ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture - 01

Sets, Binomial Theorem

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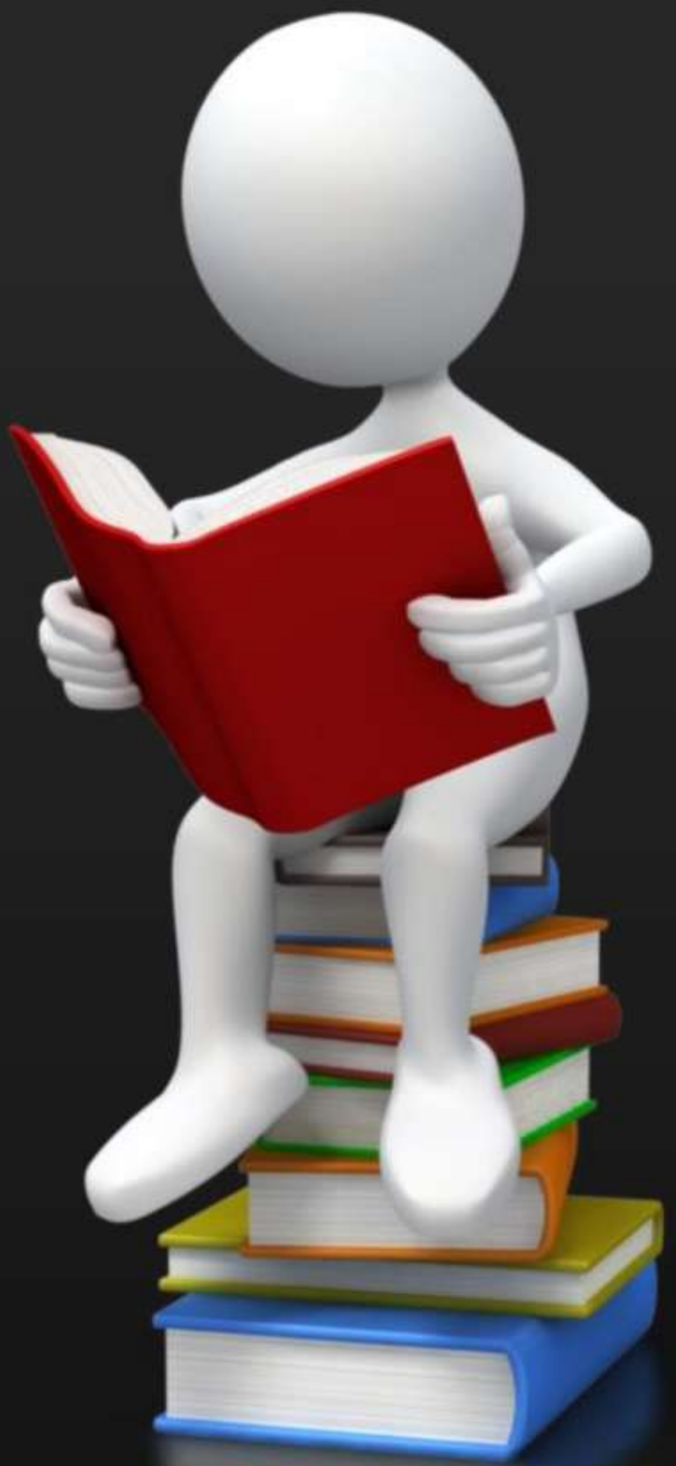
Topics *to be covered*

1. Sets
2. Binomial theorem
3. Counting of functions
4.



Subsets

Let A and B be any two sets, then A is called a subset of B, if every element of A is also an element of B. It is written as $A \subseteq B$ and read as ' A is a subset of B ' or ' A is contained in B '. Thus $A \subseteq B$ if $x \in A$ implies $x \in B$.

For example ; If $A=\{1,2,3\}$ and $B=\{1,2,3,4\}$, then $A \subseteq B$, as every element of A is also an element of B.

If atleast one element of A does not belong to B, then A is not a subset of B. Symbolically, it is written as $A \nsubseteq B$.

For example ; (i) $\{3,5\} \nsubseteq \{1,4,5\}$, as 3 does not belong to $\{1,4,5\}$

(ii) $N \subseteq Z \subseteq Q \subseteq R, T \subseteq R$ and $Q \nsubseteq T$.

$N \subseteq Z$ as all natural number are integers

Note:

- Every set is a subset of itself.
- The empty set is a subset of every set.

Let A and B be two sets, then

- $A \subseteq B$, if $a \in A \Rightarrow a \in B$.
- If $A \subseteq B$, then $n(A) \leq n(B)$.
- If $n(A) > n(B)$, then A cannot be a subset of B.
- If $A \subseteq B$ and $B \subseteq A$, then $A = B$ and vice-versa.

  or

  $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
- Total number of subsets of a finite set containing n elements is 2^n.
- Every set has only one improper subset.
- Every proper subset is a subset but a subset may no be a proper subset.

Illustration: List all the subsets and all the proper subsets of the set $\{-1,0,1\}$.

Soln.: Let $A = \{-1,0,1\}$

Subset of A having no element is : ϕ

Subset of A having one elements are : $\{-1\}, \{0\}, \{1\}$.

Subsets of A having two elements are : $\{-1,0\}, \{0,1\}, \{-1,1\}$.

Subsets of A having three elements : $\{-1,0,1\}$.

Thus, all the subsets of A are :

$$\phi, \{-1\}, \{0\}, \{1\}, \{-1,0\}, \{0,1\}, \{-1,1\}, \{-1,0,1\}$$

Proper subsets of A are :

$$\phi, \{-1\}, \{0\}, \{1\}, \{-1,0\}, \{0,1\}, \{-1,1\}$$

Illustration: If $A = \{1,2,\{2,3\},4\}$, state which of the following statements are true?

(i) $\{2,3\} \subset A$

(ii) $\{1,2\} \subset A$

Soln.: (i) False, because $\{2,3\}$ is an element of A not subset.

(ii) True

Operation on Sets

Union of Sets

The union of two sets A and B, denoted by $A \cup B$ is the set consisting of all the elements of A and B (Common elements being taken only once).

$A \cup B = \{x : x \in A \text{ or in } x \in B\}$ and read as 'A union B'.

Properties of Union of Sets

$A \cup B = B \cup A$ (Commutative law)

$(A \cup B) \cup C = A \cup (B \cup C)$ (Associative law)

$A \cup \phi = A$ (Law of identity element, ϕ is the identity of $\cup$)

$A \cup A = A$ (Idempotent law)

$U \cup A = U$(Law of U)

Intersection of Sets

The intersection of two sets A and B, denoted by $A \cap B$, is the set consisting of all those elements which are common to both A and B.

$A \cap B = \{x: x \in A$ and $x \in B\}$ and read as ' A intersection B'.

For example ; (i) If $A = \{1,2,5,6\}$ and $B = \{2,3,4,5\}$, then $A \cap B = \{2,5\}$

Properties of Intersection of Sets

$A \cap B = B \cap A$ (Commutative law)
$(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)
$\phi \cap A = \phi, U \cap A = A$ (Law of ϕ and U)
$A \cap A = A$ (Idempotent law)
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ (Distributive law) i.e., $\cap$ distributes over $\cup$.

Disjoint Sets
If A and B are two sets such that $A \cap B = \phi$, then A and B are called disjoint sets.

