > n(A) \end{cases}$

$${}^{n}C_r = \frac{n!}{(n-r)!\, r!}$$

if $n(A)=4$ & $n(B)=3$

$\therefore$ no of onto func from A to B

$n=4$ & $m=3$

$\Downarrow$

$m-1=2$

$^nC_0=1$

$^nC_n=1$

$$\sum_{r=0}^{2}$$

$$\Downarrow$$

$$\boxed{r=0}+\boxed{r=1}+\boxed{r=2}$$

$$=\sum_{r=0}^{m-1} {}^mC_r\,(-1)^r\,(m-r)^n$$

$$=\sum_{r=0}^{2} {}^3C_r\,(-1)^r\,(3-r)^4$$

$$= {}^3C_0(-1)^0(3-0)^4+{}^3C_1(-1)^1(3-1)^4+{}^3C_2(-1)^2(3-2)^4$$

$$=(1)(1)(3^4)-3(2)^4+3(1)^4=81-48+3=84-48$$

$$=\underline{36}$$

QUESTION

$n(A)=3=p$ | $n(B)=q$ |

Let A and B be finite sets such that $n(A)=3$. If the total number of relations that can be defined from A to B is 4096 , then $n(B)=$

$\Downarrow$

2^{pq}

(A) 5

(B) 4

(C) 6

(D) 8

$2^{pq}=4096$

$2^{3q}=2^{12}$

$3q=12$

$q=4$

$2^5 = 32$

$2^6 = 64$

$2^7 = 128$

$2^8 = 256$

$2^9 = 512$

$2^{10} = 1024$

$2^{11} = 1024 \times 2 = 2048$

$2^{12} = 2048 \times 2 = 4096$

one-one func

Methods:-

(1) Trial & error method

$f : R \rightarrow B$

Domain = R

irrationals

rationals

+ve integers

-ve integers

0

② graphical method :-

⇓

If the given func are

standard func [we know the graph of it]

⇓

we conduct parallel line test

Parallel line test

we draw the graph in the given domain

& then draw a straight line parallel to x-axis

⇓

(1) If this straight line meets the graph at 2 or more points, Then it is not one-one

(2) If the straight line meets the graph at only one point, Then it is one-one.

Graphs:-

1. $f(x) = x^2$
2. $f(x) = x^3$
3. $f(x) = [x]$
4. $f(x) =$ signum func
5. $f(x) = \sin x$
6. $f(x) = \cos x$
7. $f(x) = \tan x$
8. $f(x) = \cot x$
9. $f(x) = \sec x$
10. $f(x) = \csc x$
11. $f(x) = ax + b$

    $\Downarrow$

    straight line

12. $f(x) = |x|$
13. $f(x) = \frac{1}{x}$

method 3 :-

if a given func is strictly increasing $(f'(x) > 0)$

or strictly decreasing $(f'(x) < 0)$ in the

given domain. Then $f(x)$ is one-one

onto func :-

method 1:-

$f : A \to B$

$B \downarrow$ codomain

Find range & check whether it is equal to codomain

→ if range = codomain
f is onto

→ if range ≠ codomain
f is not onto

QUESTION

Let $f : R \to R$ be defined by $f(x) = x + |x|$. Then $f(x)$ is

$\to$ codomain

$$f(x) = x + |x| = \begin{cases} x + x = 2x & \text{if } x \geq 0 \Rightarrow \text{ for } x > 0,\ 2x > 0 \\ x - x = 0 & \text{if } x < 0 \end{cases}$$

A Both one one and onto

B Only one-one

C Only onto

D Neither one one nor onto

one-one :-

$f(-1) = 0$

$f(-2) = 0$

not one-one

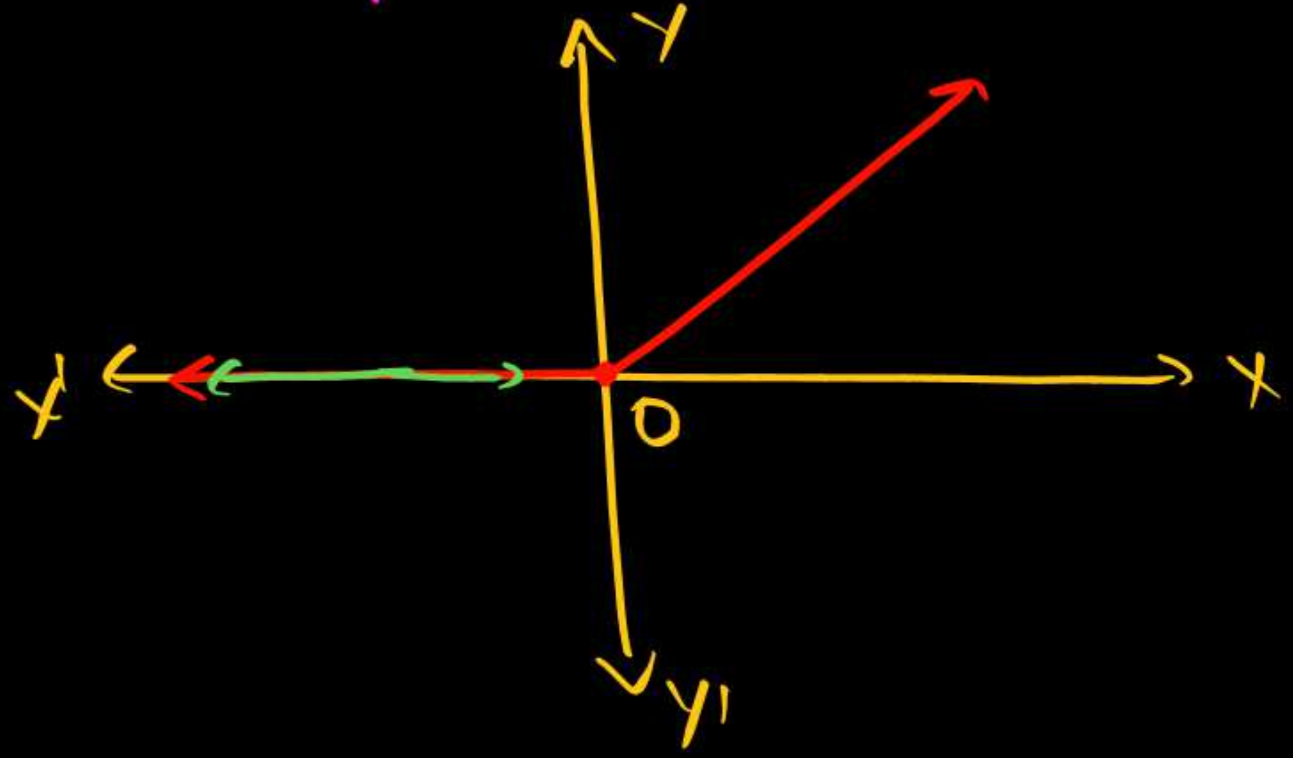
onto :-

Range is $[0, \infty)$

$\neq$ codomain

not onto

$f(x) = x + |x|$



QUESTION

Which one of the following functions is one-to-one?

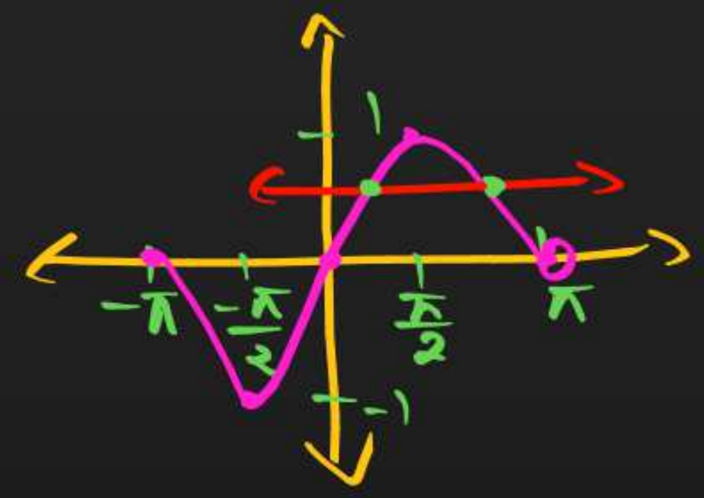
(A) $f(x) = \sin x, x \in [-\pi, \pi)$

(B) $f(x) = \sin x, x \in \left[\frac{-3\pi}{2}, \frac{-\pi}{2}\right]$

(C) $f(x) = \cos x, x \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right)$

(D) $f(x) = \sin x, x \in [\pi, 2\pi)$

(1)



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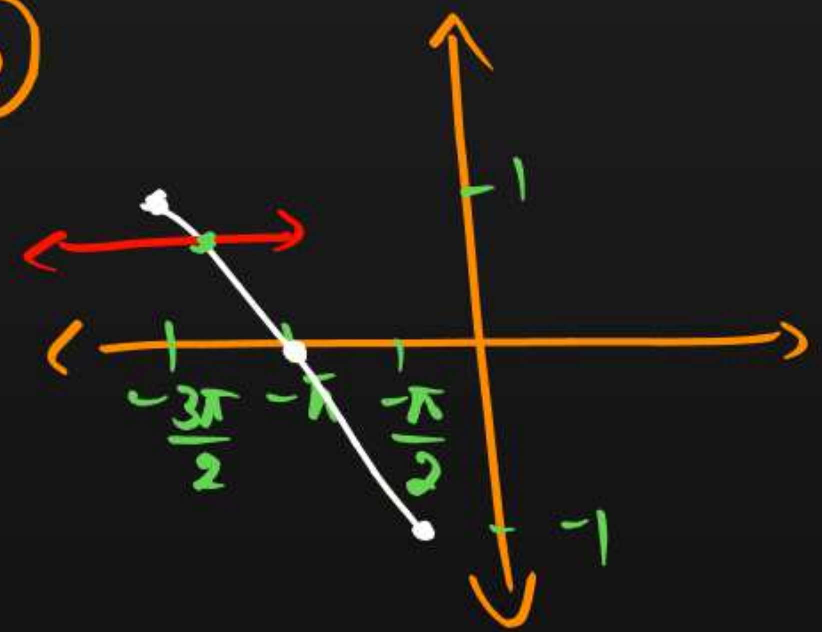

