Vernacular

Ultimate kcet crash course 2026

Maths

DPP: 1

Application of derivatives Statistics

Q1 The function $f(x)=\frac{1}{1+x^2}$ is increasing for
(A) $x\geq 1$ (B) $x\leq 0$
(C) $x\leq 1$ (D) $x\geq 0$

Q2 Find the values of *x* for which the function $f(x)=x^3+12x^2+36x+6$ is decreasing.
(A) $6<x<9$
(B) $-6<x<-2$
(C) $-6<x<0$
(D) $-4<x<4$

Q3 The function $\mathrm{f}(\mathrm{x})=\tan\mathrm{x}-\mathrm{x}$
(A) neither increases nor decreases
(B) always increases
(C) always decreases
(D) never increases

Q4 A stationary value of $f(x)=x(\ln x)^2$ is
(A) $2e^{-2}$ (B) $4e^{-2}$
(C) $2e^2$ (D) $4e^2$

Q5 Let *I* be an open interval contained in the domain of a real function '*f*', then *f*(*x*) is called strictly decreasing function in *I* if
(A) $x_1<x_2$ in $I\Rightarrow f(x_1)<f(x_2)$
(B) $x_1<x_2$ in $I\Rightarrow f(x_1)>f(x_2)$
(C) $x_1=x_2$ in $I\Rightarrow f(x_1)=f(x_2)$
(D) $x_1=x_2$ in $I\Rightarrow f(x_1)<f(x_2)$

Q6 $f(x)=\left(\sin^{-1}x\right)^2+\left(\cos^{-1}x\right)^2$ is stationary at
(A) $x=\frac{1}{\sqrt{2}}$ (B) $x=\frac{\pi}{4}$
(C) $x=1$ (D) $x=0$

Q7 $f(x)=2x^3-15x^2+36x+5$ is decreasing in
(A) (1, 2)
(B) (2, 3)
(C) $(-\infty, 2)$
(D) $(3, \infty)$

Q8 The fucntion $\tan x-4x$ for $-\frac{\pi}{3}<x<0$ is
(A) Decreasing
(B) Increasing
(C) neither increasing nor decreasing
(D) None of these

Q9 If the function $f(x)=\frac{a\sin x+2\cos x}{\sin x+\cos x}$ is increasing for all values of x, then
(A) $a<1$ (B) $a>1$
(C) $a<2$ (D) $a>2$

Q10 $f(x)=x-e^x$ is decreasing in
(A) $(-\infty, 0)$
(B) $(0, \infty)$
(C) $(-\infty, \infty)$
(D) $\emptyset$

Q11 If the function $f(x)=x^4-62x^2+ax+9$ is maximum at x = 1, then the value of a is
(A) 120 (B) –120
(C) 52 (D) 128

Q12 Twenty metres of wire is available to fence off a flower bed in the form of a circular sector. What must the radius of the circle be, if the area of the flower bed be greatest?
(A) 10 m (B) 4 m
(C) 5 m (D) 6 m

Q13 A circular sector of perimeter 60 meter with maximum area is to be constructed. The radius of the circular arc in meter must be
(A) 10m (B) 15m
(C) 5m (D) 20m

Q14 The maximum height of the curve $y=6\cos x-8\sin x$ above the *x*-axis
(A) 6 (B) 8
(C) 14 (D) 10

Q15 Maximum value of $\left(\frac{1}{x}\right)^x$ is

(A) $(e)^e$ (B) $(e)^{1/e}$

(C) $(e)^{-e}$ (D) $(1/e)^e$

Q16 Two number *x and y such that* x + y = 2 and x^2y is maximum are respectively

(A) $\frac{1}{5}, \frac{5}{3}$ (B) $\frac{4}{3}, \frac{2}{3}$

(C) 1, 1 (D) $\frac{6}{5}, \frac{4}{5}$

Q17 If the product of two positive numbers is 256 then the least value of their sum is

(A) 32 (B) 16

(C) 48 (D) 40

Q18 If 20 is divided into two parts so that their product is maximum then the numbers are

(A) 6, 14 (B) 9, 11

(C) 5, 15 (D) 10, 10

Q19 The minimum value of $\left(x^2 + \frac{250}{x}\right)$ is

(A) 75 (B) 50

(C) 25 (D) 55

Q20 Let x be a number which exceeds its square by the greatest possible quantity. Then x is equal to

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{3}{4}$

(D) none of these

Q21 The following Information Relates to a sample of size 60: $\sum x^2 = 18000, \sum x = 960$. The variance is.