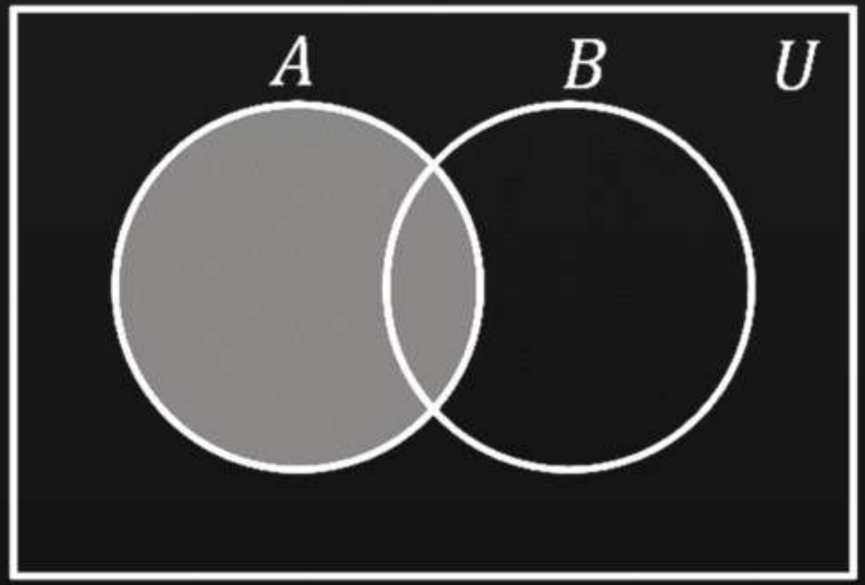
Difference of Sets

Difference of Sets

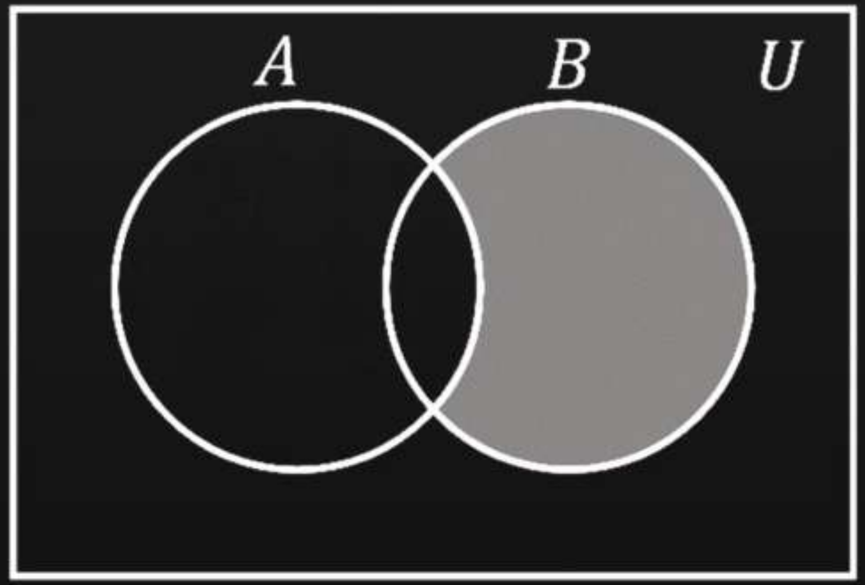
The difference of two sets $A - B$ is the set consisting of all those elements of A, which are not in B.

$A - B = \{x: x \in A \text{ and } x \notin B\}$ and $B - A = \{x: x \in B \text{ and } x \notin A\}$ For example ; (i)
$A = \{1,2,4,5,6\}$ and $B = \{4,6,7,8,9\} \therefore A - B = \{1,2,5\}$

(ii) The difference of two sets A and B can be represented by Venn diagram, as shown below :



$A–B$



$B–A$

Properties of Difference of Sets

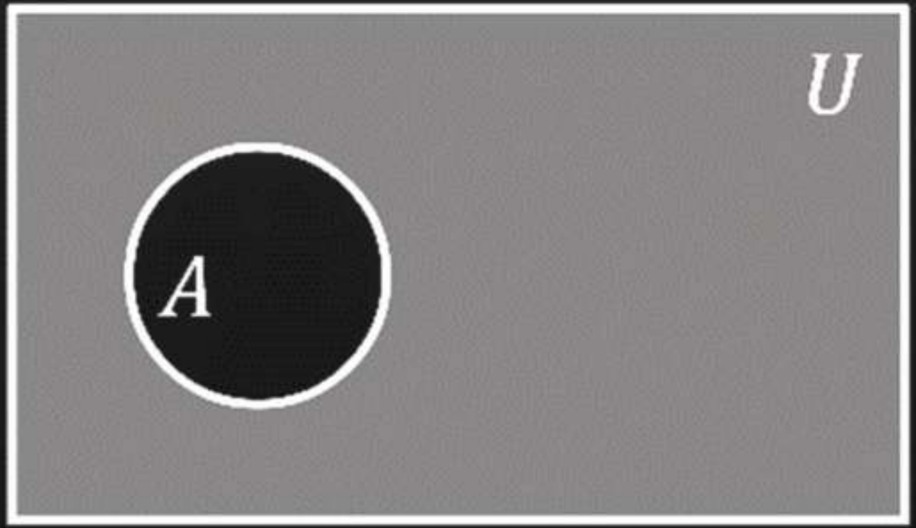
Properties of Difference of Sets

- $A - A = \phi$
- $A - \phi = A$
- $A - (B \cap C) = (A - B) \cup (A - C)$
- $A - (B \cup C) = (A - B) \cap (A - C)$

Complement of a Set

If U be the universal set and let A be a proper subset of U i.e., $A \subset U$, then complement of A with respect to U is the set of elements which belongs to U but not present in A.

denotes A'



We denote it às A^c or A' or $\bar{A}$ or $U - A$,

Properties of complement of Sets

$$A' = \{x : x \in U \text{ but } x \notin A\}$$

$$\therefore\ x \in A' \Leftrightarrow x \notin A$$

Properties of Complement of Sets

- Complement laws :
  - $A \cup A' = U$
  - $A \cap A' = \phi$
- De Morgan's law :
  - $(A \cup B)' = A' \cap B'$
  - $(A \cap B)' = A' \cup B'$

Properties of complement of Sets

- Law of double complement : $(A')' = A$
- Laws of empty set and universal set : $\phi' = U$ and $U' = \phi$

QUESTION

Write the set builder form of $A = \{-2, 2\}$ → rules to build a set

(A) $A = \{x : x \text{ is a natural number}\} = \{1, 2, 3, \ldots\}$ → infinite set

(B) $A = \{x : x \text{ is a root of the equation } x^2 + 2 = 0\} = \{\ \}$ → empty set

(C) $A = \{x : x \text{ is a real number}\} = (-\infty, \infty)$ → infinite set

(D) $A = \{x : x \text{ is a root of the equation } x^2 = 4\} = \{+2, -2\}$

QUESTION

Which of the following is a set?

A The collection of all the months of a year beginning with the letter J.

B The collection of ten most talented writers of India.

C A team of eleven best cricketers of the world.

D A collection of most dangerous animals of the world.

QUESTION

If A is the set whose elements are obtained by adding 1 to each of the positive odd integers, then the set builder form of A is

(A) $A = \{x : x \text{ is even and } x \in Z\}$

odd integers + 1 = even integers

(B) $A = \{x : x \text{ is odd and } x < 1\}$

(C) $A = \{x : x \text{ is odd and } x \in Z\}$

(D) $A = \{x : x \text{ is an integer}\}$

QUESTION

Which of the following is a $\boxed{\text{non-empty set?}}$

$x^2 - 1 = 0$

$x = \pm 1$

$x \in N$

(A) $\{x \mid x$ is a natural number and $x^2 - 1 = 0\}$ $= \{1\}$

(B) $\{x \mid x$ is a natural number and $x^2 + 3 = 0\}$ $= \{\ \}$

(C) $\{x \mid x$ is a natural number and $x^2 + 6 = 0\}$ $= \{\ \}$

(D) $\{x \mid x$ is a natural number and $x^2 = x + 3\}$ $= \{\ \}$

$x^2 - x - 3 = 0$

$x = \frac{1 \pm \sqrt{1+12}}{2} \notin N$

QUESTION

Which of the following sets is/are null sets?

