

ULTIMATE KCET

CRASH COURSE 2026

Physics

Lecture : 01

**System of particles and
Rotational motion**

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Recap

of previous lecture

1

WORK AND ITS TYPES

2

WORK-ENERGY THEOREM

3

KINETIC ENERGY AND LINEAR
MOMENTUM

4

POWER AND MCQ's+PYQ's

Topics

to be covered

- 1 UNIFORM CIRCULAR MOTION
- 2 VERTICAL CIRCULAR MOTION
- 3 COM & COLLISION
- 4 SYSTEM OF PARTICLES AND ROTATIONAL MOTION





Circular Motion

• When an object moves in a circular path with constant speed is called Uniform circular Motion

• Angular displacement

$$\theta = \frac{AB}{ON} = \frac{l}{r}$$

$$\theta = \frac{l}{r} = \frac{2\pi r}{r} = \theta = 2\pi \text{ rad}$$

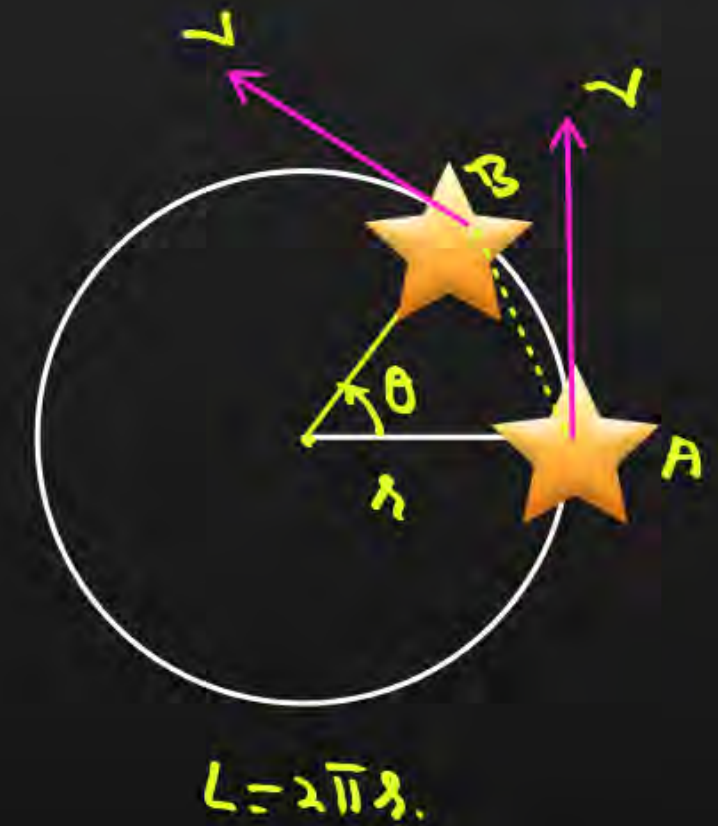
• Angular velocity

$$\omega = \frac{d\theta}{dt}$$

$$\omega = \frac{\theta}{t} = \frac{2\pi}{T}$$

$$\omega = \frac{2\pi}{T} \text{ rad/s}$$

$$v = \frac{dx}{dt}$$





Circular Motion

$$s, x \rightarrow \theta$$

$$v \rightarrow \omega$$

$$a \rightarrow \alpha$$

- Angular acceleration

$$\alpha = \frac{d\omega}{dt}$$

$$a = \frac{dv}{dt}$$

↳ rad/s^2

- Equation of motion

$$v = u + at \quad \longrightarrow \quad \omega = \omega_0 + \alpha t$$

$$s = ut + \frac{1}{2}at^2 \quad \longrightarrow \quad \theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$v^2 = u^2 + 2as \quad \longrightarrow \quad \omega^2 = \omega_0^2 + 2\alpha\theta$$



Circular Motion

- Time period

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

- Frequency

$$* f = \frac{1}{T}$$

$$\omega = 2\pi f$$



Circular Motion

Relation between Linear Velocity and Angular Velocity.

$$\vec{v} = \vec{\omega} \times \vec{r}, \quad \boxed{v = \omega r}$$

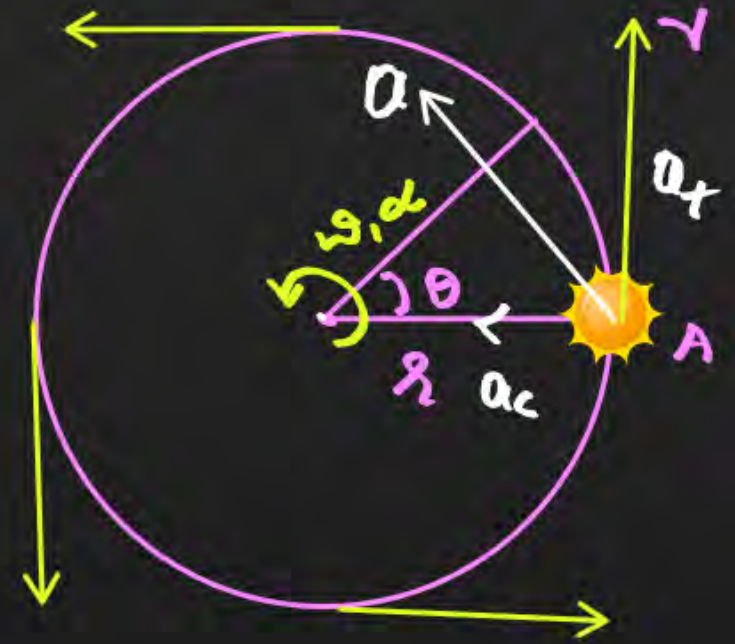
Relation between tangential acceleration and angular acceleration.

$$\vec{a}_t = \vec{\alpha} \times \vec{r}, \quad \boxed{a_t = \alpha r}$$

$$a = \sqrt{a_t^2 + a_c^2}$$

Centripetal Acceleration OR Radial Acceleration.

$$\boxed{a_c = \omega^2 r}$$



QUESTION



Two particles A and B are moving in uniform circular motion in concentric circles of radii r_A and r_B with speed v_A and v_B respectively. Their **time period of rotation is the same**. The ratio of **angular speed** of A to that of B will be:

- A $r_A : r_B$
- B $v_A : v_B$
- C $r_B : r_A$
- D **1 : 1**

$$\omega = \frac{2\pi}{T}$$

$$T_A = T_B = T$$

$$\frac{\omega_1}{\omega_2} = \frac{1}{1} = \text{constant}$$

QUESTION



A particle moves in a **circle** of radius 5 cm with constant speed and time period 0.2π s. The acceleration of the particle is

A 15 m/s^2

B 25 m/s^2

C 36 m/s^2

D 5 m/s^2

$$a_c = \omega^2 r$$

$$a_c = (10)^2 \times 5 \times 10^{-2}$$

$$a_c = 10^2 \times 5 \times 10^{-2}$$

$$a_c = 5 \text{ m/s}^2$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2\pi} = \frac{2}{0.2}$$

$$\omega = \frac{20}{2} = 10 \text{ rad/s}$$

QUESTION

A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is:

- A** 40 ms^{-2}
- B** $640\pi \text{ m}^{-2}$
- C** $160\pi \text{ ms}^{-2}$
- D** $40\pi \text{ ms}^{-2}$

$$v^2 = u^2 + 2as$$

$$v^2 = (0)^2 + 2a_t \times 2 \times 2\pi r$$

$$(80)^2 = 2 \times a_t \times 2 \times 2\pi \times \frac{20}{\pi}$$

$$6400 = 160 a_t$$

$$a_t = 40 \text{ ms}^{-2}$$

QUESTION



Two particles having mass ' M ' and ' m ' are moving in a circular path having radius R and r respectively. If their **time period are same** then the ratio of angular velocity will be:

A $\frac{r}{R}$

B $\frac{R}{r}$

C 1

D $\sqrt{\frac{R}{r}}$

$$\omega = \frac{2\pi}{T} = \text{constant}$$

$$\frac{\omega_1}{\omega_2} = \frac{1}{1}$$

QUESTION

A body is whirled in a horizontal circle of radius 20 cm . It has an angular velocity of 10 rad/s . What is its linear velocity at any point on its circular path?

