

## TEST - 01

## ULTIMATE KCET CRASH COURSE 2026

## MATHS

- Q1** If  $A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$  and  $A + A^T = I$ , where  $I$  is the unit matrix of  $2 \times 2$  and  $A^T$  is the transpose of  $A$ , then the value of  $\theta$  is equal to  
 (A)  $\pi/6$  (B)  $\pi/3$   
 (C)  $\pi$  (D)  $3\pi/2$
- Q2** If  $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$  then  $A^2 - 5A$  is equal to  
 (A)  $I$  (B)  $-I$   
 (C)  $7I$  (D)  $-7I$
- Q3** If  $A$  is a matrix of order  $m \times n$  and  $B$  is a matrix such that  $AB^T$  and  $B^T A$  are both defined, the order of the matrix  $B$  is  
 (A)  $m \times m$  (B)  $n \times n$   
 (C)  $n \times m$  (D)  $m \times n$
- Q4** The system of linear equations  $x + y + z = 6$ ,  $x + 2y + 3z = 10$  and  $x + 2y + az = b$  has no solution when \_\_\_\_\_  
 (A)  $a = 3, b \neq 10$   
 (B)  $b = 3, a \neq 10$   
 (C)  $a = 2, b \neq 3$   
 (D)  $b = 2, b = 3$
- Q5** If  $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$  and  $|A^3| = 27$ , then  $\alpha =$  \_\_\_\_\_  
 (A)  $\pm 2$  (B)  $\pm \sqrt{5}$   
 (C)  $\pm 1$  (D)  $\pm \sqrt{7}$
- Q6** Evaluate  $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$ .  
 (A) 0 (B) 3  
 (C) 1 (D) 2
- Q7** If  $\vec{a} = p\hat{i} - \hat{j} + 8\hat{k}$  and  $\vec{b} = 2\hat{i} + 4\hat{j} + q\hat{k}$  and  $(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b}) = \vec{0}$ , then  $p - q$  is equal to  
 (A)  $63/2$  (B)  $63/4$   
 (C)  $-63/2$  (D)  $-63/4$
- Q8** If  $|\vec{a}| = 5$ ,  $|\vec{b}| = 2$  and  $\vec{a} \cdot \vec{b} = 6$  then the value of  $|\vec{a} \times \vec{b}|$   
 (A) 6 (B) 10  
 (C) 16 (D) 8
- Q9** If  $\vec{a}, \vec{b}, \vec{c}$  are unit vectors and  $\theta$  is the angle between them, then  $|\vec{a} - \vec{b}| =$   
 (A)  $2 \cos \theta/2$  (B)  $2 \sin \theta/2$   
 (C)  $2 \cos \theta$  (D)  $2 \sin \theta$
- Q10** The angle between a line whose direction ratios are in the ratio  $2 : 2 : 1$  and a line joining  $(3, 1, 4)$  to  $(7, 2, 12)$  is  
 (A)  $\cos^{-1}(2/3)$  (B)  $\cos^{-1}(-2/3)$   
 (C)  $\tan^{-1}(2/3)$  (D) None of these
- Q11** If the line joining  $(2, 3, -1)$  and  $(3, 5, -3)$  is perpendicular to the line joining  $(1, 2, 3)$  and  $(3, 5, l)$  then  $l =$   
 (A) -3 (B) 2  
 (C) 5 (D) 7
- Q12** Find the shortest distance between the lines  $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$  and  $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$   
 (A)  $\frac{3\sqrt{3}}{2}$  units (B)  $\frac{3}{\sqrt{2}}$  units  
 (C)  $3/2$  units (D)  $\frac{1}{\sqrt{2}}$  units



**Q13** Solution set of the inequations  $2x - 1 \leq 3$  and  $3x + 1 \geq -5$  is  $[-k, k]$ , then  $k =$

- (A) 1 (B) 2  
(C) 3 (D) 4

**Q14** Solve  $4x + 3 \geq 2x + 17$ ,  $3x - 5 < -2$ .

- (A)  $(-1, 7)$   
(B)  $(7, \infty)$   
(C)  $(-\infty, -1)$   
(D) No solution