(A) $\{0\}$

(B) ϕ

(C) $\{\ \}$

(D) Both (B) and (C)

QUESTION

Which of the following is a singleton set? $\xrightarrow{}$ a set with only one element

I = set of integers

A $\{x : |x| < 1, x \in I\} = \{0\}$

B $\{x : |x| = 5, x \in I\} = \{+5, -5\}$

C $\{x : x^2 = 1, x \in I\} = \{+1, -1\}$

D $\{x : x^2 + x + 1 = 0, x \in R\} = \{\ \}$

$|x| < 1$

$x \in (-1, 1)$

since $x \in I$

integer b/w -1 to 1 is '0'

$|x| = a$

$x = \pm a$

$|x| = 5$

$x = \pm 5$

$$x^2+x+1=0$$

$$x=\frac{-1\pm\sqrt{1-4}}{2}$$

$$x=\frac{-1\pm\sqrt{-3}}{2}$$

$$x=\frac{-1\pm\sqrt{3}\,i}{2}\notin R$$

$\{\ \}$ $\hookrightarrow$ empty set

$\{0\}$ $\rightarrow$ singleton set

consisting the element 0.

QUESTION

Which of the following is a finite set?

(A) Set of integers Z.

(B) Set of prime numbers.

(C) Number of months in a year.

(D) Set of parallel lines.

QUESTION

Which of the following sets are equal?

A $A = \{a, b, c, d\}, B = \{d, c, b, a\}$

B $A = \{4,8,12,16\}, B = \{8,4,16,18\}$

C $A = \{x: x \text{ is a multiple of } 10\}, B = \{10,15,20,25,30, \dots\}$

D None of these

⊛ Equal sets :-

A & B are said to be equal set if they have same elements

$\Downarrow$

$A = B$

Ex: $A = \{a, b, c\}$

$B = \{a, a, b, b, b, c, c\}$

Here $A = B$

⊛ Equivalent sets :-

A & B are said to be equivalent sets if both the sets have same no of elements

$\Downarrow$

$n(A) = n(B)$

$A = \{1, 2, 3\}$

$B = \{a, b, c\}$

$\therefore n(A) = n(B)$

QUESTION

Which of the following sets are equal?

$A = \{x : x \in N, x < 3\}$, $B = \{1,2\}$ $C = \{3,1\}$, $D = \{x : x \in N, x \text{ is odd}, x < 5\}$

$E = \{2, 1\}$, $F = \{1, 1, 3\}$

$A = \{1, 2\}$

$B = \{1, 2\}$

$C = \{3, 1\}$

$D = \{1, 3\}$

$E = \{2, 1\}$

$F = \{1, 1, 3\}$

A $A = C = D$

B $A = B = E$

C $C = D = E$

D Both (B) and (C)

QUESTION

Which of the following are equal sets?

(A) $A = \{p, q, r, s\}, B = \{s, r, q, p\}$

(B) $A = \{3, 9, 12, 15\}, B = \{9, 3, 12, 18\}$

(C) $A = \{x : x \text{ is a multiple of } 8\}, B = \{16, 30, 32, 40....\}$

(D) All of these

QUESTION

Which of the following has only one subset?

A $\{\}$

B $\{4\}$ $\Rightarrow$ Subsets are $\Rightarrow \{\}, \{4\}$

C $\{4, 5\}$ $\Rightarrow$ subsets are $\Rightarrow \{4\}, \{5\}, \{4, 5\}, \{\}$

D $\{0\}$ $\Rightarrow$ subsets are $\Rightarrow \{\}, \{0\}$

* (1) Every set is a subset of itself

(2) Empty set is a subset of every set.

(3) Empty set has only one subset.

$\in$

$\downarrow$

belongs to

$\Downarrow$

This symbol is used to indicate that an element is present in the set.

$\subset$

$\downarrow$

contained in

$\Downarrow$

This symbol is used to tell that one set is a subset of other set

① $A = \{1, 2\}$ $\quad B = \{1, 2, 3\}$

$A \subset B$

② $A = \{1, 2\}$ $\quad B = \{1, \{2\}, 3\}$

$A \not\subset B$

③ $A = \{1, 2\}$ $\quad B = \{1, \{2\}, 3, 2\}$

$A \subset B$

<u>Subset</u>:-

A is a subset of B if each element of A is an element of B.

If $A = \{1, 2\}$ & $B = \{1, 2, \{1, 2\}, 3\}$, Then which of the following is <u>incorrect</u>

(A) ~~$A = B$~~

(B) $A \in B$

(C) $A \subset B$

(D) $A \neq B$

$A = \{1, 2\}$

subsets are

$\{\ \} = \phi$

$\{1\}$

$\{2\}$

$\{1, 2\}$

Power set :- A set which consists of all the subsets

$P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

$\phi \subset A$	$\phi \in P(A)$
$\{1\} \subset A$	$\{1\} \in P(A)$
$\{2\} \subset A$	$\{2\} \in P(A)$
$\{1, 2\} \subset A$	$\{1, 2\} \in P(A)$

QUESTION

If $A = \{a, \{b\}\}$, then

A $\{a\} \subset A$

B $\{b\} \subset A$ $\{b\} \in A$

C Both (A) and (B)

D None of these

QUESTION

If $A = \{a, \{b\}, b\}$, Then

A. $\{a\} \subset A$

B. $\{b\} \subset A$

C. Both (A) and (B)

D. None of these

QUESTION

If $A = \{2, 4, 6, \{1, 3\}\}$ and $B = \{1, 3\}$, then

$A = \{2, 4, 6, \{1, 3\}, 3\}$

$3 \in A$.

$3 \notin A$ $\quad A = \{2, 4, 6, B\}$ $\quad \{ B = \{1, 3\}$

(A) $3 \in A$

$3 \in B$

$B \in A$

(B) $\{3\} \subset A$

$\{3\} \subset B$

$\{1, 3\} \in A$

(C) $B \subset A$

(D) None of these

$A = \{3, \{5\}, 6, \{1,2\}\}$

$\in \rightarrow$ belongs to

$\subset \rightarrow$ subset

$\neq \rightarrow$ not equal

$\not\subset \rightarrow$ not a subset

$\notin \rightarrow$ Does not belong to

1. $3 \underline{\in} A$
2. $\{5\} \underline{\in} A$
3. $5 \underline{\notin} A$
4. $6 \underline{\in} A$
5. $\{6\} \underline{\subset} A$
6. $1 \underline{\notin} A$
7. $\{1\} \underline{\not\subset} A$
8. $\{1,2\} \underline{\in} A$
9. $\{3\} \underline{\subset} A$
10. $\{3,5,6\} \underline{\not\subset} A$

QUESTION

Here $a = 1 > 0$ | $b = -1$ | $c = 2$ & $b^2 - 4ac$
$= 1 - 8 = -7 < 0$

If $A = \{x: x^2 - x + 2 > 0\}$ and $B = \{x: x^2 - 4x + 3 \leq 0\}$, then $A \cap B$ is