A 20 m/s

B $\sqrt{2} \text{ m/s}$

C 10 m/s

D 2 m/s

$$v = \omega r$$

$$v = 10 \times 20 \times 10^{-2}$$

$$v = 200 \times 10^{-2}$$

$$v = 2 \text{ m/s}$$

QUESTION

The angular speed of a flywheel making 120 revolution/ minute is:

$$f = \frac{N}{t} = \frac{120}{60} = 2 \text{ Hz} = 2 \text{ rps}$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$\omega = 2\pi \times 2$$

$$\omega = 4\pi \text{ rad/s}$$

- A** $4\pi \text{ rad/s}$
- B** $\pi \text{ rad/s}$
- C** $4\pi^2 \text{ rad/s}$
- D** $2\pi \text{ rad/s}$

QUESTION



$$f = \frac{N}{T} = \frac{120}{60} = 2 \text{ rps}$$

An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rpm , the acceleration of a point on the tip of the blade is:

A 1600 ms^{-2}

B 47.4 ms^{-2}

C 23.7 ms^{-2}

D 50.55 ms^{-2}

$$a = \omega^2 r$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$a = (4\pi)^2 \times 30 \times 10^{-2}$$

$$\omega = 2\pi \times 2$$

$$a = 16\pi^2 \times 30 \times 10^{-2}$$

$$\omega = 4\pi \text{ rad/s}$$

$$a = 1732.60 \times 10^{-2}$$

$$a = 47.32 \text{ m/s}^2$$



Vertical Circular Motion

- **Motion in a vertical circle**, Tension at any position of angular displacement (θ) along a vertical circle is given by

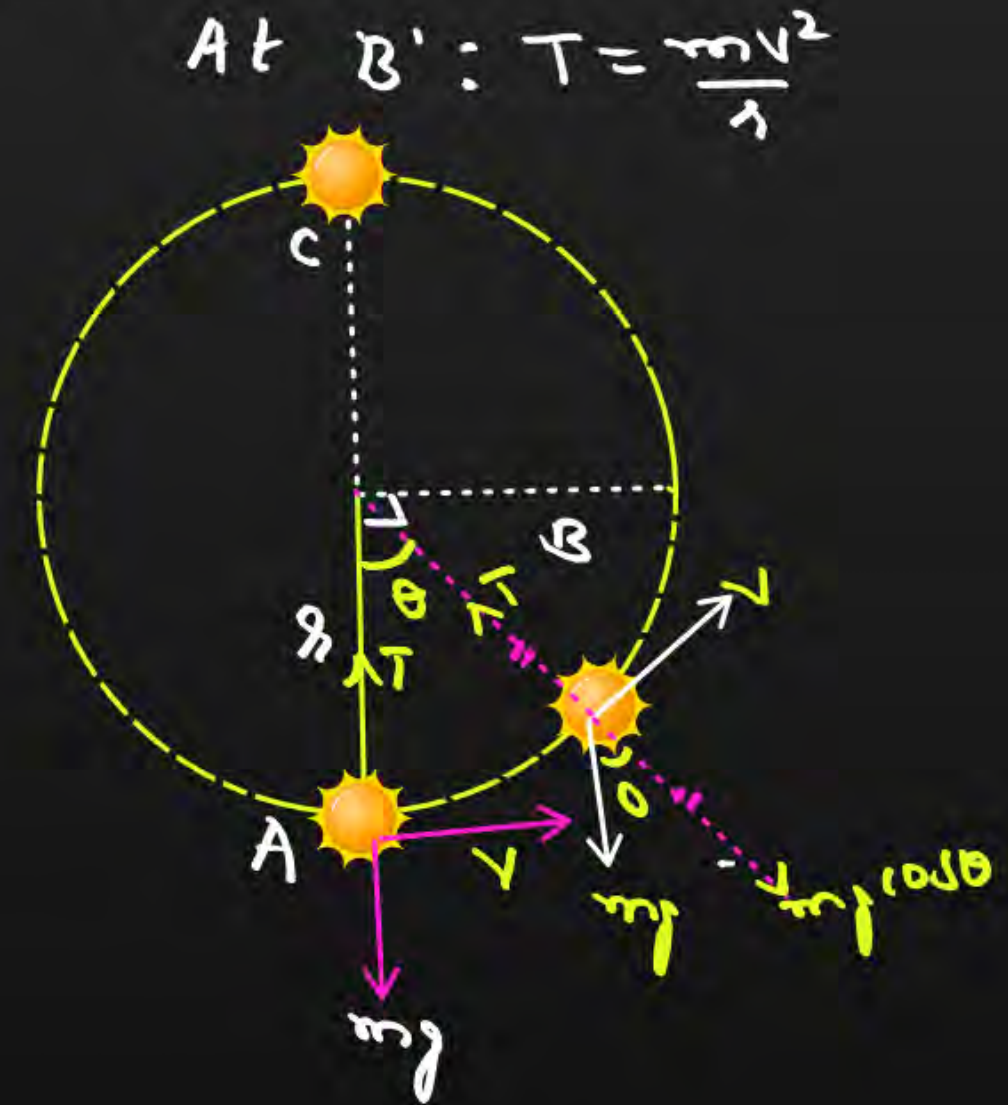
$$T = \frac{mv^2}{r} + mg \cos \theta \quad *$$

- Tension at the lowest point ($\theta = 0^\circ$) is given by, $\cos 0 = 1$

$$T = \frac{mv^2}{r} + mg$$

- Tension at the highest point ($\theta = 180^\circ$) is given by, $\cos 180 = -1$

$$T = \frac{mv^2}{r} - mg$$





Vertical Circular Motion

- Minimum speed at the highest point,

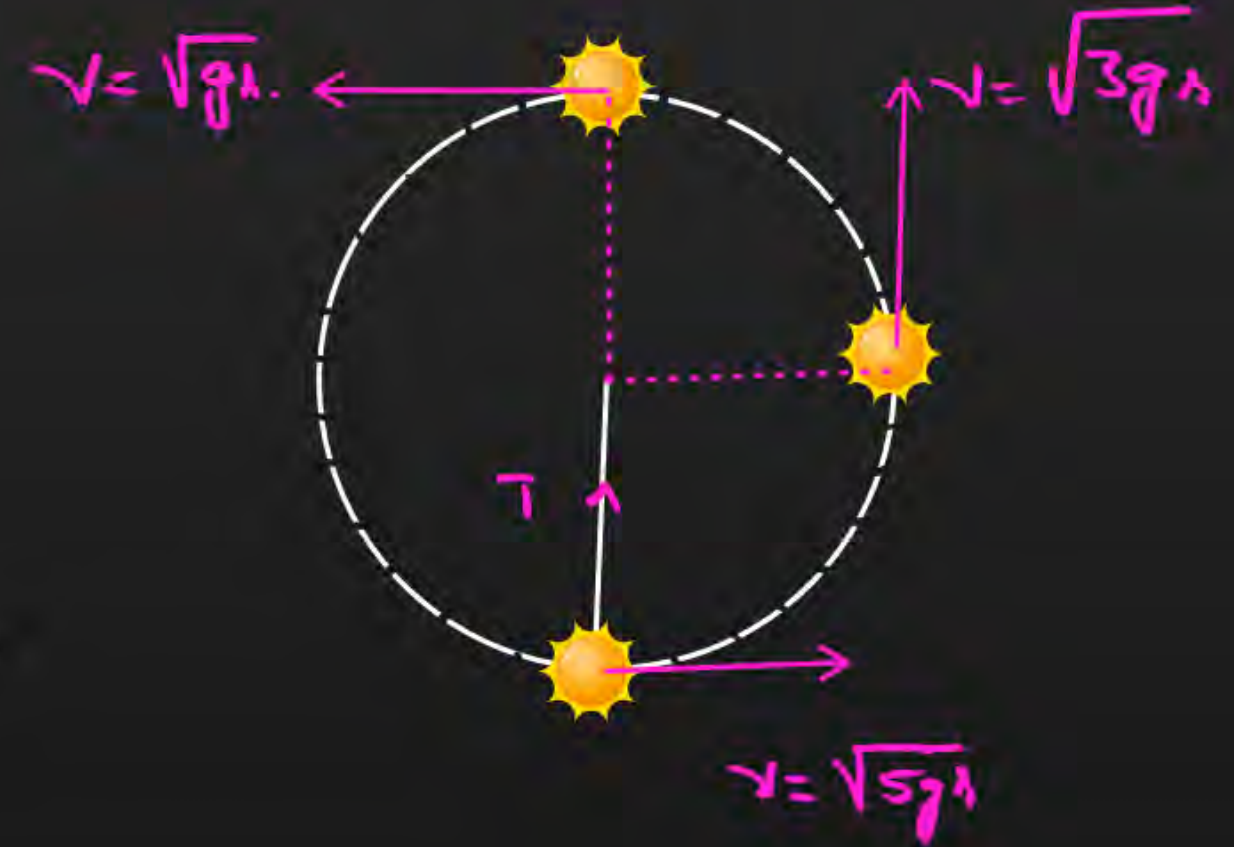
$$v = \sqrt{gR}$$

- Minimum speed at the lowest point for looping the loop,

$$v = \sqrt{5gR}$$

- When the string is horizontal, $\theta = 90^\circ$, minimum velocity,

$$v = \sqrt{3gR}$$



QUESTION



What is the minimum velocity with which a body of mass m must enter a vertical loop of radius R so that it can complete the loop?