**Q15** Find the domain of the function

$$f(x) = \sqrt{\log_{1/2} \left( \frac{5x-x^2}{4} \right)}$$

- (A)  $(-\infty, 5)$   
(B)  $(0, 1] \cup [4, 5)$   
(C)  $(0, 5)$   
(D)  $(4, \infty)$

**Q16** The domain of the function

$$f(x) = \frac{\log(2x-3)}{\sqrt{x-1}} + \sqrt{5-2x}$$

- (A)  $[1, 5/2]$  (B)  $[3/2, 5/2]$   
(C)  $[1, 5/2]$  (D)  $(3/2, 5/2]$

**Q17** The domain of the function  $f$  defined by

$$f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$$

- (A)  $(-\infty, -1) \cup (1, 4]$   
(B)  $(-\infty, -1] \cup (1, 4]$   
(C)  $(-\infty, -1) \cup [1, 4]$   
(D)  $(-\infty, -1) \cup [1, 4)$

**Q18** If  $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} - \hat{j} - 4\hat{k}$  and  $\vec{c} = \hat{i} + \hat{j} + \hat{k}$

are such that  $\vec{a} + \lambda \vec{b}$  is perpendicular to  $\vec{c}$ , then the value of  $\lambda$  is

- (A) 1 (B)  $\pm 1$   
(C) -1 (D) 0

**Q19** The vectors  $\vec{AB} = 4\hat{i} + 3\hat{k}$  and  $\vec{AC} = 5\hat{i} + 2\hat{j} - 4\hat{k}$  are the sides of a  $\triangle ABC$ .

The length of the median through A is

- (A)  $\sqrt{18}$  (B)  $\sqrt{72}$   
(C)  $\sqrt{43/2}$  (D)  $\sqrt{\frac{33}{2}}$

**Q20** If the number of terms in the binomial expansion of  $(5x + 3)^{4n}$  is 13, then the value of  $n$  is

- (A) 3 (B) 4  
(C) 5 (D) 6

**Q21** The value of  $\tan^{-1} \left[ 2 \cos \left( 2 \sin^{-1} \frac{1}{2} \right) \right]$

- (A)  $-\pi/4$  (B)  $\pi/4$   
(C)  $\pi/2$  (D) None of these

**Q22**  $\tan \left\{ \frac{\pi}{4} + \frac{1}{2} \cos^{-1} \frac{a}{b} \right\} + \tan \left\{ \frac{\pi}{4} - \frac{1}{2} \cos^{-1} \frac{a}{b} \right\}$

- $=$   
(A)  $2a/b$  (B)  $b/a$   
(C)  $2b/a$  (D)  $a/b$

**Q23** If  $\tan^{-1} 2$  and  $\tan^{-1} 3$  are two angles of a triangle, then the third angle is

- (A)  $\pi/2$  (B)  $\pi/3$   
(C)  $\pi/4$  (D)  $\pi/6$

**Q24**  $\sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8}$  is equal to

- (A) 1 (B)  $3/2$   
(C) 2 (D)  $1/4$

**Q25**  $\sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 150^\circ =$

- (A)  $\sqrt{3}/2$  (B) 1  
(C)  $1/2$  (D)  $1/4$

**Q26** If  $\frac{\sin^2 x - 2 \cos^2 x + 1}{\sin^2 x + 2 \cos^2 x - 1} = 4$ , then the value of  $\tan^2 x$  is

- (A) 3 (B) 4  
(C) 5 (D) 6



**Q27** The value  
 ${}^{20}C_3 + {}^{19}C_3 + {}^{18}C_3 + {}^{17}C_3 + {}^{16}C_3$  is  
 $+ {}^{16}C_4$   
 (A)  ${}^{20}C_4$  (B)  ${}^{21}C_4$   
 (C)  ${}^{21}C_3$  (D)  ${}^{20}C_3$

**Q28** If A and B are non-empty sets, then  
 $(A \cap B) \cup (A - B)$  is equal to  
 (A) B (B) A  
 (C) A' (D) B'

**Q29** If  $A = \{x : x \text{ is a factor of } 18\}$ ,  $B = \{x : x \text{ is a factor of } 24\}$  then  $A \cap B =$   
 (A)  $\{1, 2\}$  (B)  $\{1, 3\}$   
 (C)  $\{1, 6\}$  (D)  $\{1, 2, 3, 6\}$

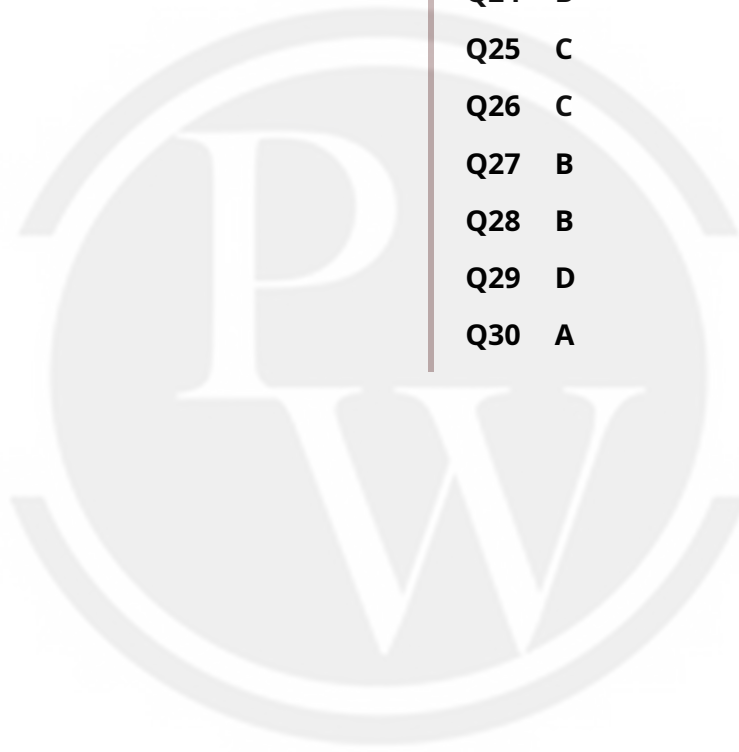
**Q30** If  $n(A) = 2$  and total number of possible relations from set A to set B is 4096 then  $n(B)$  is  
 (A) 6 (B) 12  
 (C) 512 (D) 24



# Answer Key

Q1 A  
Q2 D  
Q3 D  
Q4 A  
Q5 D  
Q6 A  
Q7 A  
Q8 D  
Q9 B  
Q10 A  
Q11 D  
Q12 B  
Q13 B  
Q14 D  
Q15 B

Q16 D  
Q17 A  
Q18 A  
Q19 C  
Q20 A  
Q21 B  
Q22 C  
Q23 C  
Q24 B  
Q25 C  
Q26 C  
Q27 B  
Q28 B  
Q29 D  
Q30 A



# Hints & Solutions

Note: scan the QR code to watch video solution

## Q1 Text Solution:

$$\text{Given, } A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\Rightarrow A^T = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$\text{Also, } A + A^T = I$$

$$\Rightarrow \begin{bmatrix} 2\cos 2\theta & 0 \\ 0 & 2\cos 2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing, we get

$$2\cos 2\theta = 1 \Rightarrow \cos 2\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

### Video Solution:



## Q2 Text Solution:

$$\text{Given, } A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \\ = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

$$\therefore A^2 - 5A = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} = -7I$$

### Video Solution:



## Q3 Text Solution:

$$O(A) = m \times n$$

$$O(B) = ? \quad \therefore O(B) = m \times n$$

$$\text{Let } O(B) = p \times q$$

$$AB' = m \times n - q \times p$$

$$\Rightarrow n = q$$

$$\text{Also } B'A = q \times p - m \times n$$

$$\Rightarrow p = m$$

### Video Solution:



## Q4 Text Solution:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & a \end{vmatrix} = 0$$

$$1(2a - 6) - 1(a - 3) + 1(2 - 2) = 0$$

$$2a - 6 - a + 3 = 0$$

$$a - 3 = 0$$

$$a = 3$$

By option verification

$$b \neq 0$$

$$\text{if } b = 10$$

System will have infinite solution

### Video Solution:



**Q5 Text Solution:**

$$\text{Given, } |A^3| = 27 \Rightarrow |A|^3 = 27 \quad [\because |A^n| = |A|^n]$$

$$\Rightarrow |A| = 3 \Rightarrow \begin{vmatrix} \alpha & 2 \\ 2 & \alpha \end{vmatrix} = 3$$

$$\Rightarrow \alpha^2 - 4 = 3 \Rightarrow \alpha^2 = 7 \Rightarrow \alpha = \pm\sqrt{7}$$

**Video Solution:****Q6 Text Solution:**

$$\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix} = \cos 15^\circ \cos 75^\circ - \sin 75^\circ \sin 15^\circ$$

$$= \cos(15^\circ + 75^\circ) = \cos(90^\circ) = 0$$

**Video Solution:****Q7 Text Solution:**

$$(\vec{a} + 3\vec{b}) \times (3\vec{a} - \vec{b}) = \vec{0} \text{ (given)}$$

$$\text{or } 3(\vec{a} \times \vec{a}) - (\vec{a} \times \vec{b}) + 9(\vec{b} \times \vec{a}) - 3(\vec{b} \times \vec{b}) = \vec{0}$$

$$\text{or } -(\vec{a} \times \vec{b}) - 9(\vec{a} \times \vec{b}) = \vec{0}$$

$$\text{or } -10(\vec{a} \times \vec{b}) = \vec{0} \text{ and } \vec{a} \times \vec{b} = \vec{0}$$

$$\text{or } \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ p & -1 & 8 \\ 2 & 4 & q \end{vmatrix} = 0\hat{i} + 0\hat{j} + 0\hat{k}$$

By comparison, we get

$$-q - 32 = 0, \quad 4p + 2 = 0 \Rightarrow q = -32 \text{ or } p = -\frac{1}{2}$$

$$\therefore p - q = -\frac{1}{2} + 32 = \frac{63}{2}$$

**Video Solution:****Q8 Text Solution:**

$$|\vec{a} \times \vec{b}| = |\vec{a}| |\vec{b}| \sin \theta$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$6 = 5 \times 2 \cos \theta$$

$$\cos \theta = \frac{6}{10}$$

$$\cos \theta = \frac{3}{5}$$

$$\therefore \sin \theta = \frac{4}{5}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = 5 \times 2 \times \frac{4}{5}$$

$$|\vec{a} \times \vec{b}| = 8$$

**Video Solution:**

**Q9 Text Solution:**

$\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  are unit vectors.

$$\text{Now, } |\vec{a} - \vec{b}| = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b} = 1$$

$$+ 1 - 2|\vec{a}||\vec{b}|\cos\theta$$

$$= 2 - 2\cos\theta = 2(1 - \cos\theta) = 4\sin^2\frac{\theta}{2}$$

$$\text{Hence, } |\vec{a} - \vec{b}| = 2\sin\frac{\theta}{2}$$

**Video Solution:****Q10 Text Solution:**

Let  $a_1 = 2x$ ,  $b_1 = 2x$  and  $c_1 = x$

Clearly,  $a_2 = 7 - 3 = 4$ ,  $b_2 = 2 - 1 = 1$ ,

$$c_2 = 12 - 4 = 8$$

$$\therefore \cos\theta = \frac{2x \times 4 + 2x \times 1 + x \times 8}{\sqrt{4x^2 + 4x^2 + x^2} \sqrt{16 + 1 + 64}} = \frac{18}{3x \times 9} = \frac{2}{3}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

**Video Solution:****Q11 Text Solution:**

DR's of the given lines are 1, 2, -2 and 2, 3, -3.