A. $[1, 3]$

B. $(-\infty, \infty)$

C. $(1, 3)$

D. $(-\infty, 1) \cup (3, \infty)$

$\Downarrow$

WKT

if $a > 0$

& $D < 0$

Then $ax^2 + bx + c > 0$

$\forall x \in R$

$A = R = (-\infty, \infty) \to (1)$

$(x-3)(x-1) \leq 0$

$+3$

$-3 \; -1$

$+ \; - \; +$

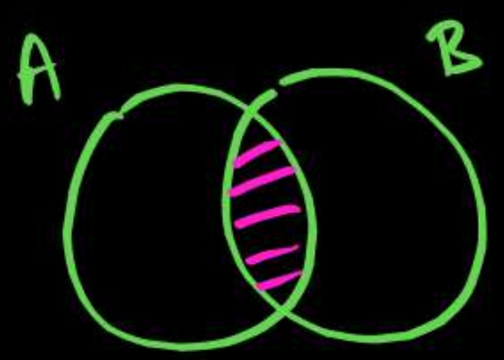
$-\infty \quad 1 \quad 3 \quad \infty$

$x \in [1, 3] \to (2)$

$(1) \cap (2)$

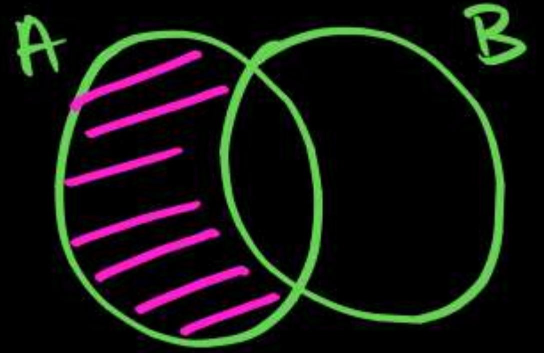
$x \in [1, 3]$

1



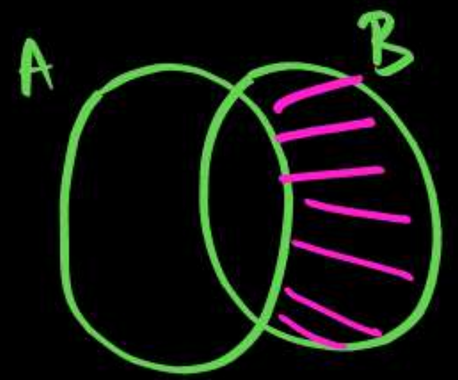
$A \cap B$

2



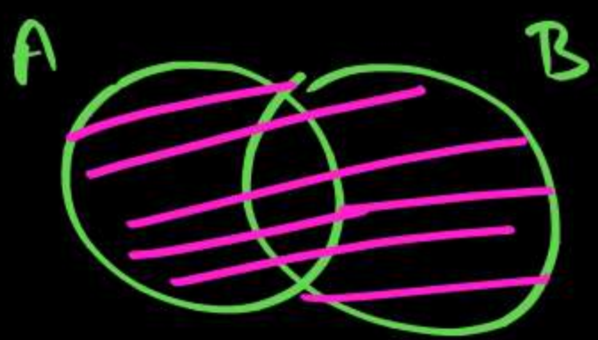
$A - B$

3



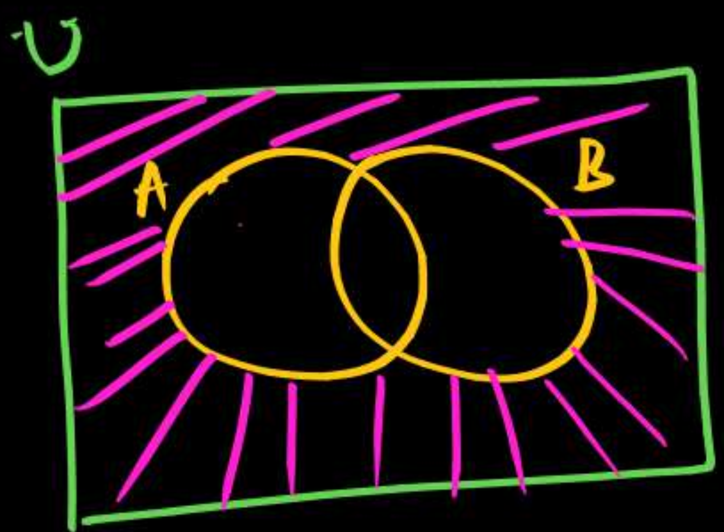
$B - A$

4



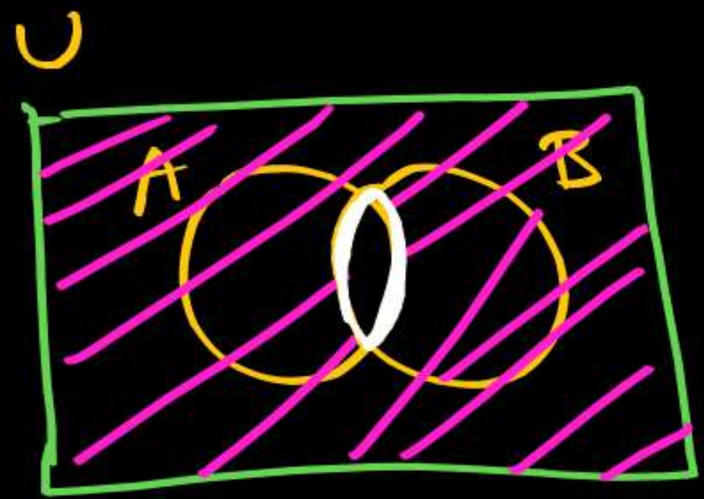
$A \cup B$

5



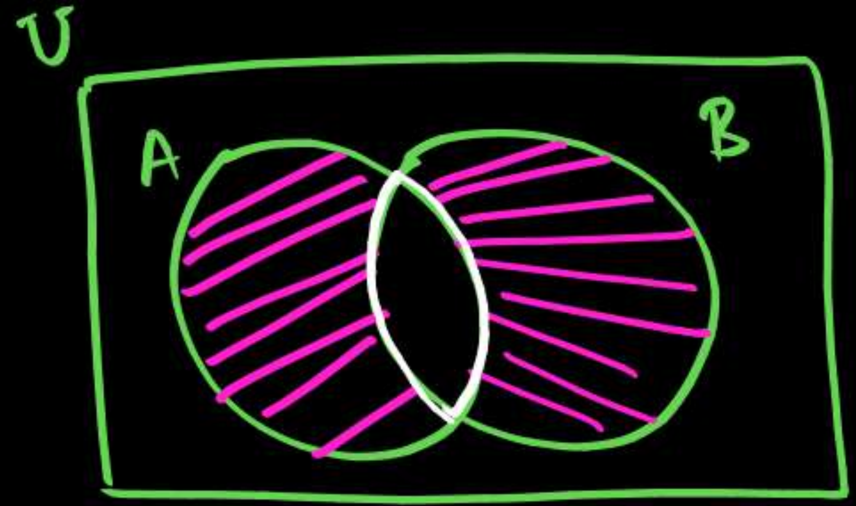
$U - A \cup B = (A \cup B)' = A' \cap B'$

6



$U - A \cap B - (A \cap B)' = A' \cup B'$

7



$(A-B) \cup (B-A)$

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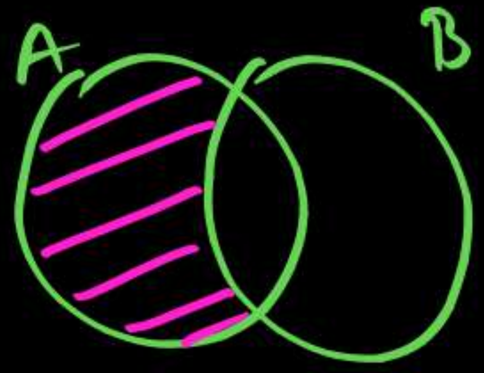
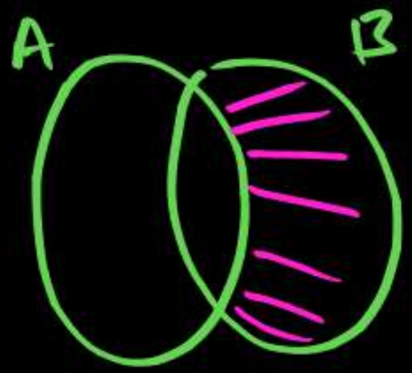
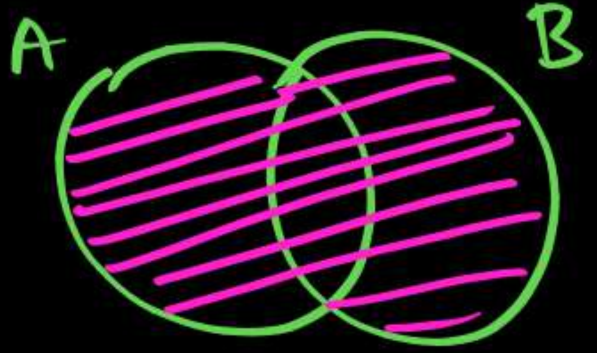
$(A \cup B) - (A \cap B)$