(A) 6.63 (B) 16

(C) 22 (D) 44

Q22 The arithmetic mean and standard deviation of a series of 20 items were calculated by a student as 20 cms and 5 cms respectively But while calculating then, an item 13 was misread at 30 Find the correct standard deviation.

(A) 3.66 cm (B) 4.66 cm

(C) 6.66 cm (D) 7.66 cm

Q23 In an experiment with 15 observations on x, the following results were available : $\Sigma x^2 = 2830$, Σx = 170. One observation 20 was found to be wrong and was replaced by the correct value 30 . Then the corrected standard deviation is

(A) $\sqrt{188.66}$ (B) $\sqrt{177.33}$

(C) $\sqrt{8.33}$ (D) $\sqrt{78.00}$

Q24 If the median of the data 6,7,x-2,x,18,21 written in ascending order is 16 , then the standard deviation of that data is

(A) $\sqrt{\frac{151}{5}}$ (B) $\sqrt{\frac{94}{3}}$

(C) $\sqrt{\frac{65}{2}}$ (D) $\sqrt{\frac{100}{3}}$

Q25 Standard deviation of first *n* odd natural numbers is

(A) $\sqrt{n}$ (B) $\sqrt{\frac{(n+2)(n+1)}{3}}$

(C) $\sqrt{\frac{n^2-1}{3}}$ (D) n

Q26 Find the standard deviation for the following data:

x_i	3	8	13	18	23
f_i	7	10	15	10	6

(A) 6.12 (B) 5.12

(C) 3.12 (D) 7.12

Q27 The mean of 6 distinct observations is 6.5 and their variance is 10.25 . If 4 out of 6 observations are 2,4 , 5 and 7, then the remaining two observations are

(A) 8, 13 (B) 1, 20

(C) 10, 11 (D) 3, 18

Q28 If each of the observations $x_1, x_2, ..., x_n$ is increased by 'a', where a is a negative or positive number, then the variance is

(A) same

(B) Increased by a^2 times

(C) Decreased by a^2 times

(D) None of these

Q29 The standard deviation of a variable x is σ. The standard deviation of the variable $\frac{ax+b}{c}$ where a, b, c are constants is

(A) (a/c) (B) |a/c|

(C) $\left|\frac{a^2}{c^2}\right|\sigma$ (D) (a) σ

Q30 Mean and standard deviation of 100 items are 50 and 4, respectively. Find the squares of items

(A) 251600 (B) 250000

(C) 215650 (D) None of these

Answer Key

Q1	(B)	Q16	(B)
Q2	(B)	Q17	(A)
Q3	(B)	Q18	(D)
Q4	(B)	Q19	(A)
Q5	(B)	Q20	(A)
Q6	(A)	Q21	(D)
Q7	(B)	Q22	(B)
Q8	(A)	Q23	(D)
Q9	(D)	Q24	(B)
Q10	(B)	Q25	(C)
Q11	(A)	Q26	(A)
Q12	(C)	Q27	(C)
Q13	(B)	Q28	(A)
Q14	(D)	Q29	(B)
Q15	(B)	Q30	(A)

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

$f\left(x\right)=\frac{1}{1+x^2}\quad \therefore f'\left(x\right)=\frac{-2x}{\left(1+x^2\right)^2}$

$f'\left(x\right)>0$ as $f\left(x\right)$ is increasing

$\therefore \frac{-2x}{\left(1+x^2\right)^2}>0$ for $x<0$

Also for $x=0,\ \frac{-2x}{\left(1+x^2\right)^2}=0$

So, $\frac{-x}{\left(1+x^2\right)^2}>0$ for $x<0$

Video Solution:

