ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture – 02

Integrals

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Recap *of previous lecture*

1. Indefinite Integrals
2.
3.
4.

Topics *to be covered*

1. Indefinite Integrals – continue
2.
3.
4.

Integrals
(6 m)

Indefinite
(3m)

1. Integration by Parts
2. Integration of Particular func
3. 1. By substitution
   2. By Trigonometric
   3. Partial fraction

Definite Integrals
(3m)

1. $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$
2. $\int_{-a}^{a}$ even or odd or $\int_0^{2a}$ last 2 Properties
3. $\int_a^b = \int_a^c + \int_c^b$

QUESTION

#Q. The value of $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$ is

Put $\sqrt{x} = t$

$\frac{1}{\sqrt{x}}dx = 2dt$

(A) $-\frac{1}{2}\cos\sqrt{x} + c$

$= 2\int \sin t\, dt$

(B) $\frac{1}{2}\sin\sqrt{x} + c$

$= -2\cos t + c$

(C) $2\sin\sqrt{x} + c$

$= -2\cos\sqrt{x} + c$

(D) $-2\cos\sqrt{x} + c$

QUESTION

#Q. The value of $\int \frac{x\,dx}{\sqrt{1-x^2}\cos^2\sqrt{1-x^2}}$ is

(A) $\tan\sqrt{1-x^2}+c$

(B) $-\tan\sqrt{1-x^2}+c$

(C) $\ln\tan\sqrt{1-x^2}+c$

(D) $\sec\sqrt{1-x^2}+c$

Put $\sqrt{1-x^2}=t$

$\frac{1}{2\sqrt{1-x^2}}(-2x)\,dx=dt$

$\frac{x\,dx}{\sqrt{1-x^2}}=-dt$

$I=\int\frac{-dt}{\cos^2 t}=\int-\sec^2 t\,dt$

$=-\tan t+c=-\tan\sqrt{1-x^2}+c$

QUESTION

$$\int a^x dx = \frac{a^x}{\log a} + c$$

#Q. The value of $\int \frac{4^{x+1}+3^{x-1}}{12^x} dx$ is

(A) $4\frac{4^x}{\ln 4} + \frac{1}{3}\frac{3^x}{\ln 3} + c$

(B) $-4\frac{3^{-x}}{\ln 3} - \frac{1}{3}\frac{4^{-x}}{\ln 4} + c$

(C) $4\frac{3^x}{\ln 3} + \frac{1}{3}\frac{4^x}{\ln 4} + c$

(D) $12\ln(4^x 3^x) + c$

$$\int \frac{4^x \cdot 4}{12^x} + \frac{3^x \cdot 3^{-1}}{12^x} dx$$

$$\int \left(\frac{1}{3}\right)^x \cdot 4\, dx + \frac{1}{3}\int \left(\frac{1}{4}\right)^x dx$$

$$4\int 3^{-x} dx + \frac{1}{3}\int 4^{-x} dx$$

$$\frac{4(3^{-x})}{-\log 3} + \frac{1}{3}\frac{(4^{-x})}{(-\log 4)} + c$$

$$\frac{-4(3^{-x})}{\log 3} - \frac{1}{3}\frac{4^{-x}}{\log 4} + c$$

QUESTION

#Q. The value of $\int \frac{dx}{(1+e^{x})(1+e^{-x})}$ is

(A) $\frac{(1+e^{x})^{3}}{3}+c$

(B) $\frac{(1+e^{x})^{-1}}{-1}+c$

(C) $\ln(1+e^{x})+c$

(D) $\tan^{-1}e^{x}+c$

$\int \frac{dx}{(1+e^{x})\left(1+\frac{1}{e^{x}}\right)}$

$\int \frac{e^{x}}{(1+e^{x})(1+e^{x})}dx$

$\int \frac{e^{x}}{(1+e^{x})^{2}}dx$

Put $1+e^{x}=t$

$I=\int \frac{dt}{t^{2}}$

$=-\frac{1}{t}+c$

$=-\frac{1}{1+e^{x}}+c=-(1+e^{x})^{-1}+c$

QUESTION

#Q. If $\int \frac{dx}{e^{2x}+e^{-2x}} = A + B\tan^{-1} e^{2x}$ the A, B are

(A) Real, $\frac{1}{2}$

(B) Real, $-\frac{1}{2}$

(C) Imaginary, $\frac{1}{2}$

(D) Imaginary, $-\frac{1}{2}$

$\int \frac{1}{e^{-2x}\left[(e^{2x})^2+1\right]}dx$

$\int \frac{e^{2x}}{1+(e^{2x})^2}dx$

Put $e^{2x} = t$

$\frac{1}{2}\int \frac{dt}{1+t^2}$

$I = \frac{1}{2}\tan^{-1}(e^{2x}) + c$

$A = C = \text{Real}$

$B = \frac{1}{2}$

QUESTION

$a \log x = \log x^a$

#Q. The value of $\int e^{2 \ln \tan x} dx$ is

$\Downarrow$

$\int e^{\log_e \tan^2 x} dx$

(A) $\tan x + x + c$

(B) $\sec x + x + c$

$= \int \tan^2 x \, dx$

(C) $\tan x - x + c$

$= \int \sec^2 x - 1 \, dx$

(D) $\sec x - x + c \tan x - \tan x$

$= \tan x - x + c$

QUESTION

#Q. The value of $\int \frac{e^{\tan^{-1}\sqrt{x}}}{\sqrt{x}+x^{\frac{3}{2}}}dx$ is