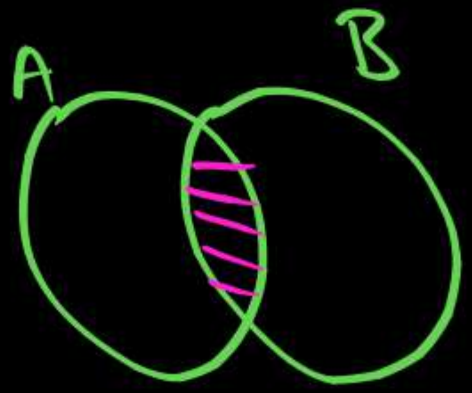
$A-B$



$B-A$



$A \cup B$



$A \cap B$

QUESTION

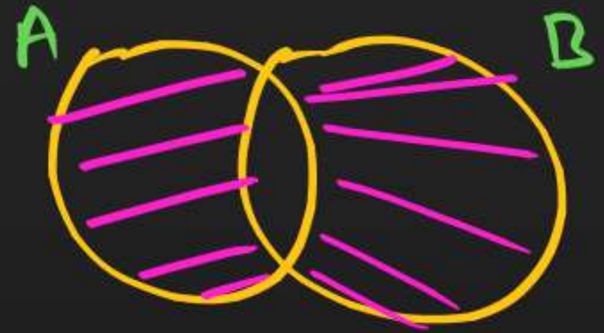
Which one of the following is equal to $(A - B) \cup (B - A)$?

A $(A \cup B) \cup (A - B)$

B $(A \cup B) \cup (A \cap B)$

C $(A \cup B) - (A \cap B)$

D $(A - B) \cap (B - A)$



$(A-B) \cup (B-A)$ ⇒ symmetric difference

$= (A \cup B) - (A \cap B)$

$\Downarrow$

$A \Delta B$

1. $A \cup B = B \cup A$
2. $A \cap B = B \cap A$
3. $A \cap (B \cap C) = (A \cap B) \cap C$
4. $A \cup (B \cup C) = (A \cup B) \cup C$
5. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
6. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
7. $(A \cup B) \cap A = A$
8. $(A \cap B) \cup A = A$
9. if $A \subset B$
   1. $A \cup B = B$
   2. $A \cap B = A$
10.
    1. $U' = \phi$
    2. $\phi' = U$
    3. $A' = U - A$
    4. $(A')' = A$
    5. $(A \cap B)' = A' \cup B'$
    6. $(A \cup B)' = A' \cap B'$

QUESTION

If A and B are non-empty sets, then $(A' \cap B')' \cap B'$ is equal to

(A) $A \cap B'$

(B) ϕ

(C) $A \cup B'$

(D) $A' \cap B$

① $A \cap A' = \phi$

② $A \cup A' = U$

$(C \cap D)' = C' \cup D'$

$[(A')' \cup (B')'] \cap B'$

$(A \cup B) \cap B'$

$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

Distributive.

$(A \cap B') \cup (B \cap B')$

$(A \cap B') \cup \phi$

$A \cap B'$

QUESTION

If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 3, 4\}$ and $B = \{4, 5, 6, 7\}$ be any two sets, then $A \cap (A \cup B)'$ is equal to

(A) A

(B) B

(C) ϕ

(D) None of these

$A \cap [A' \cap B']$

$[A \cap A'] \cap B'$

$\phi \cap B'$

$= \phi$

QUESTION

The set $A = \{x : |2x + 3| < 7\}$ is equal to the set

(A) $D = \{x : 0 < x + 5 < 7\}$

(B) $B = \{x : -3 < x < 7\}$

(C) $E = \{x: -7 < x < 7\}$

(D) $C = \{x : -13 < 2x < 4\}$

$2x+3 \in (-7,7)$

$2x \in (-10,4)$

$x \in (-5,2)$

$-5 < x < 2$

Add 5

$0 < x+5 < 7$

QUESTION

Write the set builder form of $A = \{-1, 1\}$.

(A) $A = \{x : x \text{ is an integer}\}$

(B) $A = \{x : x \text{ is a root of the equation } x^2 + 1 = 0\}$

(C) $A = \{x : x \text{ is a real number}\}$

(D) $A = \{x : x \text{ is a root of the equation } x^2 = 1\}$

QUESTION

If A and B are finite sets and $A \subset B$, then

$A \cap B = A$

$\Downarrow$

$n(A \cap B) = n(A)$

$A \cup B = B$

$\Downarrow$

$n(A \cup B) = n(B)$

A. $n(A \cup B) = n(B)$ ✓

B. $n(A \cap B) = n(B)$

C. $n(A \cap B) = \phi$

D. $n(A \cup B) = n(A)$

if A & B are non empty sets

Then

① $\max\{n(A), n(B)\} \le n(A \cup B) \le n(A) + n(B)$

max no of elements in $A \cup B$

min no of elements in $A \cup B$

Ex: if $A = \{1, 3, 5\}$ $B = \{2, 4, 3, 5, 6\}$

$n(A) = 3$ $n(B) = 5$

$\max\{3, 5\} \le n(A \cup B) \le 3 + 5$

$5 \le n(A \cup B) \le 8$

(2)

$$0 \leq n(A\cap B) \leq \min\{n(A), n(B)\}$$

Ex:- if $A = \{1,3,5\}$ & $B = \{1,2,3,5,6\}$

$n(A) = 3$ $n(B) = 5$

$$0 \leq n(A\cap B) \leq \min\{3,5\}$$

$$0 \leq \underline{n(A\cap B)} \leq 3.$$

QUESTION

If $A = \{1, 2, 3, 4, 5, 6\}$, then the number of subsets of A which contains atleast two elements is

min 2 elements

(A) 63

(B) 57

(C) 58

(D) 64

method 1:-

$${}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$$

$$= 15 + 20 + 15 + 6 + 1$$

$$= 57$$

method 2:- Total no of subsets − (no of subsets consisting of 1 & no element)

$$= 2^6 - ({}^6C_0 + {}^6C_1)$$

$$= 64 - (1+6) = \underline{57}$$

In Binomial theorem

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + \dots + {}^nC_n$$

$$2^6$$

There is one subset consisting of '0' elements (empty set) ← ${}^6C_0 = 1$

${}^6C_1 = 6$

$\Downarrow$

There are 6 subsets consisting '1' element. (singleton sets)

${}^6C_2 = 15$

${}^6C_3 = 20$

$\Downarrow$

There are 20 subsets consisting of 3 elements

${}^6C_4 = 15$

${}^6C_5 = 6$

${}^6C_6 = 1$

${}^6C_r$

$\Downarrow$

no of subsets consisting of 'r' elements in a set of 6 elements.

