Relations and Functions

If $n(A) = m$ and $n(B) = n$

1. Then $n(A \times B) = mn$
   $\therefore$ Number of relations form A to $B = 2^{mn}$.
   [Relations are subsets of cartesian products]
2. $n(B \times A) = mn$
   $\therefore$ Number of relations from B to $A = 2^{nm}$.
3. $n(A \times A) = m^2$
   $\therefore$ Number of relations from A to $A = 2^{m^2}$

If $n(A) = m$ and $n(B) = n$:

1. Number of functions from A to $B = [n(B)]^{n(A)} = n^m$
2. Number of functions from B to $A = [n(A)]^{n(B)} = m^n$
3. Number of functions from A to $A = [n(A)]^{n(A)} = m^m$
4. Number of relations from A to B which are not functions $= 2^{mn} - n^m$.

If $n(A) = m$ and $n(B) = n$

1. Number of one–one functions from A to $B = \begin{cases} {}^nP_m & \text{if } n(A) \le n(B) \\ 0 & \text{if } n(A) > n(B) \end{cases}$

Note:

One–one functions exists only if $n(\text{Domain}) \le n(\text{codomain})$

Here n(Domain) means number of elements in Domain and n(codomain) means number of elements in Codomain

2. Number of functions from A to B which are not one–one [if $n(A) \le n(B)$]

= Number of many one functions from A to $B = n^m - {}^nP_m$

If $n(A) = n$ and $n(B) = m$

Number of onto function from A to B $= \begin{cases} \sum_{r=0}^{m-1} {}^{m}C_r (-1)^r (m-r)^n & \text{if } n(B) \le n(A) \\ 0 & \text{if } n(B) > n(A) \end{cases}$

Special case of onto function:

If $n(A) = n$, where $n \ge 2$ and $n(B) = 2$

Then no. of onto functions from A to B = $2^n - 2$

Note:

Onto function exists only if n(codomain) $\le n$(Domain)

Here n(codomain) means number of elements in codomain and n(Domain) means number of elements in Domain.

- Number of Bijections from A to B $= \begin{cases} n! & \text{if } n(A) = n(B) = n \\ 0 & \text{if } n(A) \ne n(B) \end{cases}$

Note:

Bijective function exists only if n(Domain) = n(Codomain)