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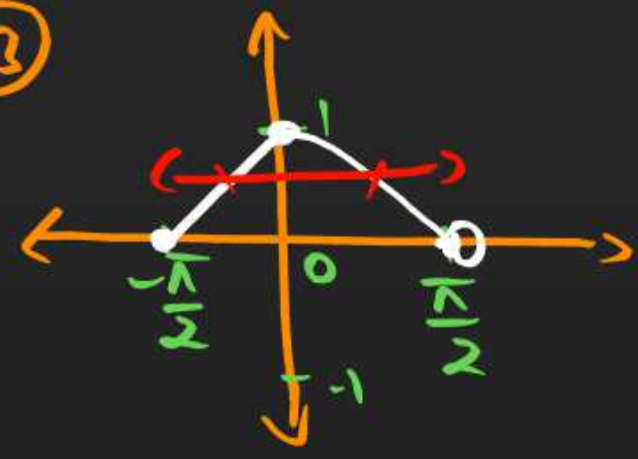

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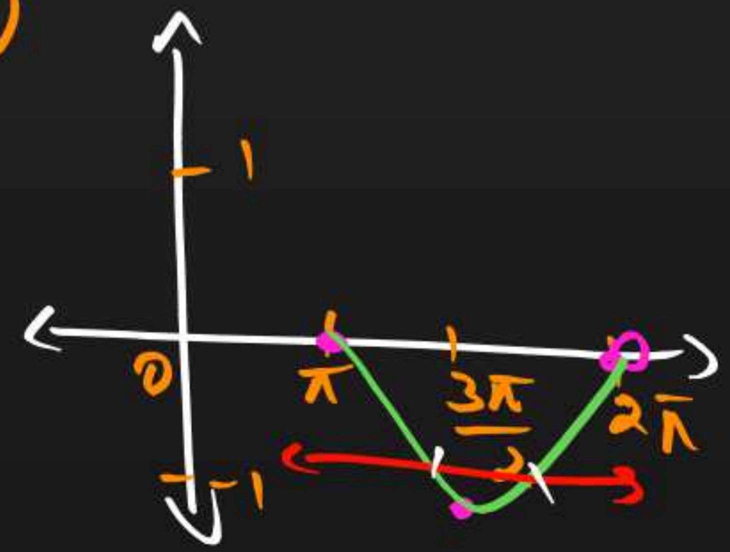


QUESTION

$f: N \to Z$

A function f from the set of natural numbers to integers is defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$$ Then, f is

A one-one but not onto

B onto but not one-one

C one-one and onto both

D neither one-one nor onto

$f(1) = \frac{1-1}{2} = 0$

$f(3) = \frac{3-1}{2} = 1$

$f(5) = \frac{5-1}{2} = 2$

output $= \{0, 1, 2, 3, \ldots\}$

$f(2) = -\frac{2}{2} = -1$

$f(4) = -2$

$f(6) = -3$

output $= \{-1, -2, -3, \ldots\}$

one-one

if odd inputs

output $= \{0, 1, 2, \ldots\} \to ①$

if even inputs

output $= \{-1, -2, \ldots\} \to ②$

$① \cup ②$

Range $= Z$

= codomain.

onto

QUESTION

$f: N \to N$ → codomain

A function f from the set of natural numbers to Natural no is defined by

$$f(n) = \begin{cases} \frac{n-1}{2}, & \text{when } n \text{ is odd} \\ -\frac{n}{2}, & \text{when } n \text{ is even} \end{cases}$$ Then, f is

Not a function

(A) one-one but not onto

(B) onto but not one-one

(C) one-one and onto both

(D) neither one-one nor onto

$n = 2$

$f(2) = -1 \notin N$ (codomain)

$\Rightarrow n = 2$ does not have output

$\therefore f$ is not a func

$f : A \to B$ is said to be a func^{n}

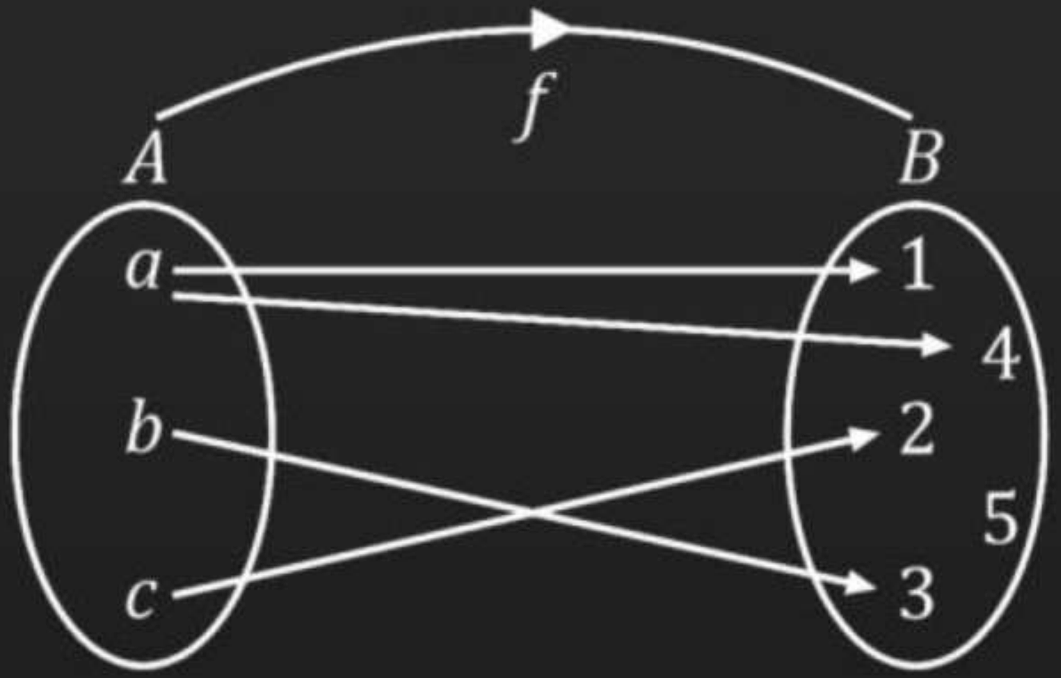
if (1) each element of A should have output in B

