ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture - 01

vectors and 3D

By – Guru sir

Recap of previous lecture

1. Determinants
2.
3.
4.

Topics *to be covered*

1. Vector Algebra
2.
3.
4.

① $\vec{a}\cdot\vec{a} = |\vec{a}|^2$

② $\vec{a}\times\vec{a} = \vec{0}$

③ $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\,\vec{a}\cdot\vec{b}$

$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

या

$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$

④ $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\,\vec{a}\cdot\vec{b}$

या

$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

To find angle b/w 2 vectors

① Dot Product :-

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$$

② Cross Product :-

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|}$$

Question

The vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i} - 2\hat{j} + 4\hat{k}$ are the sides of a $\triangle ABC$. The length of the median through A is

(A) $\sqrt{18}$

(B) $\sqrt{72}$

(C) $\sqrt{33}$

(D) $\sqrt{288}$

Question

$\vec{c} \to$ unit vector

$\Rightarrow |\vec{c}| = 1$

$\to |\vec{a}| = 1 \mid |\vec{b}| = 1$

Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if

$\to |\vec{a} + \vec{b}| = 1$

$$|\vec{a} + \vec{b}|^2 = 1$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta = 1$$

$$1 + 1 + 2(1)(1)\cos\theta = 1$$

$$2 + 2\cos\theta = 1$$

$$\cos\theta = \frac{-1}{2}$$

$$\theta = \frac{2\pi}{3}$$

(A) $\theta = \frac{\pi}{4}$

(B) $\theta = \frac{\pi}{3}$

(C) $\theta = \frac{2\pi}{3}$

(D) $\theta = \frac{\pi}{2}$

$\cos\theta = \frac{1}{2}$ | $\cos\theta = -\frac{1}{2}$

$\theta = \frac{\pi}{3}$ | $\theta = \frac{2\pi}{3}$

$\cos\theta = \frac{\sqrt{3}}{2}$ | $\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}$ | $\theta = \frac{5\pi}{6}$

$\cos\theta = -ve$

$\theta \in 2^{nd}$ Quadrant

$\theta = \pi - \alpha$

$\alpha \rightarrow$ acute angle

Question

$\rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then angle between $\vec{a}$ and $\vec{b}$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) π

$$\vec{a} + \vec{b} + \vec{c} = \vec{0}$$

$$\vec{a} + \vec{b} = -\vec{c}$$

$$|\vec{a} + \vec{b}|^2 = |-\vec{c}|^2 = |\vec{c}|^2$$

$$|\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta = 1$$

$$1 + 1 + 2(1)(1)\cos\theta = 1$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}$$

$\boxed{\text{Boards}}$

if $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$

then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a} = -\frac{3}{2}$

Ans

$|\vec{a}+\vec{b}+\vec{c}|^2$

$= |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2 + 2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$

Question

If $\boxed{\vec{a}, \vec{b} \text{ and } \vec{c} \text{ are unit vectors,}}$ such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, then $3\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+1\vec{c}\cdot\vec{a}=$

A 1

B –1

C 3

D –3

$\Downarrow$

$\vec{a}+\vec{b}=-\vec{c}$

$|\vec{a}+\vec{b}|^2=|-\vec{c}|^2=|\vec{c}|^2$

$|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=1$

$1+1+2\vec{a}\cdot\vec{b}=1$

$\vec{a}\cdot\vec{b}=-\frac{1}{2}$

Similarly

$\vec{a}+\vec{c}=-\vec{b}$

$\Downarrow$

$\vec{a}\cdot\vec{c}=-\frac{1}{2}$

Similarly

$\vec{b}+\vec{c}=-\vec{a}$

$\Downarrow$

$\vec{b}\cdot\vec{c}=-\frac{1}{2}$

$$\therefore\ 3\vec{a}\cdot\vec{b}+2\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$$

$$=3\left(-\frac{1}{2}\right)+2\left(-\frac{1}{2}\right)+\left(-\frac{1}{2}\right)$$

$$=\frac{-3-2-1}{2}=-\frac{6}{2}=\underline{\underline{-3}}$$

Note:

if $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that

$$\vec{a}+\vec{b}+\vec{c}=\vec{0}$$

Then (1) $\vec{a}\cdot\vec{b}=-\frac{1}{2}$

(2) $\vec{b}\cdot\vec{c}=-\frac{1}{2}$

(3) $\vec{a}\cdot\vec{c}=-\frac{1}{2}$

Question

If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $\underbrace{\vec{a} \cdot \vec{b}}_{-\frac{1}{2}} + \underbrace{\vec{b} \cdot \vec{c}}_{-\frac{1}{2}} + \underbrace{\vec{c} \cdot \vec{a}}_{-\frac{1}{2}} =$

$= -\frac{3}{2}$

A $\frac{3}{2}$

B $-\frac{3}{2}$

C $\frac{2}{3}$

D $\frac{1}{2}$

Question

If $\vec{a} \cdot \vec{b} = -|\vec{a}||\vec{b}|$, then the angle between $\vec{a}$ and $\vec{b}$ is

$\cos\theta = -1$

$\theta = \pi$

(A) $90°$

(B) $60°$

(C) $45°$

(D) $180°$

Question

$\alpha = 150^\circ$ $\beta = 60^\circ$

A space vector makes the angles 150° and 60° with the positive direction of X and Y-axes. The angle made by the vector with the positive direction of Z-axis is

(A) 60°

(B) 90°

(C) 120°

(D) 180°

$l = \cos\alpha = \cos 150^\circ = -\cos 30 = \frac{-\sqrt{3}}{2} \Rightarrow l^2 = \frac{3}{4}$