A $\sqrt{2gR}$

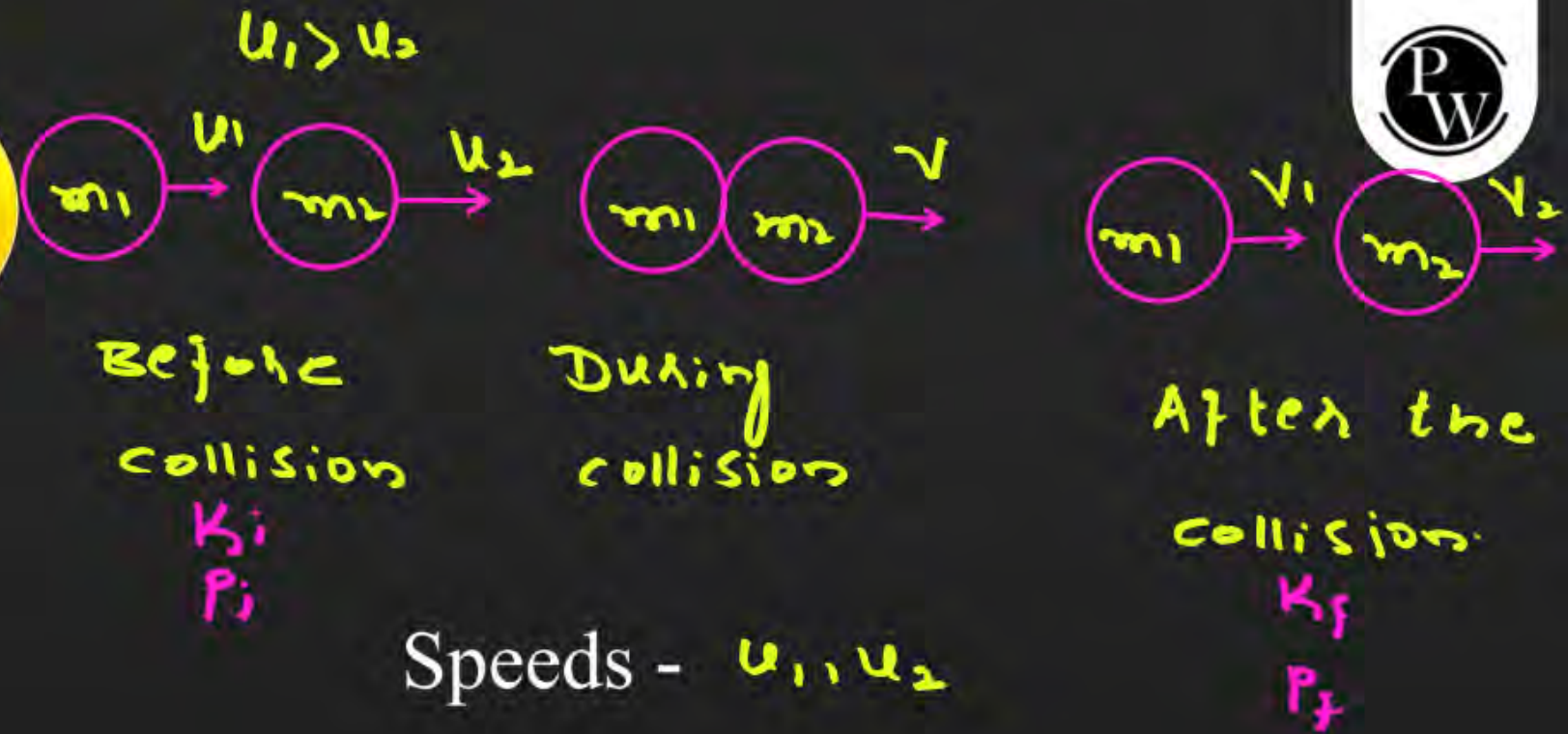
B $\sqrt{3gR}$

C $\sqrt{5gR}$

D \sqrt{gR}



Collision of bodies



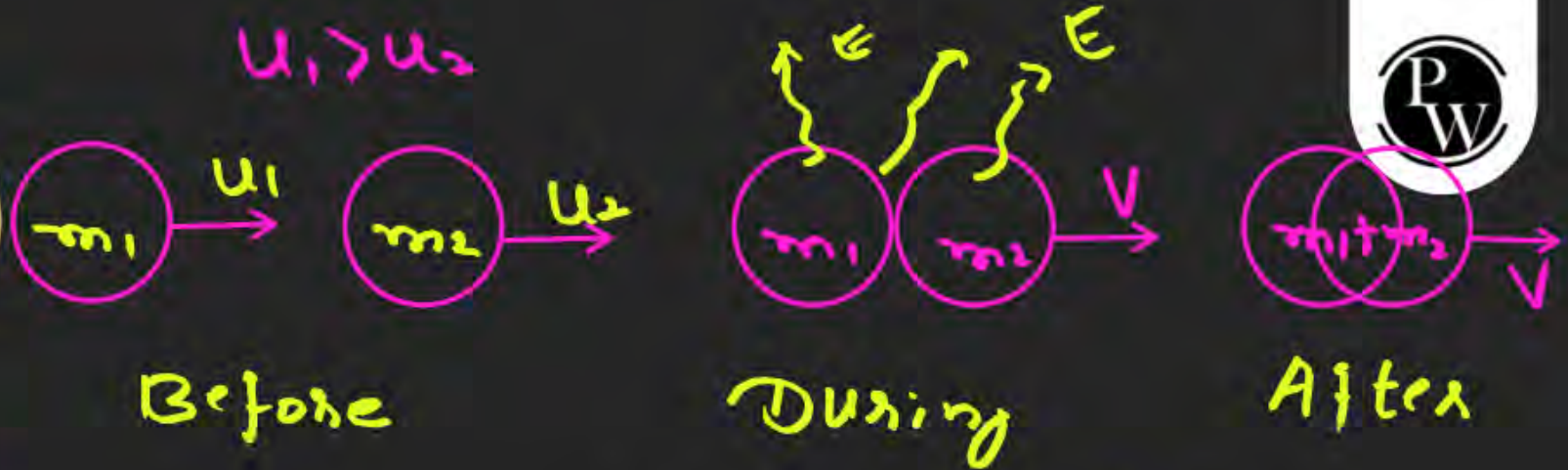
- Elastic collision in one dimension:
- Before collision: Masses - m_1, m_2 Speeds - u_1, u_2
- After collision: Masses - m_1, m_2 Speeds - v_1, v_2
- Velocities after collision are given by: Both Energy & momentum are conserved

$$* v_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2}$$

$$* v_2 = \frac{(m_2 - m_1)u_2 + 2m_1u_1}{m_1 + m_2}$$



Collision of bodies



- **Perfectly inelastic collision in one dimension:**

- Before collision: Masses - m_1 & m_2

Speeds - u_1 & u_2

- After Collision : Masses - m_1 & m_2

Speeds - $v_1 = v_2 = v$

- After collision they stick together moving with velocity, **Momentum is conserved**
Energy (K.E) is not conserved

$$P_i = P_f$$

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

$$* \quad v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$



Collision of bodies

- Kinetic energy lost in inelastic collision is, $K = \frac{1}{2}mv^2$

$$\ast \Delta K = \frac{1}{2} \left[\frac{m_1 m_2}{m_1 + m_2} \right] (u_1 - u_2)^2 (1 - e^2)$$



Collision of bodies

- Coefficient of restitution,

*

$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}} = \frac{v_2 - v_1}{u_1 - u_2}$$

For perfectly elastic collision, $e = 1$.

For perfectly inelastic collision, $e = 0$.