(2) each element of A should have only one output

ie, no element of A should have more than one output in B

QUESTION

The diagram given below shows that



A f is a function from A to B

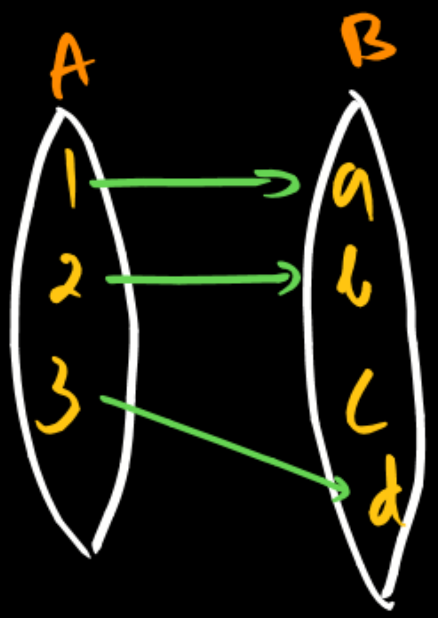
B f is a one-one function from A to B

C f is an onto function from A to B

D f is not a function

$f(a) = 1$ & $f(a) = 4$

The element $a \in A$
have 2 outputs.
$\therefore f$ is not a func^{n}

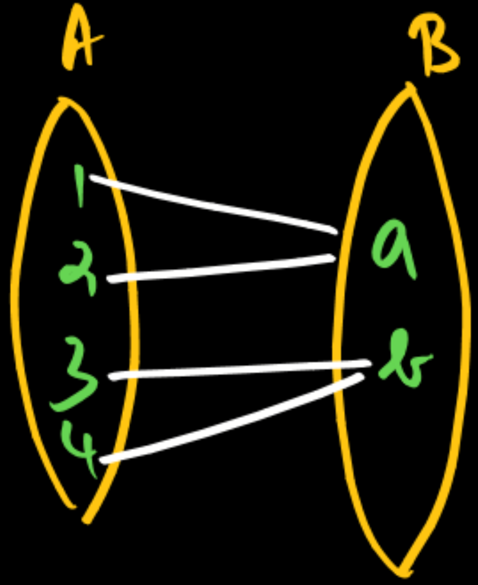


one-one
but
not onto

codomain = $\{a, b, c, d\}$

Range = $\{a, b, d\}$

Range $\neq$ codomain



$f(1) = a$

$f(2) = a$

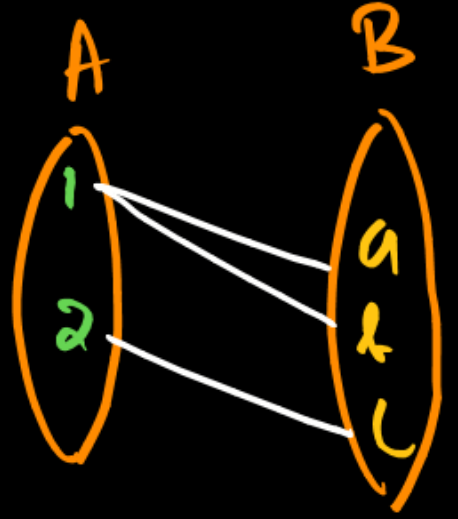
$f(3) = b$

$f(4) = b$

Function ✓

one - one ×

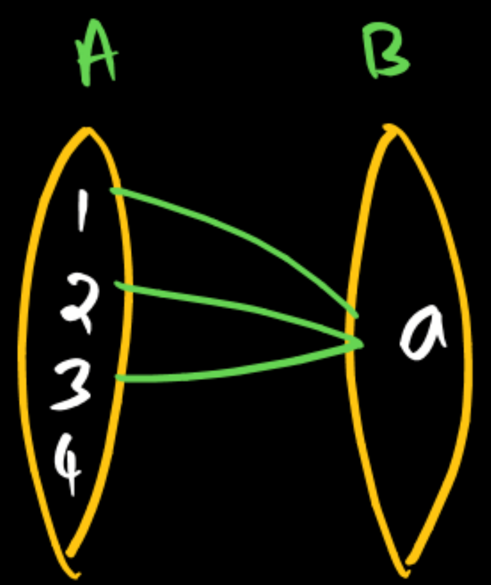
onto ✓



$f(1) = a$ | $f(2) = c$

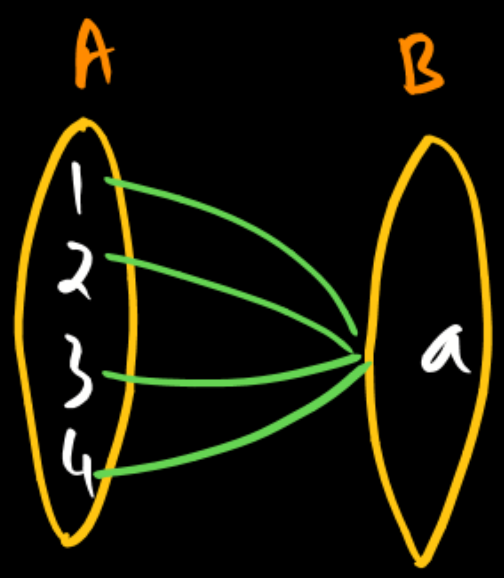
$f(1) = b$

Function ×



Not a func

$x=4$ doesnot have output



Function ✓

one-one ✗

onto ✓

QUESTION

$\sqrt{x^2} = |x|$

$f(x) = x + \sqrt{x^2}$ is a function from R to R, then $f(x)$ is

$$f(x) = x + |x| = \begin{cases} 2x & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$$

A Injective

B surjective

C bijective

D none of these

neither one-one
nor onto

$f : R \to R$

$f(x) = x|x|$

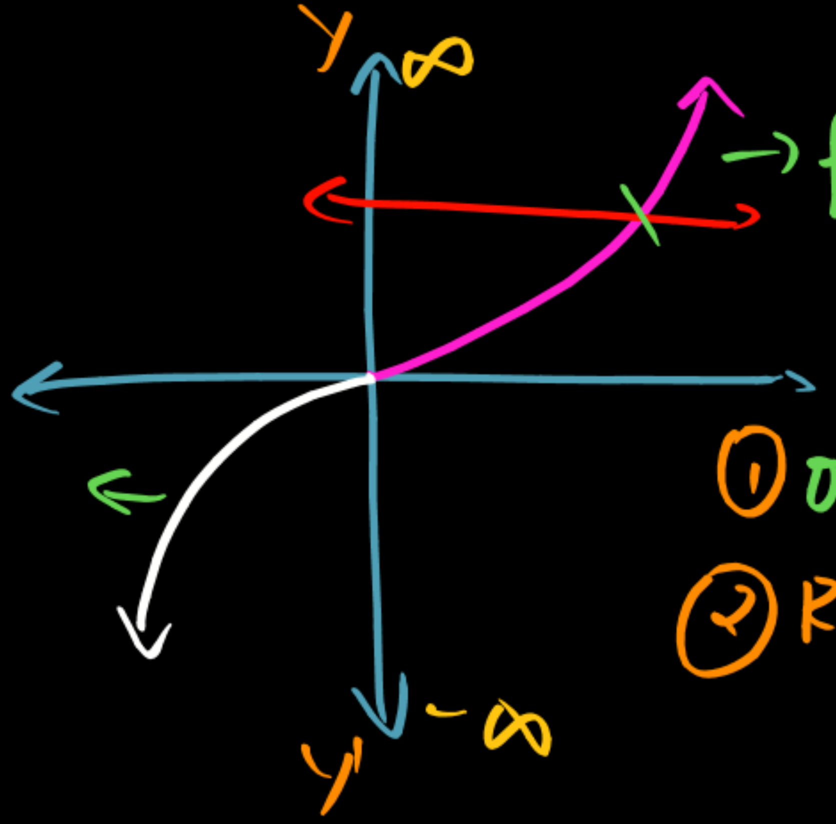
(A) one-one

(B) onto

(C) ~~Bijective~~

(D) none of the above

$$f(x) = x|x| = \begin{cases} x(x) = x^2 & \text{if } x \geq 0 \\ x(-x) = -x^2 & \text{if } x < 0 \end{cases}$$



$\to f(x) = x^2$ if $x \geq 0$

$f(x) = -x^2$ if $x < 0$ $\leftarrow$

① one-one ✓

② Range $= (-\infty, \infty) = R =$ Codomain

f is onto

Range of $f(x) = x - [x]$

Soln:-

Case 1:-

if $x \in Z$

$\rightarrow x = 2$

$f(2) = 2 - [2] = 2 - 2 = 0$

$\rightarrow x = -3$

$f(-3) = -3 - [-3] = -3 + 3 = 0$

if input $= Z$

output $= \{0\} \rightarrow 1$

Case 2:-

if input $= R - Z$

$\rightarrow x = 2.6$

$f(2.6) = 2.6 - [2.6]$

$= 2.6 - 2$

$= 0.6 \in (0,1)$

$\rightarrow x = -3.5$

$f(-3.5) = -3.5 - [-3.5]$

$= -3.5 - (-4)$

$= -3.5 + 4 = 0.5$

$\in (0,1)$

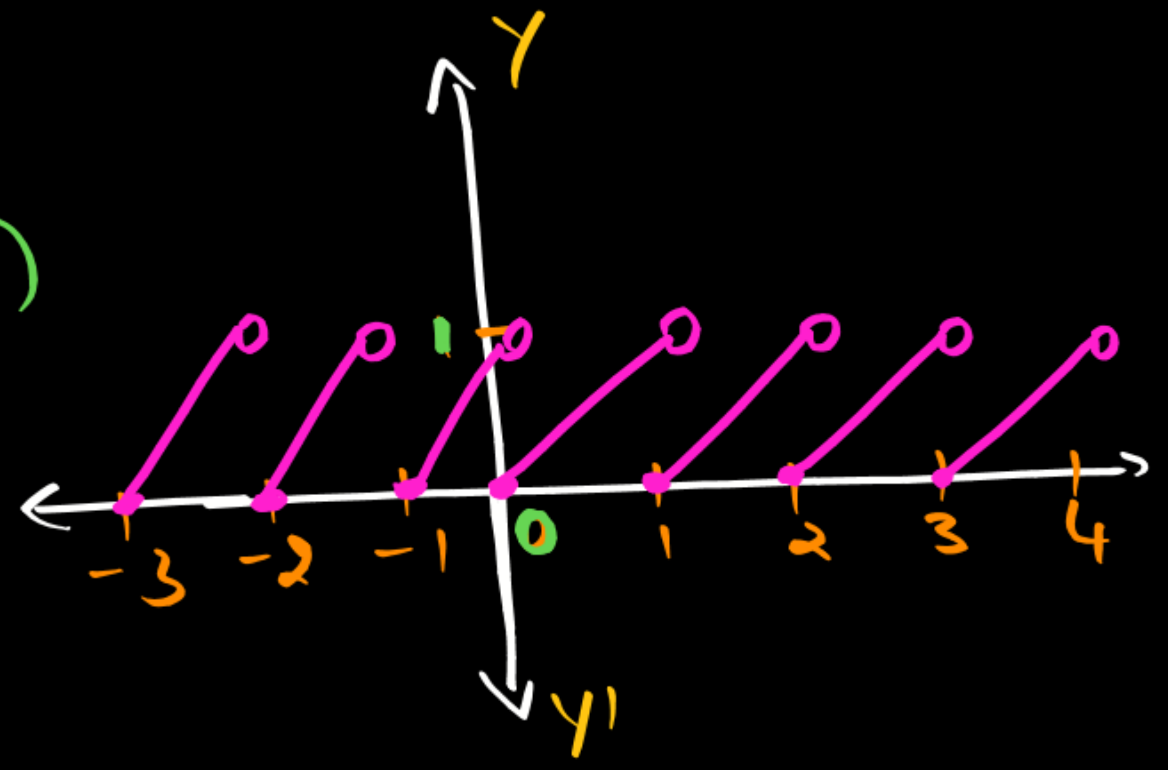
$\therefore$ if input $= R - Z$

output $= (0,1) \rightarrow$ ②

$\therefore$ Range $=$ ① $\cup$ ②

$= \{0\} \cup (0,1)$

$= [0,1)$



(*) if $f : R \to [0,1)$
given by $f(x) = x - [x]$

(A) one-one

(B) ~~onto~~

(C) bijective

(D) None of these

one-one :-

$f(3) = 3 - [3] = 3 - 3 = 0$

$f(5) = 5 - [5] = 5 - 5 = 0$

not one-one

onto :-

Range $= [0,1)$

$=$ codomain

onto

QUESTION

The function $f: R \to R$, given by $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is

(A) one-one onto

(B) one-one

(C) onto

(D) neither one-one nor onto

one-one :-

$f(3.1) = [3.1] = 3$

$f(3.5) = [3.5] = 3$

not one-one

onto :-

Range $= Z$

$\neq$ codomain (R)

$\therefore f$ is not onto

QUESTION

If $f:[0,\infty) \to [0,\infty)$ and $f(x) = \frac{x}{1+x}$, then f is

A one-one and onto

B one-one but not onto

C onto but not one-one

D neither one-one nor onto

one-one:

method 1

$f(0) = 0$

$f(1) = \frac{1}{2}$

$f(2) = \frac{2}{3}$

$f(1/2) = \frac{1/2}{1+1/2} = \frac{1/2}{3/2} = 1/3$

Different input with outputs

$\therefore$ one-one

method 2

$f(x) = \frac{x}{1+x}$

$f'(x) = \frac{(1+x)(1) - x(1)}{(1+x)^2}$

$= \frac{1+x-x}{(1+x)^2}$

$= \frac{1}{(1+x)^2} > 0$

$f'(x) > 0$ [f is strictly increasing]

$\therefore f$ is one-one

onto :-

$f : [0, \infty) \to [0, \infty)$

Domain $\leftarrow$ $[0,\infty)$, $x \geq 0$

given $f(x) = \dfrac{x}{1+x}$

Range :-

consider

$f(x) = y$

$\dfrac{x}{1+x} = y$

$x = y + xy$

$x - xy = y$

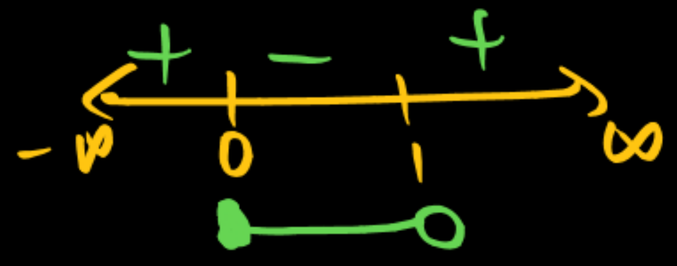
$x = \dfrac{y}{1-y}$

here $x \geq 0$

$\dfrac{y}{1-y} \geq 0$

$\dfrac{y}{-(y-1)} \geq 0$

$\dfrac{y}{y-1} \leq 0$

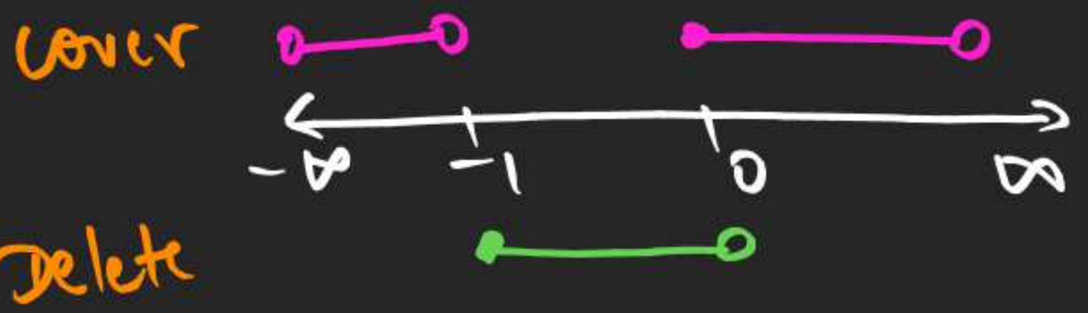


$y \in [0, 1)$

Range $= [0, 1)$

$\neq$ codomain $[0, \infty)$

Not onto



QUESTION

If the function $f : R - \{1, -1\} \to A$ defined by $f(x) = \frac{x^2}{1-x^2}$, is surjective, then A is equal to:

A $[0, \infty)$

B $R - (-1, 0)$

C $R - \{-1\}$

D $R - [-1, 0)$

Consider

$f(x) = y$

$\frac{x^2}{1-x^2} = y$

$x^2 = y - yx^2$

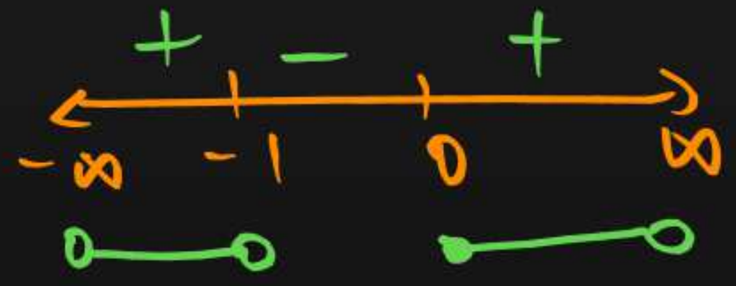
$x^2(1+y) = y$

$x^2 = \frac{y}{1+y}$

$x^2 = \frac{y}{1+y}$

WKT $x^2 \geq 0$

$\Rightarrow \frac{y}{y+1} \geq 0 \quad y+1 \to y \neq -1$



$y \in (-\infty, -1) \cup [0, \infty)$

$y \in (-\infty, -1) \cup [0, \infty)$

$\Downarrow$

$R - [-1, 0)$

$\Downarrow$

Range $= A$

usually Difficult

1. Functions → 1 Q
2. ITF → 1 Q
3. Differentiation (implicit func) → 1 Q
4. AOD → 1 Q (Max & Min)

5. Integrals → 2 Q
   1. substitution
   2. Properties
6. Probability → 2 Q
   1. class 11
   2. Bayes' theorem
7. Trigonometry → 1 or 2 Q
   11th

Easily

$\Downarrow$

45 to 50

QUESTION

$f: R \to R$

Which of the following is one-one?

A. e^x

B. e^{x^2} $\Rightarrow f(1) = e^1 = e$

$f(-1) = e^{(-1)^2} = e^1 = e$

C. $\sin x$

D. None of these

(1) $f(x) = e^x$

$f'(x) = e^x > 0$

$\Downarrow$

strictly increasing

$\therefore f$ is one-one

$f(x) = a^x$

Range $= (0, \infty)$

$\therefore e^x > 0$

$2^x > 0$

$3^x > 0$

QUESTION

The function $f : A \to R$ where $A = \left\{x \in R, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}\right\}$ defined by $f(x) = \cos x$ is

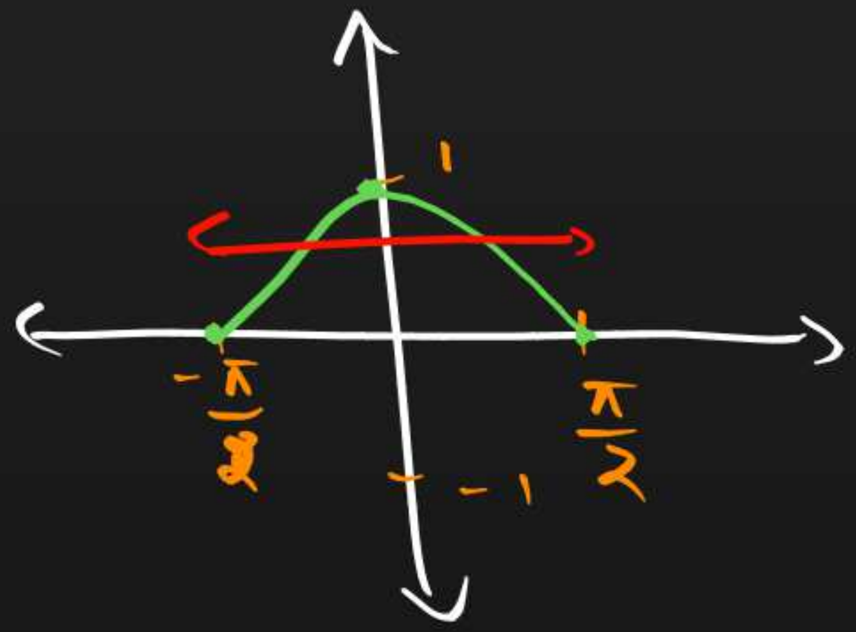
A Injective

B Surjective

C Bijective

D Neither injective nor surjective

$f : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to R \qquad f(x) = \cos x$



(1) not one-one

(2) Range : $[0, 1]$
$\neq R$ (codomain)

$\therefore f$ is not onto

QUESTION

$f : [-1, 1] \to [1, 2] \quad f(x) = 1 + x^2$

A function $f: A \to B$, where $A = \{x: -1 \le x \le 1\}$ and $B = \{y: 1 \le y \le 2\}$ is defined by the rule $y = f(x) = 1 + x^2$. Which of the following statements is true?

