ARJUNA 2.0 JEE 2026

Rotational Motion

Physics | Lecture : 4

By- Rajwant Singh (RJ)



Topics to be covered

1. Additive and Negative systems
2.
3.
4.

Revised ✓ 85%

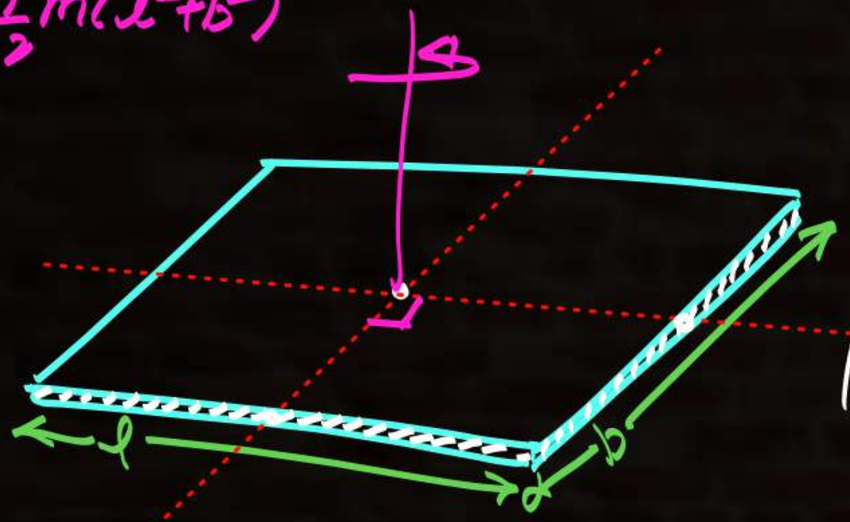
RJ special (COM lec-1).

Scholarx

- Rocket propulsion + Questions
- Variable max System + Ques
- COM frame + Questions

Rectangle (l×b)

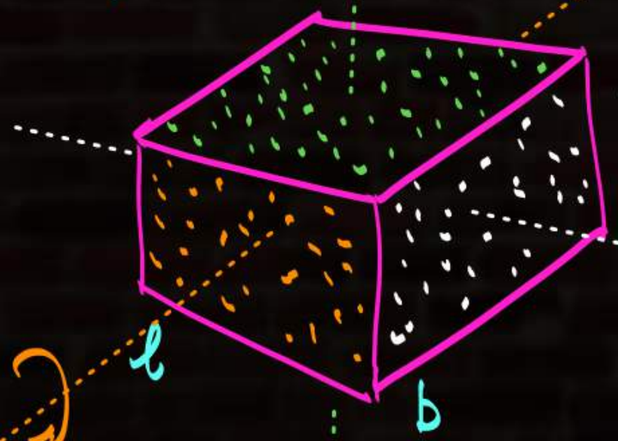
$I_z = \frac{1}{12}m(l^2+b^2)$



$\frac{1}{12}Mb^2$ (I_x)

$\frac{1}{12}Ml^2$ (I_y)

#

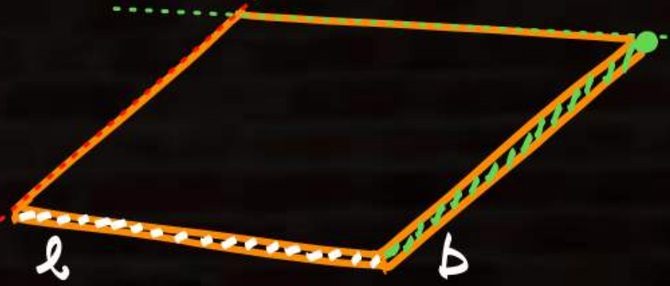


$\frac{1}{12}M(l^2+b^2)$

$\frac{1}{12}M(b^2+h^2)$

$\frac{1}{12}M(l^2+h^2)$

#

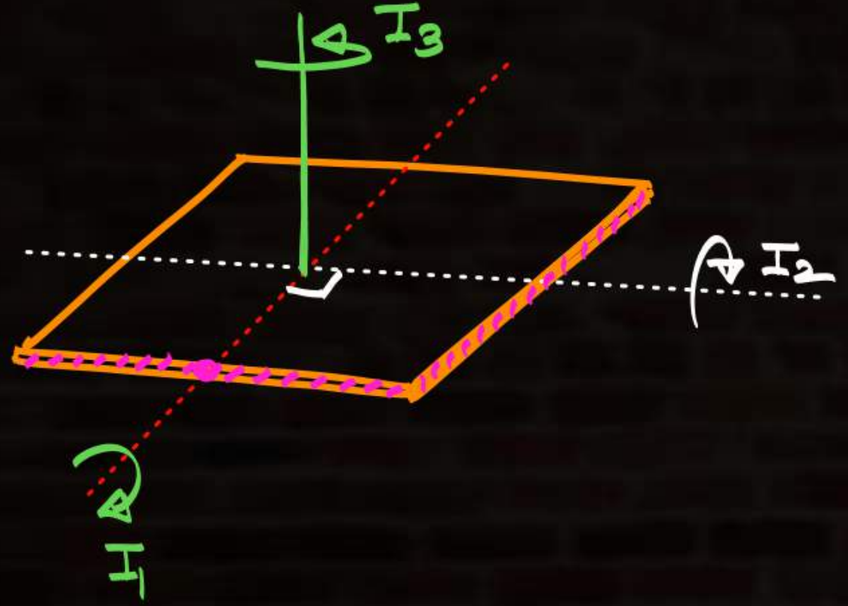


$\frac{1}{3}Mb^2$

$\frac{1}{3}Ml^2$

Square plate:

Mass = M (l × l)