(A) $e^{\tan^{-1}\sqrt{x}} + c$

(B) $2e^{\tan^{-1}\sqrt{x}} + c$

(C) $\frac{(\tan^{-1}\sqrt{x})^3}{3} + c$

(D) $\tan^{-1}\sqrt{x}\, e^{\sqrt{x}} + c$

Put $\tan^{-1}\sqrt{x} = t$

$\frac{1}{1+x}\left(\frac{1}{2\sqrt{x}}\right)dx = dt$

$\frac{1}{\sqrt{x}+x^{3/2}}\,dx = 2\,dt$

$I = 2\int e^t\,dt$

$= 2e^t + c$

$= 2e^{\tan^{-1}\sqrt{x}} + c$

7.2 34

$$I = \int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x}\,dx$$

x'y & ÷ by $\sec^2 x$

$$I = \int \frac{\sqrt{\tan x}\ \sec^2 x}{\sin x \cdot \cos x\ \sec^2 x}\,dx$$

$$= \int \frac{\sqrt{\tan x}}{\tan x}\sec^2 x\ dx$$

$$= \int \frac{1}{\sqrt{\tan x}}\sec^2 x\,dx$$

Put $\tan x = t$

$$\int \frac{1}{\sqrt{t}}dt = 2\int \frac{1}{2\sqrt{t}}dt = 2\sqrt{t} + c = 2\sqrt{\tan x} + c$$

if $\tan x$ is given

$\Downarrow$

$\sec^2 x$ will also be there in most of the cases.

$$\overline{\sin x \cdot \cos x\ \sec^2 x}$$

$$\frac{\sin x}{\cos x} = \tan x$$

QUESTION

#Q. The value of $\int \frac{\sec x \operatorname{cosec} x}{\ln(\tan x)} dx$ is

A) $\ln \tan x + c$

B) $\ln \sec x + c$

C) $\ln(\ln \tan x) + c$

D) $\ln(\ln \sec x) + c$

Put $\log(\tan x) = t$

$\frac{1}{\tan x} \sec^2 x \, dx = dt$

$\frac{\cos x}{\sin x} \sec^2 x \, dx = dt$

$\operatorname{cosec} x \cdot \sec x \, dx = dt$

$I = \int \frac{dt}{t} = \underline{\log(\log \tan x) + c}$

QUESTION

#Q. The value of $\int \frac{\tan^m x}{\sin x \cos x} dx$ is

(A) $\frac{(\tan x)^{m+1}}{m+1} + c$

(B) $\frac{(\ln \sec x)^{m+1}}{m+1} + c$

(C) $\frac{(\tan x)^m}{m} + c$

(D) $\frac{(\cot x)^m}{m} + c$

$\times$ by & $\div$ by $\sec^2 x$

$$\int \frac{\tan^m x}{\sin x \cos x} \frac{\sec^2 x}{\sec^2 x}$$

$$I = \int \frac{\tan^m x \cdot \sec^2 x}{\tan x} dx$$

$$= \int \tan^{m-1} x \sec^2 x \, dx$$

Put $\tan x = t$

$$I = \int t^{m-1} dt$$

$$= \frac{t^m}{m} + c$$

$$= \frac{(\tan x)^m}{m} + c$$

QUESTION

#Q. The value of $\int \frac{1}{x(\ln x)^{10}} dx$ is

(A) $\frac{1}{9\ln x} + c$

(B) $\frac{1}{9(\ln x)^9} + c$

(C) $-\frac{1}{9}(\ln x)^{-9} + c$ ✓

(D) $-\frac{1}{10}(\ln x)^{-10} + c$

Put $\log x = t$

$\frac{1}{x} dx = dt$

$\int t^{-10} dt$

$= \frac{t^{-9}}{-9} + C = \frac{-1}{9(\log x)^9} + c$

QUESTION

#Q. The value of $\int \sqrt{\frac{1-x}{1+x}}\,dx$ is

(A) $\cos^{-1}(x)+\sqrt{1-x^2}+c$

(B) $\cos^{-1}(x)-\sqrt{1-x^2}+c$

(C) $\sin^{-1}(x)-\sqrt{1-x^2}+c$

(D) $\sin^{-1}(x)+\sqrt{1-x^2}+c$

Put $x=\cos\theta \rightarrow \theta=\cos^{-1}x$ | $\sin\theta=\sqrt{1-x^2}$

$dx=-\sin\theta\, d\theta$

$I=-\int \tan\frac{\theta}{2}\cdot(\sin\theta)\,d\theta$

$I=-2\int \frac{\sin\theta/2}{\cancel{\cos\frac{\theta}{2}}}\sin\frac{\theta}{2}\cancel{\cos\frac{\theta}{2}}$

$=-\int 2\sin^2\theta/2\, d\theta$

$=-\int 1-\cos\theta\, d\theta$

$I=-[\theta-\sin\theta]+c$

$=-\cos^{-1}x+\sqrt{1-x^2}+c$

$\Downarrow$

$=-\left(\frac{\pi}{2}-\sin^{-1}x\right)+\sqrt{1-x^2}+c$

$=\sin^{-1}x+\sqrt{1-x^2}+\left(c-\frac{\pi}{2}\right)$

$=\sin^{-1}x+\sqrt{1-x^2}+c_1$

QUESTION

#Q. The value of $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}\,dx$ is

$\rightarrow \sqrt{\frac{1-x}{1+x}} = \tan\frac{\theta}{2}$

Put $x = \cos\theta \rightarrow \theta = \cos^{-1}x$ & $\sin\theta = \sqrt{1-x^2}$