Q2 Text Solution:

$f\left(x\right)=x^3+12x^2+36x+6$

$f'\left(x\right)=3x^2+24x+36$

$=3\left[x^2+8x+12\right]$

$f'\left(x\right)=3\left(x+2\right)\left(x+6\right)$

Since $\mathrm{f}\left(x\right)$ is decreasing

$\left(x\right)<0$

$\Rightarrow\ x\in\left(-6,\ -2\right)$

Video Solution:

Q3 Text Solution:

$\left(\mathrm{x}\right)=\sec^2\mathrm{x}-1\ge 0,\forall\mathrm{x}\ \therefore \mathrm{f}\left(\mathrm{x}\right)$ is increasing $\forall\mathrm{x}$.

Video Solution:

Q4 Text Solution:

$f'\left(x\right)=0\ \Rightarrow x=1,\ e^{-2}$ and $f\left(1\right)=0,$ $f\left(e^{-2}\right)=4e^{-2}$

Video Solution:

Q5 Text Solution:

By the definition of decreasing function

$x_1<x_2\ in\ I\ \Rightarrow f(x_1)>f(x_2)$

Video Solution:

Q6 Text Solution:

$f'\left(x\right)=0\ \Rightarrow \sin^{-1}x=\cos^{-1}x\ \ \Rightarrow x=\frac{1}{\sqrt{2}}$

Video Solution:

Q7 Text Solution:

For the function to be decreasing f'(x)<0

Video Solution:

Q8 Text Solution:

f(x) = tan x - 4x

$f'(x) = \sec^2 x - 4$

Given $-\frac{\pi}{3} < x < 0$

$\sec\left(-\frac{\pi}{3}\right) > \sec x > \sec 0$

$2 > \sec x > 1$

$1 < \sec^2 x < 4$

$-3 < \sec^2 x - 4 < 0$

$-3 < f'(x) < 0$

$\Rightarrow f'(x) = -ve$

$\therefore$ f(x) is decreasing

Video Solution:

Q9 Text Solution:

$$\text{We have, } f'\left(x\right)$$

$$= \frac{(a\cos x - 2\sin x)(\sin x + \cos x)}{-(a\sin x + 2\cos x)(\cos x - \sin x)}$$

$$= \frac{-2\sin^2 x + a\cos^2 x + a\sin^2 x - 2\cos^2 x}{(\sin x + \cos x)^2}$$

$$= \frac{a-2}{(\sin x + \cos x)^2}$$

Since, f(x) is increasing for all values of x.

$\Rightarrow f'\left(x\right) > 0 \Rightarrow a - 2 > 0 \Rightarrow a$

$> 2 \quad \left(\because (\sin x + \cos x)^2 > 0\right)$

Video Solution:

Q10 Text Solution:

For a function to be decreasing

$f'(x) < 0$ *or* $f'(x) \leq 0$

Now, $f'(x) = 1 - e^x$

So by definition,

$1 - e^x < 0 \Rightarrow 1 < e^x \Rightarrow \log(1) < x$

$0 < x$ *or*, $x > 0$

$\Rightarrow x \in (o, \infty)$

Video Solution:

Q11 Text Solution:

$f(x) = x^4 - 62x^2 + ax + 9$

For maxima, $f'(x) = 4x^3 - 124x + a = 0$

At x = 1, $4(1)^3 - 124(1) + a = 0 \Rightarrow a = 120$

Video Solution:

Q12 Text Solution:

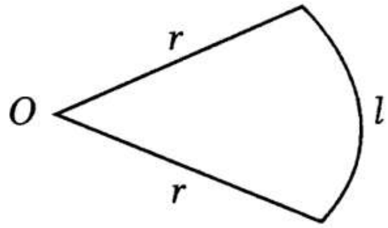
Let r is the radius of circle and l be the length of an arc of that circular sector.

$\therefore\ 20 = 2r + l$

$\Rightarrow\ l = 20 - 2r$

Area, $A = \frac{1}{2}lr = \frac{1}{2}(20 - 2r)r$

For the maximum area, $\frac{dA}{dr} = 0$



i.e., $\frac{1}{2}[20-4r]=0 \Rightarrow 4r=20 \Rightarrow r=5$
Radius must be 5 m if the area of the flower bed to be greatest.