$m = \cos\beta = \cos 60 = \frac{1}{2} \Rightarrow m^2 = \frac{1}{4}$

WKT $l^2 + m^2 + n^2 = 1$

$\frac{3}{4} + \frac{1}{4} + n^2 = 1$

$1 + n^2 = 1$

$n^2 = 0$

$n = 0$

$n = 0$

$\cos\gamma = 0$

$\gamma = 90^\circ$

if two vectors

are perpendicular to each other

Then their product equals to zero

Question

If $\vec{a} \perp \vec{b}$ and $(\vec{a}+\vec{b}) \perp (\vec{a}+m\vec{b})$, then $m =$

$\vec{a}\cdot\vec{b}=0$

$(\vec{a}+\vec{b})\cdot(\vec{a}+m\vec{b})=0$

$\boxed{\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}}$

$\Downarrow$

dot product is commutative

(A) -1

(B) 1

(C) $\dfrac{-|\vec{a}|^2}{|\vec{b}|^2}$

(D) 0

$\vec{a}\cdot\vec{a}+m\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+m\vec{b}\cdot\vec{b}=0$

$|\vec{a}|^2+m(0)+(0)+m|\vec{b}|^2=0$

$m|\vec{b}|^2=-|\vec{a}|^2$

$m=\dfrac{-|\vec{a}|^2}{|\vec{b}|^2}$

* Area of parallelogram:-

① if $\vec{a}$ & $\vec{b}$ are adjacent sides

Then Area $= |\vec{a} \times \vec{b}|$

② If $\vec{d_1}$ & $\vec{d_2}$ are diagonals of a parallelogram

then Area $= \frac{1}{2} |\vec{d_1} \times \vec{d_2}|$

* Area of Triangle:-

① If $\vec{a}$ & $\vec{b}$ are adjacent sides

Then

Area $= \frac{1}{2} |\vec{a} \times \vec{b}|$

Question

The area of the parallelogram whose adjacent sides are $\hat{i}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ is

$$\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$=\hat{i}(-1)-\hat{j}(-1)+\hat{k}(1)$$

$$|\vec{a}\times\vec{b}|=\sqrt{3}$$

(A) 3

(B) $\sqrt{2}$

(C) 4

(D) $\sqrt{3}$

Question

$|\vec{a}| = \sqrt{1+4+4} = \sqrt{9} = 3$

If $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$, $|\vec{b}| = 5$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then the area of the triangle formed by these two vectors as two sides is

A 15

B $\frac{15\sqrt{3}}{2}$

C $\frac{15}{2}$

D $\frac{15}{4}$ ✓

$$\text{Area} = \frac{1}{2}|\vec{a} \times \vec{b}|$$

$$= \frac{1}{2}|\vec{a}||\vec{b}|\sin\theta$$

$$= \frac{1}{2}(3)(5)\frac{1}{2}$$

$$= \frac{15}{4}$$

Question

$\vec{a}\times\vec{a}=\vec{0}$ | $\vec{a}\times\vec{b}=-(\vec{b}\times\vec{a})$

$\vec{b}\times\vec{b}=\vec{0}$

If the area of the parallelogram with $\vec{a}$ and $\vec{b}$ as two adjacent sides is 15 sq. units then the area of the parallelogram having $3\vec{a}+2\vec{b}$ and $\vec{a}+3\vec{b}$ as two adjacent sides in sq. units is [2021]

(A) 45

(B) 75

(C) 105

(D) 120

Area = 15

$|\vec{a}\times\vec{b}|=15$

consider

$|(3\vec{a}+2\vec{b})\times(\vec{a}+3\vec{b})|$

$=|3(\vec{a}\times\vec{a})+9(\vec{a}\times\vec{b})+2(\vec{b}\times\vec{a})+6(\vec{b}\times\vec{b})|$

$=|3(0)+9(\vec{a}\times\vec{b})-2(\vec{a}\times\vec{b})+6(0)|$

$=7|(\vec{a}\times\vec{b})|$

$=7(15)=\underline{105}$

Question

If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at angle $\frac{\pi}{3}$ $\to \theta$, then the value of $|\vec{a}+\vec{b}|$ is

(A) equal to 1

(B) greater than 1

(C) equal to 0

(D) less than 1

WKT

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

$$= 1 + 1 + 2(1)(1)\cos\frac{\pi}{3}$$

$$= 2 + 2\left(\frac{1}{2}\right)$$

$$|\vec{a}+\vec{b}|^2 = 3$$

$$|\vec{a}+\vec{b}| = \sqrt{3} > 1$$

Question

$|\vec{a}| = \sqrt{1+4+9} = \sqrt{14}$ | $|\vec{a}|^2 = 14$

Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$. If $\vec{b}$ is a vector such that $\vec{a} \cdot \vec{b} = |\vec{b}|^2$ and $|\vec{a} - \vec{b}| = \sqrt{7}$, then $|\vec{b}| =$ ______.

(A) 14

(B) 21

(C) 7

(D) $\sqrt{7}$

WKT

$|\vec{a} - \vec{b}| = \sqrt{7}$

$|\vec{a} - \vec{b}|^2 = 7$

$|\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b} = 7$

$14 + |\vec{b}|^2 - 2|\vec{b}|^2 = 7$

$7 - |\vec{b}|^2 = 0$

$|\vec{b}|^2 = 7$

$|\vec{b}| = \sqrt{7}$

Question

If $2\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|$ then the angle between $\vec{a}\&\vec{b}$ is

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} = \frac{\vec{a}\cdot\vec{b}}{2(\vec{a}\cdot\vec{b})}$$

(A) 30°

(B) 0°

$$\cos\theta = \frac{1}{2}$$

(C) 90°

$$\theta = \frac{\pi}{3}$$

(D) 60°

Question

$\frac{3}{\sqrt{3}} = \frac{\sqrt{3}\sqrt{3}}{\sqrt{3}} = \sqrt{3}$

If $\vec{a}$ and $\vec{b}$ are unit vectors then what is the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\vec{a} - \vec{b}$ to be unit vector?

