Ultimate kcet crash course 2026

maths

DPP: 1

Application of Integrals

Q1 The area of the region bounded by the curve $x = 2y + 3$ and the lines $y = 1$ and $y = -1$ is ____
(A) 4 sq. units (B) 3/2 sq. units
(C) 6 sq. units (D) 8 sq. units

Q2 The area of the region bounded by the curve $y = x^2$ and the line $y = 16$ is
(A) $\frac{128}{3}$ sq. units
(B) $\frac{32}{3}$ sq. units
(C) $\frac{256}{3}$ sq. units
(D) $\frac{64}{3}$ sq. units

Q3 The area of the region bounded by the lines $y = mx$ $x = 1$, $x = 2$ and x-axis is 6 sq. units then m is _______
(A) 3 (B) 1
(C) 2 (D) 4

Q4 The area bounded by $y = x^3$, $y = 8$ and $x = 0$ is _______
(A) 12 sq. units (B) 2 sq. units
(C) 6 sq. units (D) 14 sq. units

Q5 In the interval $\left(0, \frac{\pi}{2}\right)$, area lying between the curves y = tan x and y = cot x and the X-axis is
(A) 4 log 2 sq. units
(B) 3 log 2 sq. units
(C) log 2 sq. units
(D) 2 log 2 sq. units

Q6 The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ in sq. units is
(A) 1 (B) 2
(C) $2\sqrt{2}$ (D) 4

Q7 The area, in square unit, bounded by the curves $y = x^3, y = x$ and the ordinates $x = 1, x = 2$ is
(A) $\frac{17}{12}$sq units (B) $\frac{12}{13}$sq units
(C) $\frac{2}{7}$sq units (D) $\frac{7}{2}$sq units

Q8 Find the Area of the region bounded by the parabola $y^2 = 4ax$, its axis and two ordinates $x = 5$ and $x = 8$.
(A) $\frac{4\sqrt{a}}{3}\left(16\sqrt{2} - 5\sqrt{5}\right) sq.\ units$
(B) $15\sqrt{a}\ sq.\ Units$
(C) $\frac{13}{3}\sqrt{a}\ sq.\ Units$
(D) $\frac{6}{15}\sqrt{a}\ sq.\ Units$

Q9 The Area of the region $\{(x,y) : y^2 \le x, x^2 + y^2 \le 2\}$
(A) $\frac{\pi}{4} - \frac{1}{3}$
(B) $\frac{\pi}{4} + \frac{1}{3}$
(C) $\frac{\pi}{4} + \frac{1}{6}$
(D) $\frac{\pi}{2} + \frac{1}{3}$

Q10 Find the area bounded by the parabola $y^2 = 4ax$ and the line x = a and x = 4a.
(A) $\frac{25}{4}a^2$ sq. units
(B) $\frac{74}{3}a^2$ sq. units
(C) $\frac{56}{3}a^2$ sq. units
(D) none of these

Q11 The area (in square units) of the region bounded by $y^2 = 9x$ and $y = 3x$ is
(A) 2 (B) $\frac{1}{4}$
(C) $\frac{1}{2}$ (D) 1

Q12 The area bounded by the parabola $y^2 = 4x$ and the line $y = 2x - 4$ is ______

(A) 9 sq. units (B) 5 sq. units
(C) 4 sq. units (D) 2 sq. units

Q13 AOB is a positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $OA = a$, $OB = b$. The area between the arc AB and chord AB of the ellipse is.

(A) pab sq. units
(B) (p – 2) ab sq. units
(C) $\frac{ab(\pi+2)}{2}$ sq. units
(D) $\frac{ab(\pi-2)}{4}$ sq. units

Q14 Area lying between the prabola $y^2 = 4ax$ and its latus rectum is

(A) $\frac{1}{2} a sq. Units$
(B) $\frac{1}{3} a^2 sq. Units$
(C) $\frac{8}{3} a\, sq. Units$
(D) $\frac{8}{3} a^2\ sq. Units$

Q15 The area of the region bounded ty the curve $y = \sin x$ and x -axis $[0, 2\pi]$ is

(A) 4 sq units (B) 3 sq units
(C) 2 sq. units (D) 1 sq units

Q16 The area of the figure bounded by the curves $y = \cos x$ and $y = \sin x$ and the ordinates $x = 0$ and $x = \frac{\pi}{4}$ is

(A) $\sqrt{2} - 1$ (B) $\sqrt{2} + 1$
(C) $\frac{1}{\sqrt{2}}\left(\sqrt{2} - 1\right)$ (D) $\frac{1}{\sqrt{2}}$

Q17 The area bounded by $y = x^2 + 3$ and $y = 2x + 3$ is (in sq. units)

(A) $\frac{12}{7}$ (B) $\frac{4}{3}$
(C) $\frac{3}{4}$ (D) $\frac{8}{3}$

Q18 Find the area of the region bounded by the parabola $y^2 = 4ax$, its axis and two ordinates $x = 5$ and $x = 8$.

(A) $\frac{4\sqrt{a}}{3}\left(16\sqrt{2} - 5\sqrt{5}\right)$ sq. units
(B) $15\sqrt{a}$ sq. units
(C) $\frac{13}{3}\sqrt{a}$ sq. units
(D) $\frac{6}{15}\sqrt{a}$ sq. units

Q19 Find the area under the curve $y = \sqrt{a^2 - x^2}$ included between the lines x=0 and x=a.

(A) πa^2 sq. units
(B) $\frac{\pi}{3} a^2$ sq. units
(C) $\frac{\pi}{4} a^2$ sq. units
(D) $4\pi a^2$ sq. units

Q20 The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$, $x = 3$ is

(A) $\frac{7}{2}$ sq. units (B) $\frac{9}{2}$ sq. units
(C) $\frac{11}{2}$ sq. units (D) $\frac{13}{2}$ sq. units

Q21 Area of the region bounded by $y = |x|$, $x \le 5$ in the first quadrant is

(A) $\frac{11}{2}$ sq. units
(B) $\frac{17}{2}$ sq. units
(C) $\frac{25}{2}$ sq. units
(D) $\frac{27}{2}$ sq. units

Q22 The area enclosed by the circle $x^2 + y^2 = 25$ is

(A) 50π sq. units
(B) 25π sq. units
(C) 5π sq. units
(D) 10π sq. units

Q23 Area of region bounded by $y^2 = 4x$ and $y = 2$ is

(A) 1/3 sq. unit (B) 8/9 sq. unit
(C) 2/3 sq. unit (D) 1/9 sq. unit

Q24 The area bounded by the curve y =x^4, X - axis and lines x = -2, x = 2 is

(A) $\frac{314}{5}$ *sq. units*

(B) $\frac{316}{5}$ *sq. units*

(C) $\frac{64}{5}$ *sq. units*

(D) $\frac{66}{5}$ *sq. units*

Q25 Find the area bounded by the circle $x^2 + y^2 = 16$ and the line *y=x* in the first quadrant.

(A) (2π) sq. units

(B) (3π) sq. units

(C) (4π) sq. units

(D) (5π) sq. units

Q26 The area Included between curves $y = x^2 - 3x + 2$ and $y = -x^2 + 3x - 2$ is

(A) $\frac{1}{6}$ *sq. Units* (B) $\frac{1}{2}$ *sq. Units*

(C) 1 sq. unit (D) $\frac{1}{3}$ *sq. unit*

Q27 Find the area bounded by the circle $x^2 + y^2 = 16$ and the line *y=x* in the first quadrant.

(A) (2π) sq. units

(B) (3π) sq. units

(C) (4π) sq. units

(D) (5π) sq. units

Q28 Find the area of region bounded by the curves y = x^2 + 2, y = x, x = 0 and x = 3.

(A) 21 sq. units (B) 20 sq. units

(C) 15/2 sq. units (D) 21/2 sq. units

Q29 The total area (in sq. units) enclosed by the curves y = x^3 and y = x is equal to

(A) 1/4 (B) 1/3

(C) 1/2 (D) 1

Q30 Area of the region bounded by $y = |5\sin x|$ from $x = 0$ to $x = 4\pi$ and x-axis is

(A) 10 sq. units (B) 20 sq. units

(C) 40 sq. units (D) 80 sq. units

Answer Key

Q1	C
Q2	D
Q3	D
Q4	A
Q5	C
Q6	B
Q7	A
Q8	A
Q9	D
Q10	C
Q11	C
Q12	A
Q13	D
Q14	D
Q15	A
Q16	A
Q17	B
Q18	A
Q19	C
Q20	A
Q21	C
Q22	B
Q23	C
Q24	C
Q25	A
Q26	D
Q27	A
Q28	D
Q29	C
Q30	C

Vernacular

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

The required area under the curve is given by

$$A = \int_c^d f(y)dy = \int_{-1}^{1} (2y+3)dy = 6 \text{ sq. units}$$

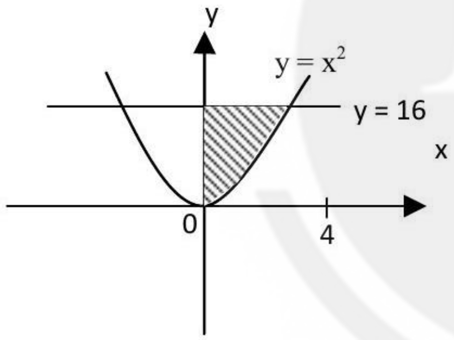
Video Solution:

Q2 Text Solution:

$$A = 2\int_0^{16} \text{x}d\text{y}$$

$$= 2\int_0^{16} \sqrt{\text{y}}d\text{y} = 2 \cdot \frac{2}{3}\text{y}^{\frac{3}{2}}\Big|_0^{16}$$

$$= \frac{4}{3}\left(16\sqrt{16} - 0\right) = \frac{256}{3}$$



Video Solution:

Q3 Text Solution:

The area bounded by the given curve is

$$A = \int_a^b f(x)dx$$

$$\Rightarrow 6 = \int_1^2 mx\, dx$$

$$\Rightarrow 6 = \frac{m}{2}\left(x^2\right)\Big|_1^2$$

\ 12 = m(4 – 1)

Þ m =4

Video Solution:

Q4 Text Solution:

The required area under the curve is given by

$$A = \int_c^d f(y)dy = \int_0^8 y^{1/3}dy$$

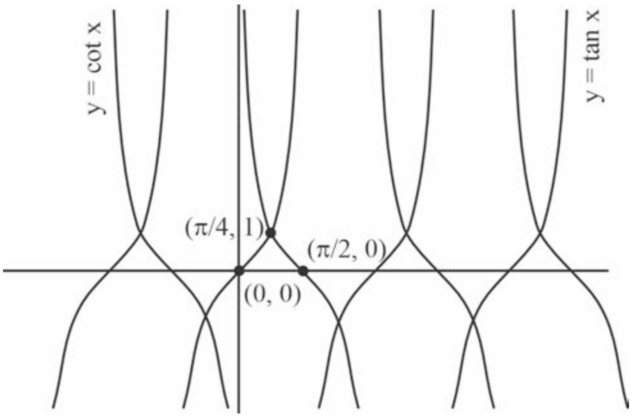
$$\therefore A = \frac{3}{4}y^{4/3}\Big|_0^8$$

$$\therefore A = \frac{3}{4}(16) = 12 \text{ sq. units}$$

Video Solution:

Q5 Text Solution:

Given curves y = tan x and y = cot x



Now, the area lying between given curves

$$= \int_0^{\pi/4} \tan x\, dx + \int_{\pi/4}^{\pi/4} \cot x\, dx$$

$$= |\log\ \sec x|_0^{\pi/4} + \log|\sin x|_{\pi/4}^{\pi/2}$$

$$= \log\ \sec\frac{\pi}{4} - 0 + \log(1) - \log|\sin \pi/4|$$

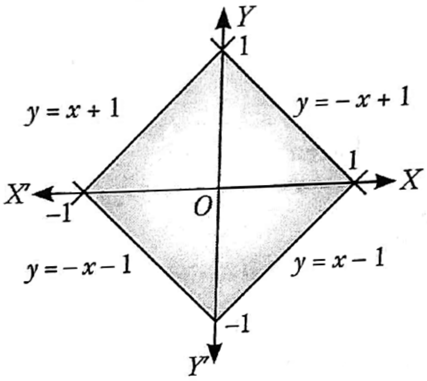
$$= \log\sqrt{2} - \log\frac{1}{\sqrt{2}}$$

$$= \frac{1}{2}\log 2 + \frac{1}{2}\log 2 = \log\ 2 \text{ sq. units}$$

Video Solution:

Q6 Text Solution:

(B)



The lines are y = x – 1, x ≥ 0

y = –x –1, x < 0, y = x –1, x > 0, y = –x + 1, x ≥ 0

Area $= 4 \times \int_0^1 \left(1 - x\right) dx = \left[\frac{4(1-x)^2}{-2}\right]_0^1 =$ sq.

$$-2\left[0 - 1\right] = 2$$

units

Video Solution:

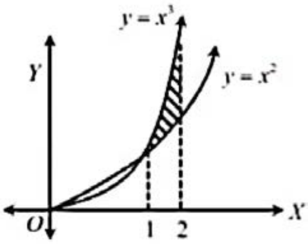
Q7 Text Solution:

The required area

$= \int_1^2 (x^3 - x^2)\,dx = \frac{x^4}{4} - \frac{x^3}{3} \int_1^2$

$= \left(4 - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right) = \frac{48-32-3+4}{12}$

$= \frac{17}{12}$ sq. units



Video Solution:

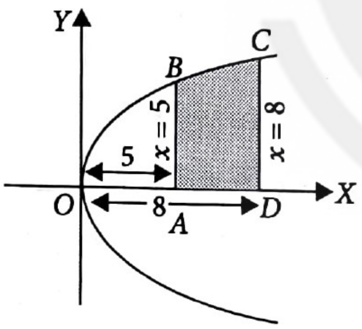
Q8 Text Solution:

Equation of the parabola is $y^2 = 4ax$

Its axis is $y = 0$ and vertex is (0,0).

It is symmetric about x-axis

Required area ABCDA



$= \int_5^8 y\,dx$

$= 2\sqrt{a} \cdot \int_5^8 \sqrt{x}\,dx$

$= 2\sqrt{a} \cdot \frac{2}{3}\left[x^{\frac{3}{2}}\right]_5^8 = \frac{4\sqrt{a}}{3}\left[8^{\frac{3}{2}} - 5^{\frac{3}{2}}\right]$

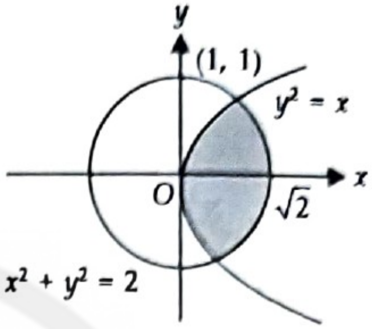
$= \frac{4\sqrt{a}}{3}\left[\left(\left(2\sqrt{2}\right)^2\right)^{\frac{3}{2}} - \left(\left(\sqrt{5}\right)^2\right)^{\frac{3}{2}}\right]$

$= \frac{4\sqrt{a}}{3}\left(16\sqrt{2} - 5\sqrt{5}\right)$ sq. units

Video Solution:

Q9 Text Solution:

The Curves $y^2 = x, x^2 + y^2 = 2$ meet at $x = 1$.



Required are $= 2\int_0^1 \left(\sqrt{2 - y^2} - y^2\right)$ dy

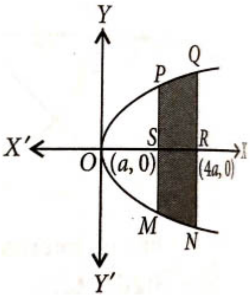
$= 2\left[\frac{y\sqrt{2-y^2}}{2} + \sin^{-1}\frac{y}{\sqrt{2}} - \frac{y^3}{3}\right]_0^1$

$= 2\left[\frac{1}{2} + \frac{\pi}{4} - \frac{1}{3}\right] = \frac{\pi}{2} + \frac{1}{3}$

Video Solution:

Q10 Text Solution:

(C)



Requred area = 2 × area of curve PSRQP

$$= 2\int_a^{4a} \sqrt{4ax}\,dx = 4\sqrt{a}\left[\frac{x^{3/2}}{3/2}\right]_a^{4a}$$

$$= \frac{8}{3}\sqrt{a}\left(8a^{3/2} - a^{3/2}\right)$$

$= \frac{56a^2}{3}$ sq. units

Video Solution:

Q11 Text Solution:

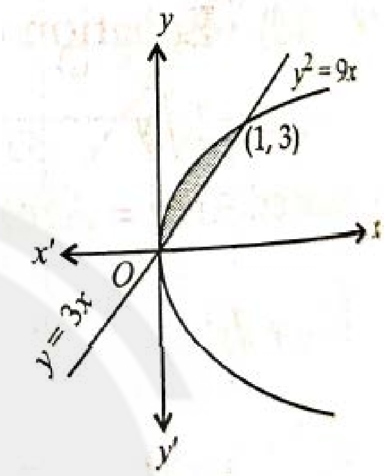
$$= \int_0^1 \left(\sqrt{9x} - 3x\right)dx = 3\int_0^1 \left(\sqrt{x} - x\right)dx$$

$$= 3\left[\frac{x^{3/2}}{3/2} - \frac{x^2}{2}\right]_0^1$$

$= 3\left(\frac{2}{3} - \frac{1}{2}\right) = 3\left(\frac{1}{6}\right) = \frac{1}{2}$ sq. unit

Intersecting points of $y^2 = 9x$ and $y = 3x$ are (0, 0) and (1, 3).

Required area



Video Solution:

Q12 Text Solution:

The required area under the curve is given by

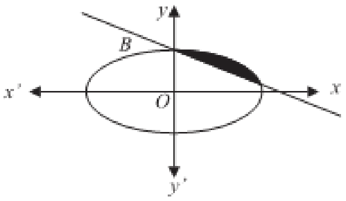
$$A = \int_c^d f(y)dy = \int_{-2}^{4}\left\{\left(\frac{y}{2} + 2\right) - \frac{y^2}{4}\right\}dy$$

$$= \left[y\frac{2}{4} + 2y - \frac{y^3}{12}\right]\Bigg|_{-2}^{4}$$

A = 9 sq. units

Video Solution:

Q13 Text Solution:



$$= \frac{b}{a}\int_0^a \sqrt{a^2-x^2}\,dx - \frac{b}{a}\int_0^a (a-x)\ dx$$

$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$- \frac{b}{a}\left[ax - \frac{x^2}{2}\right]_0^a$$

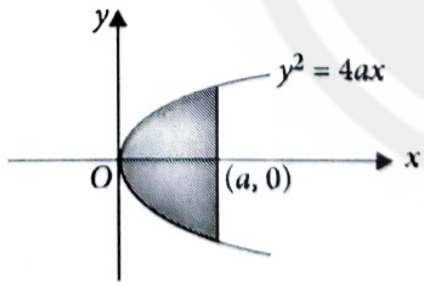
$$= \frac{b}{a}\left[\frac{a^2}{2}\sin^{-1} 1\right] - \frac{b}{a}\left[a^2 - \frac{a^2}{2}\right]$$

$$= \frac{ab}{2}\frac{\pi}{2} - ba\left(\frac{1}{2}\right) = \frac{ab}{4}\left(\pi - 2\right) \text{ sq. units}$$

Video Solution:

Q14 Text Solution:

We have, $y^2 = 4ax$ parabola with vertex (0, 0) and focus (a, 0) and latus rectum 4a. Required area = area of shaded region



$$A = 2\int_0^a 2\sqrt{a}\sqrt{x}dx$$

$$= 2\left[2\sqrt{a}x^{3/2} \times \frac{2}{3}\right]_0^a$$

$$= \frac{8}{3}a^2 \text{ sq. units}$$

Video Solution:

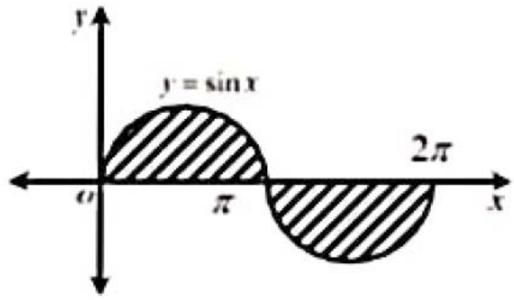
Q15 Text Solution:

If y = 0 then $\sin x = 0 \Rightarrow x = 0, x, 2x$

Required area $= \int_0^{\pi} \sin x dx + \int_{\pi}^{2\pi}(-\sin x)dx$

$= [-\cos x]_2^{\pi} + [\cos x]_{\pi}^{2\pi} - (1+1) + (1+1)$

$= 4$ sq , units



Video Solution:

Q16 Text Solution:

Clearly two curves intersect at $x = \frac{\pi}{4}$

$$\therefore A = \int_0^{\pi/4} (\cos x - \sin x)dx$$

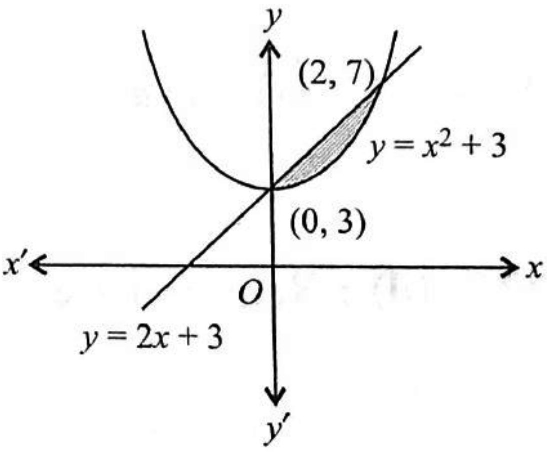
$$= \sin x + \cos x]_0^{\pi/4} = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - 1$$

$$= \sqrt{2} - 1$$

Video Solution:

Q17 Text Solution:

(B)



Point of intersection of $y = x^2 + 3$ and $y = 2x + 3$ are (0, 3) and (2, 7)

$\therefore$ Required area

$\left|\int_0^2 \left(x^2 - 2x\right) dx\right|$

$= \left|\frac{8}{3} - 4\right| = \frac{4}{3}$ sq. units

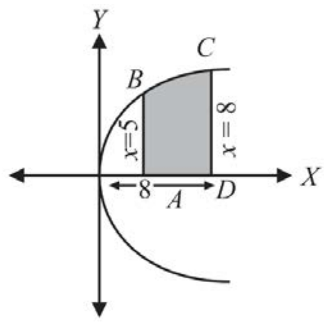
Video Solution:

Q18 Text Solution:

Equation of the parabola is $y^2 = 4ax$

Its axis is $y = 0$ and vertex is (0, 0)

It is symmetric about x-axis.



Required area

$$ABCDA = \int_5^8 y dx = 2\sqrt{a} \int_5^8 \sqrt{x} dx$$

$\left[y = 2\sqrt{a}\sqrt{x} \text{ as } y > 0\right]$

$$= 2\sqrt{a} \cdot \frac{2}{3}\left[x^{3/2}\right]_5^8 = \frac{4\sqrt{a}}{3}\left[8^{3/2} - 5^{3/2}\right]$$

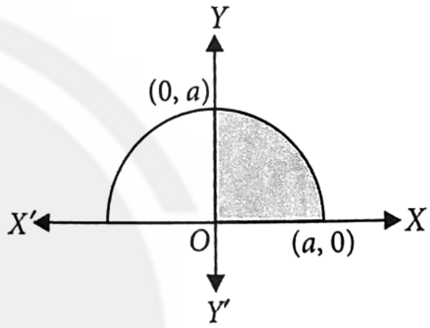
$$= \frac{4\sqrt{a}}{3}\left[\left(\left(2\sqrt{2}\right)^2\right)^{3/2} - \left(\left(\sqrt{5}\right)^2\right)^{3/2}\right]$$

$= \frac{4\sqrt{a}}{3}\left(16\sqrt{2} - 5\sqrt{5}\right)$ sq. units

Video Solution:

Q19 Text Solution:

Given curve is $y = \sqrt{a^2 - x^2}$ and $x = 0$, $x = a$



$$\therefore \text{Required area} = \int_0^a \sqrt{a^2 - x^2}\, dx$$

$$= \left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)\right]_0^a$$

$$= \left[0 + \frac{a^2}{2}\sin^{-1}\left(1\right) - 0\right] = \frac{a^2}{2} \times \frac{\pi}{2}$$

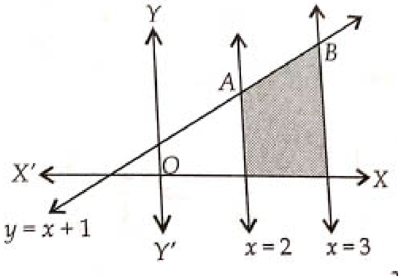
$$= \frac{\pi}{4}a^2 \text{ sq. units}$$

Video Solution:

Q20 Text Solution:

(A)

We have y = x + 1 and lines x = 2, x = 3. Points intersection are A(2, 3) and B(3, 4).



∴ Requried area of the shaded region

$$= \int_{2}^{3} (x+1)dx$$

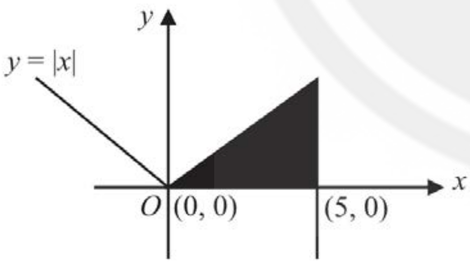
$$= \left[\frac{x^2}{2} + x\right]_2^3 = \left[\frac{9}{2} + 3 - \frac{4}{2} - 2\right] = \frac{7}{2} \text{sq. units}$$

Video Solution:

Q21 Text Solution:

We have, $y = -x$, if $x < 0$ ...(i)

$y = x$, if $x \geq 0$ ...(ii)



Required area = area of shaded region

$$= \int_0^5 x dx = \frac{x^2}{2}\Big|_0^5 = \frac{25}{2} \text{ sq. units}$$

Video Solution:

Q22 Text Solution:

We have $x^2 + y^2 = 25$ a circle of radius 5.

∴ Required area = 4 (area of shaded region)

$$= 4\int_0^5 \sqrt{25 - x^2}\, dx.$$

$$= 4\left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_0^5$$

$$= 4\left[\frac{25}{2} \cdot \times \frac{\pi}{2}\right] = 25\pi \text{ sq.unite.}$$

Video Solution:

Q23 Text Solution:

$$A = \frac{1}{4}\int_{-}^{2} y^2 dy = \frac{2}{3}$$

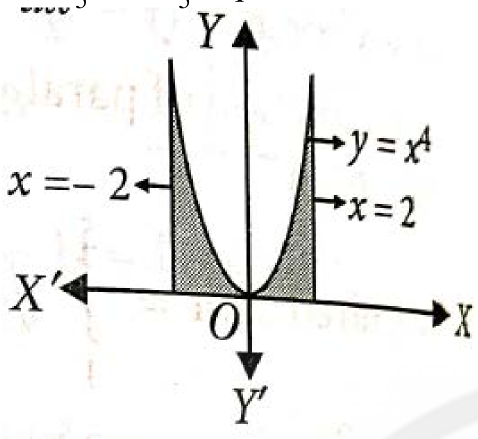
Video Solution:

Q24 Text Solution:

Required area is $A = \int_{-2}^{2} y dx$

$= \int_{-2}^{2} x^4 dx = \left[\frac{x^5}{5}\right]_{-2}^{2}$

$= \frac{32+32}{5} = \frac{64}{5}$ sq. units



Video Solution:

Q25 Text Solution:

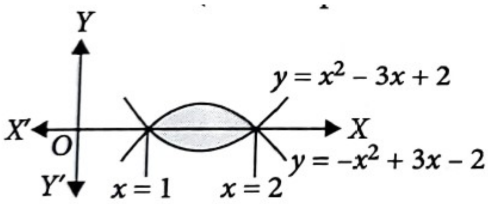
$$A = \int_0^{2\sqrt{2}} \sqrt{16-y^2} - y\,dy$$

$$= \frac{y}{2}\sqrt{16-y^2} + 8\sin^{-1}\left(\frac{y}{4}\right) - \frac{y^2}{2}\Big]_0^{2\sqrt{2}}$$

$$= 4 + 8\left(\frac{\pi}{4}\right) - 4 = 2\pi$$

Video Solution:

Q26 Text Solution:



Required area $= 2\int_1^2 \left(-x^2 + 3x - 2\right) dx$

($\because$ both portions are same)

$$= 2\left[-\frac{x^3}{3} + \frac{3x^2}{2} - 2x\right]_1^2$$

$$= 2\left[-\frac{8}{3} + 6 - 4 - \left(-\frac{1}{3} + \frac{3}{2} - 2\right)\right]$$

$$= 2\left[-\frac{8}{3} + 2 + \frac{5}{6}\right] = \frac{1}{3} \text{ sq. unit}$$

Video Solution:

Q27 Text Solution:

$$A = \int_0^{2\sqrt{2}} \sqrt{16-y^2} - y\,dy$$

$$= \frac{y}{2}\sqrt{16-y^2} + 8\sin^{-1}\left(\frac{y}{4}\right) - \frac{y^2}{2}\Big]_0^{2\sqrt{2}}$$

$$= 4 + 8\left(\frac{\pi}{4}\right) - 4 = 2\pi$$

Video Solution:

Q28 Text Solution:

Given curve is y = x2 + 2 and a straight line y = x

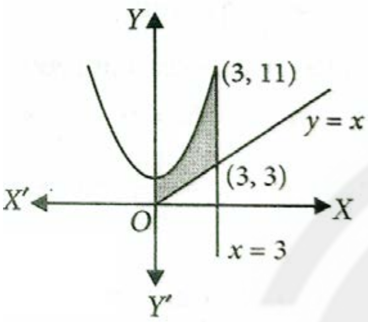
Point of intersection of the curve and x = 3 is (3, 11)

∴ Required area = Area of shaded region

$$= \int_0^3 (x^2+2)\,dx - \int_0^3 x\,dx$$

$$= \left[\frac{x^3}{3}+2x\right]_0^3 - \left[\frac{x^2}{2}\right]_0^3$$

$$=(9+6)-\left(\frac{9}{2}\right) = 15 - \frac{9}{2} = \frac{21}{2} \text{ sq. units}$$



Video Solution:

Q29 Text Solution:

From $y = x^3$ and $y = x$,

we get, $x^3 = x$

$\Rightarrow\ x^3 - x = 0$

$\Rightarrow\ x(x-1)(x+1) = 0$

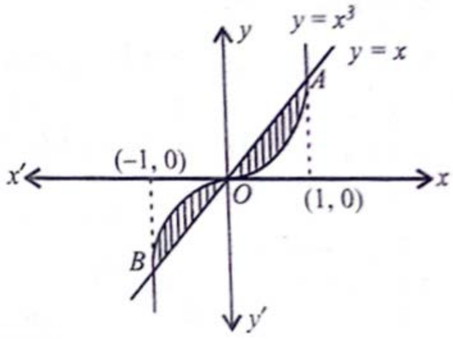
$\Rightarrow\ x = 0,\ 1 \text{ or } -1$

Since, shaded region is symmetric.

∴ Required area = 2 × Area of OAO

$$= 2 \times \int_0^1 (x - x^3)\,dx = 2 \times \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = 2 \times \left[\frac{1}{2} - \frac{1}{4}\right]$$

$$= 2 \times \frac{1}{4} = \frac{1}{2} \text{ sq. unit}$$



Video Solution:

Q30 Text Solution:

Area

$= \int_0^{4\pi} |5\sin x|\,dx = 5\int_0^{4\pi} |\sin x|\,dx = 20 \int_0^{\pi} |\sin x|\,dx = 40 \int_0^{\pi/2} \sin x\,dx = 40$ sq. units

Video Solution: