ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture - 01

Trigonometry

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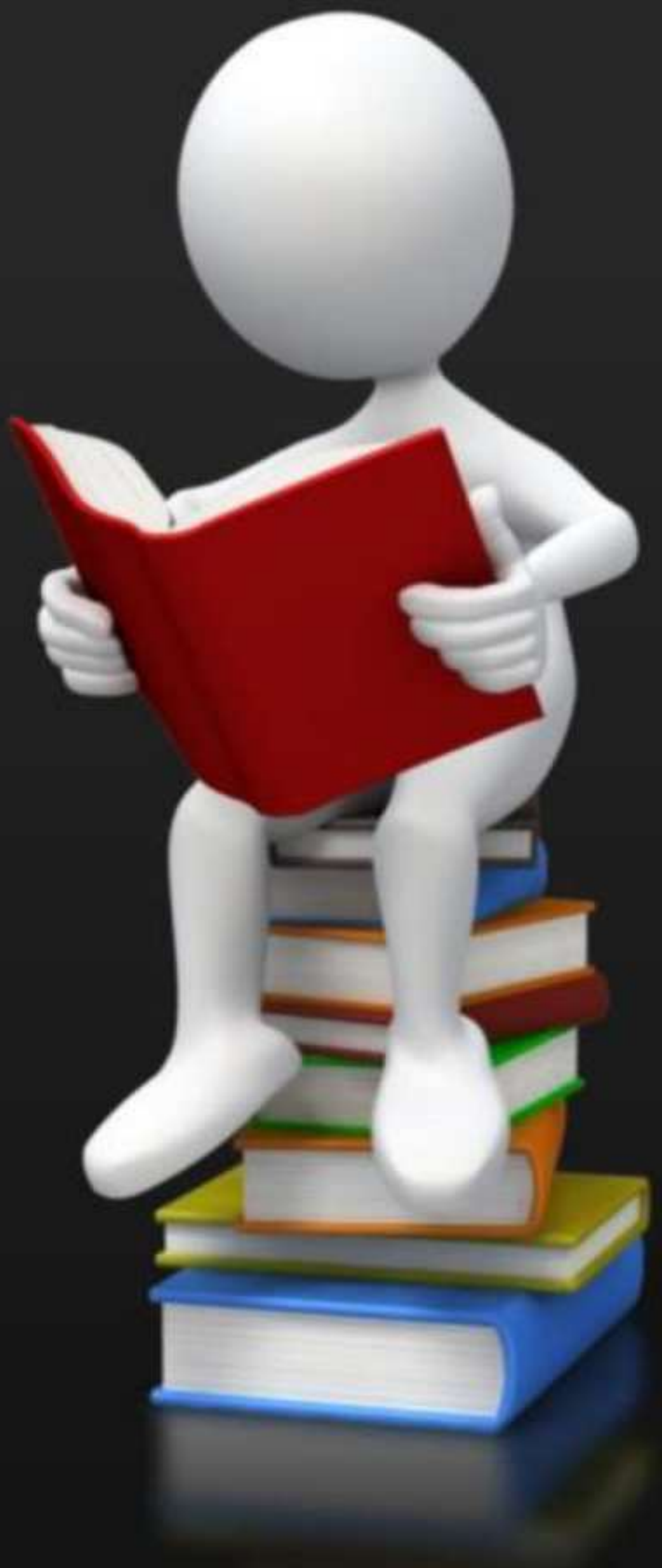
Topics *to be covered*

1. Trigonometry $\rightarrow$ class 11^{th}
2.
3.
4.



Rank is the

Biproduct

of

Hardwork, consistency & Determination

Trigonometry

Differentiation

&

Integration

Direct

MCQ's

(A) $\sin x$

(B) $\cos x$

(C) $\tan x$

(D) $\sec x$

simplest form

$$\frac{d}{dx}\left[\frac{\sin^2 x}{1+\cos x}\right]$$

$$= \frac{d}{dx}\left[\frac{1-\cos^2 x}{1+\cos x}\right]$$

$$= \frac{d}{dx}\left[\frac{(1-\cos x)(1+\cos x)}{1+\cos x}\right]$$

$$= \frac{d}{dx}(1-\cos x) = 0 + \sin x = \underline{\sin x}$$

60m

15 to 18m $\Downarrow$ Class 11th

42 to 45m $\Downarrow$ Class 12th

$$\frac{d}{dx}[\operatorname{cosec}x - \cot x]$$

(A) $-\operatorname{cosec}^2\frac{x}{2}$

(B) $\sec^2\frac{x}{2}$

(C) $-\frac{\operatorname{cosec}^2 x}{2}$

(D) $\frac{\sec^2 x/2}{2}$

$$\frac{d}{dx}\left[\frac{1}{\sin x} - \frac{\cos x}{\sin x}\right]$$

$$\frac{d}{dx}\left[\frac{1-\cos x}{\sin x}\right]$$

$$\frac{d}{dx}\left[\frac{2\sin^2 x/2}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right]$$

$$\frac{d}{dx}\left[\tan\frac{x}{2}\right]$$

$$= \sec^2 x/2\left(\frac{1}{2}\right)$$

whenever you are given problems of differentiation (or) Integration of trigonometric func

⇓

First reduce into simplest form

⇓

last step should be differentiation

List of Formula

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$$

Remark :

I. $\cos^2 x = \frac{1+\cos 2x}{2}$ or $1 + \cos 2x = 2\cos^2 x$

$\sin^2 x = \frac{1-\cos 2x}{2}$ or $1 - \cos 2x = 2\sin^2 x$

II. $\cos^2 \frac{x}{2} = \frac{1+\cos x}{2}$ or $1 + \cos x = 2\cos^2 \frac{x}{2}$

$\sin^2 \frac{x}{2} = \frac{1-\cos x}{2}$ or $1 - \cos x = 2\sin^2 x$

III. $\sin 2x = 2\sin x \cos x$

$\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

IV. $\sin 3x = 3 \sin x - 4 \sin^3 x$

$\cos 3x = 4 \cos^3 x - 3 \cos x$

$\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

V. $\sin (A + B) = \sin A \cos B + \cos A \sin B$

$\sin (A - B) = \sin A \cos B - \cos A \sin B$

$\cos (A + B) = \cos A \cos B - \sin A \sin B$

$\cos (A - B) = \cos A \cos B + \sin A \sin B$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \cot(A + B) = \frac{\cot A \cot B - 1}{\cot A + \cot B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B} \qquad \cot(A - B) = \frac{\cot A \cot B + 1}{\cot B - \cot A}$$

VI. $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

$2\cos A\sin B = \sin(A+B) - \sin(A-B)$

$2\cos A\cos B = \cos(A+B) + \cos(A-B)$

$2\sin A\sin B = -[\cos(A+B) - \cos(A-B)]$

VII. $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

1st 60mins

$\Downarrow$

Trigonometry

for

differentiation

&

Integration

2hrs

$\Downarrow$

MCQ's

QUESTION

If α, β are complementary angles, then $\cos^2\alpha + \cos^2\beta$ is equal to

$\alpha + \beta = 90$

$\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2$

$\alpha = \beta = 45$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

A 0

B 1

C –1

D 2

$1+\tan^2\theta = \sec^2\theta$

$$\boxed{1 = \sec^2\theta - \tan^2\theta}$$

$$1 = \begin{cases} (1)\ \sin^2\theta + \cos^2\theta \\ (2)\ \sec^2\theta - \tan^2\theta \\ (3)\ \operatorname{cosec}^2\theta - \cot^2\theta \end{cases}$$

QUESTION

$\sec\theta - \tan\theta = 3 \Rightarrow \theta$ lies in the quadrant. $\rightarrow (1)$

(A) I

(B) II

(C) III

(D) IV

WKT

$\sec^2\theta - \tan^2\theta = 1$

$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$(\sec\theta + \tan\theta)(3) = 1$

$\sec\theta + \tan\theta = \frac{1}{3} \rightarrow (2)$

$(1) + (2)$

$2\sec\theta = \frac{10}{3}$

$\sec\theta = \frac{5}{3} \Rightarrow \sec\theta = +ve$

$(1) - (2)$

$-2\tan\theta = \frac{8}{3}$

$\tan\theta = -\frac{4}{3} \Rightarrow \tan\theta = -ve$

$\sec\theta = +ve$ & $\tan\theta = -ve$

$\theta \in 4^{th}$ Quad

QUESTION

$\tan^2\theta + \sec\theta = 5 \Rightarrow \sec\theta =$

$\Downarrow$

$\sec^2\theta - 1 + \sec\theta = 5$

A 3

$\sec^2\theta + \sec\theta - 6 = 0$

B 2 ✓

$t^2 + t - 6 = 0$

-6

$+3 \quad -2$

$(t+3)(t-2) = 0$

C 1

$t = -3$ या $t = 2$

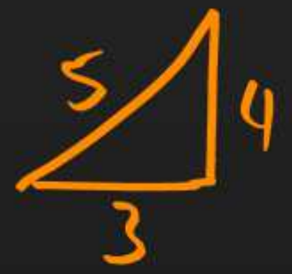
D –1

$\sec\theta = -3 \qquad \sec\theta = 2$

QUESTION

$\tan\theta = -ve$
in 2nd & ~~4th quad~~

If θ is not in 4^{th} quadrant
$\tan\theta = -4/3 \Rightarrow 5\sin\theta + 10\cos\theta + 9\sec\theta + 16\operatorname{cosec}\theta + 4\cot\theta =$



$5\left(+\frac{4}{5}\right)+10\left(\frac{-3}{5}\right)+9\left(-\frac{5}{3}\right)+16\left(+\frac{5}{4}\right)+4\left(\frac{-3}{4}\right)$

$+4-6-15+20-3$

$=24-24$

$=0$

(A) -6

(B) $2/5$

(C) $4/5$

(D) 0

QUESTION

$\tan 20° + \tan 40° + \tan 60° + \ldots\ldots + \tan 180° =$

$\cancel{\tan 20} + \cancel{\tan 40} + \tan 60 + \ldots + \cancel{\tan 140} + \cancel{\tan 160} + \tan 180 \to 0$

A 0

B 1

C 2

D 3

$\tan 160 = \tan(180 - 20) = -\tan 20$

$\tan 140 = -\tan 40$

$= 0$

QUESTION

$$\frac{\sin(-660°)\tan(1050°)\sec(-420°)}{\cos(225°)\mathrm{cosec}(315°)\cos(510°)} =$$

(A) $\sqrt{3}/4$

(B) $\sqrt{3}/2$

(C) $2/\sqrt{3}$

(D) $4/\sqrt{3}$

$$\frac{+\sin 60(-\tan 30)(\sec 60)}{(-\cos 45)(-\mathrm{cosec}\, 45)(-\cos 30)}$$

$$\frac{-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{3}}\right)(2)}{-\left(\frac{1}{\sqrt{2}}\right)(\sqrt{2})\left(\frac{\sqrt{3}}{2}\right)} = +\frac{2}{\sqrt{3}}$$

4th $660 = \underline{720} - 60$

4th $1050 = \underline{1080} - 30$

1st $420 = \underline{360} + 60$

3rd $225 = \underline{180} + 45$

4th $315 = \underline{360} - 45$

2nd $510 = \underline{540} - 30$

Multiples of 18

18
36
54
72
90
108
126

QUESTION

$\cot 25 = \frac{1}{p}$

$$\tan 25^\circ = p \Rightarrow \frac{\tan 245^\circ + \tan 335^\circ}{\tan 205^\circ - \tan 115^\circ} =$$

(A) $\frac{p^2-1}{p^2+1}$

(B) $\frac{p^2+1}{p^2-1}$

(C) $\frac{1-p^2}{1+p^2}$

(D) $\frac{1+p^2}{1-p^2}$

$$\frac{\cot 25 - \tan 25}{\tan 25 + \cot 25}$$

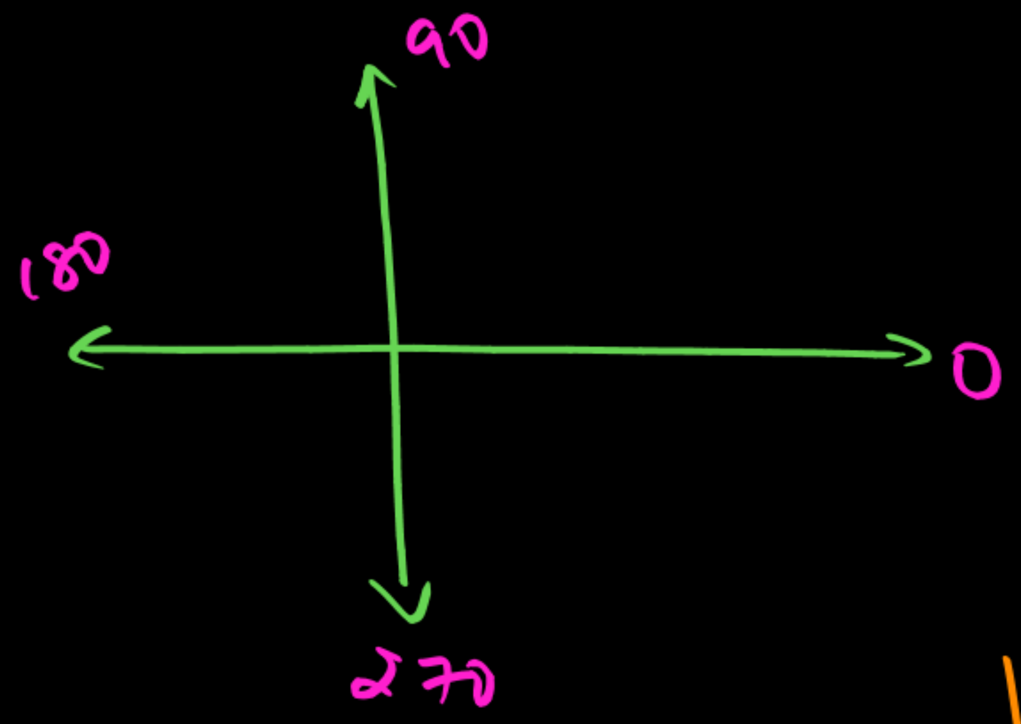
$$= \frac{\frac{1}{p} - p}{p + \frac{1}{p}} = \frac{1-p^2}{p^2+1}$$

3rd $245 = 270 - 25$ | $\tan(270-\theta) = \cot\theta$

4th $335 = 360 - 25$

3rd $205 = 180 + 25$

2nd $115 = 90 + 25$



1st Quad $\Downarrow$ All trigonometric func are +ve	**2nd Quad** $\Downarrow$ $\sin\theta$ & $\operatorname{cosec}\theta$ are +ve

If $f(x) = A\cos x + B\sin x$

or

$f(x) = A\cos x - B\sin \mathrm{x}$

Then minimum value of $f(x)$ is $-\sqrt{A^2 + B^2}$ and

Maximum value of $f(x)$ is $\sqrt{A^2 + B^2}$

Other uses of $A\cos x + B\sin x$ or $A\cos x - B\sin x$ is

(i) Finding range (ii) Finding interval in which given function is one-one

QUESTION

The domain of $f(x) = \sqrt{1 - \cos x}$

(A) $[0, 1]$

(B) $[-1, 1]$

(C) R

(D) $[0, 2\pi]$

$1 - \cos x \geq 0$

$1 \geq \cos x$

$\Downarrow$

$\cos x \leq 1$

This is true

$\forall x \in R$ (Domain)

$\cos x$

Domain $= R$

$\Downarrow$

$x \in R$

Range $= [-1, 1]$

$\Downarrow$

$\cos x \in [-1, 1]$

$\Downarrow$

$-1 \leq \cos x \leq 1$

Range of $f(x) = \sqrt{1-\cos x}$

Soln:-

WKT

$-1 \le -\cos x \le 1$

Add 1

$0 \le 1-\cos x \le 2$

Take sqrt

$0 \le \sqrt{1-\cos x} \le \sqrt{2}$

$0 \le f(x) \le \sqrt{2}$

Range $= [0, \sqrt{2}]$

$-1 \le \cos x \le 1$

$\times^{ly}$ by -1

$1 \ge -\cos x \ge -1$

$\Downarrow$ Rewrite

$-1 \le -\cos x \le 1$

Find Domain of $f(x) = \sqrt{\cos x - 1}$ → Range $= \{0\}$

<u>Soln:</u>

$$\cos x - 1 \geq 0$$

$$\cos x \geq 1$$

$\cos x > 1$ ⇓ Not Possible

$\cos x = 1$ ⇓ $x = \{2n\pi, n \in Z\}$

QUESTION

The domain of $f(x) = \frac{1}{\sqrt{1-\cos x}}$

(A) $R - \{n\pi, n \in Z\}$

(B) $R - \{2n\pi, n \in Z\}$

(C) $R - \left\{(2n+1)\frac{\pi}{2}, n \in Z\right\}$

(D) None of the above

$1 - \cos x > 0$

$1 > \cos x$

$\cos x < 1$

$\Downarrow$

$\cos x \neq 1$ but $\cos x \in [-1, 1)$

$\Downarrow$

$x \neq 2n\pi, n \in z$

Domain $= R - \{2n\pi, n \in z\}$

60m

1st PU

⇓

15 to 18 m

⇓

Easy & scoring

1. sets
2. R & F
3. complex NOS
4. Binomial theorem
5. statistics
6. 3D ✓
7. limits
8. sequence & series
9. Inequalities ✓
10. conic (lengthy but more easy) → Depend on PYQ's
11. straight lines.

⇓

11 to 13 marks

QUESTION

The Range of $f(x) = \frac{1}{2-\cos x}$ is → $2-\cos x \neq 0$

$\therefore$ Domain $= R$

Since $\cos x \in [-1, 1]$

A $[-1, 1]$

B $\left[\frac{1}{3}, 1\right]$

C $\left[1, \frac{1}{3}\right]$

D $[0, 1]$

WKT

$-1 \le -\cos x \le 1$ ↓ Add 2

$1 \le 2-\cos x \le 3$

Reciprocal

$1 \geq \frac{1}{2-\cos x} \geq \frac{1}{3}$

$1 \geq f(x) \geq \frac{1}{3}$

$\frac{1}{3} \le f(x) \le 1$

$1 \le 3$

Reciprocal

$1 \geq \frac{1}{3}$

Range of $f(x) = \frac{1}{\cos x - 2}$

Soln:

$-1 \le \cos x \le 1$

$-3 \le \cos x - 2 \le -1$

Reciprocal

$-\frac{1}{3} \ge \frac{1}{\cos x - 2} \ge -1$

$\Downarrow$

$-1 \le f(x) \le -\frac{1}{3}$

Range $= \left[-1, -\frac{1}{3}\right]$

$-3 \le -1$

Reciprocal

$-\frac{1}{3} \ge -1$

$\Downarrow$

$-1 \le -\frac{1}{3}$

① if

$a \le x \le b$

where a & b are +ve

Then

$$\frac{1}{b} \le \frac{1}{x} \le \frac{1}{a}$$

Ex:

$$2 \le x \le 5$$

$$\Downarrow$$

$$\frac{1}{5} \le \frac{1}{x} \le \frac{1}{2}$$

② if $a \le x \le b$

where a & b are −ve

Then

$$\frac{1}{b} \le \frac{1}{x} \le \frac{1}{a}$$

Ex:

$$-5 \le x \le -2$$

$$-\frac{1}{2} \le \frac{1}{x} \le -\frac{1}{5}$$

③ if $a \le x \le b$

where $a \to -ve$

&

$b = +ve$

Ex: if

$-4 \le x \le 5$

$x = -ve$ | $x = +ve$

$-4 \le x < 0$ | $0 < x \le 5$

Reciprocal | Reciprocal

$-\frac{1}{4} \ge \frac{1}{x} > -\frac{1}{0}$ | $\frac{1}{0} > \frac{1}{x} \ge \frac{1}{5}$

$\Downarrow$ | $\Downarrow$

$-\infty < \frac{1}{x} \le -\frac{1}{4}$ | $\frac{1}{5} \le \frac{1}{x} < \infty$

$\Downarrow$

$\frac{1}{x} \in (-\infty, -\frac{1}{4}] \cup [\frac{1}{5}, \infty)$

QUESTION

#Q. Range of $f(x) = \frac{1}{1-2\cos x}$ is

(A) $\left[\frac{1}{3}, 1\right]$

(B) $\left[\frac{-1}{3}, 1\right]$

(C) $\left[-1, \frac{1}{3}\right]$

(D) $(-\infty, -1) \cup \left[\frac{1}{3}, \infty\right)$

$-2 \le -2\cos x \le 2$ Add 1

$-1 \le 1-2\cos x \le 3$

$-1 \le 1-2\cos x < 0$ $\quad$ $0 < 1-2\cos x \le 3$

Reciprocal

$-1 \ge \frac{1}{1-2\cos x} > \frac{-1}{0}$ $\quad$ $\frac{1}{0} > \frac{1}{1-2\cos x} \ge \frac{1}{3}$

$\Downarrow$ $\quad$ $\Downarrow$

$-1 \ge f(x) > -\infty$ $\quad$ $\infty > f(x) \ge \frac{1}{3}$

$-\infty < f(x) \le -1$ or $\frac{1}{3} \le f(x) < \infty$

$\Downarrow$

$f(x) \in (-\infty, -1] \cup \left[\frac{1}{3}, \infty\right)$

QUESTION

The range of $f(x) = 1 + 3\cos 2x$ is

WKT

$-1 \leq \cos 2x \leq 1$

$-3 \leq 3\cos 2x \leq 3$

$-2 \leq 1 + 3\cos 2x \leq 4$

$-2 \leq f(x) \leq 4$

Range $= [-2, 4]$

QUESTION

The range of $f(x) = \sin x \cos x$ is

$$f(x) = \frac{\sin 2x}{2}$$

WKT

$$-1 \le \sin 2x \le 1$$

$$-\frac{1}{2} \le \frac{\sin 2x}{2} \le 1$$

$$-\frac{1}{2} \le f(x) \le \frac{1}{2}$$

QUESTION

The range of $f(x) = \sin^2 x \cos^2 x$ is

$= (\sin x \cos x)^2$

$= \left(\frac{\sin 2x}{2}\right)^2$

$= \frac{\sin^2 2x}{4}$

WKT

$-1 \le \sin 2x \le 1$

$0 \le \sin^2 2x \le 1$

$0 \le \frac{\sin^2 2x}{4} \le \frac{1}{4}$

Range $= \left[0, \frac{1}{4}\right]$

1 if $-1 \le x \le 1$

Then $0 \le x^2 \le 1$

QUESTION

(a, b) is different from $\{a, b\}$

$[x] \to$ GIF

The range of $f(x) = \sin[x]$, $\underbrace{\frac{-\pi}{4} < x < \frac{\pi}{4}}_{\text{given Domain}}$ is

(A) $\{0\}$

(B) $\{0, -1\}$

(C) $\{0, \pm\sin 1\}$

(D) $\{0, -\sin 1\}$

$$-\frac{\pi}{4} < x < \frac{\pi}{4}$$

$$-\frac{3.14}{4} < x < \frac{3.14}{4}$$

$$-0.78 < x < 0.78$$

$$-0.78 < x < 0.78$$

consider GIF

$$[-0.78] < [x] < [0.78]$$

$$-1 \le [x] \le 0 \text{ integers}$$

$\therefore [x] = -1$ & $[x] = 0$

$\sin[x] = \sin(-1) = -\sin 1$ | $\sin[x] = \sin 0 = 0$

QUESTION

#Q. If $A = \cos^2\theta + \sin^4\theta$ then

(A) $A \in \left[\frac{3}{4}, 2\right]$

(B) $A \in \left[\frac{1}{4}, 1\right]$

(C) $A \in \left[\frac{3}{4}, 1\right]$

(D) $A \in [-1, 1]$

$A = 1 - \sin^2\theta + \sin^4\theta$

$= \sin^4\theta - \sin^2\theta + 1$

Put $\sin^2\theta = t$

$= t^2 - t + 1$

$= t^2 - t + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1$

$= \left(t - \frac{1}{2}\right)^2 - \frac{1}{4} + 1$

$= \left(t - \frac{1}{2}\right)^2 + \frac{3}{4}$

$A = \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$

WKT

$0 \leq \sin^2\theta \leq 1$

$-\frac{1}{2} \leq \sin^2\theta - \frac{1}{2} \leq \frac{1}{2}$

on squaring

$0 \leq \left(\sin^2\theta - \frac{1}{2}\right)^2 \leq \frac{1}{4}$

$\frac{3}{4} \leq \left(\sin^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4} \leq 1$

Range $= \left[\frac{3}{4}, 1\right]$

QUESTION

$$\frac{1}{\sin\theta}-\frac{\sqrt{3}}{\cos\theta}=$$

(A) $\frac{4\cos\left(\frac{\pi}{3}-\theta\right)}{\sin 2\theta}$

(B) $\frac{4\sin\left(\frac{\pi}{3}-\theta\right)}{\sin 2\theta}$

(C) $\frac{4\cos\left(\frac{\pi}{3}+\theta\right)}{\sin 2\theta}$ ✓

(D) $\frac{4\sin\left(\frac{\pi}{3}+\theta\right)}{\sin 2\theta}$

$A=1$, $B=\sqrt{3}$

$$\frac{(1)\cos\theta-\sqrt{3}\sin\theta}{\sin\theta\cos\theta}$$

x^{ly} & $\div$ by $\sqrt{1+3}=2$

$$\frac{2\left[\frac{1}{2}\cos\theta-\frac{\sqrt{3}}{2}\sin\theta\right]}{\left(\frac{\sin 2\theta}{2}\right)}$$

$$\frac{4}{\sin 2\theta}\left[\sin\frac{\pi}{6}\cos\theta-\cos\frac{\pi}{6}\sin\theta\right]$$

$$\frac{4}{\sin 2\theta}\left[\sin\left(\frac{\pi}{6}-\theta\right)\right]$$

$A\sin\theta \pm B\cos\theta$

$\Downarrow$

x^{ly} & $\div$ by $\sqrt{A^2+B^2}$

$$=\frac{4}{\sin 2\theta}\left[\cos\left[\frac{\pi}{2}-\left(\frac{\pi}{6}-\theta\right)\right]\right]$$

since $\sin x=\cos\left(\frac{\pi}{2}-x\right)$

$$=\frac{4}{\sin 2\theta}\left[\cos\left(\frac{\pi}{3}+\theta\right)\right]$$

QUESTION

$\sin\theta\cos\theta = \frac{\sin 2\theta}{2}$

$\sqrt{3}$ cosec $20 - \sec 20$

A 2

B 4

C 3

D 1

$$\frac{\sqrt{3}}{\sin 20} - \frac{1}{\cos 20}$$

$$= \frac{\sqrt{3}\cos 20 - (1)\sin 20}{\sin 20 \cos 20}$$

$$= \frac{2\left[\frac{\sqrt{3}}{2}\cos 20 - \frac{1}{2}\sin 20\right]}{\frac{\sin 2(20)}{2}}$$

$$= \frac{4}{\sin 40}\left[\sin 60 \cos 20 - \cos 60 \sin 20\right]$$

$$= \frac{4}{\sin 40}\left[\sin(60-20)\right]$$

$$= \frac{4}{\sin 40}(\sin 40)$$

$$= 4$$

QUESTION

$\alpha \in \left(\pi, \frac{3\pi}{2}\right) \rightarrow 3^{rd}$ Quad $\quad \beta \in \left(\frac{\pi}{2}, \pi\right) \rightarrow 2^{nd}$ Quad $\quad$ Find $\tan(\alpha+\beta)$

If $\cot \alpha = \frac{1}{2}$ and $\sec \beta = -\frac{5}{3}$, where $\alpha \in]\pi, \frac{3\pi}{2}[$ and $\beta \in]\frac{\pi}{2}, \pi[$, then find the value of $\tan(\alpha+\beta)$.

$\tan\alpha = 2$

$\beta \in 2^{nd}$ Quad

$\tan\beta = -\frac{4}{3}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

$$= \frac{2 - \frac{4}{3}}{1 - 2\left(-\frac{4}{3}\right)}$$

$$= \frac{6-4}{3+8} = \frac{2}{11}$$

(A) $-\frac{2}{11}$

(B) $-\frac{3}{11}$

(C) $\frac{2}{11}$ ✓

(D) $\frac{1}{11}$

QUESTION

$1+\cos\left(\pi-\frac{3\pi}{8}\right)$ $\quad$ $1+\cos\left(\pi-\frac{\pi}{8}\right)$

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right) \text{ equals}$$

(A) $\frac{1}{2}$

(B) $\cos\frac{\pi}{8}$

(C) $\frac{1}{8}$

(D) $\frac{1+\sqrt{2}}{2\sqrt{2}}$

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{3\pi}{8}\right)\left(1-\cos\frac{\pi}{8}\right)$$

$$=\left(1-\cos^2\frac{\pi}{8}\right)\left(1-\cos^2\frac{3\pi}{8}\right)$$

$$=\left(\sin^2\frac{\pi}{8}\right)\left(\sin^2\frac{3\pi}{8}\right)$$

$$=\left(\sin^2\frac{\pi}{8}\right)\left(\cos^2\frac{\pi}{8}\right)=\sin^2\theta\cdot\cos^2\theta$$

$$=\left(\frac{\sin 2\theta}{2}\right)^2$$

$$\frac{5\pi}{8}=\frac{8\pi-3\pi}{8}$$
$$=\pi-\frac{3\pi}{8}$$

$$\frac{7\pi}{8}=\frac{8\pi-\pi}{8}=\pi-\frac{\pi}{8}$$

(1) $\frac{\pi}{8}=\frac{180}{8}=\frac{45}{2}=22.5$

(2) $3\left(\frac{\pi}{8}\right)=3(22.5)=67.5$
$$=90-22.5$$

$$\sin\frac{3\pi}{8}=\sin 67.5=\sin(90-22.5)$$
$$=\cos 22.5$$

$= \sin^2\theta \cos^2\theta \qquad \theta = \frac{\pi}{8}$

$= \left(\frac{\sin 2\theta}{2}\right)^2$

$= \left[\frac{\sin 2\left(\frac{\pi}{8}\right)}{2}\right]^2 = \left(\frac{\sin \frac{\pi}{4}}{2}\right)^2$

$= \left(\frac{1/\sqrt{2}}{2}\right)^2 = \frac{1/2}{4} = \frac{1}{8}.$

QUESTION

The value of $\cos(\overset{x}{35^\circ + A})\cos(\overset{y}{35^\circ - B}) + \sin(\overset{x}{35^\circ + A})\sin(\overset{y}{35^\circ - B})$ is equal to

$\cos[x - y]$

$= \cos[(35 + A) - (35 - B)]$

$= \cos[A + B]$

A) $\sin(A + B)$

B) $\sin(A - B)$

C) $\cos(A + B)$

D) $\cos(A - B)$

QUESTION

$$\overset{A}{}\quad \overset{B}{}\quad \overset{A}{}\quad \overset{B}{}$$

The value of $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$ is equal to

(A) $\sin(x+y)$

(B) $\sin(x-y)$

(C) $\cos(x+y)$

(D) $\cos(x-y)$

$\cos[A+B]$

$\cos\left[\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right]$

$\cos\left[\frac{\pi}{2}-(x+y)\right]$

$=\underline{\sin(x+y)}$

QUESTION

The value of $2 \operatorname{cosec} 2x + \operatorname{cosec} x$ is equal to

$$\frac{2}{\sin 2x} + \frac{1}{\sin x}$$

$$= \frac{2}{2\sin x \cos x} + \frac{1}{\sin x}$$

$$= \frac{1}{\sin x \cdot \cos x} + \frac{1}{\sin x}$$

$$= \frac{1+\cos x}{\sin x \cdot \cos x}$$

$$= \frac{\cancel{2}\cos^{\cancel{2}} \frac{x}{2}}{\cancel{2}\sin\frac{x}{2}\cos\cancel{\frac{x}{2}}} \times \frac{1}{\cos x}$$

$$= \frac{\cos x/2}{\sin x/2} \times \sec x$$

$$= \sec x \cdot \cot\left(\frac{x}{2}\right)$$

(A) $\tan x \,.\, \sec(x/2)$

(B) $\sec x \,.\, \cot(x/2)$

(C) $\sec x \,.\, \tan(x/2)$

(D) $\tan x \,.\, \cot(x/2)$

QUESTION

$(\operatorname{cosec} x - \cot x) =$

A $\tan x/2$

B $\cot x/2$

C $\tan x$

D $\cot x$

QUESTION

$$\frac{1}{1-\sin x}=$$

A $\sec x+\tan x$

B $\sec^2 x+\sec x\tan x$

C $\cos x$

D $\tan x\sec x$

$$=\frac{1}{1-\sin x}\times\frac{1+\sin x}{1+\sin x}$$

$$=\frac{1+\sin x}{1^2-\sin^2 x}$$

$$=\frac{1+\sin x}{\cos^2 x}$$

$$=\sec^2 x+\sec x\cdot\tan x$$

$$\frac{d}{dx}\left(\frac{1}{1-\sin x}\right)$$

$$\frac{d}{dx}(\sec^2 x+\sec x\tan x)$$

$$=2\sec^2 x\cdot\tan x+\sec^3 x+\sec x\cdot\tan^2 x$$

$$\int\frac{1}{1-\sin x}dx=\int\sec^2 x+\sec x\tan x\,dx$$

$$=\tan x+\sec x+C$$

$$\frac{d}{dx}\left(\underbrace{\sec^2 x}_{\downarrow} + \underbrace{\sec x \cdot \tan x}_{\downarrow}\right)$$

$$2\sec x(\sec x \tan x) + \sec x(\sec^2 x) + \tan x(\sec x \tan x)$$

$$2\sec^2 x \tan x + \sec^3 x + \underline{\sec x \cdot \tan^2 x}$$

Converting into simplest forms

(2) $\dfrac{1}{1+\sin x}$

$\Downarrow$

$\dfrac{1-\sin x}{1-\sin^2 x}$

$\dfrac{1-\sin x}{\cos^2 x}$

$\sec^2 x - \sec x \tan x$

(3) $\dfrac{1}{1+\cos x}$

$\Downarrow$

$\dfrac{1-\cos x}{1-\cos^2 x}$

$\dfrac{1-\cos x}{\sin^2 x}$

$\operatorname{cosec}^2 x - \operatorname{cosec} x \cdot \cot x$

(4) $\dfrac{1}{1-\cos x}$

$\dfrac{1+\cos x}{1-\cos^2 x}$

$\dfrac{1+\cos x}{\sin^2 x}$

$\operatorname{cosec}^2 x + \operatorname{cosec} x \cdot \cot x$

$$\frac{\cos x}{1+\cos x}$$

$$\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}} = \frac{1}{2}\left[1-\tan^2\frac{x}{2}\right] = \frac{1}{2}\left[1-\left(\sec^2\frac{x}{2}-1\right)\right]$$

$$= \frac{1}{2}\left[2-\sec^2\frac{x}{2}\right]$$

$$\Downarrow$$

$$\int \frac{\cos x}{1+\cos x}\,dx = \int 1-\frac{1}{2}\sec^2 x/2\,dx$$

$$= x-\frac{1}{2}\,\frac{(\tan x/2)}{1/2}+c$$

$$= x-\tan\frac{x}{2}+c$$

$$\frac{1+\cos x-1}{1+\cos x}$$

$$1-\frac{1}{1+\cos x}$$

Rationalise

$$1-\frac{1-\cos x}{\sin^2 x}$$

$$1-\operatorname{cosec}^2 x+\operatorname{cosec} x\cot x$$

$$\frac{\cos x}{1+\cos x}\times\frac{1-\cos x}{1-\cos x}$$

$$\frac{\cos x-\cos^2 x}{\sin^2 x}$$

$$\cot x\cdot\operatorname{cosec} x-\cot^2 x$$

$$\cot x\cdot\operatorname{cosec} x-(\operatorname{cosec}^2 x-1)$$

$$1-\operatorname{cosec}^2 x+\cot x\operatorname{cosec} x$$

$$\int\frac{\cos x}{1+\cos x}\,dx = \int 1-\operatorname{cosec}^2 x+\cot x\operatorname{cosec} x\,dx$$

$$= x+(\cot x-\operatorname{cosec} x)+c$$

QUESTION

$$\frac{1}{1+\cos x}=$$

(A) $\operatorname{cosec}^2 x$

(B) $\operatorname{cosec} x + \cot x$

(C) $\operatorname{cosec}^2 x - \operatorname{cosec} x \,.\, \cot x$

(D) $\cot x$

QUESTION

$$\frac{\sin x}{1+\cos x} =$$

(A) $\sec^2 x + \sec x \tan x$

(B) $1 - \sec^2 x - \sec x \tan x$

(C) $1 - \sec^2 x + \sec x \tan x$

(D) $\tan\left(\frac{x}{2}\right)$

$$\frac{\sin x}{1+\cos x} \times \frac{1-\cos x}{1-\cos x}$$

$$\frac{\sin x - \sin x \cos x}{\sin^2 x}$$

$$= \operatorname{cosec} x - \cot x$$

$$= \tan\left(\frac{x}{2}\right)$$

QUESTION

$$\frac{\cos x}{1+\cos x} =$$

A $1 - \operatorname{cosec}^2 x + \operatorname{cosec} x \cot x$

B $1 + \operatorname{cosec}^2 x - \operatorname{cosec} x \cot x$

C $\tan x + \sec x$

D $\tan^2 x + \sec^2 x$

QUESTION

$$\frac{\cos x}{1-\cos x}=$$

(A) $\operatorname{cosec}^2 x + \cot x \operatorname{cosec} x - 1$

(B) $1/2\,[1 + \sec 2x - \tan 2x]$

(C) $\sec 2x - \tan 2x$

(D) $\sec x + \tan x$

$$\frac{\cos x}{1-\cos x}\times\frac{1+\cos x}{1+\cos x}=\frac{\cos x(1+\cos x)}{\sin^2 x}$$

$$=\frac{\cos x+\cos^2 x}{\sin^2 x}$$

$$=\cot x\cdot\operatorname{cosec} x+\cot^2 x$$

$$=\cot x\operatorname{cosec} x+\underline{\operatorname{cosec}^2 x-1}$$

QUESTION

$$\frac{\sin x}{\sin(x+a)} =$$

(A) $\cos a + \sin a$

(B) $\cos a - \cot(x+a)\sin a$

(C) $\sin a + \tan(x+a)$

(D) $\cos a + \sin a$

$$\frac{\sin[x+a-a]}{\sin(x+a)} = \frac{\sin[\overset{A}{(x+a)} - \overset{B}{a}]}{\sin(x+a)}$$

$\rightarrow \sin(A-B) = \sin A\cos B - \cos A\sin B$

$$= \frac{\sin(x+a)\cos a}{\sin(x+a)} - \frac{\cos(x+a)\sin a}{\sin(x+a)}$$

$$= \cos a - \cot(x+a)\sin a$$

$$\int \frac{\sin x}{\sin(x+a)}\,dx$$

$$= \int \cos a - \cot(x+a)\sin a\,dx$$

$$= \cos a\,(x) - \log\sin(x+a)\sin a + c$$

$\cos a$ & $\sin a$ are constant

$\int \cot x\,dx = \log(\sin x) + c$

$\int \tan x\,dx = \log(\sec x) + c$

$$\frac{d}{dx}\left[\frac{\sin x}{\sin(x+a)}\right]$$

$$= \frac{d}{dx}\left[\cos a - \cot(x+a)\sin a\right]$$

$$= 0 - [-\operatorname{cosec}^2(x+a)\sin a$$

$$= \sin a \cdot \operatorname{cosec}^2(x+a)$$

Thank

You