$dx = -\sin\theta\, d\theta$

(A) $\frac{x^2}{4} + c$

(B) $\frac{x}{2}\cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + c$

(C) $\frac{\sin^{-1}x}{2} + c$

(D) $\frac{1}{2}\cos^{-1}x + c$

$I = -\int \tan^{-1}\left(\tan\frac{\theta}{2}\right)(\sin\theta)$

$= -\int \frac{\theta}{2}\sin\theta\, d\theta$

$= -\frac{1}{2}\left[\theta(-\cos\theta) - (1)(-\sin\theta)\right] + c$

$= \frac{\theta}{2}\cos\theta - \frac{1}{2}\sin\theta$

$= \frac{x}{2}\cos^{-1}x - \frac{1}{2}\sqrt{1-x^2} + c$

If $y = \cos^{-1}\left(\frac{1-x}{1+x}\right)$ Find $\frac{dy}{dx}$

Put $x = \tan^2\theta$

$\tan\theta = \sqrt{x}$

$\theta = \tan^{-1}\sqrt{x}$

$$y = \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$y = \cos^{-1}(\cos 2\theta) = 2\theta$$

$$y = 2\tan^{-1}\sqrt{x}$$

$$y_1 = \frac{2}{1+x}\frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}+x^{3/2}}$$

$$I = \int \cos^{-1}\left(\frac{1-x}{1+x}\right) dx$$

Put $x = \tan^2\theta$

$dx = 2\tan\theta \sec^2\theta$

$$I = 2\int \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) \tan\theta \sec^2\theta \, d\theta$$

$$= 2\int \cos^{-1}(\cos 2\theta) \tan\theta \sec^2\theta \, d\theta$$

$$= 2\int \underset{u}{2\theta} \, \underbrace{\tan\theta \sec^2\theta \, d\theta}_{v}$$

$$I = 4\left[\theta \int \tan\theta \sec^2\theta \, d\theta - \int (1) \int \tan\theta \sec^2\theta \, d\theta\right]$$

Put $\tan\theta = u$

$\sec^2\theta \, d\theta = du$

$$= 4\left[\theta \int u \, du - \int \int u \, du \, d\theta\right]$$

$$= 4\left[\theta\left(\frac{u^2}{2}\right) - \int \left(\frac{u^2}{2}\right) d\theta\right]$$

$$= 4\left[\frac{\theta(\tan^2\theta)}{2} - \frac{1}{2}\int \tan^2\theta \, d\theta\right]$$

$$= 4\left[\frac{\theta(\tan^2\theta)}{2} - \frac{1}{2}\int (\sec^2\theta - 1) \, d\theta\right]$$

$$= 4\left[\frac{\theta(\tan^2\theta)}{2} - \frac{1}{2}(\tan\theta - \theta)\right] + C$$

$$I = 4\left[\frac{\theta(\tan^2\theta)}{2} - \frac{1}{2}(\tan\theta - \theta)\right] + C$$

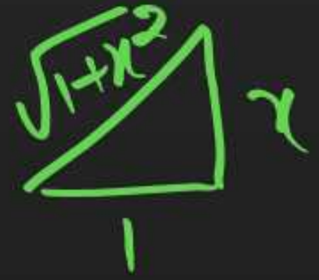
$$x = \tan^2\theta \quad \Big| \quad \theta = \tan^{-1}\sqrt{x}$$

$$\tan\theta = \sqrt{x}$$

$$I = 2\left(\tan^{-1}\sqrt{x}\right)x - 2\left(\sqrt{x}\right) + 2\tan^{-1}\sqrt{x} + C$$

QUESTION

#Q. The value of $\int \sin\left[\tan^{-1}(x)\right]dx$ is



(A) $\dfrac{1}{\sqrt{1+x^2}}+c$

(B) $\dfrac{1}{\sqrt{1-x^2}}+c$

(C) $\sqrt{1+x^2}+c$

(D) $\sqrt{1-x^2}+c$

$I=\int \sin\left[\sin^{-1}\dfrac{x}{\sqrt{1+x^2}}\right]dx$

$I=\int \dfrac{x}{\sqrt{1+x^2}}dx$

Put $1+x^2=t$

$x\,dx=\dfrac{dt}{2}$

$I=\int \dfrac{1}{2\sqrt{t}}dt$

$=\sqrt{t}+c$

$=\sqrt{1+x^2}+c$

QUESTION

#Q. If $\int \frac{1+x+x^2}{x^2(1+x)}dx = \frac{k}{x} + l\ln|1+x| + c$ then

(A) $k = 1, l = 1$

(B) $k = -1, l = 1$

(C) $k = 1, l = -1$

(D) $k = -1, l = -1$

$I = \int \frac{(1+x)}{x^2(1+x)} + \frac{x^2}{x^2(1+x)}\, dx$

$= \int \frac{1}{x^2} + \frac{1}{1+x}\, dx$

$= -\frac{1}{x} + (1)\log|1+x| + c$

$\Rightarrow k = -1 \;\&\; l = 1$

QUESTION

#Q. The value of $\int \frac{1}{\sqrt{x}(3+x)}dx$ is

Put $\sqrt{x} = t \Rightarrow x = t^2$

$\frac{1}{\sqrt{x}}dx = 2dt$

(A) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{x}}{3}\right) + c$

(B) $\frac{2}{\sqrt{3}}\ln\left|\sqrt{x} + \sqrt{3+x}\right| + c$ $= 2\int \frac{dt}{3+t^2}$

(C) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{\frac{x}{3}}\right) + c$ $I = 2\int \frac{1}{(\sqrt{3})^2 + t^2}dt$

(D) $\frac{1}{\sqrt{3}}\ln\left|\sqrt{x} - \sqrt{3+x}\right| + c$

$I = \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + c$

$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{\frac{x}{3}}\right) + c$

QUESTION

#Q. The value of $\int \frac{10^{x/2}}{\sqrt{10^{-x}-10^{x}}}\,dx$ is

(A) $\frac{1}{\ln 10}\sin^{-1}(10^x)+c$

(B) $2\sqrt{10^{-x}+10^{x}}+c$

(C) $\frac{1}{\ln 10}\ln\left|10^x+\sqrt{10^{2x}+1}\right|+c$

(D) $\frac{1}{\ln 10}\ln\left|10^x+\sqrt{10^{x}}-1\right|+c$

$$I=\int \frac{10^{x/2}}{\sqrt{10^{-x}(1-(10^x)^2)}}\,dx$$

$$=\int \frac{10^{x/2}}{10^{-x/2}\sqrt{1-(10^x)^2}}\,dx$$

$$=\int \frac{10^{x}}{\sqrt{1-(10^x)^2}}\,dx$$

Put $10^x=t$

$10^x\,dx=\frac{dt}{\log 10}$

$$I=\frac{1}{\log 10}\int \frac{1}{\sqrt{1-t^2}}\,dt$$

$$=\frac{1}{\log 10}\sin^{-1}(10^x)+c$$

$$I = \int \frac{1}{\sin x + \cos x} dx$$

$\sin x + \cos x$

$\Downarrow$

$A \sin x + B \cos x$

$A = 1$ $\quad$ $B = 1$

x^{ly} & $\div ly$ $\sqrt{A^2+B^2} = \sqrt{2}$

$$I = \frac{1}{\sqrt{2}} \int \frac{1}{\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x} dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sin\left(x + \frac{\pi}{4}\right)} dx$$

$$= \frac{1}{\sqrt{2}} \int \operatorname{cosec}\left(x + \frac{\pi}{4}\right) dx$$

$$= \frac{1}{\sqrt{2}} \log\left|\operatorname{cosec}\left(x + \frac{\pi}{4}\right) - \cot\left(x + \frac{\pi}{4}\right)\right| + c$$

WKT $\operatorname{cosec}\theta - \cot\theta = \tan \theta/2$

$$= \frac{1}{\sqrt{2}} \log\left|\tan\left(\frac{x}{2} + \frac{\pi}{8}\right)\right| + c$$

QUESTION

$\sqrt{1+\sin x} = \sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}} = \sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}$

$= \cos\frac{x}{2}+\sin\frac{x}{2}$

#Q. The value of $\int \frac{dx}{\sqrt{1+\sin x}}$ is

$I = \frac{1}{\sqrt{2}}\int \frac{1}{\sin\frac{x}{2}\frac{1}{\sqrt{2}}+\cos\frac{x}{2}\frac{1}{\sqrt{2}}}dx$

$= \frac{1}{\sqrt{2}}\int \operatorname{cosec}\left(\frac{x}{2}+\frac{\pi}{4}\right)dx$

$= \frac{1}{\sqrt{2}}\log\left|\tan\left(\frac{x}{4}+\frac{\pi}{8}\right)\right|+C$

(A) $\frac{1}{\sqrt{2}}\ln\tan\left(\frac{x}{4}+\frac{\pi}{8}\right)+c$

(B) $\sqrt{2}\ln\tan\left(\frac{x}{4}+\frac{\pi}{8}\right)+c$

(C) $\frac{1}{\sqrt{2}}\ln\tan\left(\frac{x}{4}\right)+c$

(D) $\sqrt{2}\ln\tan\left(\frac{x}{8}\right)+c$

$$I = \int \frac{1}{\sqrt{3}\cos x - \sin x} dx$$

$$I = \frac{1}{2}\int \frac{1}{\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x} dx$$

$$= \frac{1}{2}\int \frac{1}{\sin\left(\frac{\pi}{3} - x\right)} dx$$

$$= \frac{1}{2}\int \operatorname{cosec}\left(\frac{\pi}{3} - x\right) dx$$

↳ coefficient of $x = -1$

$$= -\frac{1}{2}\log\left|\tan\left(\frac{\pi}{6} - \frac{x}{2}\right)\right| + c$$

method 2

$$I = \frac{1}{2}\int \frac{1}{\cos x \frac{\sqrt{3}}{2} - \sin x \frac{1}{2}} dx$$

$$= \frac{1}{2}\int \frac{1}{\cos\left(x + \frac{\pi}{6}\right)} dx$$

$$= \frac{1}{2}\int \sec\left(x + \frac{\pi}{6}\right) dx$$

$$= \frac{1}{2}\log\left|\sec\left(x + \frac{\pi}{6}\right) + \tan\left(x + \frac{\pi}{6}\right)\right| + c$$