Since, lines are perpendicular

$$\therefore a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\Rightarrow 1 \times 2 + 2 \times 3 - 2(\lambda - 3) = 0 \Rightarrow \lambda = 7$$

**Video Solution:****Q12 Text Solution:**

The given lines are

$$\vec{r} = (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r}$$

$$= (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

We know that

S.D. between the lines

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \text{ and } \vec{r} = \vec{a}_2 + \mu \vec{b}_2 \text{ is}$$

$$\text{given by } d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

On comparing, we get

$$\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$$

$$\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}, \vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = \hat{i} - 3\hat{j} - 2\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 0 \end{vmatrix} = -3\hat{i} + 0 \cdot \hat{j}$$

$$+ 3\hat{k}$$

$$\Rightarrow |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 3^2} = 3\sqrt{2}$$

$$\Rightarrow (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 1(-3) - 3(0) - 2(3) = -9$$

$$\therefore d = \frac{|-9|}{3\sqrt{2}} = \frac{3}{\sqrt{2}} \text{ units}$$

**Video Solution:**

**Q13 Text Solution:**

$$2x - 1 \leq 3 \Rightarrow 2x \leq 4 \Rightarrow x \leq 2$$

$$3x + 1 \geq -5 \Rightarrow 3x \geq -6 \Rightarrow x \geq -2$$

Hence,  $x \in [-2, 2] \therefore k = 2$

**Video Solution:**

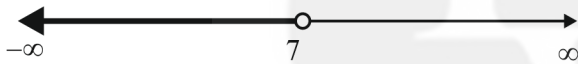


**Q14 Text Solution:**

We have given,  $4x + 3 \geq 2x + 17$   
 $\Rightarrow 4x - 2x \geq 17 - 3 \Rightarrow 2x \geq 14$   
 $\Rightarrow x \geq \frac{14}{2}$  [Dividing by 2 on both sides]  
 $\Rightarrow x \geq 7$



Also, we have  $3x - 5 < -2$   
 $\Rightarrow 3x < -2 + 5 \Rightarrow 3x < 3$   
 $\Rightarrow x < 1$  ... (ii)



On combining (i) and (ii), we see that solution is not possible because nothing is common between these two solutions (i.e.,  $x < 1, x \geq 7$ ).

**Video Solution:**



**Q15 Text Solution:**

$$f(x) = \sqrt{\log_{1/2} \left( \frac{5x-x^2}{4} \right)}$$

$$\therefore \frac{5x-x^2}{4} > 0 \Rightarrow x(5-x) > 0$$

$$\Rightarrow x(x-5) < 0$$

$$\therefore x \in (0, 5) \quad \dots (i)$$

$$\text{Also } \log_{\frac{1}{2}} \left( \frac{5x-x^2}{4} \right) \geq 0$$

$$\Rightarrow \frac{5x-x^2}{4} \leq \left( \frac{1}{2} \right)^0 \Rightarrow 5x - x^2 \leq 4$$

$$\Rightarrow x^2 - 5x + 4 \geq 0 \Rightarrow x \in (-\infty, 1]$$

$$\cup [4, \infty) \quad \dots (ii)$$

Using (i) and (ii), we get

$$x \in (0, 1] \cup [4, 5)$$

**Video Solution:**



**Q16 Text Solution:**

$$f(x) = \frac{\log(2x-3)}{\sqrt{x-1}} + \sqrt{5-2x}$$

$$x - 1 > 0 \Rightarrow x > 1, 2x - 3 > 0 \Rightarrow x > \frac{3}{2}$$

$$\text{and } 5 - 2x \geq 0 \Rightarrow x \leq \frac{5}{2} \therefore \frac{3}{2} < x \leq \frac{5}{2}$$

Hence, domain of  $f(x)$  is  $\left( \frac{3}{2}, \frac{5}{2} \right]$

**Video Solution:**



**Q17 Text Solution:**

$$\text{Given, } f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$$

$f(x)$  is defined when,

$$4-x \geq 0 \text{ and } x^2-1 > 0$$

$$\Rightarrow x \leq 4 \text{ and } x^2 > 1$$

$$\Rightarrow x \leq 4 \text{ and } x \in (-\infty, -1) \cup (1, \infty)$$

$$\therefore x \in (-\infty, -1) \cup (1, 4]$$

**Video Solution:****Q18 Text Solution:**

$$\text{Given } (\vec{a} + \lambda \vec{b}) \cdot \vec{c} = 0$$

$$\vec{a} + \lambda \vec{b} = (2\hat{i} + \hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} - 4\hat{k})$$

$$= (2 + \lambda)\hat{i} + (1 - \lambda)\hat{j} + (1 - \lambda)\hat{k}$$

$$(\vec{a} + \lambda \vec{b}) \cdot \vec{c} = (2 + \lambda) \cdot 1 + (1 - \lambda) \cdot 1 + (1 - 4\lambda) \cdot 1$$

$$0 = 2 + \lambda + 1 - \lambda + 1 - 4\lambda$$

$$= 4 - 4\lambda$$

$$4\lambda = 4$$

$$\lambda = 1$$

**Video Solution:****Q19 Text Solution:**

$$P = \frac{AB+AC}{2}$$

$$= \left(\frac{4+5}{2}\right)\hat{i} + \left(\frac{0+2}{2}\right)\hat{j} + \left(\frac{3-4}{2}\right)\hat{k}$$

$$= \frac{9}{2}\hat{i} + \hat{j} - \frac{1}{2}\hat{k}$$

$$|AP| = \sqrt{\frac{81}{4} + 1 + \frac{1}{4}}$$

$$|AP| = \sqrt{\frac{81+4+1}{4}} = \sqrt{\frac{86}{4}} = \sqrt{\frac{43}{2}}$$

**Video Solution:****Q20 Text Solution:**

A binomial expansion  $(ax + b)^k$  has  $K + 1$  distinct terms.

$$\text{Here } K = 4n$$

$$\Rightarrow 4n + 1 = 13$$

$$n = 3$$

**Video Solution:****Q21 Text Solution:**

$$\tan^{-1} \left[ 2 \cos \left( 2 \sin^{-1} \frac{1}{2} \right) \right]$$

$$= \tan^{-1} \left[ 2 \cos \left\{ 2 \sin^{-1} \left( \sin \frac{\pi}{6} \right) \right\} \right]$$

$$\left( \because \sin \frac{\pi}{6} = \frac{1}{2} \right)$$

$$= \tan^{-1} \left[ 2 \cos \left( 2 \times \frac{\pi}{6} \right) \right] = \tan^{-1} \left[ 2 \cos \frac{\pi}{3} \right]$$

$$= \tan^{-1} \left[ 2 \times \frac{1}{2} \right]$$

$$= \tan^{-1} (1) \quad \left( \because \cos \frac{\pi}{3} = \frac{1}{2} \right)$$

$$= \tan^{-1} \left( \tan \frac{\pi}{4} \right)$$

$$= \frac{\pi}{4} \quad \left( \because \tan \frac{\pi}{4} = 1 \right)$$

**Video Solution:**

**Q22 Text Solution:**

Let  $\cos^{-1}\left(\frac{a}{b}\right) = \cos \theta$ . Then  $\cos \theta = \frac{a}{b}$

$$\begin{aligned} & \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right) + \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right) \\ &= \frac{1 + \tan \frac{\theta}{2}}{1 - \tan \frac{\theta}{2}} + \frac{1 - \tan \frac{\theta}{2}}{1 + \tan \frac{\theta}{2}} = \frac{(1 + \tan \frac{\theta}{2})^2 + (1 - \tan \frac{\theta}{2})^2}{1 - \tan^2 \frac{\theta}{2}} \\ &= 2 \left( \frac{1 + \tan^2 \frac{\theta}{2}}{1 - \tan^2 \frac{\theta}{2}} \right) = \frac{2}{\cos \theta} = \frac{2b}{a}. \end{aligned}$$

**Video Solution:****Q23 Text Solution:**

Let  $A = \tan^{-1} 2$ ,  $B = \tan^{-1} 3$

We know that  $A + B + C = 180^\circ \Rightarrow C = 180^\circ - A - B$

$$\Rightarrow \tan C = \tan(180^\circ - A - B) = -\tan(A + B)$$

$$= - \left[ \frac{\tan A + \tan B}{1 - \tan A \tan B} \right] = - \left[ \frac{2+3}{1-6} \right] = 1$$

$$\Rightarrow C = \tan^{-1} 1 = \frac{\pi}{4}$$

**Video Solution:****Q24 Text Solution:**

$$\begin{aligned} & \sin^4 \frac{\pi}{8} + \sin^4 \frac{3\pi}{8} + \sin^4 \frac{5\pi}{8} + \sin^4 \frac{7\pi}{8} \\ &= \frac{1}{4} \left[ (2 \sin^2 \frac{\pi}{8})^2 + (2 \sin^2 \frac{3\pi}{8})^2 \right] \\ &+ \frac{1}{4} \left[ (2 \sin^2 \frac{\pi}{8})^2 + (2 \sin^2 \frac{3\pi}{8})^2 \right] \\ &= \frac{1}{4} \left[ (1 - \cos \frac{\pi}{4})^2 + (1 - \cos \frac{3\pi}{4})^2 \right] \\ &+ \frac{1}{4} \left[ (1 - \cos \frac{\pi}{4})^2 + (1 - \cos \frac{3\pi}{4})^2 \right] \\ &= \frac{1}{4} \left[ \left(1 - \frac{1}{\sqrt{2}}\right)^2 + \left(1 + \frac{1}{\sqrt{2}}\right)^2 \right] \\ &+ \frac{1}{4} \left[ \left(1 - \frac{1}{\sqrt{2}}\right)^2 + \left(1 + \frac{1}{\sqrt{2}}\right)^2 \right] \\ &= \frac{1}{4}(3) + \frac{1}{4}(3) = \frac{3}{2} \end{aligned}$$

**Video Solution:****Q25 Text Solution:**

$$\begin{aligned} & \sin 780^\circ \sin 480^\circ + \cos 120^\circ \sin 150^\circ \\ &= \sin(720^\circ + 60^\circ) \sin(450^\circ + 30^\circ) \\ &+ \cos(180^\circ - 60^\circ) \sin(180^\circ - 30^\circ) \\ &= \sin(60^\circ) \cos(30^\circ) - \cos(60^\circ) \sin(30^\circ) \\ &= \frac{\sqrt{3}}{2} \frac{\sqrt{3}}{2} - \frac{1}{2} \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2} \end{aligned}$$

**Video Solution:**

**Q26 Text Solution:**

$$\frac{\sin^2 x - 2 \cos^2 x + 1}{\sin^2 x + 2 \cos^2 x - 1} = 4$$

$$\Rightarrow \sin^2 x - 2 \cos^2 x + 1 = 4 \sin^2 x + 8 \cos^2 x - 4$$

$$\Rightarrow 10 \cos^2 x + 3 \sin^2 x - 5 = 0 \Rightarrow 10 + 3 \tan^2 x - 5(1 + \tan^2 x) = 0$$

$$\Rightarrow 2 \tan^2 x = 5$$

**Video Solution:****Q27 Text Solution:**

$$\therefore {}^n C_r + {}^n C_{r+1} = {}^{n+1} C_{r+1}$$

$${}^{16} C_3 + {}^{16} C_4 = {}^{16+1} C_{3+1} = {}^{17} C_4$$

$${}^{17} C_4 + {}^{17} C_3 = {}^{18} C_4$$

$${}^{18} C_4 + {}^{18} C_3 = {}^{19} C_4$$

$${}^{19} C_4 + {}^{19} C_3 = {}^{20} C_4$$

$${}^{20} C_4 + {}^{20} C_3 = {}^{21} C_4$$

**Video Solution:****Q28 Text Solution:**

$$(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B')$$

$$= A \cap (B \cup B') = A \cap U = A$$

**Video Solution:****Q29 Text Solution:**

$$A = \{1, 2, 3, 6, 9, 18\}$$

$$B = \{1, 2, 3, 6, 8, 12, 24\}$$

$$A \cap B = \{1, 2, 3, 6\}$$

**Video Solution:****Q30 Text Solution:**

$$2^{n(A \times B)} = 4096$$

$$2^{n(A \times B)} = 2^{12}$$

$$n(A \times B) = 12$$

$$n(A) n(B) = 12$$

$$n(B) = \frac{12}{2}$$

$$n(B) = 6$$

**Video Solution:**