A. f is injective but not surjective

B. f is surjective but not injective

C. f is both injective and surjective

D. f is neither injective nor surjective

(1) $f(-1) = 1 + (-1)^2 = 2$

$f(1) = 1 + 1^2 = 2$

not one-one

onto :-

Range :-

$-1 \le x \le 1$

$0 \le x^2 \le 1$

$1 \le 1 + x^2 \le 2$

$f(x) \in [1, 2]$

Range = codomain

onto

QUESTION

Set A has three elements and set B has four elements. The number of injections that can be defined from A to B is

one-one

$n(A) = 3 \mid n(B) = 4$

$^4P_3 = 4! = 24$

A 144

B 12

C 24

D 64

injections → one-one

Surjection → onto

Bijectione → one-one & onto

if $f : A \to B$

Then $\underline{\underline{no}}$ of Bijections from A to B $= \begin{cases} n! & \text{if } n(A) = n(B) = n \\ 0 & \text{if } n(A) \neq n(B) \end{cases}$

PYQ

if $n(A) = 6$

no of func which are not bijections from A to A $= 6^6 - 6!$

$\downarrow$

$[n(A)]^{n(A)} - n(A)!$

QUESTION

The number of bijective functions from set A to itself when A contains 106 elements is

A) 106

B) $(106)^2$

C) 106!

D) 2^{106}

QUESTION

$n(A)=4$ $\rightarrow n(B)=8$

If $A = \{1, 3, 5, 7\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$, then the number of one-to-one functions from A into B is

(A) 1340

(B) 1860

(C) 1430

(D) 1680

$$^8P_4 = \frac{8!}{4!} = \frac{8\times7\times6\times5\times\cancel{4!}}{\cancel{4!}}$$

$$= 56\times30$$

$$= \underline{1680}$$

QUESTION

Number of onto (surjective) functions from A to B if $n(A) = 6$ and $n(B) = 3$ are

$n = 6$ $\quad$ $m-1 = 2$

(A) $2^6 - 2$

(B) $3^6 - 3$

(C) 340

(D) 540

$$\sum_{r=0}^{2} {}^3C_r (-1)^r (3-r)^6$$

$${}^3C_0 (-1)^0 (3)^6 + {}^3C_1 (-1)^1 (2)^6 + {}^3C_2 (-1)^2 (1)^6$$

$$3^6 - 3(2)^6 + 3$$

$$729 - 3(64) + 3$$

$$732 - 192 = \underline{540}$$

QUESTION

If the set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is

$n(A) = 5$ & $n(B) = 6$

no. of bijections = 0 since $n(A) \neq n(B)$

(A) 720

(B) 120

(C) 0

(D) None of these

QUESTION

The number of surjections from $A = \{1, 2, ..., n\}, n \geq 2$ onto $B = \{a, b\}$ is

(A) ${}^{n}P_{2}$

(B) $2^n - 2$

(C) $2^n - 1$

(D) None of these

QUESTION

The number of non-bijective mappings possible from $A = \{1, 2, 3\}$ to $B = \{4, 5\}$ is

$n(A) \neq n(B)$

$\Downarrow$

func which are not bijective

$\Downarrow$

no of func from A to B − no of bijections from A to B

$2^3 - 0$

$= 8$

(A) 9

(B) 8

(C) 12

(D) 6

Constant func

$\Downarrow$

A func whose output remains always same

Ex:- $f(x) = 2$

$f(1) = 2$

$f(0) = 2$

$f(5) = 2$

QUESTION

$n(B) = 12$

The number of constant functions possible from R to $B = \{2, 4, 6, 8, \ldots 24\}$ is

if $f : R \to B$

A 24

B 12

C 8

D 6

no of constant func =

$$\left.\begin{array}{l} f(x) = 2 \\ f(x) = 4 \\ f(x) = 6 \\ f(x) = 8 \\ \vdots \\ f(x) = 24 \end{array}\right\} \text{Total 12 func}$$

QUESTION

The number of relations from $A = \{2, 6\}$ to $B = \{1, 3, 5, 7\}$ that are not functions from A to B is

(A) 240

(B) 16

(C) 128

(D) 200

$2^{2\times 4} - 4^2$

$2^8 - 16$

$256 - 16$

240

QUESTION

The number of possible many to one functions from $A = \{6, 36\}$ to $B = \{1, 2, 3, 4, 5\}$ is

$\Downarrow$

No of func − no of one-one func

(A) 32

$5^2 - {}^5P_2$

(B) 25

$25 - \frac{5!}{3!}$

(C) 5

$25 - \frac{120}{6}$

(D) 20

$25 - 20 = 5$

Thank

You