QUESTION

#Q. The value of $\int \left(1-\frac{1}{x^2}\right)e^{x+\frac{1}{x}}dx$ is

Put $x+\frac{1}{x}=t$

$\left(1-\frac{1}{x^2}\right)dx = dt$

(A) $-e^x\left(x+\frac{1}{x}\right)+c$

(B) $e^{x+\frac{1}{x}}+c$

(C) $e^{1-\frac{1}{x^2}}+c$

(D) $e^x\left(x+\frac{1}{x}\right)+c$

$I = \int e^t \, dt$

$= e^t + c$

$= e^{x+\frac{1}{x}} + c$

QUESTION

#Q. The value of $\int \frac{dx}{1+3\sin^2 x}$ is ÷ by $\cos^2 x$

A) $\tan^{-1}(2\tan x) + c$

B) $1/2 \tan^{-1}(2\tan x) + c$

C) $1/2 \cot^{-1}(2\tan x) + c$

D) $1/2 \sin^{-1}(2\tan x) + c$

$I = \int \frac{\sec^2 x}{\sec^2 x + 3\tan^2 x} dx$

$= \int \frac{\sec^2 x}{1+4\tan^2 x} dx$

Put $\tan x = t$

$I = \int \frac{1}{1+(2t)^2} dt$

$I = \frac{\tan^{-1}(2t)}{2} + c$

$= \frac{1}{2}\tan^{-1}(2\tan x) + c$

$$I = \int \frac{1}{1+3\cos^2 x}\,dx$$

$$= \int \frac{\sec^2 x}{\sec^2 x + 3}$$

$$= \int \frac{\sec^2 x}{4+\tan^2 x}\,dx$$

$$= \int \frac{dt}{2^2+t^2}$$

$$= \frac{1}{2}\tan^{-1}\left(\frac{\tan x}{2}\right) + C$$

QUESTION

#Q. The value of $\int \frac{dx}{e^{x}+4e^{-x}}$ is

(A) $\ln(e^{x}+4e^{-x})+c$

(B) $\frac{1}{2}\ln\left(\frac{e^{x}+2}{e^{x}-2}\right)+c$

(C) $\frac{1}{2}\tan^{-1}(e^{x})+c$

(D) $\frac{1}{2}\tan^{-1}\left(\frac{e^{x}}{2}\right)+c$

$\int \frac{e^{x}}{(e^{x})^{2}+4}\,dx$

Put $e^{x}=t$

$I=\int \frac{dt}{2^{2}+t^{2}}$

$=\frac{1}{2}\tan^{-1}\left(\frac{t}{2}\right)+c$

$=\frac{1}{2}\tan^{-1}\left(\frac{e^{x}}{2}\right)+c$

QUESTION

#Q. The value of $\int x 2^x dx$ is

(A) $\frac{2^x}{\ln 2}(x + \ln 2) + c$

(B) $\frac{2^x}{\ln 2}\left[\ln\left(\frac{1}{2}e^x\right)\right] + c$

(C) $\frac{2^x}{(\ln 2)^2}[x\ln 2 - 1] + c$

(D) $\frac{2^x}{\ln x}[x - \ln 2] + c$

$\log t = \log 2(x)$

Put

$2^x = t \Rightarrow x = \log_2 t = \frac{\log_1 t}{\log_1 2}$

$2^x dx = \frac{dt}{\log 2}$

$I = \frac{1}{(\log 2)^2}\int \log t \; dt$

$I = \frac{1}{(\log 2)^2}[\log(t) - t] + c$

$I = \frac{t}{(\log 2)^2}[\log t - 1] + c$

$I = \frac{2^x}{(\log 2)^2}[x\log 2 - 1] + c$

QUESTION

#Q. The value of $\int \log_{10} x \, dx$ is

(A) $\dfrac{x(\ln x - 1)}{\ln 10} + c$

(B) $\dfrac{x(1 - \ln x)}{\ln 10} + c$

(C) $\dfrac{x(x - \ln x)}{\ln 10} + c$

(D) $\dfrac{x(1 + \ln x)}{\log_{10} e} + c$

$I = \dfrac{1}{\log_e 10} \int \log x (1) \, dx$

$= \dfrac{1}{\log_e 10} [\log x (x) - x] + c$

$= \dfrac{x}{\log 10} [\log x - 1] + c$

QUESTION

#Q. The value of $\int \sin 2x \ln \cos x dx$ is

(A) $\cos^2 x\left[\frac{1}{2}+\ln \cos x\right]+c$

(B) $\cos^2 x \cdot 1 \ln \cos x+c$

(C) $\cos^2 x\left[\frac{1}{2}-\ln \cos x\right]+c$

(D) $\frac{1}{2}-\ln \cos x+c$

$I = 2\int \sin x \cos x \log \cos x \, dx$

Put $\cos x = t$

$\sin x \, dx = -dt$

$= -2\int t \log t \, dt$

$= -2\left[\log t\left(\frac{t^2}{2}\right) - \frac{t^2}{4}\right] + c$

$= \frac{t^2}{2} - t^2 \log t + c$

$= t^2\left[\frac{1}{2} - \log t\right]$

$= \cos^2 x\left[\frac{1}{2} - \log \cos x\right] + c$

QUESTION

#Q. The value of $\int \frac{dx}{\cos x+\sqrt{3}\sin x}$ is

(A) $\ln\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+c$

(B) $\ln\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+c$

(C) $\frac{1}{2}\ln\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)+c$

(D) $\frac{1}{2}\ln\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)+c$

$$I=\frac{1}{2}\int\frac{1}{\sin x\frac{\sqrt{3}}{2}+\cos x\frac{1}{2}}dx$$

$$=\frac{1}{2}\int\operatorname{cosec}\left(x+\frac{\pi}{6}\right)dx$$

$$=\frac{1}{2}\log\left|\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)\right|+c$$

$$I = \int \frac{1}{1-\tan x}\,dx$$

$$= \int \frac{\cos x}{\cos x - \sin x}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{\cos x}{\cos x\frac{1}{\sqrt{2}} - \sin x\frac{1}{\sqrt{2}}}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{\cos x}{\sin\left(\frac{\pi}{4}-x\right)}\,dx$$

WRT $\cos x = \cos(-x)$

$$I = \frac{1}{\sqrt{2}}\int \frac{\cos\left[\overset{A}{\left(\frac{\pi}{4}-x\right)} - \overset{B}{\left(\frac{\pi}{4}\right)}\right]}{\sin\left(\frac{\pi}{4}-x\right)}$$

$$= \frac{1}{\sqrt{2}}\int \frac{\cos\left(\frac{\pi}{4}-x\right)\cos\frac{\pi}{4}}{\sin\left(\frac{\pi}{4}-x\right)} + \frac{\cancel{\sin\left(\frac{\pi}{4}-x\right)}\sin\frac{\pi}{4}}{\cancel{\sin\left(\frac{\pi}{4}-x\right)}}\,dx$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{2}}\cot\left(\frac{\pi}{4}-x\right) + \frac{1}{\sqrt{2}}\,dx$$

$$= \frac{1}{2}\left[\log\sin\left(\frac{\pi}{4}-x\right) + x\right] + c$$

$$= \frac{1}{2}\left[\log\left|\frac{\cos x - \sin x}{\sqrt{2}}\right| + x\right] + c$$

$$= \frac{1}{2}\left[\log(\cos x - \sin x) - \log\sqrt{2} + x\right] + c$$

$$= \frac{1}{2}\left[\log(\cos x - \sin x) + x\right] + c \text{ where } c_1 = c - \frac{\log\sqrt{2}}{2}$$