Video Solution:

Q13 Text Solution:

$P=60$
$2r+r\theta=60$
$2r^2+r^2\theta=60r$
$r^2\theta=60r-2r^2$
$\frac{r^2\theta}{2}=30r-r^2$

$A=\pi r^2\left(\frac{\theta}{2\pi}\right)=\frac{r^2\theta}{2}$
$A=30r-r^2$
$\frac{dA}{dr}=30-2r$
Here $\frac{dA}{dr}=0 \Rightarrow r=15m$

Video Solution:

Q14 Text Solution:

The maximum height of the curve $y = 6\cos x - 8\sin x$ above the x-axis is 10

Video Solution:

Q15 Text Solution:

$f(x)=\left(\frac{1}{x}\right)^x \Rightarrow f'(x)$
$=\left(\frac{1}{x}\right)^x\left(\log\frac{1}{x}-1\right)$
$f'(x)=0 \Rightarrow \log\frac{1}{x}=1=\log e \Rightarrow \frac{1}{x}=e$
$\Rightarrow x=\frac{1}{e}$
Therefore maximum value of function is $e^{1/e}$.

Video Solution:

Q16 Text Solution:

$x+y=2$ and Let $P=x^2y$
$y=2-x \qquad P=x^2(2-x)$
$P=2x^2-x^3$
$\frac{dP}{dx}=4x-3x^2$
$\frac{dP}{dx}=0$
$x[4-3x]=0$
$x=0$ and $x=\frac{4}{3}$
$y=2 \quad \Rightarrow y=\frac{2}{3}$

Video Solution:

Q17 Text Solution:

The required two positive numbers are 16,16.
Thus least value of their sum is 32.

Video Solution:

Q18 Text Solution:

$x+y=20 \Rightarrow y=20-x$

$Let,\ P=xy=x(20-x)$

$P=20x-x^2$

$For\ maximum\ product\ \frac{dP}{dx}=0$

$\Rightarrow 20-2x=0$

$\Rightarrow x=10\ \&\ y=10$

Video Solution:

Q19 Text Solution:

Let $f(x)=x^2+\frac{250}{x}$

$f'(x)=2x-\frac{250}{x^2}$

For critical points

$f'(x)=0 \Rightarrow f'(x)=0 \Rightarrow 2x-\frac{250}{x^2}=0$

$\therefore$ x = 5

Now minimum value is f(5) = 75

Video Solution:

Q20 Text Solution:

Let $y=x-x^2$

For y to be maximum : $\frac{dy}{dx}=0$

$\therefore\ 1-2x=0$ or $x=\frac{1}{2}$

Video Solution:

Q21 Text Solution:

$$\text{Vareance } = \frac{\sum x_i^2}{x} - \left(\frac{\sum x_i}{x}\right)^2$$

$=\frac{18000}{60}-\left(\frac{960}{60}\right)^2$

$=300-256=44.$

Video Solution:

Q22 Text Solution:

Here $x=20 \quad \bar{x}=20 \quad \sigma_x=5$

Now $20=\frac{\sum x_i}{20} \Rightarrow \Sigma x_3=400$

Now Correct $\sum x_i=400-30+13$

$=383$

$\therefore$ correct mean $=\frac{583}{20}=19.15$

Also, $25=\frac{\sum x_i^2}{x}-(\bar{x})^2=\frac{\sum x_i^2}{20}-20^2$

$\therefore \sum x_2^2=425\times 20=8500$

So correct $\sum x_3^2=8500-30^2+(13)^2$

$=8500-900+169$

$=7769$

$\therefore$ correct variance $=\frac{7769}{20}-(19.15)^2$

$=388.45-366.72$

$=21.73$

$\therefore$ correct S.D $=\sqrt{21.73}=4.66$ cm.