$1 \rightarrow {}^6C_0$

$6 \rightarrow {}^6C_1$

$15 \rightarrow {}^6C_2$

$20 \rightarrow {}^6C_3$

$15 \rightarrow {}^6C_4$

$6 \rightarrow {}^6C_5$

$1 \rightarrow {}^6C_6$

$64 \rightarrow$ no of subsets

$${}^nC_r = \frac{n!}{(n-r)!\; r!}$$

or

$${}^nC_r = \frac{n}{r}\,\frac{n-1}{r-1}\,\frac{n-2}{r-2}\cdots$$

Ex:- $${}^6C_3 = \frac{6}{3}\,\frac{5}{2}\,4 = 20$$

$${}^{10}C_4 = \frac{10}{\cancel{4}}\,\frac{\cancel{9}^{\,3}}{\cancel{3}}\,\frac{\cancel{8}}{\cancel{2}}\,\frac{7}{1}$$

$$= 10 \times 3 \times 7$$

$$= 210$$

if $n(A) = 10$

no of subsets consisting of minimum 3 elements

Soln:

Total no of subsets − (subsets consisting of 0, 1 & 2 elements)

$$= 2^{10} - \left({}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2\right)$$

$$= 2^{10} - (1 + 10 + 45)$$

$$= 1024 - 56$$

$$= \underline{968}$$

if $n(A) = 10$

no of subsets consisting of $\underbrace{\text{maximum 3 elements}}$

$\Downarrow$

subsets consisting of 0, 1, 2 & 3 elements

Soln:

$$^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3$$

$$= 1 + 10 + \left(\frac{10}{2} \times 9\right) + \left(\frac{10}{\cancel{3}} \cdot \frac{\cancel{9}^{3}}{\cancel{2}} \cdot \frac{\cancel{8}^{4}}{1}\right)$$

$$= 1 + 10 + 45 + 120$$

$$= 56 + 120$$

$$= \underline{176}$$

QUESTION

$C \cap D' = C - D$

If $A \subseteq B$, then $B' - A'$ is equal to

(A) A'

(B) B'

(C) $A - B$

(D) ϕ

$B' \cap A$

$= A \cap B'$

Here $A \subset B$

$\Rightarrow A \not\subset B'$

$\therefore A \cap B' = \phi$

$A = \{1, 2\}$; $B = \{1, 2, 3\}$

$B' - A'$

$= B' \cap A$

$= A \cap B'$

$= A - B$

$= \phi$

If $A = \{1, 2, 4, 7\}$ & $B = \{2, 7, 9, 10, 11\}$

$A - B = \{1, 4\}$

$B - A = \{9, 10, 11\}$

QUESTION

If set A has 3 elements and the set $B = \{3,4,5\}$, then find the number of elements in $(A \times B)$.

A 8

B 7

C 9

D 10

QUESTION

Let $A = \{1, 2\}$, $B = \{\{1\}, \{2\}\}$, $C = \{\{1\}, \{1, 2\}\}$. Then which of the following relation is true?

$\rightarrow A$

$C = \{\{1\}, A\}$

$\Downarrow$

$A \in C$

(A) $A = B$ ✗

(B) $B \subseteq C$ ✗

(C) $A \in C$ ✓

(D) $A \subset C$

QUESTION

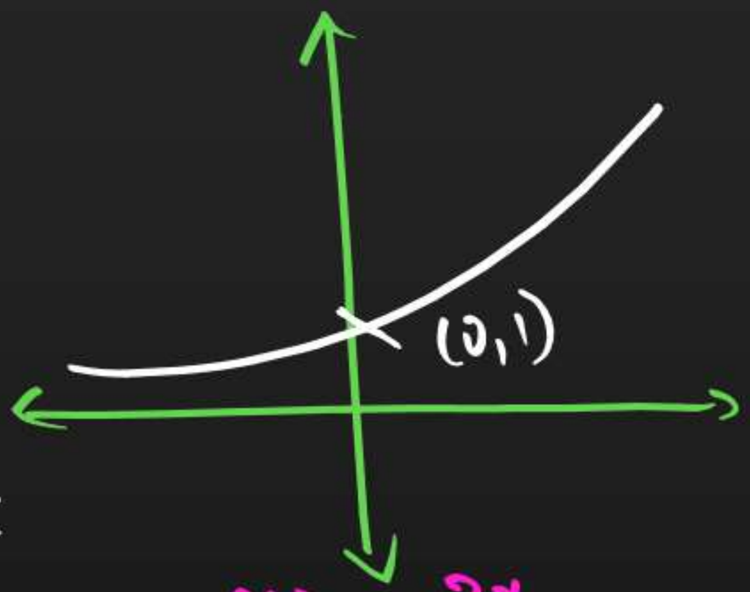
Let $A = \{(x, y) : y = e^{2x}, \forall x \in R\}$ and $B = \{(x, y) : y = e^{-2x}, \forall x \in R\}$, then $A \cap B$ is

A Not a set

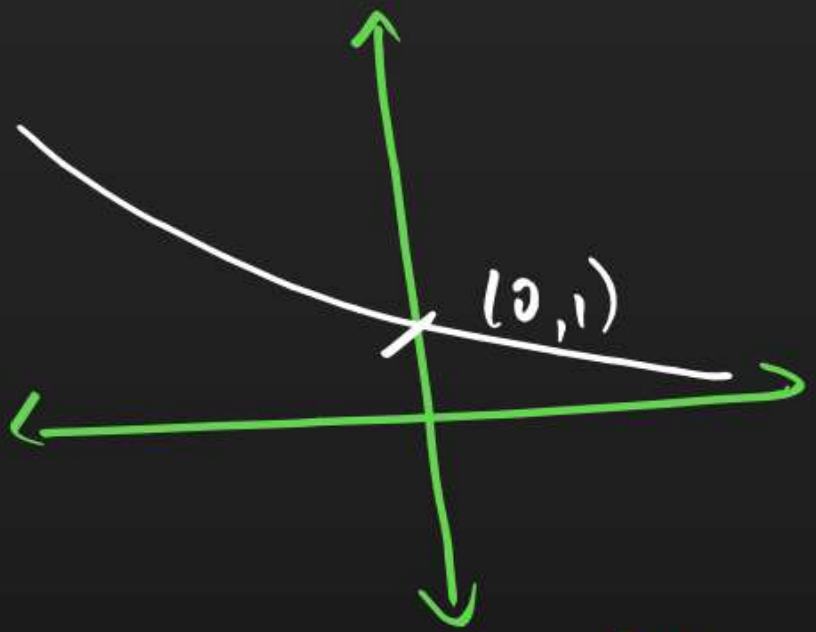
B Singleton set

C Empty set

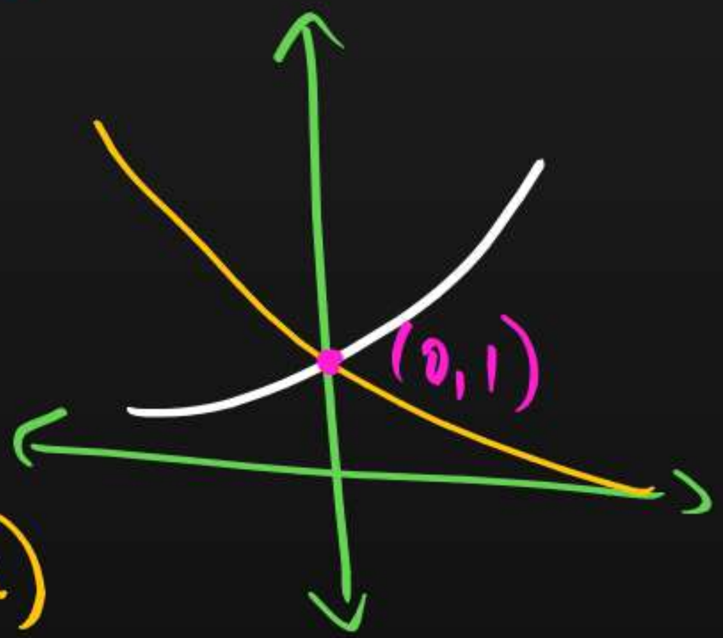
D None of these



$f(x) = e^{2x}$



$g(x) = e^{-2x}$



$f(x) \cap g(x)$

$A \cap B = \{(0,1)\}$

QUESTION

If a set A has 4 elements, then the total number of proper subsets of set A, is

A 16

B 14

C 15

D 17

$n(A) = 4$

$\therefore$ no of Proper subsets

$= 2^4 - 1$

$= 15$

if $n(A) = n$

(1) no subsets $= 2^n$

(2) no proper subsets $= 2^n - 1$

(3) no of Improper subsets $= 1$

The given set itself

Binomial theorem

$$(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \dots + {}^nC_n b^n$$

$\Downarrow$

general term

$\Downarrow$

$$T_{r+1} = {}^nC_r a^{n-r} b^r$$

In $(x+2y)^9$ Find the 4th term.