$\log \sin\left(\frac{\pi}{4}-x\right)$

$\log\left(\sin\overset{\frac{1}{\sqrt{2}}}{\frac{\pi}{4}}\cos x-\cos\overset{\frac{1}{\sqrt{2}}}{\frac{\pi}{4}}\sin x\right)$

$= \log\left[\frac{\cos x-\sin x}{\sqrt{2}}\right] \Rightarrow \log\frac{A}{B} = \log A - \log B.$

$= \log(\cos x - \sin x) - \log\sqrt{2}$

$$I = \int \frac{\cos x}{\cos x - \sin x}\,dx$$

$$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x}\,dx$$

$$= \frac{1}{2}\int \frac{\cos x + \cos x + \sin x - \sin x}{\cos x - \sin x}$$

$$= \frac{1}{2}\int \frac{(\cos x - \sin x)}{\cos x - \sin x} + \frac{(\sin x + \cos x)}{\cos x - \sin x}\,dx$$

$$= \frac{1}{2}\int 1 - \frac{\overbrace{(\sin x + \cos x)}^{f'(x)}}{\underbrace{\cos x - \sin x}_{f(x)}}\,dx$$

$$I = \frac{1}{2}\left[x - \log(\cos x - \sin x)\right] + C$$

QUESTION

#Q. The value of $\int \frac{\sin x}{\sin x-\cos x}dx$ is

A $\frac{x}{2}-\frac{1}{2}\ln(\sin x-\cos x)+c$

B $\ln\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)+c$

C $\ln\tan x+x+c$

D $\frac{x}{2}+\frac{1}{2}\ln(\sin x-\cos x)+c$

$$\frac{1}{2}\int\frac{\sin x+\sin x+\cos x-\cos x}{\sin x-\cos x}$$

$$\frac{1}{2}\int\frac{\sin x-\cos x}{\sin x-\cos x}+\frac{\sin x+\cos x}{\sin x-\cos x}dx$$

$$\frac{1}{2}\int 1+\frac{\overbrace{\cos x+\sin x}^{f'(x)}}{\underbrace{\sin x-\cos x}_{f(x)}}dx$$

$$=\frac{1}{2}\left[x+\log(\sin x-\cos x)\right]+c$$

QUESTION

#Q. If $f(0)=f'(0)=0$ and $f''(x)=\tan^2 x$ then $f(x)$ is

A. $\ln \sec x-\frac{1}{2}x^2$

B. $\ln \cos x+\frac{1}{2}x^2+c$

C. $\ln \sec x+\frac{1}{2}x^2+c$

D. $\ln(\sec x)+c$

$\int F(x)\,dx = g(x)+c$

$\int f'(x)\,dx = f(x)+c$

$\int f''(x)\,dx = \int \tan^2 x\,dx = \int \sec^2 x - 1\,dx$

$f'(x) = \tan x - x + c$

Put $x=0$

$f'(0) = 0-0+c$

$0 = 0+c$

$c=0$

$f'(x) = \tan x - x$

$\int f'(x)\,dx = \int \tan x - x\,dx$

$f(x) = \log(\sec x) - \frac{x^2}{2} + c$

Put $x=0$

$\therefore c=0$

$f(x) = \log(\sec x) - \frac{x^2}{2}$

QUESTION

#Q. The value of $\int \frac{dx}{3+2\sin x+\cos x}$ is

(A) $\tan^{-1}(\tan x)+c$

(B) $\tan^{-1}\left(\frac{\tan x}{2}\right)+c$

(C) $\tan^{-1}\left(\frac{\tan x}{3}\right)+c$

(D) $\tan^{-1}\left(\tan\frac{x}{2}+1\right)+c$

$3+\cos x = 2+(1+\cos x)$
$= 2+2\cos^2 x/2$

$I = \int \frac{1}{2+4\sin\frac{x}{2}\cos\frac{x}{2}+2\cos^2\frac{x}{2}}\,dx$

$\div$ by $\cos^2 x/2$

$I = \int \frac{\sec^2 x}{2\sec^2 x/2+4\tan\frac{x}{2}+2}$

$I = \int \frac{\sec^2 x/2}{4+2\tan^2\frac{x}{2}+4\tan\frac{x}{2}}\,dx$

$= \frac{1}{2}\int \frac{\sec^2 x/2}{\tan^2\frac{x}{2}+2\tan\frac{x}{2}+1+1}$

$= \frac{1}{2}\int \frac{\sec^2 x/2}{(\tan\frac{x}{2}+1)^2+1}$

Put $\tan\frac{x}{2}+1 = t$
$\frac{1}{2}\sec^2\frac{x}{2}\,dx = dt$

$I = \int \frac{dt}{t^2+1} = \tan^{-1}(t)+c$
$= \tan^{-1}\left(\tan\frac{x}{2}+1\right)+c$

QUESTION

#Q. The value of $\int \frac{dx}{5+4\cos x}$ is

(A) $\frac{1}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$

(B) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$

(C) $\frac{1}{3}\ln\left(\frac{\tan\frac{x}{2}+3}{\tan\frac{x}{2}-3}\right)+c$

(D) $\frac{1}{3}\ln\left(\frac{\tan x-3}{\tan x+3}\right)+c$

$5+4\cos x$
$= 1+4(1+\cos x)$
$= 1+4(2\cos^2 x/2)$
$= 1+8\cos^2 x/2$

$I = \int \frac{1}{1+8\cos^2 x/2}\,dx$

$= \int \frac{\sec^2 x/2\,dx}{\sec^2\frac{x}{2}+8}$

$I = \int \frac{\sec^2 x/2}{9+\tan^2 x/2}\,dx$

Put $\tan\frac{x}{2} = t$

$\sec^2\frac{x}{2}\,dx = 2\,dt$

$I = 2\int \frac{dt}{3^2+t^2}$

$= \frac{2}{3}\tan^{-1}\left(\frac{t}{3}\right)+c$

$= \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\tan\frac{x}{2}\right)+c$