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Given $|\sqrt{3}\vec{a} - \vec{b}| = 1$

$|\sqrt{3}\vec{a} - \vec{b}|^2 = 1$

$(\sqrt{3})^2|\vec{a}|^2 + |\vec{b}|^2 - 2\sqrt{3}|\vec{a}||\vec{b}|\cos\theta = 1$

$3(1) + 1 - 2\sqrt{3}(1)(1)\cos\theta = 1$

$3 = 2\sqrt{3}\cos\theta$

$\cos\theta = \frac{3}{2\sqrt{3}}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6} = 30^0$

Question

Suppose $\boxed{\vec{a}+\vec{b}+\vec{c}=0}$ $|\vec{a}|=3, |\vec{b}|=5, |\vec{c}|=7$, then the $\boxed{\text{angle between } \vec{a} \& \vec{b} \text{ is}}$

$$\vec{a}+\vec{b}=-\vec{c}$$

$$|\vec{a}+\vec{b}|^2=|-\vec{c}|^2=|\vec{c}|^2$$

$$|\vec{a}|^2+|\vec{b}|^2+2|\vec{a}||\vec{b}|\cos\theta=7^2$$

$$9+25+2(3)(5)\cos\theta=49$$

$$30\cos\theta=49-34=15$$

$$\cos\theta=\frac{1}{2}$$

$$\theta=\pi/3$$

(A) π

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$

Question

If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors, then $(3\vec{a}+2\vec{b})\cdot(5\vec{a}-6\vec{b})=$

$\Downarrow$

$\vec{a}\perp\vec{b}$

$\vec{a}\cdot\vec{b}=0$

&

$|\vec{a}|=|\vec{b}|=1$

$= 15(\vec{a}\cdot\vec{a}) - 18(\vec{a}\cdot\vec{b}) + 10(\vec{b}\cdot\vec{a}) - 12\,\vec{b}\cdot\vec{b}$

$= 15|\vec{a}|^2 - 18(0) + 10(0) - 12|\vec{b}|^2$

$= 15(1) - 0 + 0 - 12(1)$

$= \underline{3}$

(A) 5

(B) 3

(C) 6

(D) 12

Question

If $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ are orthogonal, then value of λ is

perpendicular

$$\vec{a} \cdot \vec{b} = 0$$

$$2 + 2\lambda + 3 = 0$$

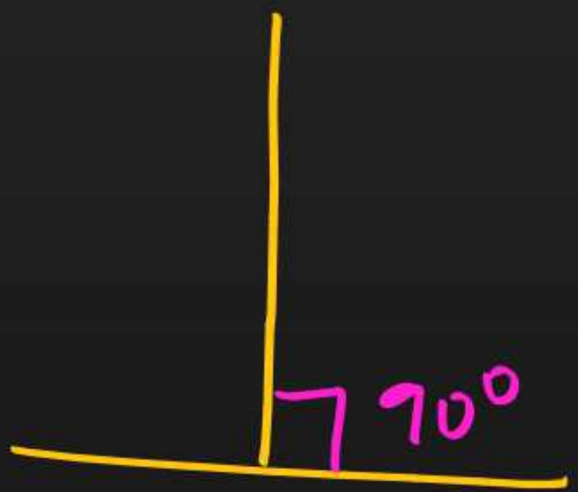
$$\lambda = -\frac{5}{2}$$

(A) $\frac{3}{2}$

(B) 1

(C) $-\frac{5}{2}$

(D) 0



Question

$|\vec{a}| = \sqrt{x^2+y^2+z^2}$

$|\vec{a}|^2 = x^2+y^2+z^2$

If $\vec{a} = \hat{i} + \lambda\hat{j} + 2\hat{k}$; $\vec{b} = \mu\hat{i} + \hat{j} - \hat{k}$ are $\boxed{\text{orthogonal}}$ and $|\vec{a}| = |\vec{b}|$, then $(\lambda, \mu) =$

$\vec{a} \cdot \vec{b} = 0$ perpendicular

$\mu + \lambda - 2 = 0$

$\mu + \lambda = 2$

$\Downarrow$

$|\vec{a}|^2 = |\vec{b}|^2$

$1 + \lambda^2 + 2^2 = \mu^2 + 1 + 1$

$\lambda^2 + 4 = \mu^2 + 1$

$\mu^2 - \lambda^2 = 3$

$(\mu+\lambda)(\mu-\lambda) = 3$

$2(\mu-\lambda) = 3$

$\mu - \lambda = \frac{3}{2}$

A $\left(\frac{1}{4}, \frac{7}{4}\right)$

B $\left(\frac{7}{4}, \frac{1}{4}\right)$

C $\left(\frac{1}{4}, \frac{9}{4}\right)$

D $\left(\frac{-1}{4}, \frac{9}{4}\right)$

$\mu + \lambda = 2 \rightarrow (1)$ & $\mu - \lambda = \frac{3}{2} \rightarrow (2)$

$(1) + (2)$

$2\mu = \frac{7}{2}$

$\mu = \frac{7}{4}$

$(1) - (2)$

$2\lambda = \frac{1}{2}$

$\lambda = \frac{1}{4}$

$(\lambda, \mu) = \left(\frac{1}{4}, \frac{7}{4}\right)$

Question

$\rightarrow \theta = \frac{2\pi}{3}$ | $\cos\frac{2\pi}{3} = -\frac{1}{2}$

If the angle between $\vec{a}$ and $\vec{b}$ is $\frac{2\pi}{3}$ and the projection of $\vec{a}$ in the direction of $\vec{b}$ is -2, then $|\vec{a}| =$