Video Solution:

Q23 Text Solution:

Correct $\Sigma x = 170 - 20 + 30 = 180,$

$\Sigma x^2 = 2830 - (20)^2 + (30)^2 = 3330$

$\therefore \sigma^2 = \frac{1}{n}\Sigma x^2 - \left(\frac{1}{n}\Sigma x\right)^2 = \frac{1}{15} \times 3330$

$- \left(\frac{1}{15} \times 180\right)^2$

$= 222 - 144 - 78$

$\Rightarrow \sigma = \sqrt{78.00}$

Video Solution:

Q24 Text Solution:

Median of $6, 7, x-2, x, 18, 21$ is 16

$\Rightarrow \frac{x-2+x}{2} = 16 \Rightarrow x = 17$

The observation are 6, 7, 15, 17, 18, 21.

Now, mean = $= \frac{6+7+15+17+18+21}{6} = \frac{84}{6} = 14$

Variance = $= \frac{6^2+7^4+15^2+17^2+18^2+21^2}{6} - (14)^2$

$= \frac{36+49+225+289+324+441}{6} - 196$

$= \frac{1364}{6} - 196 = \frac{94}{3} = 31\frac{1}{3} \Rightarrow \sigma = \sqrt{\frac{94}{3}}$

Video Solution:

Q25 Text Solution:

Lete x_i: 1, 3, 5,..... $(2n - 1)$

$\bar{x} = \frac{n^2}{n} = n$

$\sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2 = \frac{n(2n+1)(2n-1)-n^2}{3n}$

$= \frac{n^2-1}{3}$

$\therefore \sigma\sqrt{\frac{n^2-1}{3}}$

Video Solution:

Q26 Text Solution:

Let us form the following table:

X_i	f_i	$f_i x_i$	x_1^2	$f_i x_1^2$
3	7	12	9	63
8	10	80	64	640
13	15	195	169	2535
18	10	180	324	3240
23	6	138	529	3174
Total	**48**	**614**		**9652**

Now, σ

$= \frac{1}{N}\sqrt{N\sum_{i=1}^{n} f_i x_i^2 - \left(\sum_{i=1}^{n} f_i x_i\right)^2}$

$= \frac{1}{48}\sqrt{48 \times 9652 - (614)^2}$

$= \frac{1}{48}\sqrt{463296 - 376996}$

$= \frac{1}{48} \times 293.77 = 6.12$

Video Solution:

Q27 Text Solution:

Let the remaining two observations be and .

Then, mean $= \frac{2+4+5+7+a+b}{6} = 6.5$

(Given),

⇒ a+b=21

Also, variance =10.25 (Given)

⇒ $1/6(2^2+4^2+5^2+7^2+a^2+b^2) - (6.5)^2 = 10.25$

⇒ $a^2 + b^2 = 315-94 = 221)$

Now, $(a+b)^2 + (a-b)^2 = 2(a^2+b^2)$

⇒ $(a - b)^2 = 2 \times 221 - (21)^2$

[From (i) and (ii)]

⇒ $(a - b)^2 = 1$ ⇒ a - b = ±1

If a-b=1, then a=11,b=10
If a-b=-1, then a=10,b=11

Video Solution:

Q28 Text Solution:

$\sigma_{\mathrm{x}}^2 = \sigma_{\mathrm{y}}^2$

Video Solution:

Q29 Text Solution:

$y = \frac{ax+b}{c}$

$y = \frac{a}{c}x + b$

$\therefore \sigma y = \frac{a}{c}\sigma_x$

$\therefore \sigma = \left|\frac{a}{c}\right|\sigma$

Video Solution:

Q30 Text Solution:

We have given, $\bar{x} = 50,\ n = 100 \text{ and } \sigma = 4$

$\therefore \frac{\Sigma x_i}{100} = 50 \ \Rightarrow \Sigma x_i = 5000$

Also, $\sigma'^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$

$\Rightarrow \ (4)^2 = \frac{\Sigma x_i^2}{100} - (50)^2 \ \Rightarrow 16 = \frac{\Sigma x_i^2}{100}$
$- 2500$

$\Rightarrow \frac{\Sigma x_i^2}{100} = 2516 \ \ \Rightarrow \Sigma x_i^2 = 251600$

Video Solution: