ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture : 01

Sequence and Series, Straight Lines and complex numbers

By – Guru sir

Recap *of previous lecture*

1. AOD
2.
3.
4.

Topics to be covered

1. Complex Nos
2. sequence & series
3. straight lines
4.

Cube root of unity

if $x^3 = 1$

$x^3 - 1 = 0$

$x^3 - 1^3 = 0$

$(x-1)(\underbrace{x^2+x+1}) = 0$

$x - 1 = 0$

$x = 1$

$x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{3}\,i}{2}$

$\omega = \frac{-1+\sqrt{3}i}{2}$ $\quad$ $\omega^2 = \frac{-1-\sqrt{3}i}{2}$

$\therefore 1, \omega$ & ω^2 are cube root of unity.

Properties:-

① $1 + \omega + \omega^2 = 0$

② $1 + \omega = -\omega^2$

③ $1 + \omega^2 = -\omega$

④ $\omega + \omega^2 = -1$

⑤ $\omega^3 = 1$

$\omega^6 = 1$

$\vdots$

$\omega^{3n} = 1$

⑥ $\omega^4 = \omega^3 \cdot \omega = \omega$

$\omega^5 = \omega^3 \cdot \omega^2 = \omega^2$

Fourth root of unity:-

$$x^4 = 1$$

$$x^4 - 1 = 0$$

$$(x^2)^2 - 1^2 = 0$$

$$(x^2 - 1)(x^2 + 1) = 0$$

$x^2 = 1$

$x = \pm 1$

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm\sqrt{-1}$

$x = \pm i$

$\therefore 1, -1, i, -i$ are the fourth root of unity

(*) (1) $1 + (-1) + (i) + (-i) = 0$

(2) $i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^5 = i^4 \cdot i = i$

$i^6 = i^4 \cdot i^2 = -1$

$i^7 = i^4 \cdot i^3 = -i$

$i^4 = 1$

$i^8 = 1 \Rightarrow i^{4n} = 1$

$i^{12} = 1$

$\frac{1}{i} = -i$

* 1. $(1+i)^2 = 2i$

2. $(1-i)^2 = -2i$

QUESTION

$x^3 - 1^3 = 0$

$(x-1)(x^2+x+1) = 0$

If α and β are roots of the equation $x^2 + x + 1 = 0$ then $\alpha^2 + \beta^2$ is

$\Downarrow$

Roots are

ω & ω^2

$\downarrow \quad \downarrow$

$\alpha \quad \beta$

$\alpha^2 + \beta^2$

$= \omega^2 + (\omega^2)^2$

$= \omega^2 + \omega^4$

$= \omega^2 + \omega^3 \cdot \omega$

$= \omega^2 + \omega$

$= \underline{-1}$

(A) $\dfrac{-1 + i\sqrt{3}}{2}$

(B) -1

(C) 1

(D) $\dfrac{-1 - i\sqrt{3}}{2}$

$z = x + iy$

x = Real part of z

y = Imaginary part of z

QUESTION

The imaginary part of $\frac{(1+i)^2}{i(2i-1)}$ is

A) $4/5$

B) 0

C) $2/5$

D) $-4/5$

$$\frac{2i}{2i^2-i} = \frac{2i}{-2-i}$$

↓ Rationalise

$$= -\frac{2i}{2+i} \times \frac{2-i}{2-i}$$

$$= -\left[\frac{4i+4}{4+1}\right]$$

$$= \frac{-4-4i}{5}$$

$$z = -\frac{4}{5} + \left(-\frac{4}{5}\right)i$$

$$y = -\frac{4}{5}$$

$$\frac{(1+i)^{2016}}{(1-i)^{2014}} = \left(\frac{1+i}{1-i}\right)^{2014} (1+i)^2$$

$$= \left[\frac{(1+i)^2}{1+1}\right]^{2014} (2i)$$

$$= \left(\frac{2i}{2}\right)^{2014} (2i)$$

$$= i^{2014} (2i)$$

$$= i^{2012} \cdot i^2 (2i)$$

$$= (1)\, i^3 (2) = \underline{-2i}$$

$$(x+iy)(x-iy)=x^2+y^2$$

$$(3+2i)(3-2i)=3^2+2^2=13$$

$$\frac{1+i}{1-i}=\frac{1+i}{1-i}\times\frac{1+i}{1+i}=\frac{(1+i)^2}{1+1}=\frac{2i}{2}=i$$

QUESTION

If $(1+i)^3/(1-i)^3-(1-i)^3/(1+i)^3=x+iy$, then

(A) $x=0, y=-2$

(B) $x=-2, y=0$

(C) $x=1, y=1$

(D) $x=-1, y=1$

$(1+i)^6=[(1+i)^2]^3=(2i)^3$

$\frac{(1+i)^3}{(1-i)^3}-\frac{(1-i)^3}{(1+i)^3}=x+iy$

$\frac{(1+i)^6-(1-i)^6}{[(1-i)(1+i)]^3}=x+iy$

$\frac{(2i)^3-(-2i)^3}{(1+1)^3}=x+iy$

$\frac{\cancel{8}i^3+\cancel{8}i^3}{\cancel{8}}=x+iy$

$-i-i=x+iy$

$0+(-2)i=x+iy$

$x=0 \mid y=-2$

QUESTION

If $x = \frac{-1+i\sqrt{3}}{2}$, then $x^2 + x + 1 =$

$x = w$ $\quad$ $w^2 + w + 1 = 0$

A) 2

B) 1/2

C) 0

D) 1

if $z = x + iy$

$$|z| = \sqrt{x^2+y^2}$$

$$|z|^2 = x^2+y^2$$

Properties :-

1. $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$

2. $|z_1 z_2| = |z_1|\,|z_2|$

Ex :- if $z = \frac{(3+2i)(4+3i)}{(3-2i)(1+2i)}$ Find $|z|$

$$|z| = \frac{|3+2i|\,|4+3i|}{|3-2i|\,|1+2i|} = \frac{\cancel{\sqrt{9+4}}\,\sqrt{16+9}}{\cancel{\sqrt{9+4}}\,\sqrt{1+4}}$$

$$= \frac{\sqrt{25}}{\sqrt{5}} = \sqrt{\frac{25}{5}} = \sqrt{5}$$

QUESTION

The modulus of $\frac{1-i}{3+i}+\frac{4i}{5}$ is

(A) $\sqrt{5}$ units

(B) $\sqrt{11}/5$ units

(C) $\sqrt{5}/5$ units ✓

(D) $\sqrt{12}/5$ units

$$\frac{5-5i+12i+4i^2}{5(3+i)}$$

$$=\frac{(5-4)+7i}{5(3+i)}$$

$$z = \frac{1+7i}{5(3+i)}$$

$$|z| = \frac{\sqrt{1+49}}{5\sqrt{9+1}}$$

$$|z| = \frac{1}{5}\sqrt{\frac{50}{10}} = \frac{1}{5}\sqrt{5}$$

$$= \frac{\sqrt{5}}{5}$$

if $z = x + iy$

Then multiplicative inverse of z is z^{-1}

$$\therefore z^{-1} = \frac{1}{z} = \frac{1}{x+iy}$$

$$z^{-1} = \frac{1}{x+iy} \times \frac{x-iy}{x-iy}$$

$$z^{-1} = \frac{x-iy}{x^2+y^2} = \frac{\bar{z}}{|z|^2}$$

if $z = x + iy$

Then conjugate of z is

$$\bar{z} = x - iy$$

$$z = x + iy$$

$$|z| = \sqrt{x^2+y^2}$$

$$|z|^2 = x^2 + y^2$$

① if $z = 3-5i$, Find z^{-1}

Soln:

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{3+5i}{3^2+5^2} = \frac{3+5i}{34}$$

② if $z = \frac{1}{2+3i}$, find z^{-1}

Soln:

$$z = \frac{1}{2+3i} = \frac{2-3i}{4+9} = \frac{2-3i}{13}$$

$$z^{-1} = \frac{2+3i}{13\left[\frac{4}{169}+\frac{9}{169}\right]} = \frac{2+3i}{13\left[\frac{13}{169}\right]} = 2+3i$$

WKT $z^{-1} = \frac{1}{z}$

$\therefore z = \frac{1}{z^{-1}}$

$\therefore$ if $z = \frac{1}{2+3i} \rightarrow z^{-1}$

QUESTION

The multiplicative inverse of $\frac{3+4i}{4-5i}$ is

A $\left(\frac{-8}{25}, \frac{31}{25}\right)$

B $\left(\frac{-8}{25}, \frac{-31}{25}\right)$

C $\left(\frac{8}{25}, \frac{-31}{25}\right)$

D $\left(\frac{8}{25}, \frac{31}{25}\right)$

$$z = \frac{3+4i}{4-5i}$$

$$\therefore z^{-1} = \frac{4-5i}{3+4i}$$

$$= \frac{4-5i}{3+4i} \times \frac{3-4i}{3-4i}$$

$$= \frac{12-16i-15i-20}{9+16}$$

$$= \frac{-8-31i}{25}$$

$$= \left(\frac{-8}{25}\right) + \left(\frac{-31}{25}\right)i$$

if $z = x + iy$

$\bar{z} = x - iy$

Properties:-

1. $\overline{(z_1 \cdot z_2)} = \bar{z}_1 \cdot \bar{z}_2$

2. $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

3. $\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$

4. $\overline{(z_1 - z_2)} = \bar{z}_1 - \bar{z}_2$

5. $\overline{(\bar{z})} = z$

(1) $z = 2+3i$

$\bar{z} = 2-3i$

(2) $z = 2-3i$

$\bar{z} = 2+3i$

(3) $z = -2+3i$

$\bar{z} = -2-3i$

(4) $z = -2-3i$

$\bar{z} = -2+3i$

(5) $z = i-1$

$z = -1+i$

$\bar{z} = -1-i$

QUESTION

if $z \to$ complex No.

The conjugate of a complex number is $\frac{1}{i-1}$. Then, the complex number is

Then

$\bar{z} = \frac{1}{i-1}$

$\bar{z} = \frac{1}{-(1-i)}$

$\bar{z} = \frac{-1}{1-i}$

$\therefore \overline{(\bar{z})} = \overline{\frac{-1}{(1-i)}}$

$z = \frac{-1}{1+i}$

(A) $\frac{1}{i-1}$

(B) $\frac{-1}{i-1}$

(C) $\frac{1}{i+1}$

(D) $\frac{-1}{i+1}$ ✓

* if z is purely real

Then Imaginary Part of $z = 0$

i.e $z = k$

* if z is purely imaginary

Then real part of $z = 0$

ie, $z = ki$

QUESTION

If $\frac{z-1}{z+1}$ is purely imaginary, then

A $|z| = 1/2$

B $|z| = 1$

C $|z| = 2$

D $|z| = 3$

Let

$$\frac{z-1}{z+1} = ki$$

$$z-1 = ki(z+1)$$

$$z-1 = kiz + ki$$

$$z - zki = ki + 1$$

$$z(1-ki) = 1+ki$$

$$z = \frac{1+ki}{1-ki}$$

$$|z| = \frac{\sqrt{1+k^2}}{\sqrt{1+k^2}}$$

$$\underline{|z| = 1}$$

QUESTION

If $\frac{z-\sqrt{2}}{z+\sqrt{2}}$ is purely imaginary, then

A $|z| = 1$

B $|z| = \sqrt{2}$

C $|z| = 3$

D $|z| = 1/\sqrt{2}$

$$\frac{z-\sqrt{2}}{z+\sqrt{2}} = ki$$

$$z-\sqrt{2} = kiz + \sqrt{2}ki$$

$$z(1-ki) = \sqrt{2}ki + \sqrt{2}$$

$$z = \frac{\sqrt{2}(1+ki)}{1-ki}$$

$$|z| = \sqrt{2}\,\frac{|1+ki|}{|1-ki|}$$

$$|z| = \sqrt{2}\,\frac{\sqrt{1+k^2}}{\sqrt{1+k^2}}$$

$$\underline{|z| = \sqrt{2}}$$

QUESTION

It $z=-1+i$, then z lies in

$x<0 \mid y>0$

(A) I quadrant

(B) II quadrant

(C) III quadrant

(D) IV quadrant

QUESTION

If a complex number lies in the III quadrant, then find the quadrant in which its conjugate lies.

$z = -x - iy$

$\Downarrow$

$\bar{z} = -x + iy$

$x < 0 \mid y > 0$

(A) I quadrant

(B) II quadrant

(C) III quadrant

(D) IV quadrant

QUESTION

The value of $(1+i)^3+(1-i)^3 =$

$(1+i)^2(1+i)+(1-i)^2(1-i)$

(A) 1

$2i(1+i)+(-2i)(1-i)$

(B) -2

$2i[1+i-1+i]$

(C) 0

$2i(2i)$

(D) -4

$4i^2=-4$

QUESTION

The least value of $n \in N$ for which $\left[\frac{1-i}{1+i}\right]^n$ is real, is

A 8

B 4

C 2

D 1

$$\left[\frac{(1-i)^2}{(1+i)(1-i)}\right]^n$$

$$\left(\frac{-2i}{2}\right)^n$$

$$(-i)^n$$

$n=1 \Rightarrow (-i)^1 = -i$

$n=2 \Rightarrow (-i)^2 = +i^2 = -1$ is real

QUESTION

The real part of $\frac{(1+i)^2}{(3-i)}$ is

(A) $1/3$

(B) $1/5$

(C) $-1/3$

(D) $-1/5$

$\frac{2i}{3-i} \times \frac{3+i}{3+i}$

$\frac{6i-2}{9+1} = \frac{-2+6i}{10}$

$= \frac{-2}{10} + \frac{6}{10} i$

$\downarrow$

$Re(z) = -\frac{1}{5}.$

QUESTION

The value of $\left(\frac{1+i}{1-i}\right)^{1000} + \left(\frac{1-i}{1+i}\right)^{2000}$ is equal to

(A) $1 + i$

(B) 2

(C) $-i$

(D) 1

$\left\{\left[\frac{1+i}{1-i}\right]^2\right\}^{500} + \left\{\left(\frac{1-i}{1+i}\right)^2\right\}^{1000}$

$\left(\frac{2i}{-2i}\right)^{500} + \left(\frac{-2i}{2i}\right)^{1000}$

$(-1)^{500} + (-1)^{1000}$

$= 1 + 1 = 2$

QUESTION

Find the value of $|\sqrt{3}-\sqrt{-5}|$.

$|\sqrt{3}-\sqrt{5}i|$

$\sqrt{(\sqrt{3})^2+(\sqrt{5})^2}$

$=\sqrt{3+5}$

$=\sqrt{8}$

(A) $\sqrt{8}$

(B) $\sqrt{5}$

(C) $\sqrt{2}$

(D) $\sqrt{34}$

QUESTION

$|z^n| = |z|^n$

If $2^{101} z = (\sqrt{3} + i)^{104}$, then modulus of z is

$z = \frac{1}{2^{101}} (\sqrt{3} + i)^{104}$

A 4

B 8

C 16

D 32

$|z| = \frac{1}{2^{101}} |\sqrt{3} + i|^{104}$

$= \frac{1}{2^{101}} \left(\sqrt{3+1}\right)^{104}$

$= \frac{1}{2^{101}} 2^{104} = 2^3 = 8$

QUESTION

$\sqrt{8} = \sqrt{2^3} = 2^{3/2}$

If $(\sqrt{5}+\sqrt{3}i)^{33} = 2^{49}z$, then modulus of the complex number z is equal to

A 1

B $\sqrt{2}$

C $2\sqrt{2}$

D 4

$|z| = \frac{1}{2^{49}} |\sqrt{5}+\sqrt{3}i|^{33}$

$= \frac{1}{2^{49}} \left(\sqrt{5+3}\right)^{33}$

$= \frac{1}{2^{49}} (\sqrt{8})^{33}$

$= \frac{1}{2^{49}} (2^{3/2})^{33}$

$|z| = \frac{2^{\frac{99}{2}}}{2^{49}} = 2^{\frac{99}{2}-49}$

$= 2^{1/2}$

$= \underline{\sqrt{2}}$

QUESTION

If the conjugate of $\overbrace{(x + iy)(1 - 2i)}^{z}$ is $1 + i$, then

(A) $x = -\dfrac{1}{5}$

(B) $x - iy = \dfrac{1 + i}{1 - 2i}$

(C) $x + iy = \dfrac{1 - i}{1 - 2i}$

(D) $x = \dfrac{1}{5}$

Given $\bar{z} = 1 + i$

$\overline{(\bar{z})} = \overline{1 + i}$

$z = 1 - i$

$(x + iy)(1 - 2i) = 1 - i$

$x + iy = \dfrac{1 - i}{1 - 2i}$

QUESTION

$x^3 - 1 = 0$

$(x-1)(x^2+x+1) = 0$

$\hookrightarrow x = \omega \mid x = \omega^2$

$z^4 = z \mid z^5 = z^2 \mid z^6 = 1$

$z = \omega$

If $z^2 + z + 1 = 0$ where z is a complex number, then the value of $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \cdots . + \left(z^6 + \frac{1}{z^6}\right)^2$ is

$\left(\frac{z^2+1}{z}\right)^2 + \left(\frac{z^4+1}{z^2}\right)^2 + \left(1 + \frac{1}{1}\right)^2 + \left(\frac{z^2+1}{z}\right)^2 + \left(\frac{z^4+1}{z^2}\right)^2 + \left(1 + \frac{1}{1}\right)^2$

$2\left\{\left(-\frac{z}{z}\right)^2 + \left(-\frac{z^2}{z^2}\right)^2 + (2)^2\right\}$

$= 2\{1 + 1 + 4\} = 2(6) = 12$

$1 + \omega = -\omega^2$

$1 + \omega^2 = -\omega$

(A) 18

(B) 54

(C) 6

(D) 12

QUESTION

If ω is a complex cube root of unity, then $(1-\omega+\omega^2)(1-\omega^2+\omega^4)(1-\omega^4+\omega^8)(1-\omega^8+\omega^{16})$ is equal to

$(1+w^2-w)(1+w-w^2)(1+w^2-w)(1+w-w^2)$

$(-w-w)(-w^2-w^2)(-w-w)(-w^2-w^2)$

$(-2w)^2(-2w^2)^2$

$(4w^2)(4w^4)$

$16w^6$

$=16$

A) 12

B) 16

C) 14

D) None of these

QUESTION

$1+w=-w^2$

If ω is an imaginary cube root of unity then $(\underbrace{1+\omega}_{}-\omega^2)^7$ equals

$(-w^2-w^2)^7$

(A) 128ω

(B) -128ω

(C) $128\omega^2$

(D) $-128\omega^2$

$(-2w^2)^7$

$(-2)^7(w^2)^7$

$-128\,w^{14}$

$-128w^2$

QUESTION

$\omega^4 = \omega$

ω is an imaginary cube root of unity. If $(1+\omega^2)^m = (1+\omega^4)^m$, then least positive integral value of m is

$(-\omega)^m = (1+\omega)^m$

$(-\omega)^m = (-\omega^2)^m$

$\cancel{(-1)^m} \cdot \omega^m = \cancel{(-1)^m}\, \omega^{2m}$

$\omega^m = \omega^{2m}$

only if $m=3$

A 6

B 5

C 4

D 3

QUESTION

The value of $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^6+\left(\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\right)^6$ is

$w = \frac{-1+i\sqrt{3}}{2}$

$-2w = 1-i\sqrt{3}$

$w^2 = -\frac{1-i\sqrt{3}}{2}$

$-w^2 = 1+i\sqrt{3}$

(A) 0

(B) 1

(C) 2

(D) -2

$\left(\frac{-2w^2}{-2w}\right)^6+\left(\frac{-2w}{-w^2}\right)^6$

$(w^6)+\left(\frac{1}{w}\right)^6$

$1+1=2$

QUESTION

The value of $\left(\frac{-1+\sqrt{-3}}{2}\right)^{100}+\left(\frac{-1-\sqrt{-3}}{2}\right)^{100}$ is

A 2

B 0

C -1

D 5

$$\left(\frac{-1+\sqrt{3}i}{2}\right)^{100}+\left(\frac{-1-\sqrt{3}i}{2}\right)^{100}$$

$$\omega^{100}+(\omega^{2})^{100}$$

$$\omega^{100}+\omega^{200}$$

$$\omega^{99}\cdot\omega+\omega^{198}\cdot\omega^{2}$$

$$(1)\omega+(1)\omega^{2}=\omega+\omega^{2}$$

$$=-1$$

Sequence & Series

(1) n^{th} term of G.P :-

(2) Sum of n terms of a G.P

(3) Sum of Infinite terms of a G.P

(4) Relationship b/w AM & G.M

(5) Insertion of n G.M's b/w a & b

* n^{th} term of a G.P :-

$$a_n = ar^{n-1}$$

* sum of n terms of a G.P

$$S_n = \frac{a[r^n - 1]}{r-1} \text{ if } r > 1$$

or

$$S_n = \frac{a[1-r^n]}{1-r} \text{ if } r < 1$$

* sum of infinite terms of a G.P if $|r| < 1$ is

$$S_\infty = \frac{a}{1-r}$$

Here the sequence is

$a, ar, ar^2, ar^3, \ldots$

① if $1+\sin\theta+\sin^2\theta+\sin^3\theta+\ldots = 2\sqrt{3}+4$, find '$\theta$'

Soln:

$a = 1$

$r = \sin\theta$ | The sequence is in G.P

$\therefore S_\infty = 2\sqrt{3}+4$

$$\frac{1}{1-\sin\theta} = 2\sqrt{3}+4$$

$$\frac{1}{1-\sin\theta} = (2\sqrt{3}+4)\times\frac{2\sqrt{3}-4}{2\sqrt{3}-4}$$

$$\frac{1}{1-\sin\theta} = \frac{4(3)-16}{2\sqrt{3}-4}$$

$$\frac{1}{1-\sin\theta} = \frac{-4}{2\sqrt{3}-4}$$

$$1-\sin\theta = \frac{2\sqrt{3}}{-4}+\left(\frac{-4}{-4}\right)$$

$$1-\sin\theta = -\frac{\sqrt{3}}{2}+1$$

$$1-\sin\theta = 1-\frac{\sqrt{3}}{2}$$

$\sin\theta = \sqrt{3}/2$ | $\theta = \pi/3$

QUESTION

Find the values of ' k ' for which $-\frac{2}{7}, k, -\frac{7}{2}$ are in G.P.

(A) 0

(B) ± 4

(C) ± 2

(D) ± 1

$$\frac{k}{-2/7} = \frac{-7/2}{k}$$

$$k^2 = 1$$

$$\underline{k = \pm 1}$$

QUESTION

Which term of the G.P. $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$ is $\frac{1}{128}$

(A) 9^{th}

(B) 8^{th}

(C) 7^{th}

(D) 5^{th}

$$a_n = \frac{1}{128} = a r^{n-1}$$

$$\frac{1}{128} = 2\left(\tfrac{1}{2}\right)^{n-1}$$

$$\frac{1}{256} = \left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^{8} = \left(\frac{1}{2}\right)^{n-1}$$

$$n-1 = 8$$

$$n = 9$$

Insertion of 'n' G.M's b/w a & b :-

Let $a, G_1, G_2, G_3, \ldots G_n, b$ be a sequence in G.P

such that $G_1, G_2, \ldots G_n$ are 'n' geometric means

Total no of terms in the sequence $= n+2$

$$b = aR^{(n+2)-1}$$

$$b = aR^{n+1}$$

Common Ratio $R = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$a_n = aR^{n-1}$

$a_{n+2} = aR^{n+2-1}$

$\Downarrow$

$a_{n+2} = aR^{n+1}$

$\therefore a \to 1^{st}$ term.

$G_1 = 2^{nd}$ term $= aR = a\left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$G_2 \to 3^{rd}$ term $= aR^2 = a\left(\frac{b}{a}\right)^{\frac{2}{n+1}}$

$\vdots$

$G_n = (n+1)^{th}$ term $= aR^n = a\left(\frac{b}{a}\right)^{\frac{n}{n+1}}$

(*) In the problems related to Insertion of G.M's we need to majorly find common ratio

$$R = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$$

$n \to n$ G.M's which are inserted b/w a & b

(1) if $a=2$ & $b=10$

Find the 4 G.M's b/w a & b so that resultant sequence is in GP

Soln:-

There $n=4$ | $a=2$

$n+1=5$ | $b=10$

$$R=\left(\frac{b}{a}\right)^{\frac{1}{5}}=\left(\frac{10}{2}\right)^{1/5}=5^{1/5}$$

$G_1=aR=2(5)^{1/5}$ | $G_3=aR^3=2(5)^{3/5}$

$G_2=aR^2=2(5)^{2/5}$ | $G_4=aR^4=2(5)^{4/5}$

(*) if $a=3$ & $b=18$

Find the product of 4 G.M's inserted b/w a & b.

Soln:

$n=4$ | $a=3$

$n+1=5$ | $b=18$

$$R=\left(\frac{b}{a}\right)^{\frac{1}{5}}=(6)^{\frac{1}{5}}$$

$G_1=aR=3(6)^{1/5}$

$G_2=aR^2=3(6)^{2/5}$

$G_3=3(6)^{3/5}$

$G_4=3(6)^{4/5}$

$G_1\,G_2\,G_3\,G_4$

$=3^4(6)^{\frac{1+2+3+4}{5}}$

$=3^4(6)^{10/5}=3^4(6^2)=81\times 36$

$=\underline{2916}$

QUESTION

The product of three geometric means between 4 and 1/4 is

(A) 0

(B) −1

(C) 1

(D) −2

$a = 4 \mid b = 1/4$

$n = 3$

$n + 1 = 4$

$R = \left(\frac{1/4}{4}\right)^{1/4} = \left(\frac{1}{16}\right)^{1/4}$

$R = \left[\left(\frac{1}{2}\right)^4\right]^{1/4}$

$R = \frac{1}{2}$

$G_1 \quad G_2 \quad G_3$

$= 4\left(\frac{1}{2}\right) \cdot 4\left(\frac{1}{2}\right)^2 4\left(\frac{1}{2}\right)^3$

$= 4^3\left(\frac{1}{2}\right)^6$

$= \frac{64}{64} = 1$

QUESTION

The third term of a G.P. is the square of the first term. If the second term is 8 , then its sixth term is

A 128

B 126

C 124

D 120

$a_3 = (a_1)^2$

$ar^2 = a^2$

$r^2 = a$

$a = r^2$

$a = 4$

$a_2 = 8$

$ar = 8$

$r^2(r) = 8$

$r^3 = 8$

$r = 2$

$a_6 = ar^5$

$= 4(2)^5$

$= 2^7$

$= \underline{128}$

(*) if a & b are any +ve no, Then

$\boxed{A.M = \frac{a+b}{2}}$ & $\boxed{G.M = \sqrt{ab}}$

& A.M $\geq$ GM

$\Downarrow$

$\boxed{a+b \geq 2\sqrt{ab}}$

QUESTION

If A.M. between two positive numbers a and b is 15 and G.M. between a and b is 9, then the numbers are

A) 3, 27

B) 2, 27

C) 3, 26

D) −3, −27

$\frac{a+b}{2} = 15$

$a+b = 30$

$\sqrt{ab} = 9$

$ab = 81$

option verification

QUESTION

The arithmetic mean of two numbers x and y is 3 and geometric mean is 1 . Then $x^2 + y^2$ is equal to

(A) 34

(B) 31

(C) 32

(D) 33

$\frac{x+y}{2} = 3$ | $\sqrt{xy} = 1$

$x+y = 6$ | $xy = 1$

$x^2+y^2 = (x+y)^2 - 2xy$

$= 6^2 - 2(1)$

$= \underline{34}$

QUESTION

if α & β are the roots of the roots of the Quadratic eqn, Then the eqn is $x^2-(\alpha+\beta)x+\alpha\beta=0$

Two numbers x and y have arithmetic mean 9 and geometric mean 4. Then x and y are the roots of

(A) $x^2 - 18x - 16 = 0$

(B) $x^2 - 18x + 16 = 0$

(C) $x^2 + 18x - 16 = 0$

(D) $x^2 + 18x + 16 = 0$

$\frac{x+y}{2}=9$ | $\sqrt{xy}=4$

$x+y=18$ | $xy=16$

$x^2-18x+16=0$

QUESTION

Find the sum to infinity for $5, \frac{5}{3}, \frac{5}{9}, \ldots\ldots$

A. 7.5

B. 15

C. 7

D. 17

$a = 5$

$r = \frac{5/3}{5} = \frac{1}{3}$

$S_\infty = \frac{a}{1-r} = \frac{5}{1-1/3} = \frac{5}{2/3}$

$= \frac{15}{2} = \underline{7.5}$

QUESTION

$a = p \mid r = 1/p$

If sum of the infinite G.P. $p, 1, 1/p, 1/p^2, \ldots$ is $9/2$, then the value of p is

A $3/2$

B 3

C Both (A) and (B)

D None of these

$$\frac{p}{1-1/p} = \frac{9}{2}$$

$$\frac{p^2}{p-1} = \frac{9}{2}$$

$$2p^2 = 9p - 9$$

$$2p^2 - 9p + 9 = 0$$

$$2p^2 - 6p - 3p + 9 = 0 \qquad +18 \;\; -6 \;\; -3$$

$$2p(p-3) - 3(p-3) = 0$$

$$p = \frac{3}{2} \mid p = 3$$

QUESTION

The sum of an infinite number of terms of a G.P. is 15 and the sum of their squares is 45 , then the series is

(A) $5, \frac{10}{3}, \frac{20}{9}, \ldots$

(B) $2, \frac{1}{3}, \frac{1}{8}, \ldots$

(C) $5, \frac{20}{9}, \frac{40}{11}, \ldots$

(D) $1, \frac{1}{3}, \frac{1}{7}, \ldots$

$\frac{a}{1-r} = 15$ | $\frac{a^2}{1-r^2} = 45$

$\frac{a}{1-r} \cdot \frac{a}{1+r} = 45$

$\Downarrow$

$a = 15 - 15r \rightarrow (1)$

$\frac{a}{1+r} = 3$

$a = 3 + 3r \rightarrow (2)$

$15 - 15r = 3 + 3r$

$12 = 18r$

$r = 2/3$

$a = 15 - 15r$

$= 15 - 15\left(\frac{2}{3}\right)$

$a = 15 - 10 = 5$

$ar = 5\left(\frac{2}{3}\right) = \frac{10}{3}$

$ar^2 = 5\left(\frac{4}{9}\right) = \frac{20}{9}$

QUESTION

$$\frac{(ar)^2}{a^2} = r^2$$

Sum of infinite number of terms in G.P. is 20 and sum of their squares is 100 , then the common ratio of G.P is

(A) 1/5

(B) 3/5

(C) 2/5

(D) 4/5

$$a + ar + ar^2 + ar^3 + \dots = 20$$

$$\frac{a}{1-r} = 20$$

$$\Downarrow$$

$$a = 20 - 20r \rightarrow (1)$$

$$a^2 + (ar)^2 + (ar^2)^2 + \dots = 100$$

$$\frac{a^2}{1-r^2} = 100$$

$$\frac{a}{1-r} \cdot \frac{a}{1+r} = 100$$

$$20\left(\frac{a}{1+r}\right) = 100$$

$$\frac{a}{1+r} = 5$$

$$a = 5 + 5r \rightarrow (2)$$

$$20-20r=5+5r$$

$$15=25r$$

$$r=\frac{3}{5}$$

QUESTION

The fifth term of a G.P. is x, eighth term of a G.P. is y and eleventh term of a G.P. is $z, y^2 =$

A. xz

B. yz

C. x/z

D. y/z

$a_5 = x$

$ar^4 = x$

$a_8 = y$

$ar^7 = y$

$\Downarrow$

$ar^4(r^3) = y$

$xr^3 = y$

$r^3 = \frac{y}{x}$

$a_{11} = z$

$ar^{10} = z$

$ar^7 \, r^3 = z$

$\downarrow \quad \downarrow$

$y \; \frac{y}{x} = z$

$y^2 = xz$

QUESTION

If the third term of a G.P. is equal to 4 , then product of its first five terms is equal to

$a_3 = 4$

$ar^2 = 4$

$a_1 a_2 a_3 a_4 a_5$

$= a(ar)(ar^2)(ar^3)(ar^4)$

$= a^5 r^{1+2+3+4}$

$= (ar^2)^5$

$= 4^5$

$= (2^2)^5 = \underline{2^{10}}$

A 2^6

B 2^{10}

C 2^8

D None of these

* $\boxed{\frac{a}{r}, a, ar}$ are 3 consecutive terms in G.P

* $\boxed{\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2}$ are 5 consecutive terms in GP

QUESTION

If the product of five consecutive terms of a G.P. is $\frac{243}{32}$, then the middle term is

(A) 2/3

(B) 3/2

(C) 4/3

(D) 3/4

$$\frac{a}{r^2}\cdot\frac{a}{r}\cdot a\cdot ar\cdot ar^2=\left(\frac{3}{2}\right)^5$$

$$a^5=\left(\frac{3}{2}\right)^5$$

$$a=\frac{3}{2}$$

QUESTION

The sum of the 3rd and the 4th terms of a G.P. is 60 and the product of its first three terms is 1000 . If the first term of this G.P. is positive, then its 7th term is

(A) 7290

(B) 320

(C) 640

(D) 2430

$a_3 + a_4 = 60$

$ar^2 + ar^3 = 60$

$ar(r + r^2) = 60$

$10(r + r^2) = 60$

$r + r^2 = 6$

$a(ar)(ar^2) = 1000$

$a^3 r^3 = 1000$

$(ar)^3 = 10^3$

$ar = 10$

$r^2 + r - 6 = 0$

-6

$+3 \quad -2$

$(r+3)(r-2) = 0$

$r = -3 \quad | \quad r = 2$

$a = -\frac{10}{3} \quad | \quad a = 5$

$a > 0$

$\Downarrow$

$a = 5$

$a_7 = ar^6 = 5(2)^6$

$= 5(64) = \underline{320}$

Thank You