A 4

B 2

C 3

D 1

Projection $= -2$

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = -2$$

$$\frac{|\vec{a}|\cancel{|\vec{b}|}\cos\theta}{\cancel{|\vec{b}|}} = -2$$

$$|\vec{a}|\left(-\frac{1}{2}\right) = -2$$

$|\vec{a}| = 4$

① $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

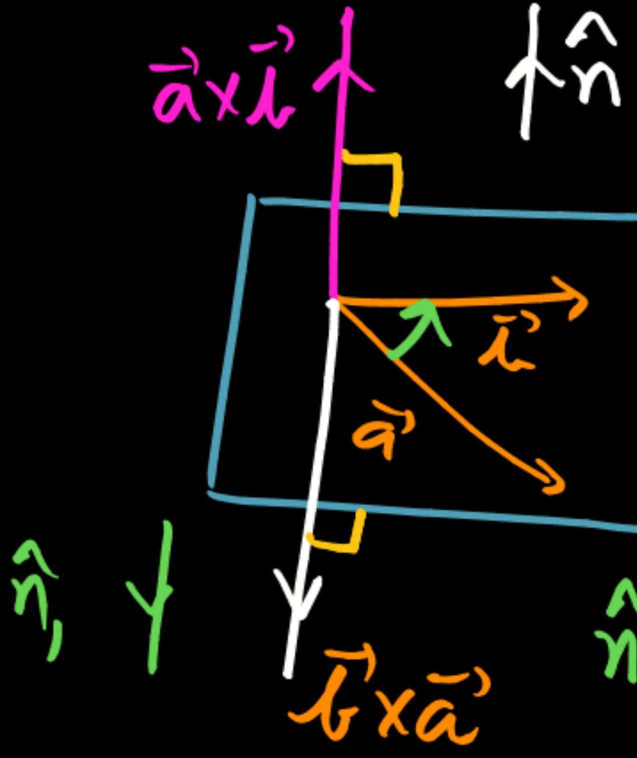
$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

② $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta \, \hat{n}$

$\hat{n} \rightarrow$ unit vector

① parallel to $\vec{a}\times\vec{b}$

② perpendicular to both $\vec{a}$ & $\vec{b}$



$$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

$$\hat{n}_1 = -\frac{(\vec{b}\times\vec{a})}{|\vec{a}\times\vec{b}|}$$

Question

A unit vector perpendicular to the plane containing the vectors $\hat{\imath}+2\hat{\jmath}+\hat{k}$ and $-2\hat{\imath}+\hat{\jmath}+3\hat{k}$ is **[2019]**

$\vec{a} = \hat{\imath}+2\hat{\jmath}+\hat{k}$, $\vec{b} = -2\hat{\imath}+\hat{\jmath}+3\hat{k}$

(A) $\dfrac{\hat{\imath}+\hat{\jmath}+\hat{k}}{\sqrt{3}}$

(B) $\dfrac{-\hat{\imath}+\hat{\jmath}-\hat{k}}{\sqrt{3}}$

(C) $\dfrac{\hat{\imath}+\hat{\jmath}-\hat{k}}{\sqrt{3}}$

(D) $\dfrac{-\hat{\imath}-\hat{\jmath}-\hat{k}}{\sqrt{3}}$

$$\Rightarrow \hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \frac{-(\vec{b}\times\vec{a})}{|\vec{a}\times\vec{b}|}$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ -2 & 1 & 3 \end{vmatrix}$$

$$= i(5) - j(5) + \hat{k}(5)$$

$$= 5(i - j + \hat{k})$$

$$|\vec{a}\times\vec{b}| = 5\sqrt{1+1+1} = 5\sqrt{3}$$

$$\hat{n} = \frac{5(i-j+\hat{k})}{5\sqrt{3}}$$

$$\hat{n} = \frac{i-j+\hat{k}}{\sqrt{3}} \quad \text{or}$$

$$\boxed{\hat{n} = \frac{-i+j-\hat{k}}{\sqrt{3}}}$$

① if $\vec{a}$ & $\vec{b}$ are ⊥r to each other

$\Downarrow$

$\vec{a} \cdot \vec{b} = 0$

② unit vector ⊥r to both $\vec{a}$ & $\vec{b}$

$$\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

or

$$\hat{n}_1 = \frac{-(\vec{b} \times \vec{a})}{|\vec{a} \times \vec{b}|}$$

(4) $\vec{a} \times \vec{b} = |\vec{a}||\vec{b}| \sin\theta \, \hat{n}$

Take magnitude on B.S

$|\hat{n}| = 1$

$\therefore |\vec{a} \times \vec{b}| = |\vec{a}| \, |\vec{b}| \sin\theta$

$$\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}| \, |\vec{b}|}$$ $\Rightarrow$ Formula to find angle b/w 2 vector, when data is given in terms of cross product

(*) if $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

&

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

Then if $\vec{a}$ is is parallel (collinear) to $\vec{b}$

Then ① $\theta = 0$ b/w $\vec{a}$ & $\vec{b}$

$\Rightarrow$ ① $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|$

② $\vec{a} \times \vec{b} = \vec{0}$

② $\dfrac{a_1}{b_1} = \dfrac{a_2}{b_2} = \dfrac{a_3}{b_3}$

③ $\vec{a} = \lambda \vec{b}$

(*) if $\vec{a} = 3\hat{i} + \lambda\hat{j} + 8\hat{k}$

& $\vec{b} = -12\hat{i} + 2\hat{j} - 32\hat{k}$

Find λ if ① $\vec{a}$ is collinear to $\vec{b}$

② $\vec{a} \perp \vec{b}$

Solⁿ: ① $\vec{a}$ is collineare to $\vec{b}$

$$\frac{3}{-12} = \frac{\lambda}{2} = \frac{8}{-32}$$

$$-\frac{1}{4} = \frac{\lambda}{2}$$

$$\lambda = -\frac{1}{2}$$

② $\vec{a} \perp \vec{b}$

$\Rightarrow \vec{a} \cdot \vec{b} = 0$

$$3(-12) + 2\lambda + 8(-32) = 0$$

$$-3(6) + \lambda - 4(32) = 0$$

$$-18 - 128 + \lambda = 0$$

$$\lambda = 146$$

(*) if $(3\hat{i}+\lambda\hat{j}+4\hat{k})\times(6\hat{i}-3\hat{j}+8\hat{k})=\vec{0}$

Find λ.

Soln.:

Here the vectors are parallel

$\therefore \dfrac{3}{6}=\dfrac{\lambda}{-3}=\dfrac{4}{8}$

$\dfrac{1}{2}=\dfrac{\lambda}{-3}$

$\lambda=-\dfrac{3}{2}$

if $\vec{a}\times\vec{b}=\vec{0}$

Then $\theta=0^\circ$

$\Rightarrow \vec{a}$ is parallel to $\vec{b}$

$\sin\frac{\theta}{2}$ in terms of $\cos\theta$

$\Downarrow$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$\cos\frac{\theta}{2}$ in terms of $\cos\theta$

$\Downarrow$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

Question

$2\sin^2\frac{\theta}{2} = 1 - \cos\theta$

If $\boxed{\vec{a} \text{ and } \vec{b} \text{ are unit vectors}}$ and θ is the angle between $\vec{a}$ and $\vec{b}$, then $\sin\frac{\theta}{2}$ is

(A) $\frac{|\vec{a}+\vec{b}|}{2}$

(B) $\frac{|\vec{a}-\vec{b}|}{2}$ ✓

(C) $|\vec{a}-\vec{b}|$

(D) $|\vec{a}+\vec{b}|$

WKT

$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

$= 1 + 1 - 2(1)(1)\cos\theta$

$= 2 - 2\cos\theta$

$= 2(1-\cos\theta)$

$= 2(2\sin^2\theta/2)$

$|\vec{a}-\vec{b}|^2 = 2^2\sin^2\frac{\theta}{2}$

$|\vec{a}-\vec{b}| = 2\sin\frac{\theta}{2}$

$\therefore \sin\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|}{2}$

Question

$2\cos^2\frac{\theta}{2} = 1 + \cos\theta$

If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then $\cos\frac{\theta}{2}$ is

We use

$|\vec{a}+\vec{b}|^2$

(A) $\frac{|\vec{a}+\vec{b}|}{2}$

(B) $\frac{|\vec{a}-\vec{b}|}{2}$

(C) $|\vec{a}-\vec{b}|$

(D) $|\vec{a}+\vec{b}|$

Question

$\Rightarrow \cos 120 = -\cos 60 = -\frac{1}{2}$

If $|\vec{a}| = 2$ and $|\vec{b}| = 3$ and the angle between $\vec{a}$ and $\vec{b}$ is $120°$, then the length of the vector $\left|\frac{1}{2}\vec{a} - \frac{1}{3}\vec{b}\right|^2$ is

(A) 2

(B) $\frac{1}{6}$

(C) 3 ✓

(D) 1

$$\left|\frac{1}{2}\vec{a} - \frac{1}{3}\vec{b}\right|^2 = \left(\frac{1}{2}\right)^2|\vec{a}|^2 + \left(\frac{1}{3}\right)^2|\vec{b}|^2 - \cancel{2}\left(\frac{1}{\cancel{2}}\right)\left(\frac{1}{3}\right)|\vec{a}||\vec{b}|\cos\theta$$

$$= \frac{1}{4}(4) + \frac{1}{9}(9) - \frac{1}{3}(2)(3)\cos 120$$

$$= 1 + 1 - 2\left(-\frac{1}{2}\right)$$

$$= 1 + 1 + 1$$

$$= 3$$

Question

If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ then

$$|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$$

$$\cancel{|\vec{a}|^2}+\cancel{|\vec{b}|^2}+2\vec{a}\cdot\vec{b} = \cancel{|\vec{a}|^2}+\cancel{|\vec{b}|^2}-2\vec{a}\cdot\vec{b}$$

(A) $\vec{a}$ and $\vec{b}$ are coincident.

(B) $\vec{a}$ and $\vec{b}$ are perpendicular.

(C) Inclined to each other at 60°.

(D) $\vec{a}$ and $\vec{b}$ are parallel.

$$2\vec{a}\cdot\vec{b} = -2\vec{a}\cdot\vec{b}$$

$$4\vec{a}\cdot\vec{b} = 0$$

$$\vec{a}\cdot\vec{b} = 0$$

$$\Downarrow$$

$$\vec{a} \perp \vec{b}$$

W.K.T

$|\vec{a}\cdot\vec{b}| = |\vec{a}||\vec{b}|\cos\theta$ & $|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$

$$\therefore |\vec{a}\cdot\vec{b}|^2 + |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2(\cos^2\theta+\sin^2\theta)$$

$$\Downarrow$$

$$|\vec{a}\cdot\vec{b}|^2 + |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2$$

Question

If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}| =$ **[2012, 2018]**

$\Downarrow$

$|\vec{a}|^2 |\vec{b}|^2 = 144$

$16 |\vec{b}|^2 = 144$

$\frac{144}{16} = 9$

$|\vec{b}|^2 = 9$

$|\vec{b}| = 3$

(A) 12

(B) 16

(C) 8

(D) 3 ✓

Question

If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 6$, then $|\vec{b}|^2$ is equal to [2023]

$\Downarrow$

$|\vec{a}|^2 |\vec{b}|^2 = 144$

$\downarrow$

$36 |\vec{b}|^2 = 144$

$|\vec{b}|^2 = 4$

A 8

B 12

C 4

D 3

Question

If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 36$ and $|\vec{a}| = 3$ then $|\vec{b}|$ is equal to [2022]

$\Downarrow$

$|\vec{a}|^2 |\vec{b}|^2 = 36$

$\downarrow$

$9 |\vec{b}|^2 = 36$

$|\vec{b}|^2 = 4$

$|\vec{b}| = 2$

(A) 9

(B) 4

(C) 36

(D) 2

Question

If $|\vec{a} \times \vec{b}| = 4$ and $|\vec{a} \cdot \vec{b}| = 2$, then $|\vec{a}|^2 |\vec{b}|^2$ is equal to

$4^2 + 2^2 = |\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2$

$= 16 + 4$

$= 20$

(A) 2

(B) 6

(C) 8

(D) 20

Question

If $\vec{a} = 2\hat{i} + 3\hat{j} - 5\hat{k}, \vec{b} = m\hat{i} + n\hat{j} + 12\hat{k}$ and $\vec{a} \times \vec{b} = \vec{0}$, then $(m, n) =$

$\Downarrow$

$\theta = 0$

$\Downarrow$

$\vec{a}$ is parallel to $\vec{b}$

A $\left(\frac{-24}{5}, \frac{-36}{5}\right)$

B $\left(\frac{-24}{5}, \frac{36}{5}\right)$

C $\left(\frac{24}{5}, \frac{-36}{5}\right)$

D $\left(\frac{24}{5}, \frac{36}{5}\right)$

$\frac{2}{m} = \frac{3}{n} = \frac{-5}{12}$

$\frac{2}{m} = -\frac{5}{12}$ | $\frac{3}{n} = -\frac{5}{12}$

$m = -\frac{24}{5}$ | $n = -\frac{36}{5}$

Question

For any vector $\vec{x}$, the value of $(\vec{x}\times\hat{i})^2+(\vec{x}\times\hat{j})^2+(\vec{x}\times\hat{k})^2$ is equal to

$\rightarrow |\vec{x}|^2\sin^2\alpha$ $\rightarrow |\vec{x}|^2\sin^2\beta$ $\rightarrow |\vec{x}|^2\sin^2\gamma$

A $|\vec{x}|^2$

B $2|\vec{x}|^2$

C $3|\vec{x}|^2$

D $4|\vec{x}|^2$

$$|\vec{x}\times\hat{i}|^2=|\vec{x}|^2|\hat{i}|^2\sin^2\alpha=|\vec{x}|^2(1)\sin^2\alpha$$

$\alpha,\beta,\gamma \rightarrow$ angle by $\vec{x}$ wrt x, y & z-axis

$$\therefore |\vec{x}|^2(\sin^2\alpha+\sin^2\beta+\sin^2\gamma)$$

$$|\vec{x}|^2(1-\cos^2\alpha+1-\cos^2\beta+1-\cos^2\gamma)$$

$$|\vec{x}|^2[3-(\cos^2\alpha+\cos^2\beta+\cos^2\gamma))$$

$$=|\vec{x}|^2(3-1)$$

$$=2|\vec{x}|^2$$

WKT

Direction cosines $\begin{cases} l = \cos\alpha \\ m = \cos\beta \\ n = \cos\gamma \end{cases}$

WKT

$$l^2 + m^2 + n^2 = 1$$

$$\Downarrow$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$\vec{OP} = \vec{r} = a\hat{i} + b\hat{j} + c\hat{k} \longrightarrow$ Position vector $\Rightarrow$ initial point

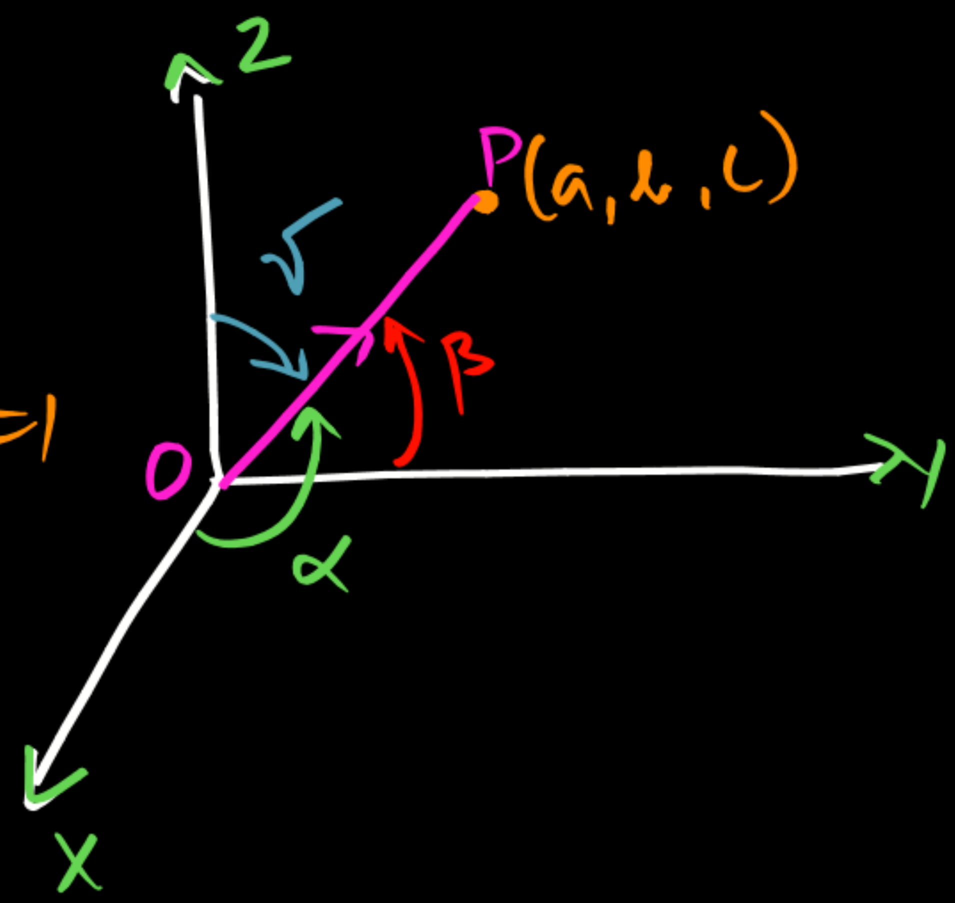
$\Downarrow$

origin

$l^2 + m^2 + n^2 = 1$

$\Downarrow$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$



① D.R's:- a, b, c

② Direction angles:- α, β, γ

③ D.c's:-

$l = \frac{a}{r} = \cos\alpha$ | $m = \frac{b}{r} = \cos\beta$ | $n = \frac{c}{r} = \cos\gamma$

Question

$\rightarrow |\vec{x}|^2 \cos^2\alpha \quad \rightarrow |\vec{x}|^2 \cos^2\beta \quad \rightarrow |\vec{x}|^2 \cos^2\gamma$

For any vector $\vec{x}$, the value of $(\vec{x} \cdot \hat{i})^2 + (\vec{x} \cdot \hat{j})^2 + (\vec{x} \cdot \hat{k})^2$ is equal to

$$(\vec{x}\cdot\hat{i})^2 = |\vec{x}|^2 |\hat{i}|^2 \cos^2\alpha = |\vec{x}|^2 \cos^2\alpha$$

(A) $|\vec{x}|^2$ ✓

(B) $2|\vec{x}|^2$

(C) $3|\vec{x}|^2$

(D) $4|\vec{x}|^2$

$$\rightarrow |\vec{x}|^2 (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$

$$|\vec{x}|^2 (1)$$

$$= \underline{|\vec{x}|^2}$$

Question

The area (in sq. units) of the parallelogram whose diagonals are along the vectors $8\hat{i} - 6\hat{j}$ and $3\hat{i} + 4\hat{j} - 12\hat{k}$, is

$\vec{d_1}$ $\vec{d_2}$

(A) 65

(B) 52

(C) 26

(D) 20

$$\text{Area} = \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$$

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & -6 & 0 \\ 3 & 4 & -12 \end{vmatrix}$$

$$= \hat{i}(72) - \hat{j}(-96) + \hat{k}(32+18)$$

$$= 72\hat{i} + 96\hat{j} + 50\hat{k}$$

$$= 2(36\hat{i} + 48\hat{j} + 25\hat{k})$$

$$|\vec{d}_1 \times \vec{d}_2| = 2\sqrt{36^2+48^2+25^2}$$

$$= 2\sqrt{4225}$$

$$= 2(65)$$

$$4225 > (60)^2$$

$$\downarrow 3600$$

$$\text{Area} = \frac{1}{2}[2(65)] = 65$$

Area of Parallelogram

① Area $= |\vec{a} \times \vec{b}|$

$\vec{a}$ & $\vec{b} \rightarrow$ adjacent sides

② Area $= \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

$\vec{d_1}$ & $\vec{d_2} \rightarrow$ diagonals

$(36)^2$

$\underline{3^2}$ $\underline{3(6)(2)}$ $\underline{6^2}$

Add

$\underline{9}$ $\underline{36}$ $\underline{36}$

$36+3=39$

Add

1296

$(48)^2$

$\underline{16}$ $\underline{64}$ $\underline{64}$

4^2 $4(8)(2)$ 8^2

2304

$$\begin{array}{r} 2304 \\ 1296 \\ 625 \\ \hline 4225 \\ \hline \end{array}$$

$(76)^2$

$7^2 \quad 7(6)(2) \quad 6^2$

Add

$49 \quad 84 \quad 36$

$84+3=87$

$49+8$
$=57$

5776

Question

If $|\vec{a}| = 10, |\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 12$, then the value of $|\vec{a} \times \vec{b}|$ is

WKT

$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2$

$|\vec{a} \times \vec{b}|^2 + 144 = (100)(4)$

$|\vec{a} \times \vec{b}|^2 = 400 - 144 = 256$

$|\vec{a} \times \vec{b}| = 16$

(A) 5

(B) 10

(C) 14

(D) 16

Question

If $|\vec{a}| = 4, |\vec{b}| = 2$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $(\vec{a} \times \vec{b})^2$ is equal to

$\rightarrow \sin\frac{\pi}{6} = \frac{1}{2}$

$\Downarrow$

$|\vec{a}|^2 |\vec{b}|^2 \sin^2\theta$

$16(4)\left(\frac{1}{2}\right)^2$

16

A. 48

B. 16

C. $\vec{a}$

D. None of these

Question

If $(\vec{a} \times \vec{b})^2 + (\vec{a} \cdot \vec{b})^2 = 676$ and $|\vec{b}| = 2$, then $|\vec{a}|$ is equal to

(A) 13

(B) 26

(C) 39

(D) None of these

Question

$\vec{a}\times\vec{a}=\vec{0}$ | $\vec{b}\times\vec{b}=\vec{0}$

If $\vec{x}=\vec{a}+\vec{b}, \vec{y}=\vec{a}-\vec{b}, |\vec{a}|=2, |\vec{b}|=3$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$, then $|\vec{x}\times\vec{y}|$ is equal to

$\sin\theta=\frac{\sqrt{3}}{2}$

$$|\vec{x}\times\vec{y}|=|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|$$
$$=|\vec{0}-(\vec{a}\times\vec{b})+(\vec{b}\times\vec{a})-\vec{0}|$$
$$=|-[-(\vec{b}\times\vec{a})]+\vec{b}\times\vec{a}|$$
$$=|2(\vec{b}\times\vec{a})|$$
$$=2|\vec{b}||\vec{a}|\sin\theta$$
$$=2(3)(\cancel{2})\frac{\sqrt{3}}{\cancel{2}}=6\sqrt{3}$$

A. $6\sqrt{3}$

B. 6

C. $4\sqrt{3}$

D. 9

Question

If $|\vec{a}| = |\vec{b}| = 1$ and $|\vec{a} + \vec{b}| = \sqrt{3}$, then the value of $(3\vec{a} - 4\vec{b}) \cdot (2\vec{a} + 5\vec{b})$ is **[2024]**

(A) -21

(B) $-21/2$

(C) 21

(D) $21/2$

Question

$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144$ and $|\vec{a}| = 4$, then $|\vec{b}|$ is equal to [2024]

(A) 12

(B) 3

(C) 8

(D) 4

Question

If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, then $\vec{a} \times \vec{b} =$ [2024]

$\vec{a} = -\vec{b} - \vec{c}$

$(-\vec{b} - \vec{c}) \times \vec{b}$

$= -(\vec{b} \times \vec{b}) - (\vec{c} \times \vec{b})$

$= \vec{0} - [-(\vec{b} \times \vec{c})]$

$= \vec{b} \times \vec{c}$

or

$\vec{b} = -\vec{a} - \vec{c}$

$\vec{a} \times \vec{b} = \vec{a} \times (-\vec{a} \times -\vec{c})$

$= -\vec{0} - (\vec{a} \times \vec{c})$

$= \vec{c} \times \vec{a}$

A $\vec{c} \times \vec{a}$

B $\vec{b} \times \vec{c}$

C $\vec{0}$

D Both (A) and (B)

Question

If $\vec{a} = 2\hat{i} - \hat{j} - m\hat{k}$ and $\vec{b} = \frac{4}{7}\hat{i} - \frac{2}{7}\hat{j} + 2\hat{k}$ are collinear, then the value of m is equal to [2023]

(A) -7

(B) -1

(C) 2

(D) 7

Question

If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}, |\vec{a}|=1, |\vec{b}|=2, |\vec{c}|=3$, then $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ is equal to **[2020]**

(A) 0

(B) -7 ✓

(C) 7

(D) 1

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$|\vec{a}+\vec{b}|^2=|-\vec{c}|^2$

$1^2+2^2+2\vec{a}\cdot\vec{b}=9$

$2\vec{a}\cdot\vec{b}=4$

$\vec{a}\cdot\vec{b}=2$

similarly

$|\vec{b}+\vec{c}|^2=|-\vec{a}|^2$

$4+9+2\vec{b}\cdot\vec{c}=1$

$2\vec{b}\cdot\vec{c}=-12$

$\vec{b}\cdot\vec{c}=-6$

similarly

$|\vec{a}+\vec{c}|=|-\vec{b}|^2$

$1+9+2\vec{a}\cdot\vec{c}=4$

$2\vec{a}\cdot\vec{c}=-6$

$\vec{a}\cdot\vec{c}=-3$

$\therefore 2+(-6)+(-3)=2-9=-7$

Question

If $\vec{a}, \vec{b}, \vec{c}$ are three vectors, such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}, |\vec{a}|=1, |\vec{b}|=2, |\vec{c}|=3$, then $4\vec{a}\cdot\vec{b}+3\vec{b}\cdot\vec{c}+2\vec{c}\cdot\vec{a}$ is equal to [2020]

A 0

B -7

C 7

D -16

$\vec{a}+\vec{b}+\vec{c}=\vec{0}$

$|\vec{a}+\vec{b}|^2=|-\vec{c}|^2$

$1^2+2^2+2\vec{a}\cdot\vec{b}=9$

$2\vec{a}\cdot\vec{b}=4$

$\vec{a}\cdot\vec{b}=2$

similarly

$|\vec{b}+\vec{c}|^2=|-\vec{a}|^2$

$4+9+2\vec{b}\cdot\vec{c}=1$

$2\vec{b}\cdot\vec{c}=-12$

$\vec{b}\cdot\vec{c}=-6$

similarly

$|\vec{a}+\vec{c}|=|-\vec{b}|^2$

$1+9+2\vec{a}\cdot\vec{c}=4$

$2\vec{a}\cdot\vec{c}=-6$

$\vec{a}\cdot\vec{c}=-3$

$4(2)+3(-6)+2(-3)$

$=8-18-6=-10-6=-16$

Question

If $\vec{a} = -3\hat{i} + n\hat{j} + 4\hat{k}$ and $\vec{b} = -2\hat{i} + 4\hat{j} + p\hat{k}$ are collinear, then **[2021]**

$$\frac{-3}{-2} = \frac{n}{4} = \frac{4}{p}$$

(A) $n = \frac{-8}{3}, p = 6$

(B) $n = 6, p = \frac{8}{3}$

(C) $n = 18, p = -24$

(D) $n = -3, p = 48$

$\frac{3}{2} = \frac{n}{4}$

$n = 6$

$\frac{3}{2} = \frac{4}{p}$

$p = \frac{8}{3}$

Thank

You