QUESTION



Two bodies A and B of the **same mass** undergo completely **inelastic** one-dimensional collision. The body A moves with velocity v_1 while the body B is at **rest** before collision. The velocity of the system after collision is v_2 . The ratio of $v_1 : v_2$ is: $u_2 = 0$

A

2 : 1

B

4 : 1

C

1 : 4

D

1 : 2

$$v = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

$$v_2 = \frac{m v_1 + m(0)}{m + m} = \frac{m v_1}{2m} = \frac{v_1}{2}$$

$$\frac{v_1}{v_2} = 2$$

QUESTION

When masses = same
velocity = Exchange

$$v_1 = -0.3 \text{ m/s}$$

$$v_2 = 0.5 \text{ m/s}$$

u_1

u_2

Two identical balls A and B having velocities of 0.5 m/s and -0.3 m/s respectively collide elastically in one dimension. The velocities of B and A after the collision respectively will be:

A -0.5 m/s and 0.3 m/s

B 0.5 m/s and -0.3 m/s

C -0.3 m/s and 0.5 m/s

D 0.3 m/s and 0.5 m/s

Body - A

$$v_1 = \frac{(m_1 - m_2)u_1 + 2m_2u_2}{m_1 + m_2}$$

Body - B

$$v_2 = \frac{(m_2 - m_1)u_2 + 2m_1u_1}{m_1 + m_2}$$

$$v_1 = \frac{(m - m)0.5 + 2m(-0.3)}{m + m}$$

$$v_2 = \frac{0(+0.3) + 2m \times 0.5}{2m}$$

$$v_1 = -\frac{2m \times 0.3}{2m}$$

$$v_2 = \frac{2m \times 0.5}{2m}$$

$$v_1 = -0.3 \text{ m/s}$$

$$v_2 = 0.5 \text{ m/s}$$



QUESTION



Two equal masses m , and m , moving along the same straight line with velocities $+3$ m/s and -5 m/s respectively collide elastically. Their velocities after the collision will be respectively :

- A** $+4$ m/s for both
- B** -3 m/s and $+5$ m/s
- C** -4 m/s and $+4$ m/s
- D** -5 m/s and $+3$ m/s

When masses = same * VEP
Velocities = Exchange

QUESTION



Two perfectly elastic particles P and Q of equal mass travelling along the line joining them with velocities 15 m/sec and 10 m/sec. After collision, their velocities respectively (in m/sec) will be

A 0, 25

B 5, 20

C 10, 15

D 20, 5

QUESTION



A ball strikes the floor and after collision rebounds back. In this state –

- A** Momentum of the ball is conserved ✗
- B** Mechanical energy of the ball is conserved ✗
- C** Momentum of ball earth system is conserved
- D** The kinetic energy of ball earth system is conserved

QUESTION



In an **inelastic collision** between two bodies, the physical quantity that is conserved :

- A** Kinetic energy
- B** Momentum
- C** Potential energy
- D** Kinetic energy and momentum

QUESTION



A collision is said to be perfectly inelastic when :

- A** Coefficient of restitution = 0
- B** Coefficient of restitution = 1
- C** Coefficient of restitution = ∞
- D** Coefficient of restitution < 1

QUESTION



Which of the following is **true** :

- A** Momentum is conserved in all collisions but kinetic energy is conserved only in inelastic collision
- B** Neither momentum nor kinetic energy is conserved in inelastic collisions.
- C** Momentum is conserved in all collisions but not kinetic energy
- D** Both momentum and kinetic energy are conserved in all collisions

QUESTION



A metal ball of mass 2 kg moving with a velocity of 36 km/h has a head on collision with a stationary ball of mass 3 kg. If after the collision, the two balls move together, the loss in kinetic energy due to collision is :

- A 140 J
- B 100 J
- C 60 J
- D 40 J

$$\begin{aligned}
 u_1 &= 36 \text{ km/h} \\
 &= \frac{36 \text{ km}}{1 \text{ h}} \\
 &= \frac{36 \times 1000 \text{ m}}{3600 \text{ s}}
 \end{aligned}$$

$$u_1 = 10 \text{ m/s}$$

$$u_1 = 36 \times \frac{5}{18} = 2 \times 5 = 10 \text{ m/s}$$

$$\Delta K = \frac{1}{2} \left(\frac{m_1 m_2}{m_1 + m_2} \right) (u_1 - u_2)^2 (1 - e^2)$$

$$\Delta K = \frac{1}{2} \left(\frac{2 \times 3}{2 + 3} \right) (10 - 0)^2 (1 - 0)$$

$$\Delta K = \frac{1}{2} \times \frac{3}{5} \times 100$$

$$\Delta K = 3 \times 20$$

$$\Delta K = 60 \text{ J}$$



Centre of mass of discrete particle system

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

The coordinates of centre of mass are given by

$$\vec{r}_{cm} = \frac{\sum_{i=1}^n m_i \vec{r}_i}{\sum_{i=1}^n m_i = M}$$

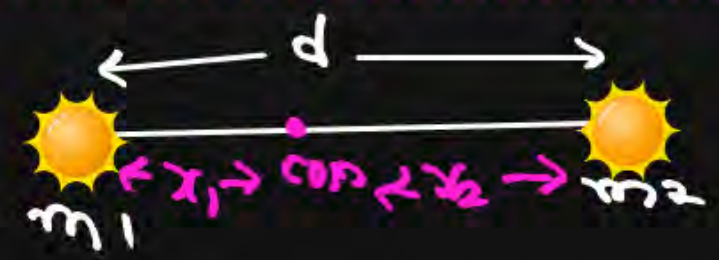
$$\vec{r}_{cm} = \frac{1}{M} \sum_{i=1}^n m_i \vec{r}_i$$

$$x_{cm} = \frac{1}{M} \sum_{i=1}^n m_i x_i, \quad x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \dots}{M}$$

$$y_{cm} = \frac{1}{M} \sum_{i=1}^n m_i y_i, \quad y_{cm} = \frac{m_1 y_1 + m_2 y_2 + \dots}{M}$$

$$z_{cm} = \frac{1}{M} \sum_{i=1}^n m_i z_i, \quad z_{cm} = \frac{m_1 z_1 + m_2 z_2 + \dots}{M}$$

The distance of the COM from the mass -



$$r_1 = \left(\frac{m_2}{m_1 + m_2} \right) d$$

$$r_2 = \left(\frac{m_1}{m_1 + m_2} \right) d$$



Centre of mass of discrete particle system



For a continuous distribution of mass, the coordinates of centre of mass are given by

$$x_{cm} = \frac{1}{M} \sum m_i x_i$$

$$y_{cm} = \frac{1}{M} \sum m_i y_i$$

$$z_{cm} = \frac{1}{M} \sum m_i z_i$$

$$x_{cm} = \frac{1}{M} \int x \cdot dm$$

$$y_{cm} = \frac{1}{M} \int y \cdot dm$$

$$z_{cm} = \frac{1}{M} \int z \cdot dm$$



Centre of mass of discrete particle system

Velocity of centre of mass is given by

$$V_{cm} = \frac{1}{M} \sum_{i=1}^n m_i v_i$$

Acceleration of centre of mass is given by

$$a_{cm} = \frac{1}{M} \sum_{i=1}^n m_i a_i$$

$$x_{cm} = \frac{1}{M} \sum_{i=1}^n m_i x_i$$

$$V = \frac{dx_{cm}}{dt} = \frac{1}{M} \sum_{i=1}^n m_i \frac{dx_i}{dt}$$



Rotational Motion

Torque

*

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = r F \sin \theta$$

Moment of inertia:

$$I = \sum_{i=1}^n m_i r_i^2$$

↳ particle

Theorem of perpendicular axes: X

Theorem of parallel axes: X

Linear

Rotation

m	→	I
s	→	θ
v	→	ω
a	→	α
F	→	τ
P	→	L
K_T	→	K_R



Rotational Motion

$$F = m$$

$$F = ma$$

Relation between torque and moment of inertia

$$\tau = I \alpha$$

Angular velocity, ω

$$v = \frac{dx}{dt}$$

$$\omega = \frac{d\theta}{dt}$$

Angular acceleration

$$a = \frac{dv}{dt}$$

$$\alpha = \frac{d\omega}{dt}$$

Equations of rotational motion

Already done



Rotational Motion

Angular momentum

$$L = I\omega$$

$$\vec{L} = \vec{r} \times \vec{p}$$

$$L = r p \sin\theta$$

$$p = mv$$

↳ Rigid body

↳ particles

Relationship between torque and angular momentum

τ
 F

L
 p

$$F = \frac{dp}{dt}$$

$$\tau = \frac{dL}{dt}$$

Relation between angular momentum and moment of inertia

L
 p

I
 ω

$$p = mv$$

$$L = I\omega$$

Kinetic energy of rotational motion,

$$K_T = \frac{1}{2} m v^2$$

$$v = \omega r$$

$$K = \frac{1}{2} m (\omega r)^2$$

$$K_R = \frac{1}{2} m \omega^2 r^2$$



Rotational Motion

Kinetic energy of a rolling body = Translational kinetic energy (TK) + rotational kinetic energy (RK)

