ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture - 02

Domain & Range

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Recap *of previous lecture*

1. Domain (Problems)
2.
3.
4.

Topics *to be covered*

1. Range (problems)
2. Domain
3.
4.

Range:-

① $f(x) = \dfrac{1}{\text{linear expression}}$

Ex: $f(x) = \dfrac{1}{5x+3}$

Soln:

Consider $f(x) = y$

$\dfrac{1}{5x+3} = y$

$1 = 5xy + 3y$

$5xy = 1 - 3y$

$x = \dfrac{1-3y}{5y} \Rightarrow x =$ expression in y

Now we need to find the value of y for which x exists

$x = \dfrac{1-3y}{5y}$

Here $y \neq 0$ $(D^r \neq 0)$

Range $= R - \{0\}$

② $f(x) = \dfrac{ax+b}{cx+d} = \dfrac{\text{linear expression}}{\text{linear expression}}$

Ex:

$f(x) = \dfrac{2x+3}{5x+4}$ find Range

x = expression in y

$\downarrow$ LHS $\quad \downarrow$ RHS

Soln:

consider

$f(x) = y$

$\dfrac{2x+3}{5x+4} = y$

$2x+3 = 5xy+4y$

$2x - 5xy = 4y - 3$

$x(2-5y) = 4y-3$

$x = \dfrac{4y-3}{2-5y}$

$2-5y \neq 0$

$y \neq \dfrac{2}{5}$

$\text{Range} = R - \left\{\dfrac{2}{5}\right\}$

③ $f(x) = \dfrac{ax^2+b}{cx^2+d}$

Ex: $f(x) = \dfrac{3x^2+4}{x^2-5}$

Soln: consider

$$f(x) = y$$

$$\frac{3x^2+4}{x^2-5} = y$$

$$3x^2+4 = x^2y - 5y$$

$$3x^2 - x^2y = -5y - 4$$

$$x^2(3-y) = -5\left(y+\frac{4}{5}\right)$$

$$x^2 = \frac{-5\left(y+\frac{4}{5}\right)}{3-y}$$

$$x^2 = \frac{-5\left(y+\frac{4}{5}\right)}{-(y-3)}$$

$$x^2 = \frac{5\left(y+4/5\right)}{y-3}$$

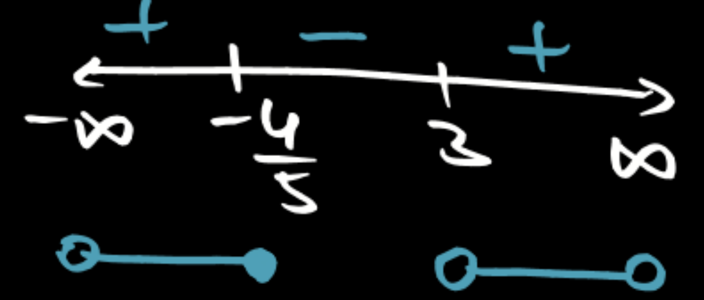
Here $x^2 \geq 0$

$$\frac{5\left(y+\frac{4}{5}\right)}{y-3} \geq 0$$

$\div$ by 5

$$\frac{y+\frac{4}{5}}{y-3} \geq 0$$

Critical Points $y = -\frac{4}{5}$ & $y = 3$



$$y \in \left(-\infty, -\frac{4}{5}\right] \cup (3, \infty)$$

↓

Range

④ $f(x) = \dfrac{ax^2+b}{cx+d}$

Ex:

Domain $\Rightarrow R - \{-\frac{1}{2}\}$ ← $f(x) = \dfrac{3x^2+2}{2x+1}$

Consider

$f(x) = y$

$\dfrac{3x^2+2}{2x+1} = y$

$3x^2+2 = 2xy+y$

$3x^2-2xy+(2-y)=0$

$3x^2-2yx+(2-y)=0$

$a=3 \mid b=-2y$ & $c=2-y$

Here x exists only if

$b^2-4ac \geq 0$

$(-2y)^2-4(3)(2-y) \geq 0$

$4y^2-24+12y \geq 0$

÷ by 4

$y^2+3y-6 \geq 0$ → Factorisation is not possible

completing the square form

$\underbrace{y^2+3y+\left(\frac{3}{2}\right)^2}-\left(\frac{3}{2}\right)^2-6 \geq 0$

$\left(y+\frac{3}{2}\right)^2-\frac{9}{4}-6 \geq 0$

$ax^2+bx+c=0$

has real roots (ie, x exists)

if $D \geq 0$

$b^2-4ac \geq 0$

$$\left(y+\frac{3}{2}\right)^2-\frac{9}{4}-6\geq 0$$

$$-\frac{9}{4}-6$$

$$=\frac{-9-24}{4}=\frac{-33}{4}$$

$$\left(y+\frac{3}{2}\right)^2-\frac{33}{4}\geq 0$$

$$\left(y+\frac{3}{2}\right)^2\geq \frac{33}{4}$$

$$\sqrt{\left(y+\frac{3}{2}\right)^2}\geq \sqrt{\frac{33}{4}}$$

$$\left|y+\frac{3}{2}\right|\geq \frac{\sqrt{33}}{2}$$

$$y+\frac{3}{2}\in\left(-\infty,-\frac{\sqrt{33}}{2}\right]\cup\left[\frac{\sqrt{33}}{2},\infty\right)$$

$$y\in\left(-\infty,-\frac{3}{2}-\frac{\sqrt{33}}{2}\right]\cup\left[\frac{\sqrt{33}}{2}-\frac{3}{2},\infty\right)\rightarrow \text{Range}$$

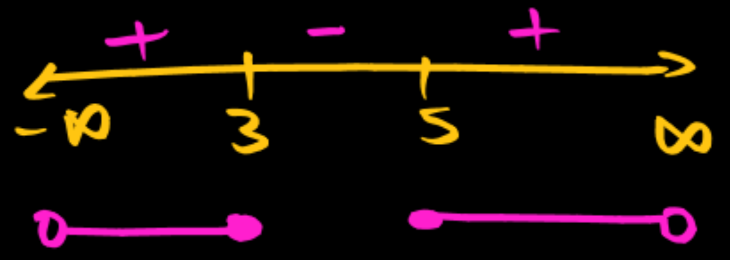
Solve

① $x^2-8x+15 \geq 0$

$+15 \to -3,\ -5$

$(x-3)(x-5) \geq 0$

Here factorisation is possible



$x \in (-\infty, 3] \cup [5, \infty)$

② Solve

$x^2-2x-5 \leq 0$ → Here factorisation is not possible

⇓

∴ we use completing the square form

$$\underbrace{x^2-2x+1^2}-1^2-5 \leq 0$$

$$(x-1)^2-1-5 \leq 0$$

$$(x-1)^2-6 \leq 0$$

$$(x-1)^2 \leq 6$$

$$\sqrt{(x-1)^2} \leq \sqrt{6}$$

$$|x-1| \leq \sqrt{6}$$

$$x-1 \in [-\sqrt{6}, \sqrt{6}]$$

$$x \in [1-\sqrt{6}, 1+\sqrt{6}]$$

QUESTION

If $[x]^2 - 5[x] + 6 = 0$, where $[\cdot]$ denotes the greatest integer function, then

(A) $x \in (2,4)$

(B) $x \in [2,4]$

(C) $x \in [2,4)$

(D) $x \in (2,4]$

QUESTION

The range of the function $f(x) = \frac{x^2+8}{x^2+4}, x \in R$, is

$x^2+4 \neq 0$ $\boxed{\therefore \text{Domain} = R}$

(A) $[-1, 3/2]$

(B) $(1,2]$

(C) $(1,2)$

(D) $[1,2]$

Consider

$f(x) = y$

$\frac{x^2+8}{x^2+4} = y$

$x^2+8 = x^2y+4y$

$x^2(1-y) = 4y-8$

$x^2 = \frac{4(y-2)}{1-y}$

WKT

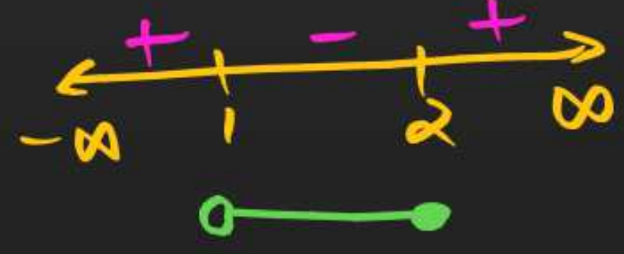
$x^2 \geq 0$

$\frac{4(y-2)}{-(y-1)} \geq 0$

$\div$ by -4

$\frac{y-2}{y-1} \leq 0$

$y-1 \to y \neq 1$ in the soln



$y \in (1,2]$

QUESTION

If R is the set of all real numbers and if $f: R - \{2\} \to R$ is defined by $f(x) = \frac{2+x}{2-x}$ for $x \in R - \{2\}$, then the range of f is

(A) R

(B) $R - \{1\}$

(C) $R - \{-1\}$

(D) $R - \{-2\}$

Consider

$$f(x) = y$$

$$\frac{2+x}{2-x} = y$$

$$2+x = 2y - xy$$

$$x(1+y) = 2y - 2$$

$$x = \frac{2y-2}{y+1}$$

$y + 1 \neq 0$

$y \neq -1$

Range $= R - \{-1\}$

(*) Find the range of $f(x) = 4x - x^2 \longrightarrow$ Domain $= R$

Soln:

consider

$$f(x) = y$$

$$4x - x^2 = y$$

$$x^2 - 4x + y = 0$$

$a = 1 \mid b = -4 \mid c = y$

Here x exists

if $b^2 - 4ac \geq 0$

$$(-4)^2 - 4(1)y \geq 0$$

$$16 - 4y \geq 0$$

$$16 \geq 4y$$

$$4y \leq 16$$

$$y \leq 4$$

$$y \in (-\infty, 4]$$

$\hookrightarrow$ Range

QUESTION

Let $f:[2,\infty) \rightarrow X$ defined by $f(x)=4x-x^2$, then f is invertible if X equals

(A) $[2,\infty)$

(B) $(-\infty,2]$

(C) $(-\infty,4]$

(D) $[4,\infty)$

Find the range of

① $f(x) = \dfrac{3x+2}{x-2}$

Soln:

$$\frac{3x+2}{x-2} = y$$

$$3x + 2 = xy - 2y$$

$$x(3-y) = -2y - 2$$

$$x = \frac{-2y-2}{3-y}$$

$$3 - y \neq 0$$

$$y \neq 3$$

Range $= R - \{3\}$

② $f(x) = \dfrac{x^2}{4+x^2}$

Soln:

Consider

$f(x) = y$

$$x^2 = 4y + x^2 y$$

$$x^2(1-y) = 4y$$

$$x^2 = \frac{4y}{1-y}$$

WKT $x^2 \geq 0$

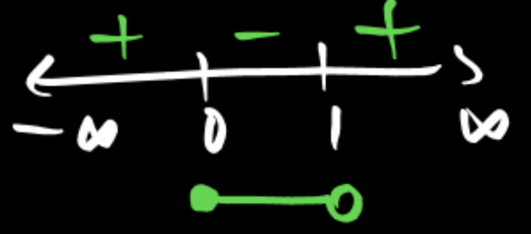
$$\frac{4y}{1-y} \geq 0$$

$$\frac{4(y)}{-(y-1)} \geq 0$$

$\div$ by -4

$$\frac{y}{y-1} \leq 0$$

$y - 1 \rightarrow y \neq 1$



$y \in [0, 1)$

③ $f(x) = \dfrac{x^2+2}{x+1}$

Consider

$f(x) = y$

$$x^2 + 2 = xy + y$$

$$x^2 - xy + (2-y) = 0$$

$a = 1 \mid b = -y \mid c = 2 - y$

$$b^2 - 4ac \geq 0$$

$$y^2 - 4(2-y) \geq 0$$

$$y^2 + 4y - 8 \geq 0$$

(next Page)

$y^2+4y-8 \geq 0$ $\hookrightarrow$ cannot be factorised

$\underbrace{y^2+4y+2^2}-2^2-8 \geq 0$

$(y+2)^2 \geq 12$

$\sqrt{(y+2)^2} \geq \sqrt{12}$

$|y+2| \geq 2\sqrt{3}$

$y+2 \in (-\infty, -2\sqrt{3}] \cup [2\sqrt{3}, \infty)$

$y \in (-\infty, -2-2\sqrt{3}] \cup [2\sqrt{3}-2, \infty)$

Find range of $f(x) = x^2+4$

Soln:

WKT

$$x^2 \geq 0$$

$$x^2+4 \geq 4$$

$$f(x) \geq 4$$

$$f(x) \in [4, \infty)$$

Range $= [4, \infty)$

2nd method

consider $f(x) = y$

$$x^2+4 = y$$

$$x^2 = y-4$$

WKT

$$x^2 \geq 0$$

$$\Rightarrow y-4 \geq 0$$

$$y \geq 4$$

$$y \in [4, \infty)$$

Find range of

$$f(x) = 3-2x^2$$

Soln:

WKT

$$x^2 \geq 0$$

$\times^{ly}$ by (-2)

$$-2x^2 \leq 0$$

Add 3

$$3-2x^2 \leq 3$$

$$f(x) \leq 3$$

$f(x) \in (-\infty, 3] \to$ Range.

open interval is a subset of closed interval

QUESTION

If the domain of the function $f(x) = x^2 - 6x + 7$ is $(-\infty, \infty)$, then the range of function is

↳ completing the square form

A $(-2, \infty)$

B $(-\infty, \infty)$

C $(-2,1)$

D $(-\infty, -2)$

$f(x) = \underbrace{x^2 - 6x + 3^2} - 3^2 + 7$

$f(x) = (x-3)^2 - 9 + 7$

$f(x) = (x-3)^2 - 2$

WKT

$(x-3)^2 \geq 0$

Add -2

$(x-3)^2 - 2 \geq -2$

$f(x) \geq -2$

$f(x) \in [-2, \infty)$

↳ Range

QUESTION

If $f(x+1) = x^2 - 3x + 2$, then what is $f(x)$ equal to?

insta id

↓

gurumathswallah

(A) $x^2 - 5x + 4$

(B) $x^2 - 5x + 6$

(C) $x^2 + 3x + 3$

(D) $x^2 - 3x + 1$

$f(x+1) = x^2 - 3x + 2$

Put $x + 1 = t$

$x = t - 1$

$f(t) = (t-1)^2 - 3(t-1) + 2$

$= t^2 + 1 - 2t - 3t + 3 + 2$

$f(t) = t^2 - 5t + 6$

Replace t by x

$f(x) = x^2 - 5x + 6$

$f(x-1) = x^2 - 3x + 2$ find $f(2x)$

Soln:

Put $x - 1 = t$

$x = t + 1$

$f(t) = (t+1)^2 - 3(t+1) + 2$

$= t^2 + 1 + 2t - 3t - 3 + 2$

$f(t) = t^2 - t$

Replace t by $2x$

$f(2x) = (2x)^2 - (2x)$

$f(2x) = 4x^2 - 2x$

QUESTION

If $f(x) = 2x - x^2$, then what is the value of $f(x+2) + f(x-2)$ when $x = 0$?

$2(x+2) - (x+2)^2 + 2(x-2) - (x-2)^2$

Put $x = 0$

$2(2) - (2)^2 + 2(-2) - (-2)^2$

$= 4 - 4 - 4 - 4$

$= -\underline{8}$

(A) -8

(B) -4

(C) 8

(D) 4

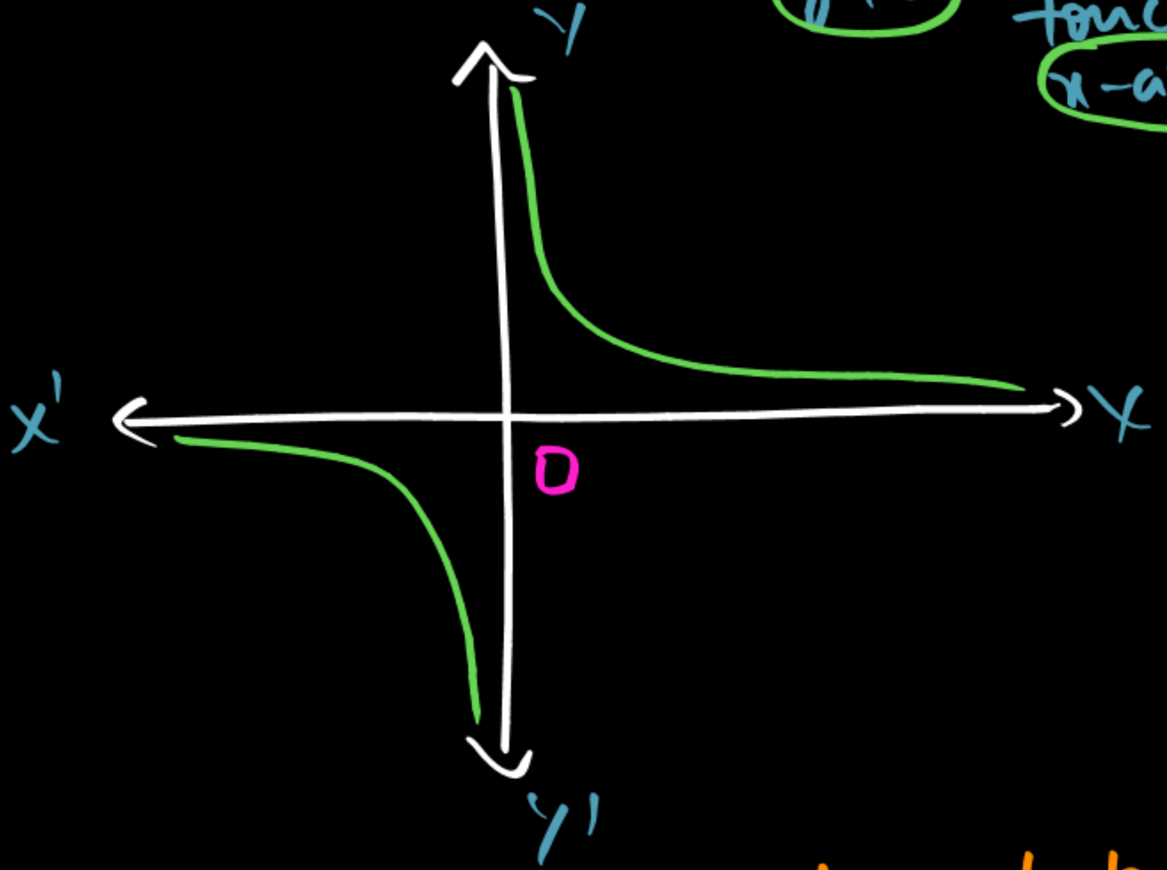
$f(x) = \frac{1}{x}$

$x \neq 0$

Domain $= R - \{0\}$ → Doesnot touch y-axis

Range $= R - \{0\}$ → Doesnot touch x-axis

$y \neq 0$



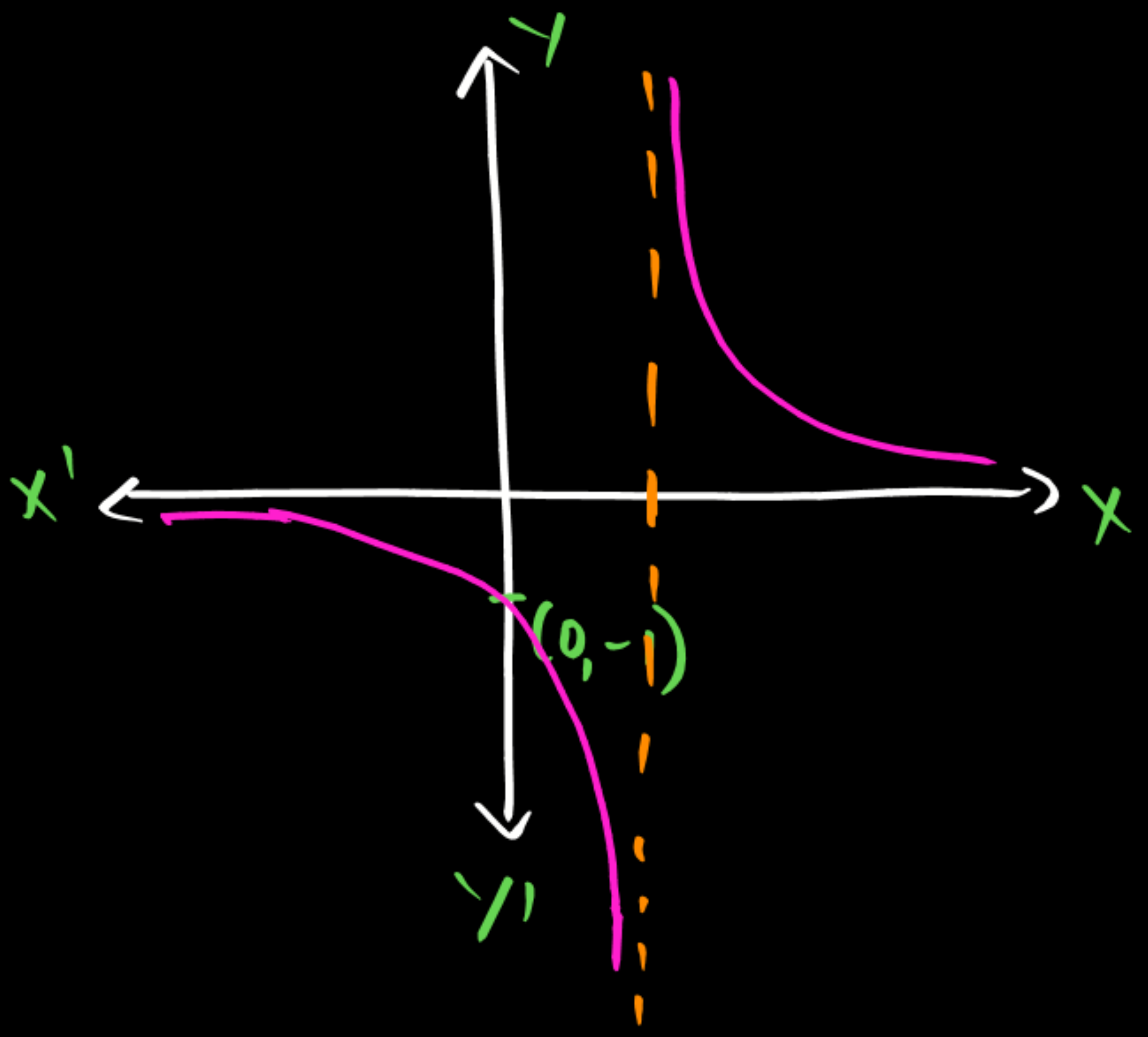
This graph doesnot touch x-axis & y-axis

$f(x) = \frac{1}{x-1}$ | $f(0) = \frac{1}{-1} = -1$

Domain $= R - \{1\}$

Range $= R - \{0\}$ Doesnot touch x-axis

$y \neq 0$



QUESTION

Domain $= R - \{1\}$

Which one of the following is correct in respect of the graph of $y = \frac{1}{x-1}$?

$f(x) = y$

$\frac{1}{x-1} = y$

$1 = xy - y$

$\frac{1+y}{y} = x$

$x = \frac{1+y}{y}$

Range :- $R - \{0\}$

(A) The domain is $\{x \in R \mid x \neq 1\}$ and the range is the set of reals. False

(B) The domain is $\{x \in R \mid x \neq 1\}$, the range is $\{y \in R \mid y \neq 0\}$ and the graph intersects y-axis at $(0, -1)$. True

(C) The domain is the set of reals and the range is the singleton set $\{0\}$. False

(D) The domain is $\{x \in R \mid x \neq 1\}$ and the range is the set of points on the y-axis. False

QUESTION

If $f(x) = \log_{10}(1 + x)$, then what is $4f(4) + 5f(1) - \log_{10} 2$ equal to ?

(A) 0

(B) 1

(C) 2

(D) 4 ✓

$4\log_{10}5 + 5\log_{10}2 - \log_{10}2$

$4\log_{10}5 + 4\log_{10}2$

$\log_{10}5^4 + \log_{10}2^4$

$= \log_{10}5^4(2^4)$

$= \log_{10}(5\times2)^4$

$= 4\log_{10}10$

$= \underline{4}$

$\log m + \log n = \log mn$

$x^m \cdot y^m = (xy)^m$

QUESTION

Suppose $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = \frac{x^2}{1+x^2}$. What is the range of the function?

A $[0,1)$

B $[0,1]$

C $(0,1]$

D $(0,1)$

$$f(x) = y$$

$$x^2 = y + x^2 y$$

$$x^2(1-y) = y$$

$$x^2 = \frac{y}{1-y}$$

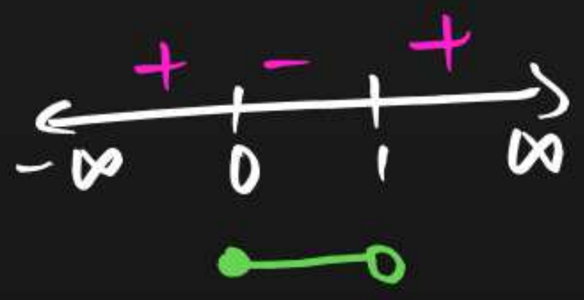
WKT $x^2 \geq 0$

$$\frac{y}{1-y} \geq 0$$

$$\frac{y}{-(y-1)} \geq 0$$

xly by -1

$$\frac{y}{y-1} \leq 0 \longrightarrow y \neq 1 \text{ in the soln}$$



$$y \in [0,1)$$

if $f(x) = A\sin x \pm B\cos x$

or

$A\cos x \pm B\sin x$

Range $= \left[-\sqrt{A^2+B^2}, \sqrt{A^2+B^2}\right]$

Ex:

$f(x) = 2\sin x + 3\cos x$

$A = 2 \quad B = 3$

$\sqrt{A^2+B^2} = \sqrt{4+9} = \sqrt{13}$

Range $= \left[-\sqrt{13}, \sqrt{13}\right]$

② if $f(x) = 4\cos x - 3\sin x + 2$ find range

$$\underset{A=4}{\downarrow} \qquad \underset{B=-3}{\downarrow}$$

$$\sqrt{A^2+B^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$\text{Range} = [-5+2,\ 5+2]$$

$$= \underline{[-3, 7]}$$

$$\int \frac{1}{\sqrt{3}\sin x - \cos x}\,dx$$

$$\int \frac{1}{\sqrt{3}\sin x - \cos x}\,dx$$

$$\int \frac{1}{\sqrt{3}\sin x - (1)\cos x}\,dx$$

$A = \sqrt{3}$ & $B = 1$

$$\sqrt{A^2+B^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$I = \int \frac{1}{2\left[\frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x\right]}\,dx$$

$\frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$ | $\frac{1}{2} = \sin\frac{\pi}{6}$

$$I = \frac{1}{2}\int \frac{1}{\sin x\cos\frac{\pi}{6} - \cos x\sin\frac{\pi}{6}}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\sin\left(x - \frac{\pi}{6}\right)}\,dx$$

$$I = \frac{1}{2}\int \operatorname{cosec}\left(x - \frac{\pi}{6}\right)dx$$

$$I = \frac{1}{2}\log\left|\operatorname{cosec}\left(x - \frac{\pi}{6}\right) - \cot\left(x - \frac{\pi}{6}\right)\right| + c$$

WKT $\operatorname{cosec}\theta - \cot\theta = \tan\frac{\theta}{2}$

$$I = \frac{1}{2}\log\left|\tan\left(\frac{x - \pi/6}{2}\right)\right| + c$$

$$I = \frac{1}{2}\log\left|\tan\left(\frac{x}{2} - \frac{\pi}{12}\right)\right| + c$$

QUESTION

If $f: \mathbb{R} \rightarrow S$ defined by $f(x) = 4\sin x - 3\cos x + 1$ is onto, then what is S equal to?

$A = 4$ $\quad B = 3$

codomain = Range = S

(A) $[-5,5]$

(B) $(-5,5)$

(C) $(-4,6)$

(D) $[-4,6]$

$\sqrt{A^2 + B^2} = \sqrt{16+9} = \sqrt{25} = 5$

$S = \text{Range} = [-5+1, +5+1]$

$= [-4, 6]$

QUESTION

What is the range of $f(x) = \cos 2x - \sin 2x$?

$f(x) = (1)\cos 2x - (1)\sin 2x$

$A = 1$ $B = 1$

A. $[2,4]$

B. $[-1,1]$

C. $(-\sqrt{2}, \sqrt{2})$

D. $[-\sqrt{2}, \sqrt{2}]$

Range $= [-\sqrt{2}, \sqrt{2}]$

QUESTION

The range of $f(x) = \cos x - \sin x$ is

A $(-1,1)$

B $[-1,1)$

C $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

D $[-\sqrt{2}, \sqrt{2}]$

QUESTION

while converting into completing the square form

$\Downarrow$

Make sure coefficient of x^2 is $\underline{+1}$

The range of $f(x) = (3x^2 + 7x + 10)$ is

$$3\left[x^2 + \frac{7}{3}x + \frac{10}{3}\right]$$

$$= 3\left[x^2 + \frac{7}{3}x + \left(\frac{7}{6}\right)^2 - \left(\frac{7}{6}\right)^2 + \frac{10}{3}\right]$$

$$= 3\left[\left(x + \frac{7}{6}\right)^2 - \frac{49}{36} + \frac{10}{3}\right]$$

$$= 3\left[\left(x + \frac{7}{6}\right)^2 + \frac{71}{36}\right]$$

$$= 3\left(x + \frac{7}{6}\right)^2 + 3\left(\frac{71}{36}\right)$$

$$= 3\left(x + \frac{7}{6}\right)^2 + \frac{71}{12}$$

$$\frac{-49}{36} + \frac{10}{3}$$

$$\frac{-49 + 120}{36}$$

$$= \frac{71}{36}$$

(A) $\left[\frac{70}{3}, \infty\right[$ $= \left[\frac{70}{3}, \infty\right)$

(B) $\left[\frac{71}{12}, \infty\right[$ $= \left[\frac{71}{12}, \infty\right)$ ✓

(C) $[0, \infty[$ $= [0, \infty)$

(D) $]-\infty, -3]$ $= (-\infty, -3]$

WKT

$$3\left(x+\frac{7}{6}\right)^2 \geq 0$$

$$3\left(x+\frac{7}{6}\right)^2+\frac{71}{12} \geq \frac{71}{12}$$

$$f(x) \geq \frac{71}{12}$$

$$\text{Range} = \left[\frac{71}{12}, \infty\right)$$

method (2)

$f(x) = 3x^2 + 7x + 10$

consider

$f(x) = y$

$3x^2 + 7x + 10 = y$

$3x^2 + 7x + (10 - y) = 0$

$a = 3 \mid b = 7 \mid c = 10 - y$

$b^2 - 4ac \geq 0$

$49 - 4(3)(10 - y) \geq 0$

$49 - 120 + 12y \geq 0$

$-71 + 12y \geq 0$

$12y \geq 71$

$y \geq \frac{71}{12}$

$y \in \left[\frac{71}{12}, \infty\right)$

QUESTION

The range of $f(x) = x + \frac{1}{x}$, is:

$[2, \infty)$

(A) $[2, \infty[$

(B) $[-2,2]$

$(-\infty, -2]$

(C) $]-\infty, -2]$

(D) None of these ✓

$f(x) = \frac{x^2+1}{x} = y$

$x^2+1 = xy$

$x^2 - xy + 1 = 0$

$a=1 \mid b=-y \mid c=1$

$b^2 - 4ac \geq 0$

$y^2 - 4 \geq 0$

$y^2 \geq 4$

$\sqrt{y^2} \geq \sqrt{4}$

$|y| \geq 2$

$y \in (-\infty, -2] \cup [2, \infty)$

QUESTION

If $f:[0,\pi/2)\rightarrow R$ is defined as $f(\theta)=\begin{vmatrix}1 & \tan\theta & 1\\ -\tan\theta & 1 & \tan\theta\\ -1 & -\tan\theta & 1\end{vmatrix}$. Then the range of f is

A $(2,\infty)$

B $(-\infty,-2]$

C $[2,\infty)$

D $(-\infty,2]$

$f(\theta)=1[1+\tan^2\theta]-\tan\theta(0)+1(\tan^2\theta+1)$

$=\sec^2\theta+\sec^2\theta$

$=2\sec^2\theta$

WKT $\sec\theta\in(-\infty,-1]\cup[1,\infty)$

$\sec^2\theta\in[1,\infty)$

$2\sec^2\theta\in[2,\infty)$

$f(\theta)\in[2,\infty)$

QUESTION

$f(x)$ is real valued function such that $2f(x) + 3f(-x) = 15 - 4x$ for all $x \in \mathrm{R}$. Then $f(2) =$

(A) -15

(B) 22

(C) 11

(D) 0

$2f(x) + 3f(-x) = 15 - 4x \rightarrow (1)$

Replace x by $-x$

$2f(-x) + \underline{3f(x)} = 15 + 4x \rightarrow (2)$

$(1) \times 2 \Rightarrow 4f(x) + 6\cancel{f(-x)} = 30 - 8x$

$(2) \times 3 \Rightarrow 9f(x) + 6\cancel{f(-x)} = 45 + 12x$

$-5f(x) = -15 - 20x$

$f(x) = 3 + 4x$

$f(x) = 3 + 4x$

$f(2) = 3 + 8$

$= 11$

QUESTION

If $f(x) = 4x - x^2$, then $f(a+1) - f(a-1) =$

A $4(2-a)$

B $2(4-a)$

C $4(2+a)$

D $2(4+a)$

QUESTION

The range of the function $f(x) = \frac{x+2}{|x+2|}$ is

output

(A) $\{0,1\}$

(B) $\{-1,1\}$

(C) R

(D) $R - \{-2\}$

Case 1:- if $x+2 \geq 0$

$|x+2| = x+2$

$f(x) = \frac{x+2}{x+2} = 1$

Case 2:- if $x+2 < 0$

$x < -2$

$|x+2| = -(x+2)$

$f(x) = \frac{x+2}{-(x+2)} = -1$

Range $= \{1, -1\}$

Ex:- $x = 3 > -2$

$f(3) = \frac{3+2}{|3+2|} = \frac{5}{|5|} = \frac{5}{5}$

$f(3) = 1$

$x = -5 < -2$

$f(-5) = \frac{-5+2}{|-5+2|} = \frac{-3}{|-3|}$

$= \frac{-3}{3} = -1$

$f(-5) = -1$

Find Domain & Range of $f(x) = \sqrt{x^2-4}$

$f(x) = \sqrt{g(x)}$

(1) $g(x) \geq 0$

(2) $f(x) \geq 0 \Rightarrow y \geq 0$

Soln:

Domain:

consider $f(x) = \sqrt{x^2-4}$

$x^2-4 \geq 0$

$x^2 \geq 4$

$\sqrt{x^2} \geq \sqrt{4}$

$|x| \geq 2$

$x \in (-\infty, -2] \cup [2, \infty)$

Range:

consider

$f(x) = y$

$\sqrt{x^2-4} = y \quad (y \geq 0)$

$x^2-4 = y^2$

$x^2 = y^2+4$

Here WKT $x^2 \geq 0$

$\Rightarrow y^2+4 \geq 0$

The above statement is true $\forall y \in R \rightarrow$ (1)

but $y \geq 0 \rightarrow$ (2)

$\therefore$ (1) $\cap$ (2)

$y \geq 0$

$y \in [0, \infty)$

$\Downarrow$

Range

Find Domain & Range of $f(x) = \sqrt{4-x^2}$

Soln:-

Domain:-

$f(x) = \sqrt{4-x^2}$

$4 - x^2 \geq 0$

$4 \geq x^2$

$x^2 \leq 4$

$\sqrt{x^2} \leq \sqrt{4}$

$|x| \leq 2$

$x \in [-2, 2]$

Range:-

Consider

$f(x) = y$

$\sqrt{4-x^2} = y \quad (y \geq 0) \rightarrow 1$

$4 - x^2 = y^2$

$x^2 = 4 - y^2$

WKT $x^2 \geq 0$

$4 - y^2 \geq 0$

$4 \geq y^2$

$y^2 \leq 4$

$\sqrt{y^2} \leq \sqrt{4}$

$|y| \leq 2$

$y \in [-2, 2] \rightarrow 2$

$\therefore 1 \cap 2$

$y \in [0, 2]$

Range

Find Domain & Range of $f(x) = \frac{1}{\sqrt{x^2-4}}$

Soln:-

Domain:-

$$f(x) = \frac{1}{\sqrt{x^2-4}}$$

$$x^2 - 4 > 0$$

$$x^2 > 4$$

$$|x| > 2$$

$$x \in (-\infty, 2) \cup (2, \infty)$$

Range:-

consider

$$f(x) = y$$

$$\frac{1}{\sqrt{x^2-4}} = y \quad (y > 0) \rightarrow ①$$

$$\sqrt{x^2-4} = \frac{1}{y}$$

$$x^2 - 4 = \frac{1}{y^2}$$

$$x^2 = \frac{1}{y^2} + 4$$

WKT

$$x^2 \geq 0$$

$$\frac{1}{y^2} + 4 \geq 0$$

The above statement is true

$$\forall\, y \in R - \{0\} \rightarrow ②$$

$$\therefore ① \cap ②$$

$$y > 0$$

$$y \in (0, \infty)$$

Find the domain & Range of $f(x) = \frac{1}{\sqrt{4-x^2}}$

Soln:-

Domain:-

$f(x) = \frac{1}{\sqrt{4-x^2}}$

$4 - x^2 > 0$

$4 > x^2$

$x^2 < 4$

$|x| < 2$

$x \in (-2, 2)$

Range:-

consider

$f(x) = y$

$\frac{1}{\sqrt{4-x^2}} = y$ $(y > 0) \rightarrow 1$

$\sqrt{4-x^2} = \frac{1}{y}$

$4 - x^2 = \frac{1}{y^2}$

$x^2 = 4 - \frac{1}{y^2}$

WKT

$x^2 \geq 0$

$4 - \frac{1}{y^2} \geq 0$

$4 \geq \frac{1}{y^2}$

cross multiplication is allowed, since y^2 is always +ve

$4y^2 \geq 1$

$y^2 \geq \frac{1}{4}$

$|y| \geq \frac{1}{2}$

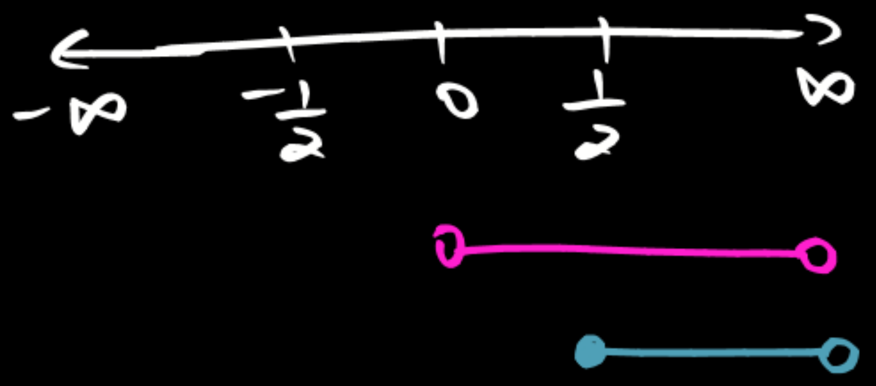
$y \in (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, \infty) \rightarrow 2$

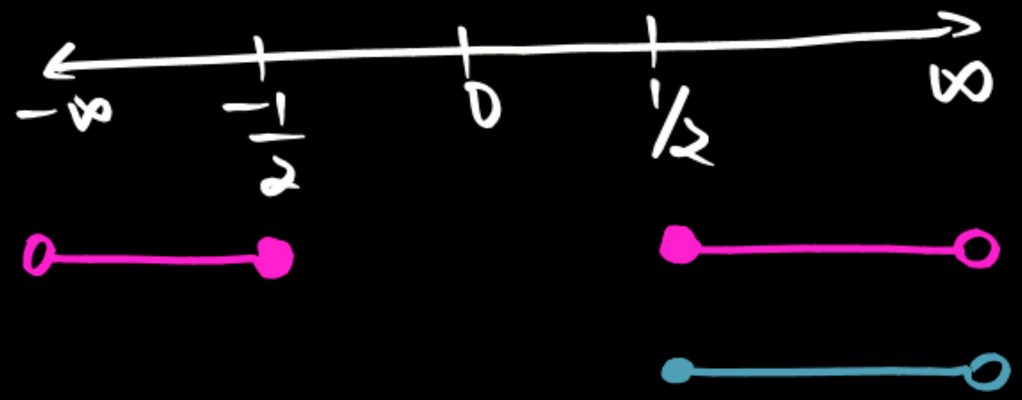
$1 \cap 2$

$y \in [\frac{1}{2}, \infty) \rightarrow$ Range

$y > 0$

$y \in \left(-\infty, -\frac{1}{2}\right] \cup \left[\frac{1}{2}, \infty\right)$





$y \in \left[\frac{1}{2}, \infty\right)$

QUESTION

The range of the function $f(x)=\sqrt{9-x^2}$ is

(A) ✗ $(0,3)$

(B) ✓ $[0,3]$

(C) ✗ $(0,3]$

(D) $[0,3)$

Consider

$f(x)=y$

$\sqrt{9-x^2}=y$ $(y\geq 0)$ $\rightarrow$ (1)

$9-x^2=y^2$

$x^2=9-y^2$

WKT

$x^2\geq 0$

$9-y^2\geq 0$

$9\geq y^2$

$y^2\leq 9$

$|y|\leq 3$

$y\in[-3,3]$ $\rightarrow$ (2)

(1) $\cap$ (2)

$y\in[0,3]$

QUESTION

Range of the function $f(x)=\sqrt{x^2+x+1}$ is equal

(A) $[0,\infty]$

(B) $\left[\frac{\sqrt{3}}{2},\infty\right)$

(C) $\left(\frac{\sqrt{3}}{2},\frac{-\sqrt{3}}{2}\right)$

(D) $(0,0)$

Consider

$$x^2+x+1 = \underbrace{x^2+x+\left(\tfrac{1}{2}\right)^2}-\left(\tfrac{1}{2}\right)^2+1$$

$$= \left(x+\tfrac{1}{2}\right)^2-\tfrac{1}{4}+1$$

$$x^2+x+1 = \left(x+\tfrac{1}{2}\right)^2+\tfrac{3}{4}$$

WKT

$$\left(x+\tfrac{1}{2}\right)^2 \geq 0$$

$$\left(x+\tfrac{1}{2}\right)^2+\tfrac{3}{4} \geq \tfrac{3}{4}$$

$$x^2+x+1 \geq \tfrac{3}{4}$$

$$\sqrt{x^2+x+1} \geq \sqrt{\tfrac{3}{4}}$$

$$f(x) \geq \tfrac{\sqrt{3}}{2}$$

$$f(x) \in \left[\tfrac{\sqrt{3}}{2},\infty\right)$$

2^{nd} method

consider $f(x) = y$

$$\sqrt{x^2+x+1} = y \quad (y \geq 0)$$

$$x^2+x+1 = y^2$$

$$x^2+x+(1-y^2) = 0$$

$a = 1 \mid b = 1 \mid c = 1-y^2$

$$b^2-4ac \geq 0$$

$$1-4(1-y^2) \geq 0$$

$$1-4+4y^2 \geq 0$$

$$4y^2 \geq 3$$

$$y^2 \geq \frac{3}{4}$$

$$|y| \geq \frac{\sqrt{3}}{2}$$

$$y \in \left(-\infty, -\frac{\sqrt{3}}{2}\right] \cup \left[\frac{\sqrt{3}}{2}, \infty\right) \rightarrow 2$$

$$1 \cap 2$$

$$y \in \left[\frac{\sqrt{3}}{2}, \infty\right)$$

QUESTION

$\log_e e = 1$ | $\log_e 4 = \log_e 2^2 = 2\log_e 2$

The range of the function $f(x) = \log_e(3x^2 + 4)$ is equal to

(A) $[\log_e 2, \infty]$

(B) $[\log_e 3, \infty)$

(C) $[2\log_e 3, \infty)$

(D) $[2\log_e 2, \infty)$ ✓

Consider

$f(x) = y$

$\log_e(3x^2+4) = y$

$3x^2+4 = e^y$

$3x^2 = e^y - 4$

$x^2 = \dfrac{e^y - 4}{3}$

WKT $x^2 \geq 0$

$\dfrac{e^y - 4}{3} \geq 0$

$e^y - 4 \geq 0$

$e^y \geq 4$

Take $\log_e$ on B·S

$\log_e e^y \geq \log_e 4$

$y \log_e e \geq 2\log_e 2$

$y \geq 2\log_e 2$

Range

$= [2\log_e 2, \infty)$

2nd method

$$f(x) = \log_e(3x^2+4)$$

WKT

$$x^2 \geq 0$$

$$3x^2 \geq 0$$

$$3x^2+4 \geq 4$$

Take $\log_e$ on BS

$$\log_e(3x^2+4) \geq \log_e 4$$

$$f(x) \geq 2\log_e 2$$

$$\text{Range} = [2\log_e 2, \infty)$$

QUESTION

$\log(-ve) = -\infty$

The range of function $f(x) = \log_e \sqrt{4 - x^2}$ is given by

A. $(0, \infty)$

B. $(-\infty, \infty)$

C. $(-\infty, \log_e 2]$

D. $(\log_e 2, \infty)$

Consider

$f(x) = y$

$\log_e \sqrt{4-x^2} = y$

$\sqrt{4-x^2} = e^y$

$4-x^2 = (e^y)^2 = e^{2y}$

$x^2 = 4 - e^{2y}$

WKT $x^2 \geq 0$

$4 - e^{2y} \geq 0$

$4 \geq e^{2y}$

$e^{2y} \leq 4$

$(e^y)^2 \leq 4$

$|e^y| \leq 2$

$-2 \leq e^y \leq 2$

Take $\log_e$ Throughout

$\log_e(-2) \leq \log_e e^y \leq \log_e 2$

$-\infty \leq y \log_e e \leq \log_e 2$

$-\infty \leq y \leq \log_e 2$

$y \in (-\infty, \log_e 2]$

QUESTION

The range of the function $f(x) = \tan\sqrt{\frac{\pi^2}{9} - x^2}$ is

(A) $[0,3]$

(B) $[0, \sqrt{3}]$

(C) $[3, \sqrt{3}]$

(D) $[\sqrt{3}, 3]$

$f(x) = \sqrt{g(x)}$

① $f(x) \geq 0$

② $g(x) \geq 0$

WKT

$x^2 \geq 0$

$-x^2 \leq 0$

$\frac{\pi^2}{9} - x^2 \leq \frac{\pi^2}{9}$

$\sqrt{\frac{\pi^2}{9} - x^2} \leq \sqrt{\frac{\pi^2}{9}}$

$\sqrt{\frac{\pi^2}{9} - x^2} \leq \frac{\pi}{3}$

WKT sqrt func ≥ 0

$0 \leq \sqrt{\frac{\pi^2}{9} - x^2} \leq \frac{\pi}{3}$

Take Tan Throughout

$\tan 0 \leq \tan\sqrt{\frac{\pi^2}{9} - x^2} \leq \tan\frac{\pi}{3}$

$0 \leq f(x) \leq \sqrt{3}$

QUESTION

If $f: R - \{2\} \rightarrow R$ is a function defined by $f(x) = \frac{x^2-4}{x-2}$, then its range is

A R

B $R - \{2\}$

C $R - \{4\}$

D $R - \{-2,2\}$

QUESTION

The range of the function $f(x) = \frac{1}{\sqrt{x^2-9}}, x \in (3, \infty)$ is

A $(-3,3)$

B $[-3,3]$

C $(3, \infty)$

D $(0, \infty)$

QUESTION

The range of the function $f(x) = \frac{x-3}{5-x}, x \neq 5$ is

A $R - \{-1\}$

B $R - \{1\}$

C $R - \{5\}$

D $R - \{-5\}$

QUESTION

The range of the function $f(x) = \frac{x^2}{x^2+1}$ is [2023]

(A) $(0,1)$

(B) $[0,1)$

(C) $(0,1]$

(D) $[0,1]$

Thank

You