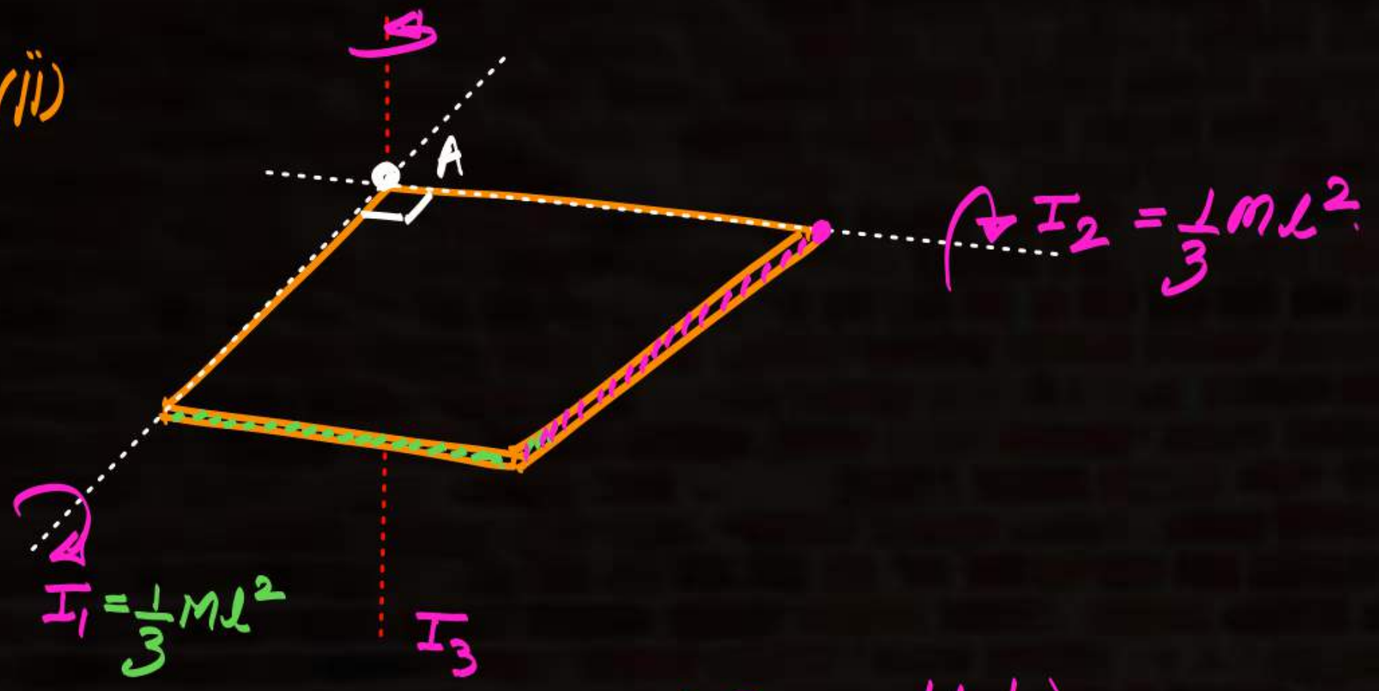


$$I_1 = \frac{1}{12} m l^2$$

$$I_2 = \frac{1}{12} m l^2$$

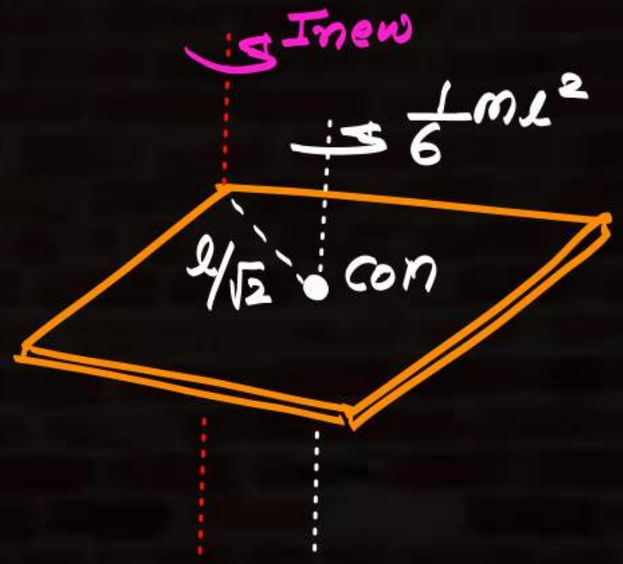
$$I_3 = I_2 + I_1 = \frac{1}{6} M l^2$$

(ii)



($\perp$ to plane of Square plate).

We Cannot apply $\perp$ Axis theorem at (A) because it is not CM.

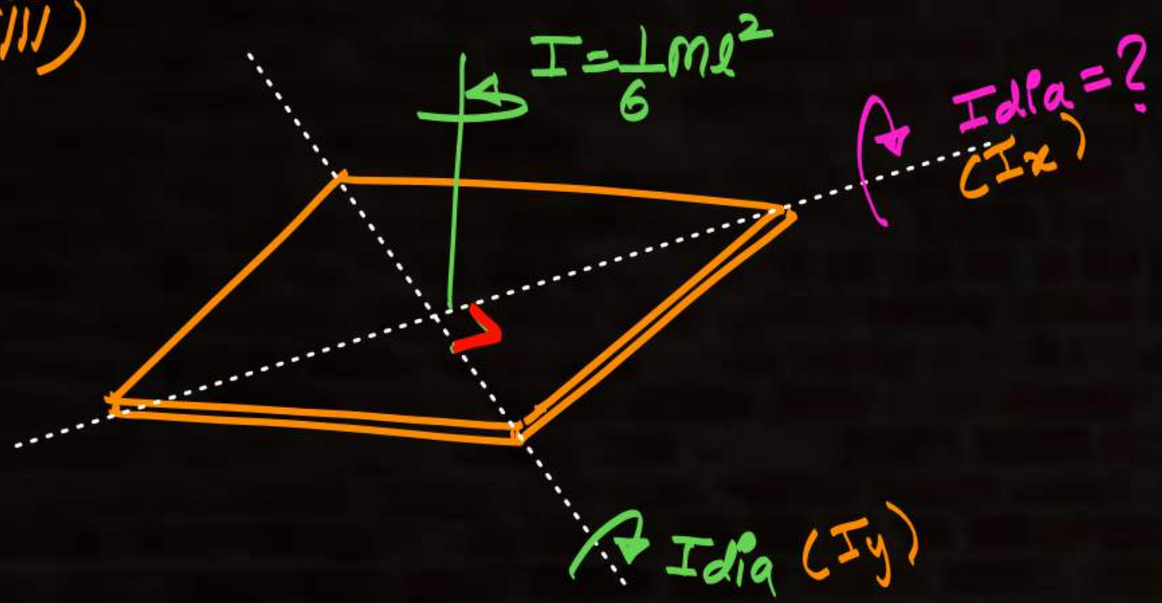


$$I_{new} = I_c + Mh^2$$

$$= \frac{1}{6}ml^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$$

$$= \frac{ml^2}{6} + \frac{ml^2}{2} = \frac{ml^2}{2}\left[\frac{4}{3}\right] = \frac{2ml^2}{3}$$

(iii)



diagonals (90°)

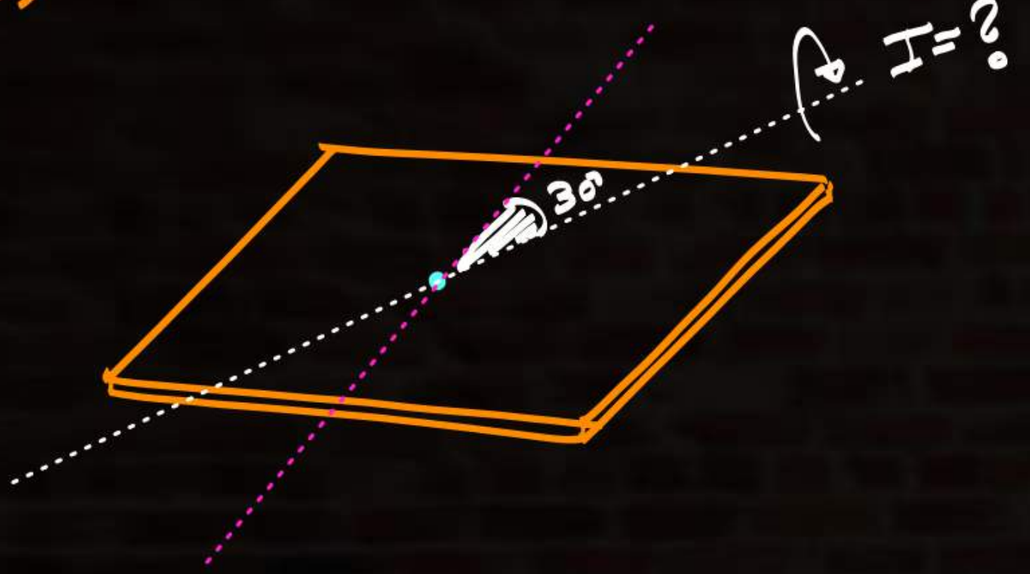
Square plate of M (l×l).

$$I_z = I_x + I_y.$$

$$\frac{ml^2}{6} = 2\,I_{dia}$$

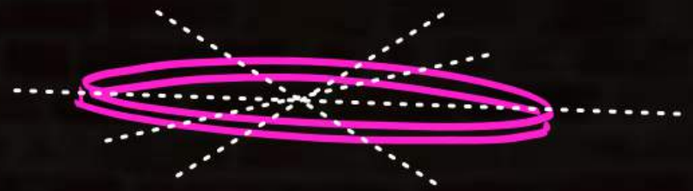
$$\boxed{\frac{ml^2}{12} = I_{dia}}$$

(iv)

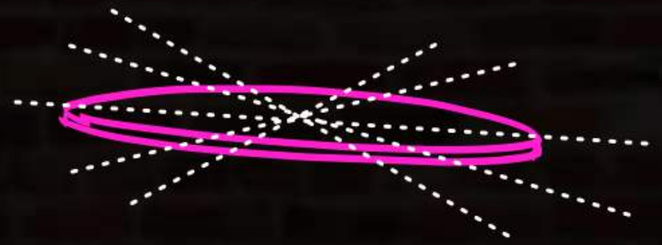


Ans = $I = \frac{1}{12}ML^2$ "Independent of θ"

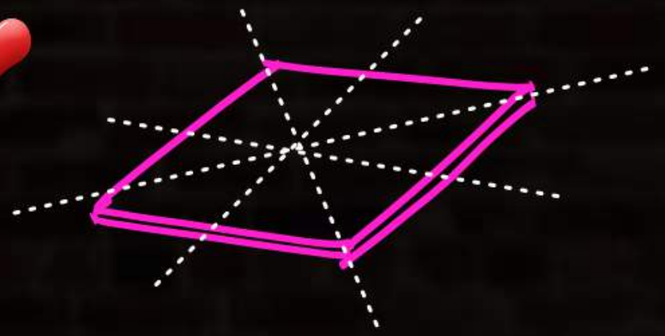
Koi Sa bhi axis agar Square Ke plane mein hai aur COM se pass Kar Raha hai toh $I = \frac{1}{12}ML^2$ hi hoga Kyunki Koi bhi axis Lo wo Square Ko do Equal parts mein Todega|



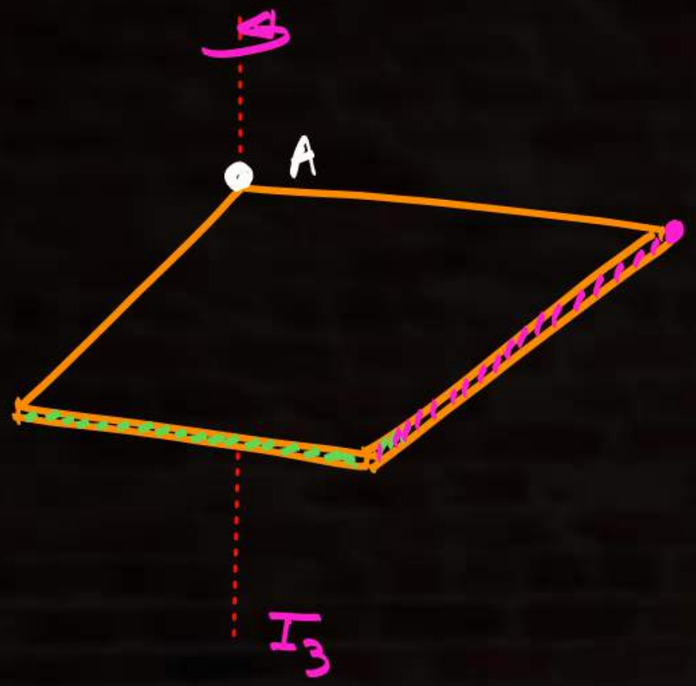
all I_{dia} will be Same $= \frac{MR^2}{2}$



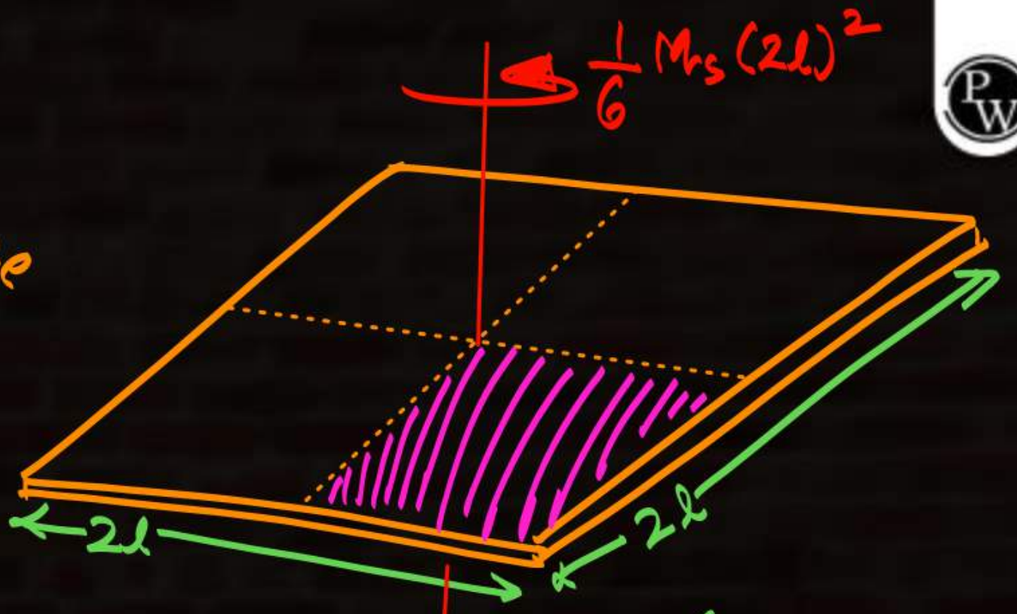
all I_{dia} will be Same $= \frac{MR^2}{4}$



(V)



Parent Square plate



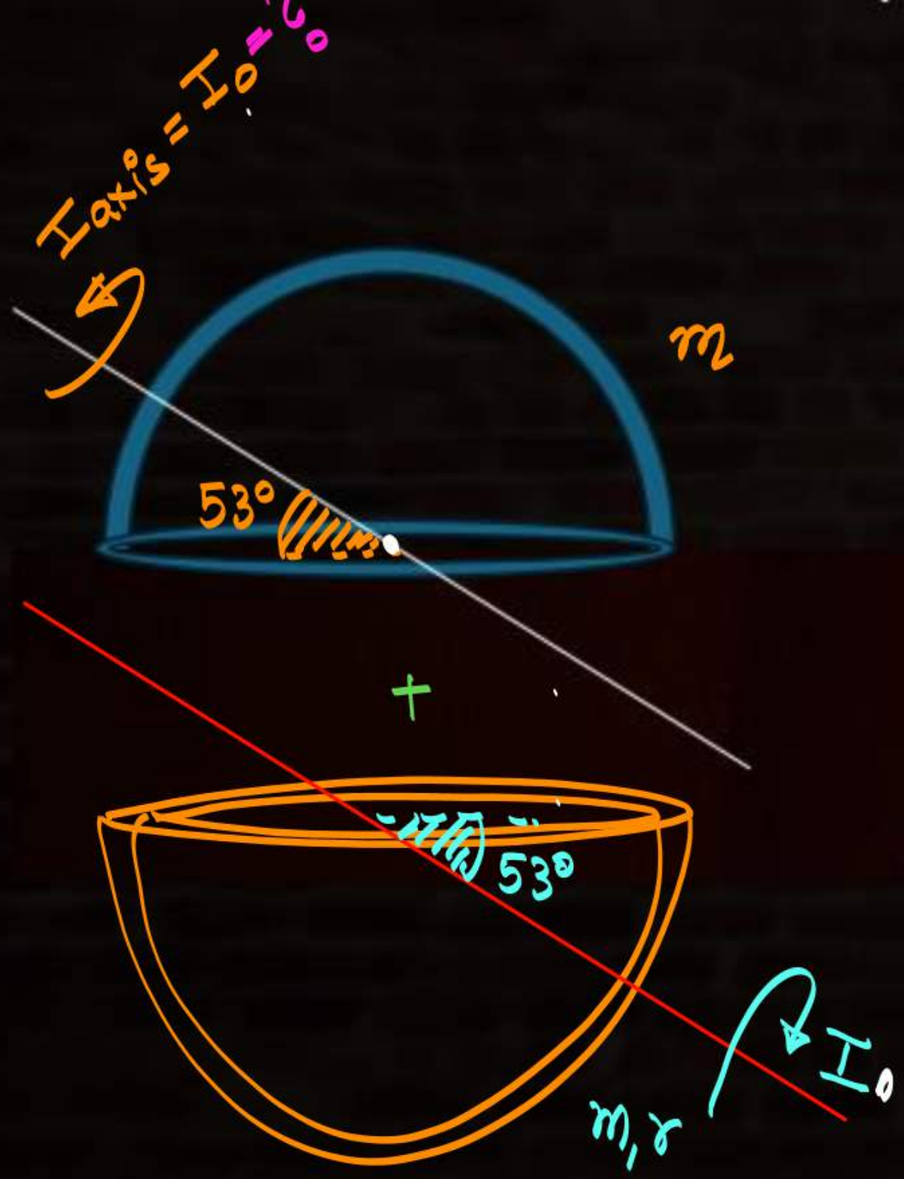
Comple Parent Plate = $\frac{1}{6} 4M (2l)^2$

$= \frac{1}{6} 4m\ 4l^2$

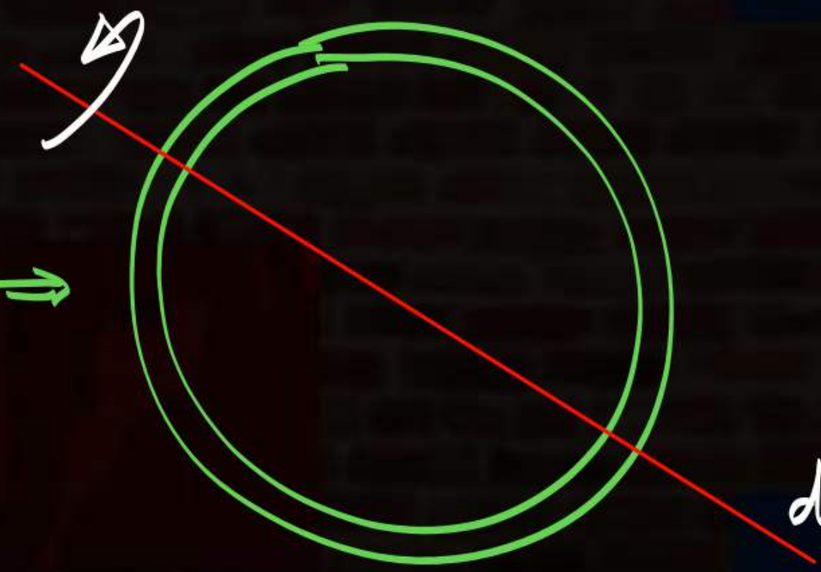
one plate $= \frac{1}{4} I_{Parent} = \frac{1}{4} \frac{1}{6} 4m\ 4l^2 = \frac{2}{3} ml^2$ //

Question 1

Hollow hemisphere of mass m and radius r



$\Rightarrow$



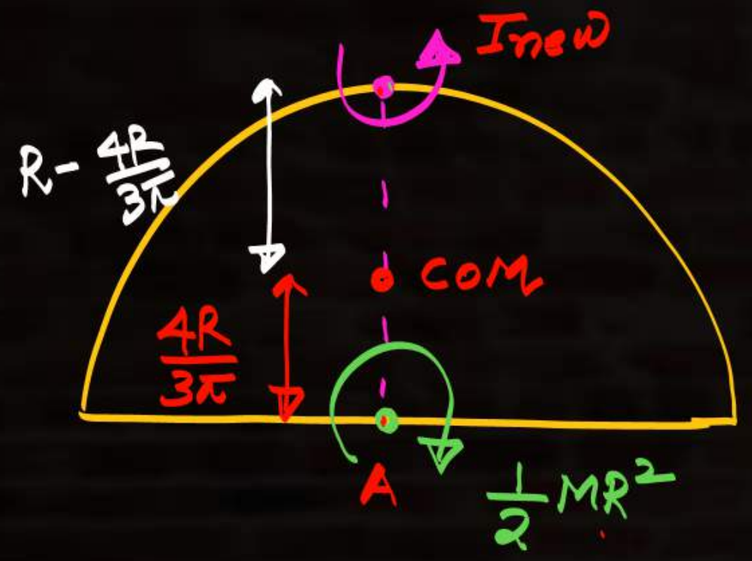
$$I_{dia} = \frac{2}{3}(2m)r^2 = I_0 + I_0$$

$$\frac{2}{3}2mr^2 = 2I_0$$

$$\boxed{I_0 = \frac{2}{3}mr^2.}$$

Example no. 1

Semi disc of Mass M & Radius = R.



Axis is at Circumference & ⊥ to plane of Semi disc.

$I_c + mh^2 = I_{new}$.

A & C

$$I_c + m\left(\frac{4R}{3\pi}\right)^2 = \frac{1}{2}MR^2$$

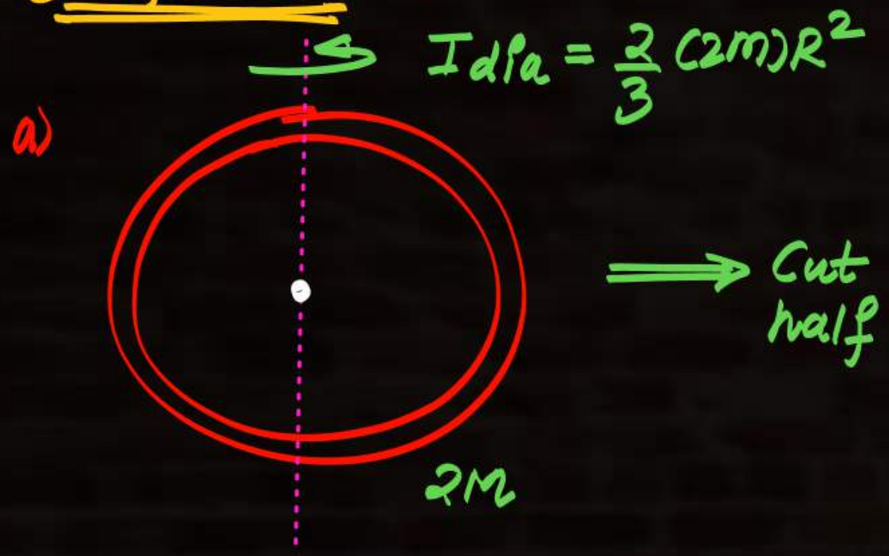
$$I_c = \frac{1}{2}mR^2 - \frac{16MR^2}{9\pi^2}$$

C → New

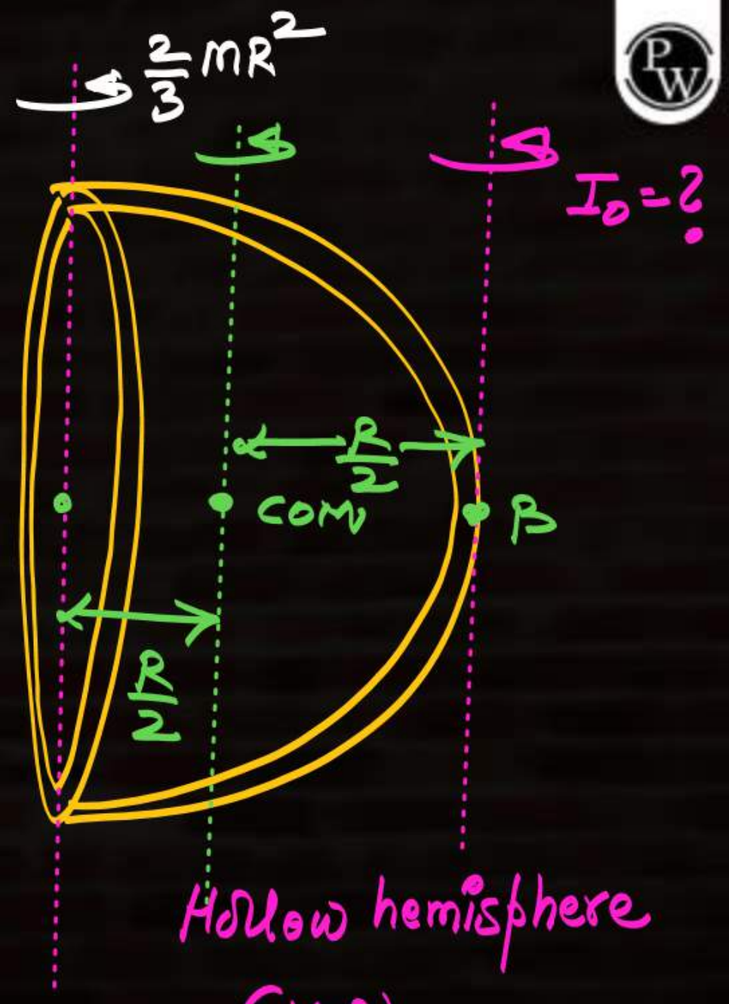
$I_c + mh^2 = I_{new}$

$$\frac{1}{2}mR^2 - \frac{16mR^2}{9\pi^2} + m\left(R - \frac{4R}{3\pi}\right)^2 = I_{new}.$$

Example no. 2







A & C

$$I_c + mh^2 = I_{new}$$

$$I_c + m\left(\frac{R}{2}\right)^2 = \frac{2}{3}mR^2$$

$$I_c = \frac{2}{3}mR^2 - \frac{mR^2}{4}$$

C & B

$$I_c + mh^2 = I_0$$

$$\frac{5}{3}mR^2 - \frac{mR^2}{4} + m\left(\frac{R}{2}\right)^2 = I_0$$

HW

Important Points:

The moment of inertia of body varies as x^2-4x+8. Find the location of COM

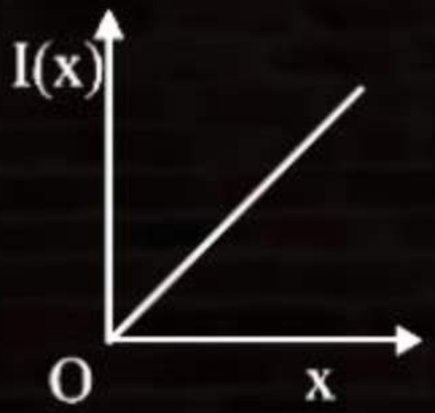
$I_{new} = I_c + Mh^2$

Kisi bhi body ka Minimum MoI hamesha CoM ke about hoga!

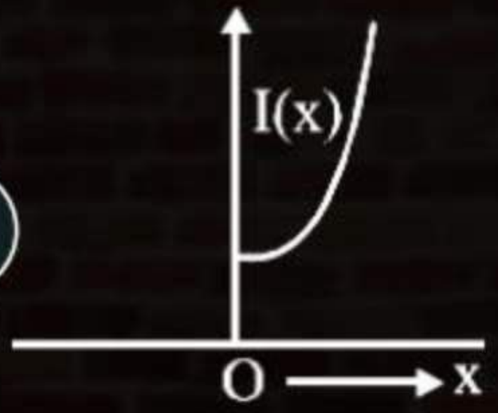
Question 2

The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is 'I(x)'. Which one of the graphs represents the variation of I(x) with x correctly? **[Main-2019]**

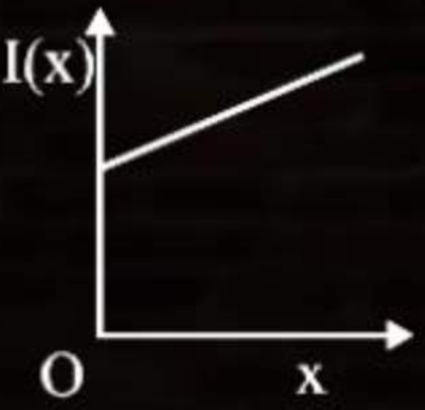
1



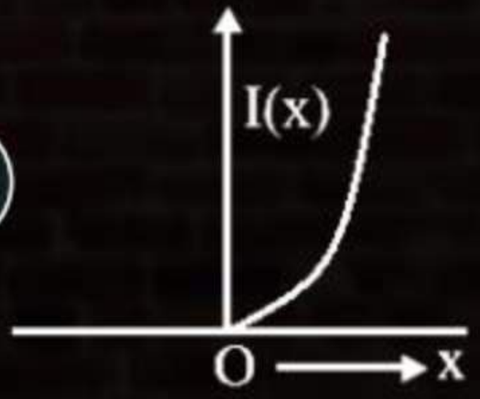
2



3



4



Important Points:

7. Additive and Negative Systems

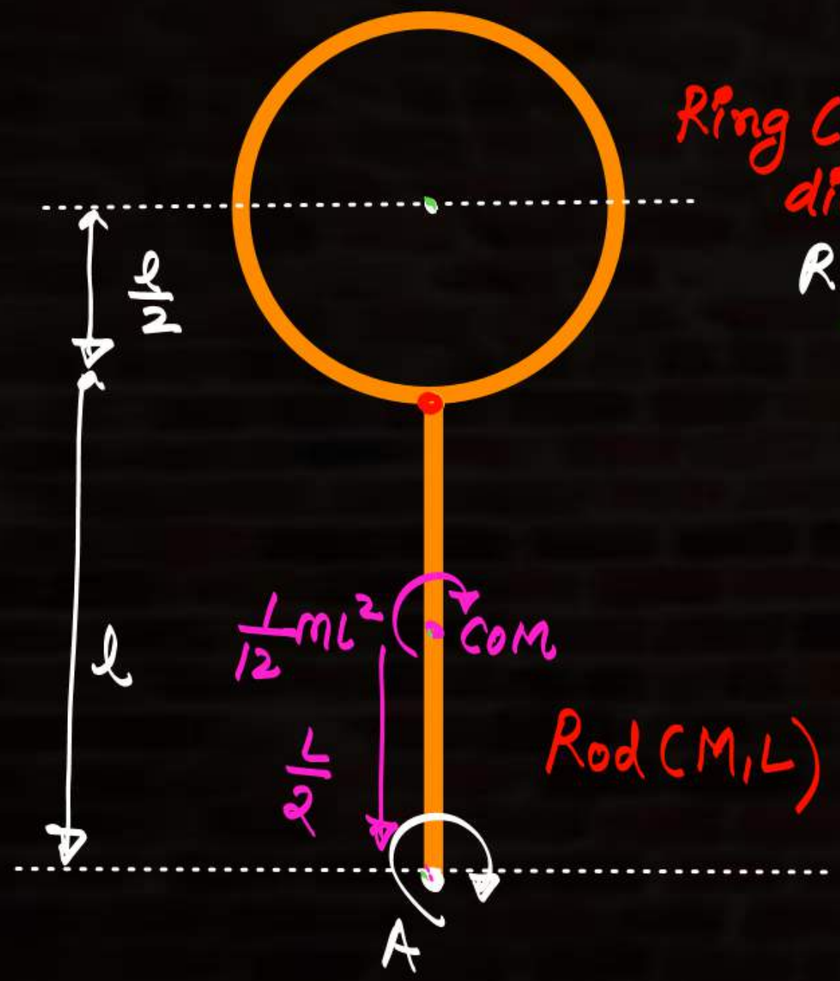
MoI → Scalar ✓ / Vector ✗

→ we can add / Subtract MoI about Common axis.

Har individual body ka MoI common axis par laaiye (IInd / I Axis theorem)

Then (add, Subtract).

Example no. 3



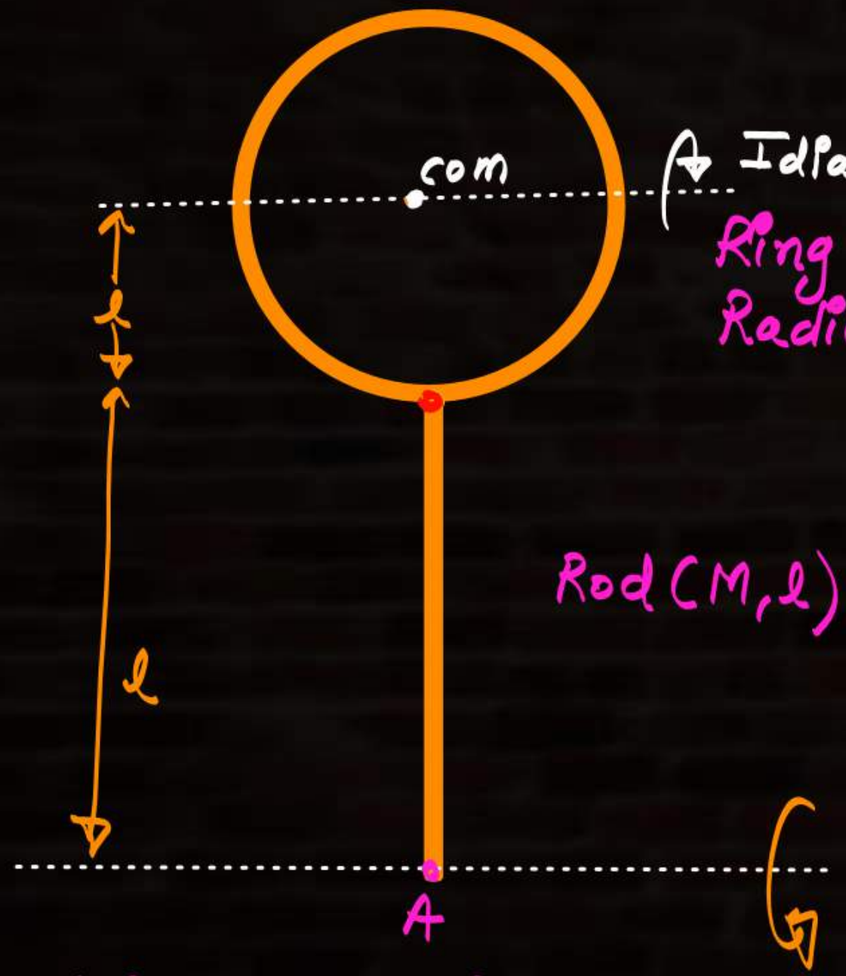
$I_{A\,system}$ (axis along Z)

$$I_{Rod_A} = I_c + mh^2$$
$$= \frac{1}{12}m\ell^2 + m\left(\frac{\ell}{2}\right)^2 = \frac{1}{3}m\ell^2$$

$$I_{Ring_A} = I_c + mh^2$$
$$= M\left(\frac{\ell}{2}\right)^2 + M\left(\frac{3\ell}{2}\right)^2$$
$$= \frac{m\ell^2}{4} + \frac{9m\ell^2}{4} = \frac{10m\ell^2}{4} = \frac{5}{2}m\ell^2$$

$$I_{sy} = I_{Rod} + I_{Ring} \;|\; \text{common Axis}$$
$$= \frac{m\ell^2}{3} + \frac{5m\ell^2}{2} = \frac{2m\ell^2 + 15m\ell^2}{6} = \frac{17m\ell^2}{6}$$

Example no. 3



$I_{dia\,ring} = \frac{M\ell^2}{2}$

Ring (M)

Radius = ℓ.

$I_{Rod\,A} = \frac{1}{3}m\ell^2$

$I_{Ring\,A} = I_c + mh^2$

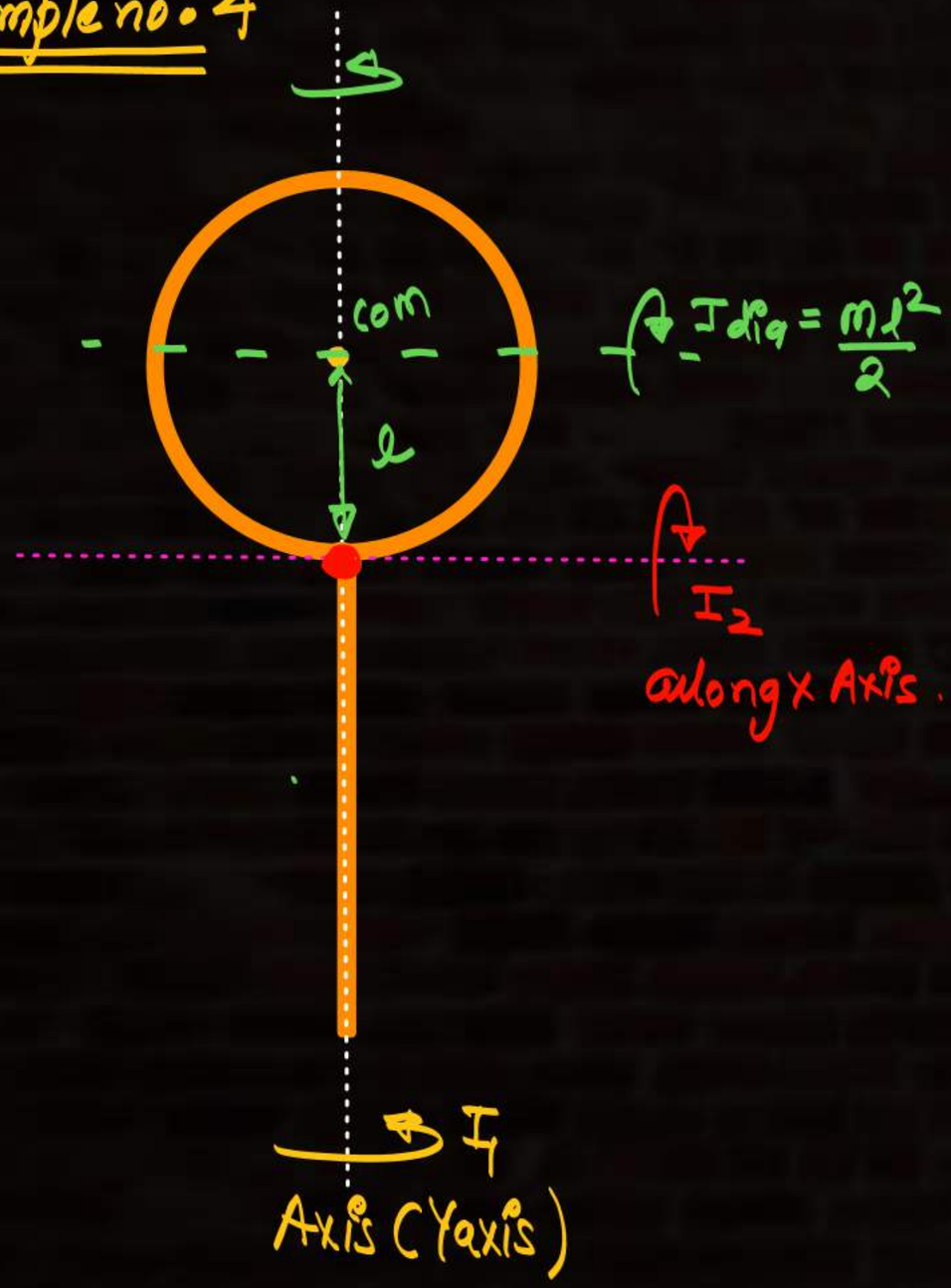
$= \frac{m\ell^2}{2} + m(2\ell)^2$

$= \frac{m\ell^2}{2} + 4m\ell^2$

$= \frac{9}{2}m\ell^2$

$$I_{Ans} = \frac{1}{3}m\ell^2 + \frac{9}{2}m\ell^2 =$$

Example no. 4



Rod (M, ℓ)

Ring $(M,$ Radius $= \ell)$.

$I_{Ring} = \frac{MR^2}{2}$

$I_{Rod} = \frac{M\ell^2}{12}$

$I_1 = I_{Rod} + I_{Ring}$ / common axis

$= 0 + \frac{M\ell^2}{2} = \frac{M\ell^2}{2}$.

$I_2 = I_{Rod} + I_{Ring}$ / common axis.

$= \frac{1}{12}M\ell^2 + (I_c + Mh^2)$

$= \frac{1}{12}M\ell^2 + \left(\frac{M\ell^2}{2} + M\ell^2\right)$

$= \frac{1}{12}M\ell^2 + \frac{3}{2}M\ell^2$.

Example no. 4



Axis is tangent to Ring (Y axis)

Ring (M) Radius = l.

Rod (M) l.

$$I_{Total} = I_{Ring} + I_{Rod}$$

$$= (I_c + mh^2)_{Ring} + (I_c + Mh)^2_{Rod}$$

$$= \left(\frac{ml^2}{2} + ml^2\right) + (0 + ml^2)$$

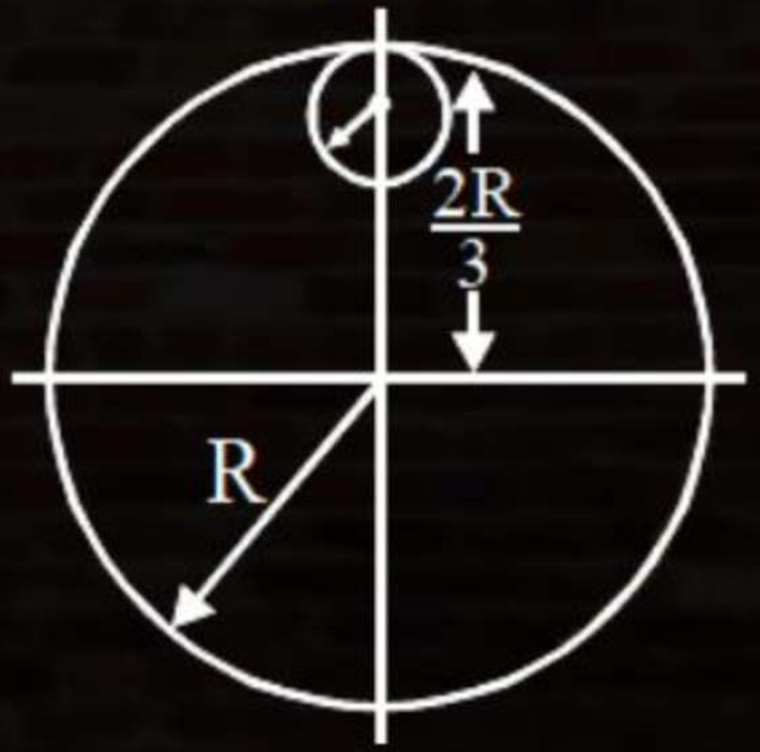
$$I_{Total} = \left(\frac{3}{2}ml^2\right)_{Ring} + (ml^2)_{Rod}$$



Question 23

From a uniform circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is

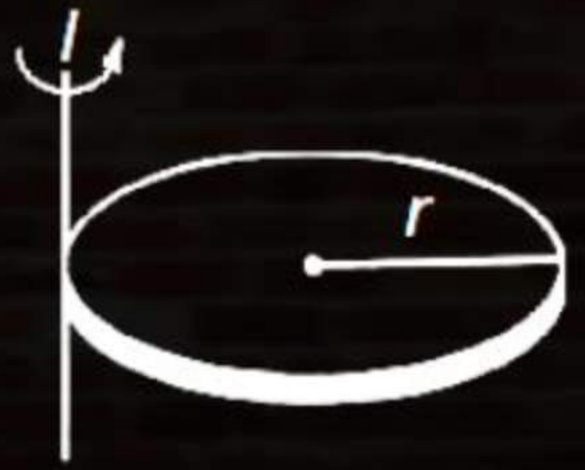
[Main-2018]



1. $\frac{37}{9}MR^2$
2. $4\ MR^2$
3. $\frac{40}{9}MR^2$
4. $10\ MR^2$

Question 25

A solid sphere of radius R has moment of inertia I about its geometrical axis. It is melted into a disc of radius r and thickness t. If it's moment of inertia about the tangential axis (which is perpendicular to plane of the disc), is also equal to I, then the value of r is equal to [2016]



1. $\frac{2}{\sqrt{15}}R$
2. $\frac{2}{\sqrt{5}}R$
3. $\frac{3}{\sqrt{15}}R$
4. $\frac{\sqrt{3}}{\sqrt{15}}R$

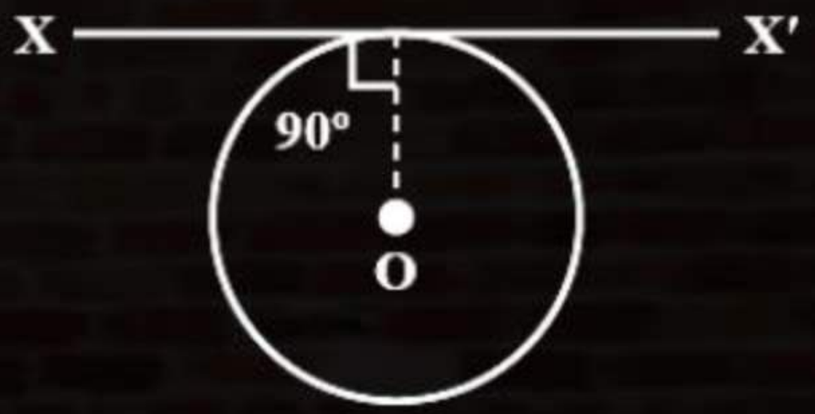
Question 26

From a solid sphere of mass M and radius R, a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of tis faces is

1. $\frac{MR^2}{32\sqrt{2}\pi}$
2. $\frac{4MR^2}{9\sqrt{3}\pi}$
3. $\frac{MR^2}{16\sqrt{2}\pi}$
4. $\frac{4MR^2}{3\sqrt{3}\pi}$

Question 27

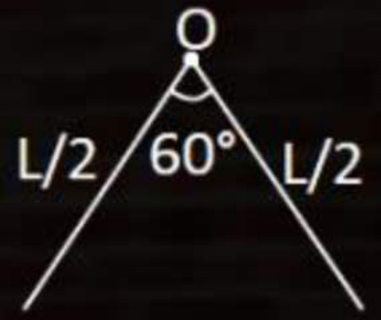
A thin wire of length L and uniform linear mass density ρ is bent into a circular loop with centre at O as shown. The moment of inertia of the loop about the axis XX' is



1. $\frac{\rho L^3}{8\pi^2}$
2. $\frac{\rho L^3}{16\pi^2}$
3. $\frac{5\rho L^3}{16\pi^2}$
4. $\frac{3\rho L^3}{8\pi^2}$

Question 28

A thin rod of length L and mass M is bent at the middle point O at an angle of 60°. The moment of inertia of the rod about an axis passing through O and perpendicular to the plane of the rod will be



1. $\frac{ML^2}{6}$
2. $\frac{ML^2}{12}$
3. $\frac{ML^2}{24}$
4. $\frac{ML^2}{3}$


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THANK YOU