$$K.E = K_T + K_R = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2 R^2$$

*

$$K.E = \frac{1}{2}mv^2 \left[1 + \frac{K^2}{R^2} \right]$$

Where K - Radius of Gyration

$$I = mK^2$$

$$K = \frac{1}{2}mv^2$$

$$K = \frac{1}{2}I\omega^2$$

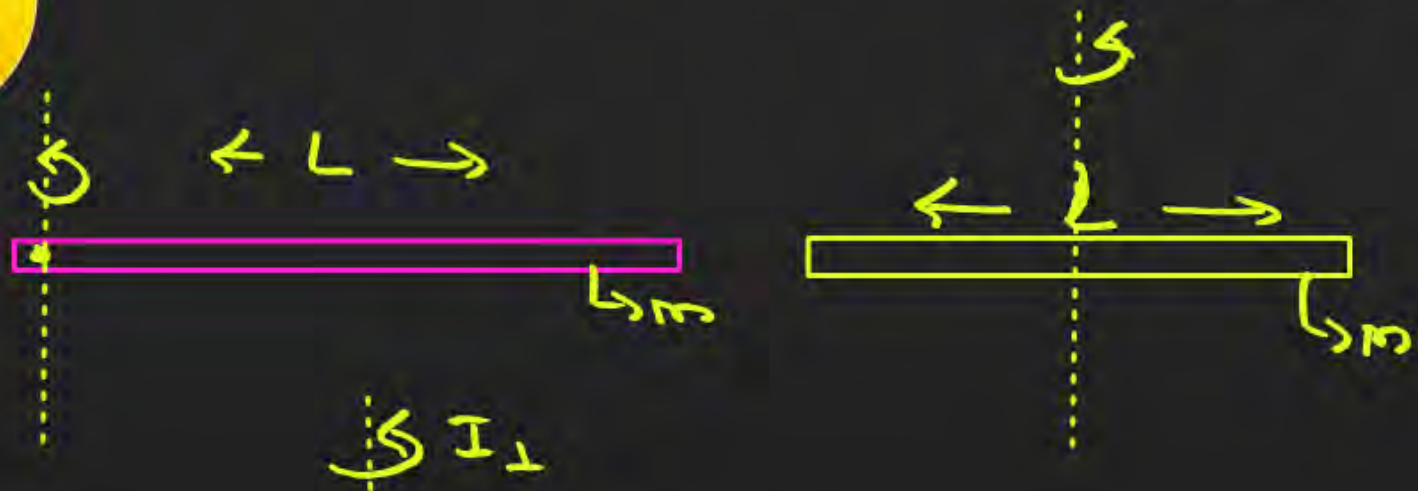


Moment of inertia of bodies

1. Rod :

$$I_{\text{end}} = \frac{ML^2}{2}$$

$$I_{\text{mid}} = \frac{ML^2}{12}$$



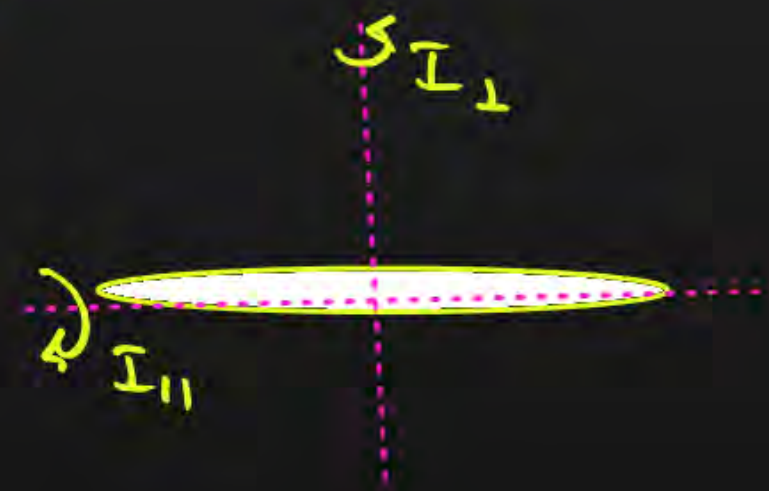
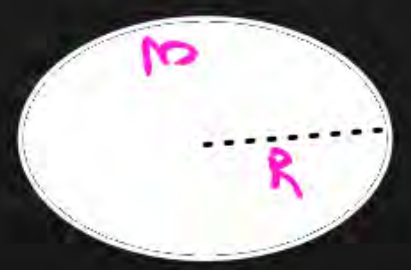
2. Ring and Hollow Cylinder

$$I_{\perp} = MR^2 \quad I_{\parallel} = \frac{MR^2}{2}$$



3. Disc and Solid Cylinder

$$I_{\perp} = \frac{MR^2}{2} \quad I_{\parallel} = \frac{MR^2}{4}$$



4. Solid Sphere

$$I = \frac{2}{5} MR^2$$

5. Hollow Sphere

$$I = \frac{2}{3} MR^2$$



QUESTION

The centre of mass of a body

- A** Depends on the choice of co-ordinate system
- B** is independent of the choice of co-ordinate system
- C** May or may not depend on the choice of co-ordinate system
- D** None of the above

$$r_{cm} = \frac{1}{M} \sum_{i=1}^n m_i r_i$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

QUESTION



The centre of mass of a system of particles **does not depend** on:

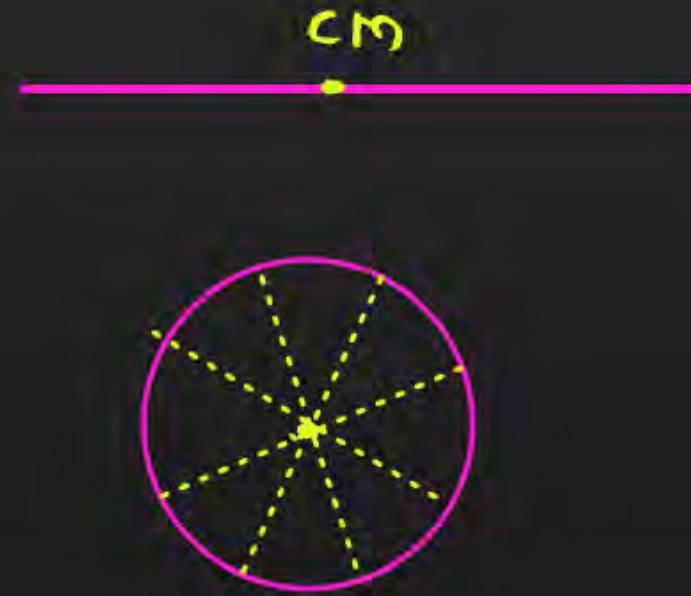
- A** Masses of the particles ✓
- B** Forces acting on the particles ✓
- C** Position of particles ✓
- D** Relative distances between the particles ✓

$$x_{cm} = \frac{1}{M} \sum_{i=1}^n m_i x_i$$

QUESTION

The centre of mass of a body:

- A** Lies always at the geometrical centre
- B** Lies always inside the body
- C** Lies always outside the body
- D** May lie within or outside the body



QUESTION



Three identical spheres, each of mass M , are placed at the corners of a right angle triangle with the mutually perpendicular sides equal to $2m$ (see figure). Taking the point of intersection of the two mutually perpendicular sides as the origin, find the position vector of centre of mass.

A $2(\hat{i} + \hat{j})$

B $(\hat{i} + \hat{j})$

C $\frac{2}{3}(\hat{i} + \hat{j})$

D $\frac{4}{3}(\hat{i} + \hat{j})$

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j}$$

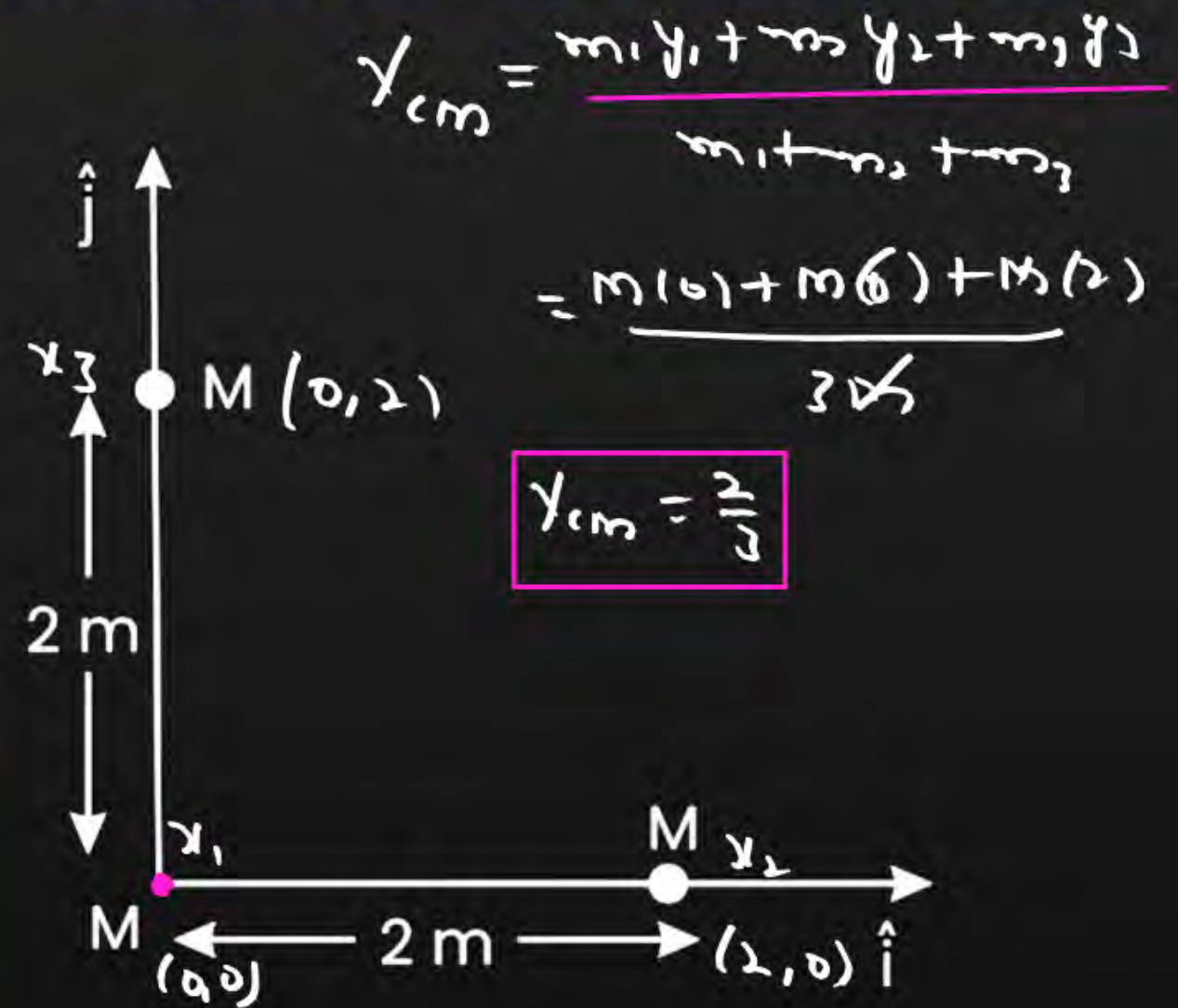
$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$x_{cm} = \frac{m(0) + M(2) + m(0)}{3M}$$

$$x_{cm} = \frac{2}{3}$$

$$\vec{r} = \frac{2}{3}\hat{i} + \frac{2}{3}\hat{j}$$

$$\vec{r} = \frac{2}{3}(\hat{i} + \hat{j})$$



$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{m(0) + m(0) + M(2)}{3M}$$

$$y_{cm} = \frac{2}{3}$$

QUESTION



Two bodies of masses 1 kg and 3 kg have position vectors $(\hat{i} + 2\hat{j} + \hat{k})$ and $(-3\hat{i} - 2\hat{j} + \hat{k})$, respectively. The centre of mass of this system has a position vector

A $-2\hat{i} + 2\hat{k}$

B $-2\hat{i} - \hat{j} + \hat{k}$

C $2\hat{i} - \hat{j} + \hat{k}$

D $-\hat{i} + \hat{j} + \hat{k}$

$$\vec{r}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

$$\vec{r}_{cm} = -2\hat{i} - \hat{j} + \hat{k}$$

$$\vec{r}_{cm} = \frac{1(\hat{i} + 2\hat{j} + \hat{k}) + 3(-3\hat{i} - 2\hat{j} + \hat{k})}{1+3}$$

$$\vec{r}_{cm} = \frac{\hat{i} + 2\hat{j} + \hat{k} - 9\hat{i} - 6\hat{j} + 3\hat{k}}{4}$$

$$= \frac{-8\hat{i} - 4\hat{j} + 4\hat{k}}{4}$$

QUESTION



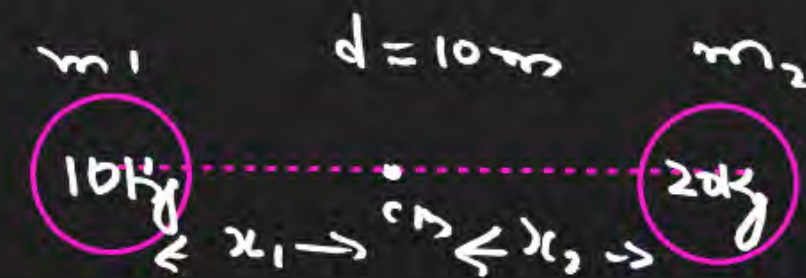
Two objects of mass 10 kg and 20 kg respectively are connected to the two ends of a rigid rod of length 10 m with negligible mass. The distance of the centre of mass of the system from the 10 kg mass is

A 10/3 m

B 20/3 m

C 10 m

D 5 m



$$x_1 = \left(\frac{m_2}{m_1 + m_2} \right) d$$

$$x_1 = \left(\frac{20}{10 + 20} \right) \times 10 = \frac{20}{30} \times 10$$

$$x_1 = \frac{20}{3} \text{ m}$$

QUESTION



11.2

Two particles of mass 5 kg and 10 kg respectively are attached to the two ends of a rigid rod of length 1 m with negligible mass. The centre of mass of the system from the 5 kg particle is nearly at a distance of

- A** 50 cm
- B** 67 cm
- C** 80 cm
- D** 33 cm

QUESTION



Figure shows a composite system of two uniform rod of mass M and $2M$ & lengths as indicated. Then the coordinates of the centre of mass of the system of rods are

A $\left(\frac{L}{2}, \frac{2L}{3}\right)$

B $\left(\frac{L}{4}, \frac{2L}{3}\right)$

C $\left(\frac{L}{6}, \frac{2L}{3}\right)$

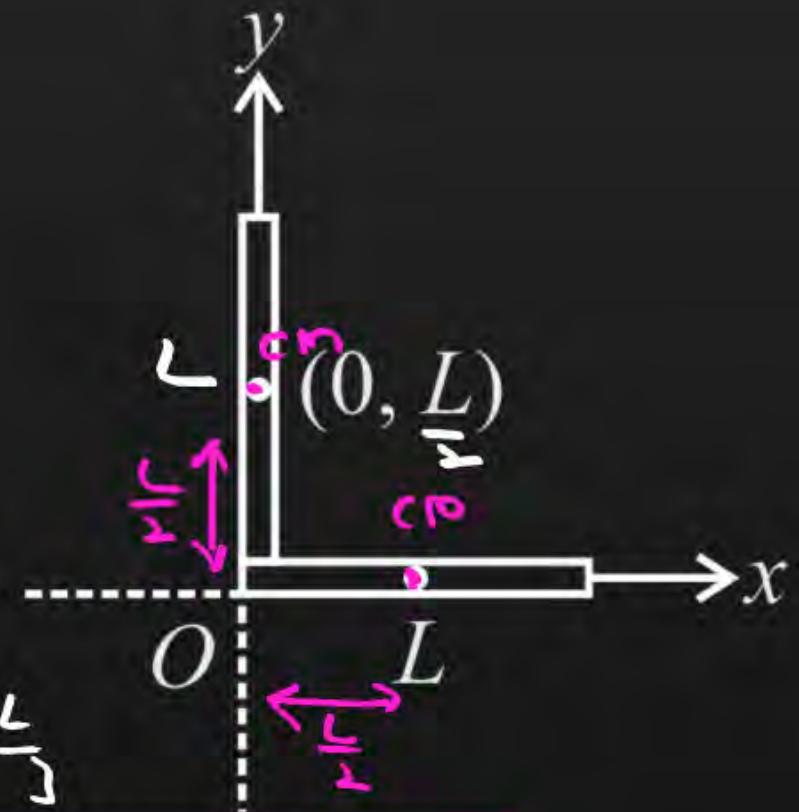
D $\left(\frac{L}{6}, \frac{L}{3}\right)$

$m_1 = m$ $m_2 = 2m$
 $x_1 = \frac{L}{2}$ $y_1 = 0$
 $x_2 = 0$ $y_2 = \frac{L}{2}$
 $x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$ $y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$
 $x_{cm} = \frac{m \frac{L}{2} + 2m(0)}{m + 2m}$ $x_{cm} = \frac{m(0) + 2m \frac{L}{2}}{3m} = \frac{L}{3}$

$x_{cm} = \frac{mL}{2 \times 3m} = \frac{L}{6}$

$y_{cm} = \frac{L}{3}$

$(x_{cm}, y_{cm}) = \left(\frac{L}{6}, \frac{L}{3}\right)$



QUESTION



Two particles which are initially at rest, move towards each other under the action of their internal attraction. If their speeds are v and $2v$ at any instant, then the speed of the centre of mass of the system will be:-

- A** v
- B** $2v$
- C** zero
- D** $1.5v$

QUESTION

If the system is released, then the acceleration of the centre of mass of the system:-