Soln:

given $(x+2y)^9$

$a = x$ | $b = 2y$ | $n = 9$

$r+1 = 4$

$r = 3$

$$T_4 = {}^9C_3 (x)^{9-3} (2y)^3$$

$$= \frac{\cancel{9}^3}{\cancel{3}} \frac{\cancel{8}^4}{\cancel{2}_1} \frac{7}{1} (2^3) x^6 y^3$$

$$= 12 \times 7 \times 8 \; x^6 y^3$$

$$= 12 \times \overset{7}{56} \; x^6 y^3$$

$$= 672 \, x^6 y^3$$

$$\frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{n-r+1}{r}$$

Ex:

$$\frac{{}^{6}C_{5}}{{}^{6}C_{4}} = \frac{6-5+1}{5}$$

$$= \frac{6-4}{5}$$

$$= \frac{2}{5}.$$

$(1+x)^{44}$

$a=1$

$b=x$

$n=44$

(2) If 21^{st} & 22^{nd} terms in the expansion of $(1+x)^{44}$ are equal, Then $x=$

(A) $\frac{8}{7}$

(B) $\frac{21}{22}$

(C) $\frac{7}{8}$

(D) $\frac{23}{24}$

$$T_{21} = T_{22}$$

$r+1=21$ $\quad$ $r+1=22$

$r=20$ $\quad$ $r=21$

$$^{44}C_{20}(1)^{44-20}x^{20} = {}^{44}C_{21}(1)^{44-21}(x)^{21}$$

$$^{44}C_{20}\,x^{20} = {}^{44}C_{21}\,x^{21}$$

$$\frac{x^{20}}{x^{21}} = \frac{^{44}C_{21}}{^{44}C_{20}} = \frac{44-21+1}{21}$$

$$\frac{1}{x} = \frac{44-20}{21}$$

$$\frac{1}{x} = \frac{24}{21}$$

$$x = \frac{21}{24}$$

$$x = \frac{7}{8}$$

③ The value of 'a' if 17^{th} & 18^{th} term of the expansion $(2+a)^{50}$ are equal

(A) 2

(B) -3

(C) 1

(D) 4

$$T_{17} = T_{18}$$

$r+1=17$ $\quad$ $r+1=18$

$r=16$ $\quad$ $r=17$

$$^{50}C_{16}\, 2^{50-16}\, a^{16} = {}^{50}C_{17}\, 2^{50-17}\, a^{17}$$

$$\frac{2^{34}}{2^{33}} \frac{a^{16}}{a^{17}} = \frac{^{50}C_{17}}{^{50}C_{16}}$$

$$\frac{2}{a} = \frac{50-17+1}{17}$$

$$\frac{2}{a} = \frac{50-16}{17}$$

$$\frac{2}{a} = \frac{34}{17}$$

$$\frac{2}{a} = 2$$

$$a=1$$

QUESTION

$^{n}C_r = {^{n}C_p} \Rightarrow r + p = n$

In the binomial expansion of $(1 + x)^{15}$, the coefficients of x^r and x^{r+3} are equal. Then r is

$T_{r+1} = {^{15}C_r}(1)^{15-r}(x)^r$

A 7

B 8

C 6

D 4

coefficient of $x^r = {^{15}C_r}$

coefficient of $x^{r+3} = {^{15}C_{r+3}}$

Here ${^{15}C_r} = {^{15}C_{r+3}}$

$15 = r + (r+3)$

$12 = 2r$

$r = 6$

QUESTION

$^nC_r = {^nC_{n-r}}$ | $^{14}C_{10} = {^{14}C_4}$

The 11^{th} term in the expansion of $\left(x + \frac{1}{\sqrt{x}}\right)^{14}$ is

(A) $999/x$

(B) $1001/x$

(C) i

(D) $x/1001$

$r+1=11$

$r=10$

$n=14$

$a=x$

$b=x^{-1/2}$

$T_{11} = {^{14}C_{10}}\, x^{14-10} \left(x^{-1/2}\right)^{10}$

$= {^{14}C_4}\, x^4\, x^{-10/2}$

$= \frac{\cancel{14}^{7}}{\cancel{4}} \cdot \frac{13}{\cancel{3}} \cdot \frac{\cancel{12}}{\cancel{2}} \cdot \frac{11}{1} (x^4)(x^{-5})$

$= 7 \times 13 \times 11 \left(\frac{1}{x}\right)$

$= \frac{91 \times 11}{x} = \frac{1001}{x}$

Term independent of x

or

constant term

$=$ coefficient of x^0

QUESTION

$r+1=13$

$r=12$

The 13^{th} term in the expansion of $\left(x^2+\frac{2}{x}\right)^n$ is independent of x, then the sum of the divisors of n is

(A) 39

(B) 36

(C) 37

(D) 38

$a=x^2$ & $b=2x^{-1}$

$T_{13}={}^nC_{12}(x^2)^{n-12}(2x^{-1})^{12}$

$={}^nC_{12}\,x^{2n-24}\,2^{12}\,x^{-12}$

$={}^nC_{12}\,2^{12}\,x^{2n-24-12}$

This is equated with x^0

$x^{2n-24-12}=x^0$

$2n-36=0$

$2n=36$

$n=18$

Divisors of $n=18$ are

1, 2, 3, 6, 9, 18

Sum = 39

QUESTION

The constant term in the expansion of $\left(x^2 - \frac{1}{x^2}\right)^{16}$ is

(A) ${}^{16}C_8$

(B) ${}^{16}C_7$

(C) ${}^{16}C_9$

(D) ${}^{16}C_{10}$

$T_{r+1} = {}^{16}C_r (x^2)^{16-r} (-x^{-2})^r$

$= {}^{16}C_r (-1)^r x^{32-2r} \cdot x^{-2r}$

$= {}^{16}C_r (-1)^r x^{32-4r} \rightarrow (1)$

$\Downarrow$

$x^{32-4r} = x^0$

$32 - 4r = 0$

$r = 8$

$\therefore$ Put $r = 8$ in (1)

$= {}^{16}C_8 (-1)^8$

$= {}^{16}C_8$.

In $(a+b)^K$

$K \to$ index

no of terms in the expansion $= K+1$

Ex: In $(a+b)^{13}$

$K = 13$

Total no terms after expansion $= 13+1 = 14$

No. of terms in the expansion of $(a^2-2ab+b^2)^{10}$

(A) 11

(B) 20

(C) 21

(D) 19

$$\Downarrow$$

$$[(a-b)^2]^{10}$$

$$(a-b)^{20}$$

$$K = 20$$

$\therefore$ no. of terms $= 21$

No of terms in the expansion of $(1+2x+x^2)^{11}$

(A) 12

(B) 23

(C) 22

(D) 19

$$(1+2x+x^2)^{11}$$
$$\Downarrow$$
$$[(x+1)^2]^{11}$$
$$(x+1)^{22}$$
$$K = 22$$

no of terms $= 23$

No of terms in the expansion of $\left[(2x-3y)^2(2x+3y)^2\right]^2$

(A) 4

(B) 5

(C) 8

(D) 16

$$\Downarrow$$

$$\left[\left[(2x-3y)(2x+3y)\right]^2\right]^2$$

$$(4x^2-9y^2)^4$$

$$k=4$$

no of terms = 5

QUESTION

The number of terms in the expansion of $(x + y + z)^{10}$ is

A 142

B 11

C 110

D 66

$[(x+y)+z]^{10}$

Let $(x+y) = t$

$(t+z)^{10}$

$= t^{10} + {}^{10}C_1 t^9 z + {}^{10}C_2 t^8 z^2 + {}^{10}C_3 t^7 z^3 + \dots + z^{10}$

$= (x+y)^{10} + {}^{10}C_1 (x+y)^9 z + {}^{10}C_2 (x+y)^8 z^2 + {}^{10}C_3 (x+y)^7 z^3 + \dots + z^{10}$

no of term $= 11$ | no of terms $= 10$ | no of terms $= 9$ | no of terms $= 8$ ...... | no of terms $= 1$

$(a+b+c)^n$

$\Downarrow$

no of terms after expansion

$= \dfrac{(n+1)(n+2)}{2}$

$11+10+9+8+\cdots+1$

$\Downarrow$

$1+2+3+\cdots+10+11$

$\Downarrow$

$\frac{11(11+1)}{2} = \frac{11(12)}{6} = 11(6)$

$= \underline{66}$

$1+2+3+4+\cdots+K$

$\Downarrow$

$\frac{K(K+1)}{2}$

QUESTION

The middle term of expansion $\left(\frac{10}{x}+\frac{x}{10}\right)^{10}$ is

(A) ${}^{8}C_{5}$

(B) ${}^{10}C_{5}$

(C) ${}^{7}C_{5}$

(D) ${}^{9}C_{5}$

$n = 10 \rightarrow$ even

middle term $= \frac{10}{2} + 1$

$= 6^{th}$ term

$r + 1 = 6$

$r = 5$

$T_6 = {}^{10}C_5 \left(\frac{10}{x}\right)^{10-5} \left(\frac{x}{10}\right)^5$

$= {}^{10}C_5 \left(\frac{10}{x}\right)^5 \left(\frac{x}{10}\right)^5$

$= {}^{10}C_5$

(*) The coefficient of middle term in the expansion of

$(1+4x+4x^2)^5$.

(A) 4032

(B) 8064

(C) 2016

(D) 1008

$$\Downarrow$$

$$[(2x+1)^2]^5$$

$$=(2x+1)^{10}$$

$n = 10 =$ even

$\therefore$ middle term $= \frac{10}{2}+1$

$= 6^{th}$ term

$$r+1=6$$

$$r=5$$

$$T_6 = {}^{10}C_5 (2x)^{10-5} (1)^5$$

$$= \frac{10}{5}\cdot\frac{9}{4}\cdot\frac{8}{3}\cdot\frac{7}{2}\cdot\frac{6}{1}\, 2^5 (x^5)$$

$$= 6 \times 42 \times 32 \ (x^5)$$

$$= 252 \times 32$$

504
756
8064

In $(a+b)^n$

if $n \to$ odd

we get 2 middle terms

$\Downarrow$

middle terms are

$\left(\frac{n+1}{2}\right)^{th}$ term & $\left(\frac{n+1}{2}+1\right)^{th}$ term

Ex: In $(a+b)^{13}$

middle terms are

$\frac{13+1}{2} = 7th$ & $\left(\frac{13+1}{2}+1\right) = 8th$ term

In the expansion of $\left(x^2+\frac{1}{x}\right)^{15}$

The sum of coefficient of middle terms

$^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

(A) $^{15}C_9$

(B) $^{16}C_9$

(C) $\cancel{^{16}C_8}$

(D) None of these

$n=15$

$\therefore$ middle terms are

$\frac{15+1}{2} = 8^{th}$ & 9^{th} term

$r+1=8$	$r+1=9$
$r=7$	$r=8$
$^{15}C_7$	$^{15}C_8$
$\downarrow$ coefficients	$\downarrow$ coefficients

$^{15}C_8 + {}^{15}C_7$

$= {}^{16}C_8$

Find $\sum_{r=0}^{1} {}^{n+r}C_n$

(A) ${}^{n+2}C_1$

(B) ${}^{n+2}C_n$

(C) ${}^{n+3}C_n$

~~(D)~~ ${}^{n+2}C_{n+1}$

$r=0$ & $r=1$

$\Downarrow$ $\Downarrow$

${}^{n+0}C_n + {}^{n+1}C_n$

${}^{n}C_n + {}^{n+1}C_n$

WKT ${}^{n}C_n = 1 = {}^{n+1}C_{n+1}$

${}^{n+1}C_{n+1} + {}^{n+1}C_n$

$= {}^{n+2}C_{n+1}$

In $(a+b)^n + (a-b)^n$ or $(a+b)^n - (a-b)^n$

If n is odd

$\therefore$ no. of terms left after simplification $= \frac{n+1}{2}$

Ex: $(a+b)^3 + (a-b)^3$

$= a^3 + b^3 + 3a^2b + 3ab^2 + a^3 - b^3 - 3a^2b + 3ab^2$

$= 2(a^3 + 3ab^2)$

$\hookrightarrow$ 2 terms are left $= \frac{3+1}{2}$

In $(a+b)^n + (a-b)^n$

if $n \rightarrow$ even

no. of terms left after simplification $= \frac{n}{2}+1$

Ex: $(a+b)^{2} + (a-b)^{2}$

$= a^2+b^2+2ab+a^2+b^2-2ab$

$= 2(\underline{a^2+b^2})$

↳ 2 terms left $= \frac{2}{2}+1$

In $(a+b)^n - (a-b)^n$

$n \rightarrow$ even

no. of terms left after simplification $= \frac{n}{2}$

Ex: $(a+b)^{2} - (a-b)^{2}$

$= a^2+b^2+2ab-a^2-b^2+2ab$

$= \underline{4ab}$

↳ 1 term is left $= \frac{2}{2}$

QUESTION

The number of terms in the expansion of $(x^2 + y^2)^{25} - (x^2 - y^2)^{25}$ after simplification is

$n = 25$

(A) 0

(B) 26

(C) 13

(D) 50

terms left $= \frac{25+1}{2} = 13$

QUESTION

The total number of terms in the expansion of $(x+a)^{51}-(x-a)^{51}$ after simplification is

$n = 51 = \text{odd}$

terms left $\Rightarrow \frac{51+1}{2} = 26$

(A) 102

(B) 25

(C) 26

(D) 23

QUESTION

The total number of terms in the expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is

$n = 100$ & In b/w +ve sign

(A) 50

(B) 202

(C) 51

(D) None of these

no of terms $= \frac{100}{2} + 1$

$= 51$

QUESTION

The total number of terms in the expansion of $(x+a)^{100} - (x-a)^{100}$ after simplification is

Here $n = 100$ & In b/w -ve sign

(A) 50

No of terms $= \frac{100}{2}$

(B) 202

$= 50$

(C) 51

(D) None of these

⊛ Middle term:-

If given $(a+b)^n$

where $n \to$ even

Then middle term

is $\left(\frac{n}{2}+1\right)^{th}$ term.

Ex:-

In $(a+b)^{14}$

$n = 14$ is even

$\therefore$ middle term $= \frac{14}{2}+1 = 8^{th}$ term

$\Downarrow$

$r+1 = 8$

$r = 7$

Thank you