- A** g/4
- B** g/2
- C** g
- D** 2g

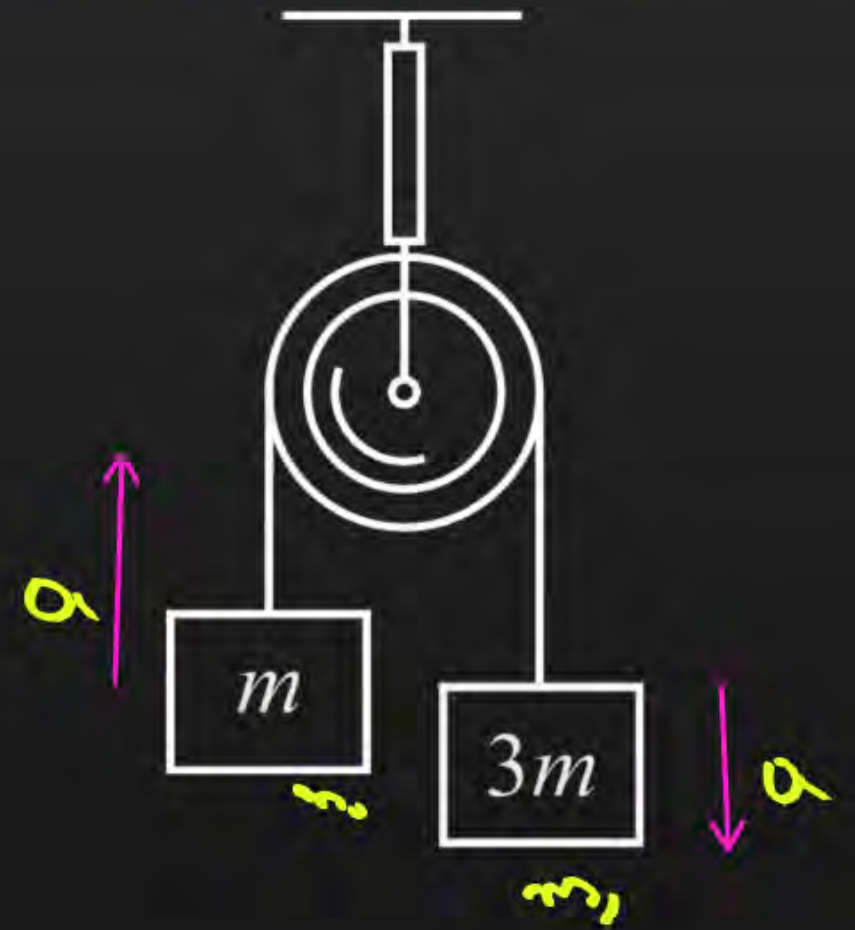
Acceleration of the system.

$$a = \frac{(3m - m)}{3m + m} \times g = \frac{2m}{4m} \times g$$

$a = \frac{g}{2}$

$$a_{cm} = \frac{m_1 a_1 + m_2 a_2}{m_1 + m_2} = \frac{m \times \frac{g}{2} + 3m \times g}{m + 3m}$$

$$a_{cm} = \frac{m \times \frac{g}{2} + 3m \times g}{4m} = \frac{\frac{m \times g}{2} + 3m \times g}{4m} = \frac{7mg}{8m} = \frac{7g}{8}$$
 $\frac{7g}{8}$



QUESTION



A 2 kg body and a 3 kg body are moving along the x -axis. At a particular instant the 2 kg body has a velocity of 3m/s and the 3 kg body has the velocity of 2m/s. The velocity of the centre of mass at that instant is :-

A 5 m/s

B 1 m/s

C 0

D $\frac{12}{5}$ m/s

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$
$$= \frac{(2 \times 3) + (3 \times 2)}{2 + 3} = \frac{6 + 6}{5} = \frac{12}{5}$$

$$v_{cm} = \frac{12}{5}$$

QUESTION



$$0.2 \hat{k} \hat{j}$$

Two objects of masses 200 gram and 500 gram possess velocities $10\hat{i}$ m/s and $3\hat{i} + 5\hat{j}$ m/s respectively. The velocity of their centre of mass in m/s is:

A $5\hat{i} - 25\hat{j}$

B $\frac{5}{7}\hat{i} - 25\hat{j}$

C $5\hat{i} + \frac{25}{7}\hat{j}$

D $25\hat{i} - \frac{5}{7}\hat{j}$

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = \frac{0.2(10\hat{i}) + 0.5(3\hat{i} + 5\hat{j})}{0.2 + 0.5}$$

$$\vec{v}_{cm} = \frac{2\hat{i} + 1.5\hat{i} + 2.5\hat{j}}{0.7} = \frac{3.5\hat{i} + 2.5\hat{j}}{0.7}$$

$$\vec{v}_{cm} = \frac{3.5}{0.7}\hat{i} + \frac{2.5}{0.7}\hat{j} = 5\hat{i} + \frac{25}{7}\hat{j}$$

$$\vec{v}_{cm} = 5\hat{i} + \frac{25}{7}\hat{j}$$

QUESTION



A bullet of mass m is fired from a rifle of mass M . if \vec{v} be the velocity of the bullet, velocity acquired by the rifle is:

A $\vec{V} = -\frac{M}{m}\vec{v}$

B $\vec{V} = -\frac{m}{M}\vec{v}$

C $\vec{V} = -\vec{v}$

D $\vec{V} = +\vec{v}$

$$P_i = P_f$$

$$0 = m\vec{v} + M\vec{V}$$

$$M\vec{V} = -m\vec{v}$$

$$\vec{V} = -\frac{m}{M}\vec{v}$$

QUESTION



A bullet weighing 50 gm leaves the gun with a velocity of 30 m/s. if the recoil speed imparted to the gun is 1 m/s, the mass of the gun is:

A 15 kg

B 30 kg

C 1.5 kg

D 20 kg

$$P_i = P_f$$

$$0 = mv + MV$$

$$0 = (50 \times 10^{-3} \times 30) + m(-1)$$

$$m = 1500 \times 10^{-3}$$

$$m = 1.5 \text{ kg}$$

QUESTION



A shell explodes and many pieces fly off in different direction. Which of the following is conserved?

- A** Kinetic energy
- B** Momentum
- C** Neither momentum nor KE
- D** Momentum and KE

$$F_{ext} = \frac{dp}{dt} \quad p = \text{const}$$

QUESTION



A bomb initially at rest explodes by itself into three equal mass fragments. The velocities of two fragments are $(3\hat{i} + 2\hat{j})$ m/s and $(-\hat{i} - 4\hat{j})$ m/s. The velocity of the third fragment is (in m/s)-

$v_i = 0$

$$P_i = P_f$$

$$0 = P_f$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 = 0$$

$$\cancel{m}(3\hat{i} + 2\hat{j}) + \cancel{m}(-\hat{i} - 4\hat{j}) + \cancel{m}\vec{v}_3 = 0$$

$$3\hat{i} + 2\hat{j} - \hat{i} - 4\hat{j} + \vec{v}_3 = 0$$

$$2\hat{i} - 2\hat{j} + \vec{v}_3 = 0$$

$$\vec{v}_3 = -2\hat{i} + 2\hat{j}$$

A $2\hat{i} + 2\hat{j}$

B $2\hat{i} - 2\hat{j}$

C $-2\hat{i} + 2\hat{j}$

D $-2\hat{i} - 2\hat{j}$

QUESTION



The moment of inertia of a body about a given axis of rotation depends upon:-

- A** The distribution of mass ✓
- B** Distance of particle of body from the axis of rotation ✓
- C** Shape of the body ✓
- D** All of the above

QUESTION



The **moment of inertia** in rotational motion is **equivalent** to:

- A** Angular velocity of linear motion
- B** **Mass of linear motion**
- C** Frequency of linear motion
- D** Current

QUESTION



Three-point masses (each of mass m) are arranged in the X-Y plane the moment of inertia of this array of masses about Y-axis is

- A** ma^2
- B** $2ma^2$
- C** $4ma^2$
- D** $6ma^2$

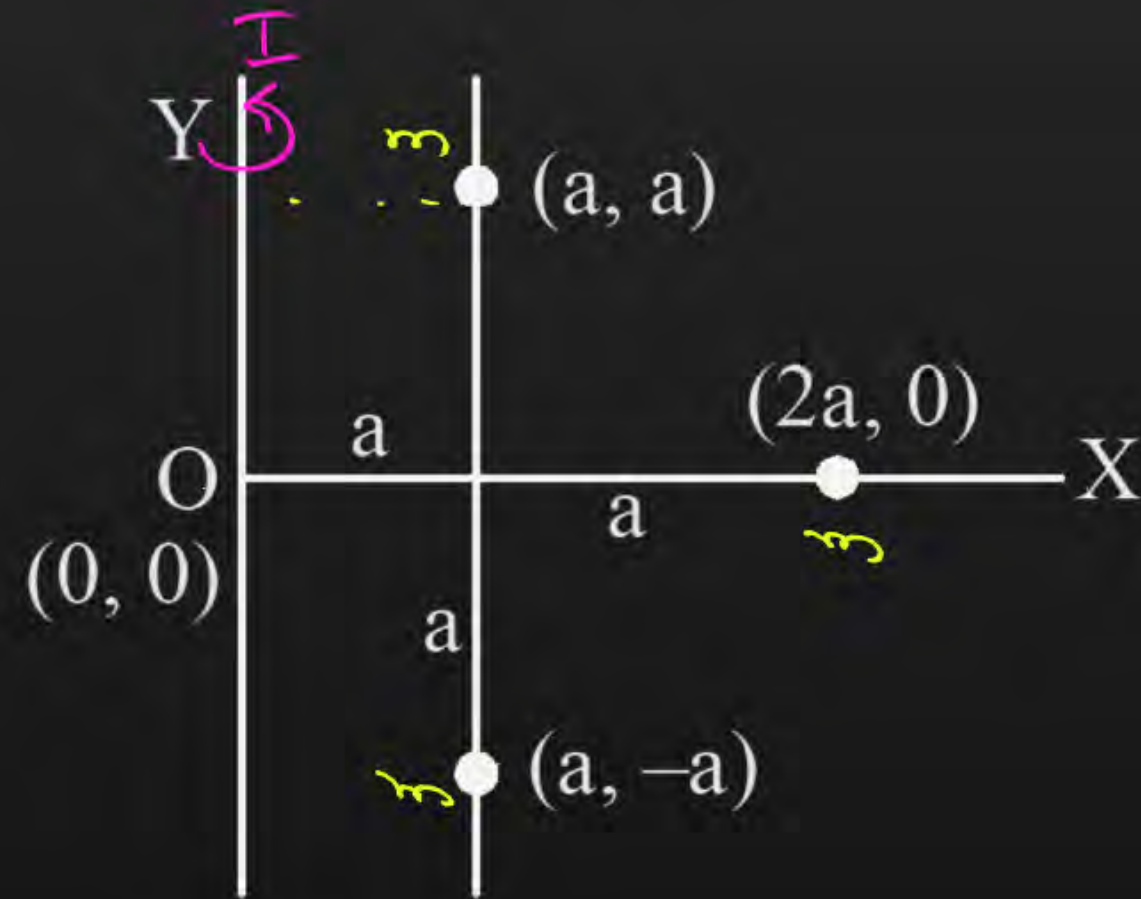
$$I = ma^2$$

$$I = I_1 + I_2 + I_3$$

$$I = ma^2 + m(2a)^2 + ma^2$$

$$I = ma^2 + 4ma^2 + ma^2$$

$$I = 6ma^2$$



QUESTION



The ratio of radius of gyration of a solid sphere of mass M and radius R about its own axis to the radius of gyration of the thin hollow sphere of same mass and radius about its axis is:

A

$$\sqrt{\frac{5}{2}}$$

B

$$\sqrt{\frac{3}{5}}$$

C

$$\sqrt{\frac{5}{3}}$$

D

$$\sqrt{\frac{2}{5}}$$

$$I_{SS} = \frac{2}{5} MR^2$$

$$I_{HS} = \frac{2}{3} MR^2$$

$$\frac{K_1}{K_2} = \sqrt{\frac{2}{5}}$$

$$I = MK^2$$

$$\frac{I_{SS}}{I_{HS}} = \frac{M K_1^2}{M K_2^2}$$

$$\frac{\frac{2}{5} MR^2}{\frac{2}{3} MR^2} = \left(\frac{K_1}{K_2}\right)^2$$

$$\frac{2}{5} \times \frac{3}{2} = \left(\frac{K_1}{K_2}\right)^2$$

QUESTION

The ratio of the radius of gyration of a thin uniform disc about an axis passing through its centre and normal to its plane to the radius of gyration of the disc about its diameter is

$$I_{\perp} = \frac{mR^2}{2}$$

$$I_{\parallel} = \frac{mR^2}{4}$$

A 2 : 1

B $\sqrt{2}$: 1

C 4 : 1

D 1 : $\sqrt{2}$

QUESTION



Find the torque of a force $F = -3\hat{i} + \hat{j} + 5\hat{k}$ acting at the point $\mathbf{r} = 7\hat{i} + 3\hat{j} + \hat{k}$

- A $-21\hat{i} + 3\hat{j} + 5\hat{k}$
- B $-14\hat{i} + 3\hat{j} - 16\hat{k}$
- C $4\hat{i} + 4\hat{j} + 6\hat{k}$
- D $14\hat{i} - 38\hat{j} + 16\hat{k}$

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 3 & 1 \\ -3 & 1 & 5 \end{vmatrix}$$

$$\vec{\tau} = \hat{i}(15-1) - \hat{j}(35+3) + \hat{k}(7+9)$$

$$\vec{\tau} = 14\hat{i} - 38\hat{j} + 16\hat{k}$$

QUESTION



H.W

Find the torque about the origin when a force of $F = 3\hat{j}\text{N}$ acting at on the particle whose position vector $\mathbf{r} = 2\hat{k}\text{ m}$

A $6\hat{j}\text{ Nm}$

B $-6\hat{i}\text{ Nm}$

C $6\hat{k}\text{ Nm}$

D $6\hat{i}\text{ Nm}$

QUESTION



A rope is wound around a hollow cylinder of mass 3kg and radius 40cm. what is the angular acceleration of the **cylinder**, if the rope is pulled with a force of 30 N?

- A** 25 m/s²
- B** 0.25 rad/s²
- C** 25 rad/s²
- D** 5 m/s²

$$I = MR^2 = 3 \times (40 \times 10^{-2})^2$$

$$I = 3 \times 1600 \times 10^{-4}$$

$$I = 0.48$$

$$\tau = rF$$

$$\tau = 40 \times 10^{-2} \times 30$$

$$\tau = 12 \text{ N-m}$$

$$F = ma$$

$$\tau = I\alpha$$

$$\alpha = \frac{\tau}{I}$$

$$\alpha = \frac{12}{0.48} = 25$$

QUESTION



The moment of inertia of a disc of radius 0.5 m about its geometric axis is $2\text{kg}\cdot\text{m}^2$. If a string is tied to its circumference and a force of 10 Newton is applied, if the disc executes rotatory motion, its angular acceleration will be: [H.W]

- A** 2.5 rad/sec^2
- B** 5 rad/sec^2
- C** 10 rad/sec^2
- D** 20 rad/sec^2

QUESTION

A flywheel rotating about a fixed axis has a kinetic energy of 360 J when its angular speed is 30 rad/s. The moment of inertia of the wheel about the axis of rotation is

- A** 0.6 kg-m²
- B** 0.15 kg-m²
- C** 0.8 kg-m²
- D** 0.75 kg-m²

K

↪

I

$$K = \frac{1}{2} I \omega^2$$

$$360 = \frac{1}{2} \times I \times (30)^2$$

$$\frac{360 \times 2}{900} = I$$

$$I = 0.8$$

QUESTION



A ring of mass m and radius r rotates about an axis passing through its centre and perpendicular to its plane with angular velocity ω . Its kinetic energy is

A

$$\frac{1}{2}mr^2\omega^2$$

B

$$mr\omega^2$$

C

$$mr^2\omega^2$$

D

$$\frac{1}{3}mr^2\omega^2$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(\omega r)^2$$

$$K = \frac{1}{2}m\omega^2 r^2$$

QUESTION



Two bodies have their moments of inertia I and $2I$ respectively about their axis of rotation. If their kinetic energies of rotation are equal, their angular momenta will be in the ratio

A $1 : 2$

B $\sqrt{2} : 1$

C $2 : 1$

D $1 : \sqrt{2}$

$$K = \frac{P^2}{2m}$$

$$K = \frac{L^2}{2I}$$

$$K_1 = K_2$$

$$\frac{L_1^2}{2I_1} = \frac{L_2^2}{2I_2}$$

$$\frac{L_1^2}{I} = \frac{L_2^2}{2I} \Rightarrow \left(\frac{L_1}{L_2}\right)^2 = \frac{1}{2} \Rightarrow \frac{L_1}{L_2} = \frac{1}{\sqrt{2}}$$

QUESTION

Two rotating bodies A and B of masses m and $2m$ with moments of inertia I_A and I_B ($I_B > I_A$) have equal kinetic energy of rotation. If L_A and L_B be their angular momenta respectively, then

- A** $L_A = \frac{L_B}{2}$
- B** $L_A = 2L_B$
- C** $L_B > L_A$
- D** $L_A > L_B$

$$\frac{L_1}{L_2} = \sqrt{\frac{I_1}{I_2}} \Rightarrow \frac{L_A}{L_B} = \sqrt{\frac{I_A}{I_B}} \quad I_B > I_A$$

$$\frac{L_A}{L_B} < 1$$

$$= \sqrt{\frac{5}{10}} = \sqrt{\frac{1}{2}}$$

$$\begin{aligned} L_A &< L_B \\ L_B &> L_A \end{aligned}$$

Thank

You