QUESTION

#Q. The value of $\int \cos\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\} dx$ is

$\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \tan\frac{\theta}{2}$

(A) $\frac{1}{8}(x^2-1)+c$

(B) $\frac{1}{2}x^2+c$

(C) $\frac{1}{2}x+c$

(D) $x+c$

Put $x = \cos\theta$

$dx = -\sin\theta\, d\theta$

$I = -\int \cos\left[2\cancel{\tan}^{-1}\left(\cancel{\tan}\frac{\theta}{2}\right)\right]\sin\theta\, d\theta$

$= -\int \cos\left(\cancel{2}\left(\frac{\theta}{\cancel{2}}\right)\right)\sin\theta\, d\theta$

$= -\int \cos\theta\sin\theta\, d\theta$

$= -\int \frac{\sin 2\theta}{2}\, d\theta$

$I = \frac{-1}{2}\left[\frac{-\cos 2\theta}{2}\right]+c$

$= \frac{\cos 2\theta}{4}+c$

$= \frac{2\cos^2\theta-1}{4}+c$

$= \frac{2x^2-1}{4}+c$

$= \frac{x^2}{2}-\frac{1}{4}+c$

$= \frac{x^2}{2}+c_1 \qquad c_1 = c-\frac{1}{4}$

QUESTION

#Q. If $\int \frac{\sin x}{\sin(x-\alpha)} dx = Ax + B \ln \sin(x-\alpha) + c$ then

(A) $A = \sin 2\alpha$

(B) $B = \cos 2\alpha$

(C) $A = \cos\alpha, B = \sin\alpha$

(D) $A = \sin\alpha, B = \cos\alpha$

$$A = \int \frac{\sin[\overset{A}{(x-\alpha)} + \overset{B}{\alpha}]}{\sin(x-\alpha)}$$

$$= \int \frac{\sin(x-\alpha)\cos\alpha}{\sin(x-\alpha)} + \frac{\cos(x-\alpha)\sin\alpha}{\sin(x+\alpha)} dx$$

$$= \int \cos\alpha + \cot(x-\alpha)\sin\alpha \, dx$$

$$= \underbrace{\cos\alpha}_{A}(x) + \log\sin(x-\alpha)\underbrace{\sin\alpha}_{B} + c$$

QUESTION

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$\frac{n+n-2}{2} = n-1$ $\quad \frac{n-(n-2)}{2} = 1$

#Q. If $I_n = \int \frac{\sin(nx)}{\sin x}\,dx$ then $I_n - I_{n-2}$ is

$I_n - I_{n-2} = \int \frac{\sin(nx) - \sin(n-2)x}{\sin x}\,dx$

$= 2\int \frac{\cos(n-1)x\,\cancel{\sin x}}{\cancel{\sin x}}\,dx$

$= 2\left(\frac{\sin(n-1)x}{n-1}\right) + c$

(A) $2\sin(n-1)x + c$

(B) $\frac{2}{n-1}\sin(n-1)x + c$

(C) $\frac{2}{(n-1)}\cos(n-1)x + c$

(D) $2\cos(n-1)x + c$

QUESTION

#Q. If $I_n = \int \cot^n x dx$ then $I_n + I_{n-2}$ is

(A) $\frac{(\cot x)^n}{n} + c$

(B) $\frac{(\cot x)^{n-1}}{n-1} + c$

(C) $\frac{(\cot x)^{n-2}}{n-2} + c$

(D) ✓ $-\frac{(\cot x)^{n-1}}{n-1} + c$

$I_n + I_{n-2} = \int \cot^n x + \cot^{n-2} x \, dx$

$= \int \cot^{n-2} x \cdot (\cot^2 x) + \cot^{n-2} x \, dx$

$= \int \cot^{n-2} x (\cot^2 x + 1) dx$

$= \int \cot^{n-2} x (\operatorname{cosec}^2 x) dx$

Put $\cot x = t$

$I = -\int t^{n-2} dt$

$= -\frac{t^{n-1}}{n-1} + c$

$= -\frac{\cot^{n-1} x}{n-1} + c$

QUESTION

#Q. If $I_n = \int \tan^n x \, dx$ then $I_n + I_{n-2}$ is

(A) $\dfrac{(\tan x)^{n-1}}{n-2} + c$

(B) $\dfrac{(\tan x)^{n+1}}{n+1} + c$

(C) $\dfrac{(\tan x)^{n-1}}{n-1} + c$

(D) $\dfrac{(\tan x)^{n}}{n} + c$

$I_n + I_{n-2} = \int \tan^{n-2}x \,(\tan^2 x + 1)\, dx$

$= \int \tan^{n-2}x \cdot \sec^2 x \, dx$

Put $\tan x = t$

$= \int t^{n-2} dt$

$= \dfrac{t^{n-1}}{n-1} + c = \dfrac{\tan^{n-1}x}{n-1} + c$

QUESTION

$1+\cos 4x = 2\cos^2 2x$

#Q. If $\int \frac{\cos 4x+1}{\cot x-\tan x}dx = k\cos 4x + c$ then k is

$\Downarrow$

$$I = \int \frac{2\cos^2 2x}{\frac{(\cos^2 x-\sin^2 x)}{\cos x \sin x}}$$

$$= \int \frac{\cos^2 2x}{\cos 2x}(2\sin x\cos x)\,dx$$

$$= \int \cos 2x \sin 2x\,dx$$

$$= \int \frac{\sin 4x}{2}\,dx$$

$$I = -\frac{1}{8}\cos 4x + c$$

$\downarrow$

$k = -\frac{1}{8}$

(A) $-1/4$

(B) $-1/2$

(C) $-1/8$

(D) $1/2$

QUESTION

#Q. $\int \frac{1+x}{x\sin(x+\log x)}dx$

Put $x+\log x = t$

$\left(1+\frac{1}{x}\right)dx = dt$

$\frac{(x+1)}{x}dx = dt$

$I = \int \frac{1}{\sin t}dt$

$= \int \operatorname{cosec} t \, dt$

$= \log(\operatorname{cosec} t - \cot t) + c$

$= \log\left(\tan\frac{t}{2}\right) + c$

$= \log\left|\tan\left(\frac{x+\log x}{2}\right)\right| + c$

(A) $\log\left|\frac{\tan(x+\log x)}{2}\right| + c$

(B) $\frac{1}{2}\log\left|\frac{\tan(x+\log x)}{2}\right| + c$

(C) $\log\left|\tan\left(\frac{x+\log x}{2}\right)\right| + c$

(D) $\frac{1}{2}\log\left|\tan\left(\frac{x+\log x}{2}\right)\right| + c$

QUESTION

#Q. $\int \frac{(x+1)^2}{x(x^2+1)}dx$ $= \int \frac{\cancel{x^2+1}}{x(\cancel{x^2+1})} + \frac{2\cancel{x}}{\cancel{x}(x^2+1)} dx$

A) $\log|x| + c$ $= \int \frac{1}{x} + \frac{2}{x^2+1} dx$

B) $\log|x| + 2\tan^{-1}x + c$ $= \log x + 2\tan^{-1}x + c$

C) $-\log|x^2+1| + c$

D) $\log|x(x^2+1)| + c$

QUESTION

#Q. The value of $\int \frac{\sin 2x}{\sin^4 x+\cos^4 x} dx$ is

(A) $\tan^{-1}(\cot^2 x) + c$

(B) $\tan^{-1}(\tan^2 x) + c$

(C) $\tan^{-1}(\sec^2 x) + c$

(D) $\tan^{-1}(\cos^2 x) + c$

$$\int \frac{2\sin x\cos x}{\cos^4 x\,[\tan^4 x+1]}$$

$$= \int \frac{2\tan x\sec^2 x}{1+(\tan^2 x)^2}\,dx$$

Put $\tan^2 x = t$

$$= \int \frac{1}{1+t^2}\,dt$$

$$= \tan^{-1} t + c$$

$$= \tan^{-1}(\tan^2 x) + c$$

QUESTION

#Q. The value of $\int \frac{\cos x + x\sin x}{x(\cancel{x}+\cos x)}\,dx$ is

(A) $\ln[x(x+\cos x)] + c$

(B) $\ln\left[\frac{x}{x+\cos x}\right] + c$

(C) $\ln\left[\frac{x+\cos x}{x}\right] + c$

(D) $\ln\left(\frac{x-\cos x}{x}\right) + c$

$$I = \int \frac{\cos x + x\sin x + x - x}{x(x+\cos x)}$$

$$= \int \frac{(\cos x + x)}{x(x+\cos x)} + \frac{x\sin x - x}{x(x+\cos x)}\,dx$$

$$= \int \frac{1}{x} + \frac{\cancel{x}(\sin x - 1)}{\cancel{x}(x+\cos x)}\,dx$$

$$= \int \frac{1}{x} - \frac{(1-\sin x) \rightarrow f'(x)}{x+\cos x \rightarrow f(x)}$$

$$I = \log x - \log(x+\cos x) + c$$

$$= \log\left(\frac{x}{x+\cos x}\right) + c$$

QUESTION

#Q. The value of $\int \frac{dx}{x^n(1+x^n)^{\frac{1}{n}}}$, where $n \in$ N is

(A) $\frac{1}{1-n}\left(1+\frac{1}{x^n}\right)^{1-\frac{1}{n}}+c$

(B) $\frac{1}{1+n}\left(1-\frac{1}{x^n}\right)^{1+\frac{1}{n}}+c$

(C) $\frac{1}{1-n}\left(1+\frac{1}{x^n}\right)^{n-1}+c$

(D) $\frac{1}{1-n}\frac{1}{(1+x^n)}+c$

$$I = \int \frac{1}{x^n\left[x^n\left(x^{-n}+1\right)\right]^{1/n}}$$

$$= \int \frac{1}{x^n (x^n)^{1/n}\left(x^{-n}+1\right)^{1/n}}$$

$$= \int \frac{1}{x^n (x)\left(x^{-n}+1\right)^{1/n}}$$

$$I = \int \frac{1}{x^{n+1}\left(x^{-n}+1\right)^{1/n}}dx$$

Put $x^{-n}+1 = t$

$$-n x^{-n-1}dx = dt$$

$$x^{-(n+1)}dx = -\frac{dt}{n}$$

$$\frac{dx}{x^{n+1}} = \frac{-dt}{n}$$

$$I = \frac{-1}{n}\int t^{-1/n}\,dt$$

$$I = -\frac{1}{n}\int t^{-1/n}\,dt$$

$$= -\frac{1}{n}\,\frac{t^{-\frac{1}{n}+1}}{\left(-\frac{1}{n}+1\right)} + c$$

$$= -\frac{1}{n}\,\frac{t^{1-\frac{1}{n}}}{\left(1-\frac{1}{n}\right)} + c$$

$$= -\frac{1}{\cancel{n}}\,\frac{1}{\frac{(n-1)}{\cancel{n}}}\left(x^{-n}+1\right)^{1-\frac{1}{n}} + c$$

$$= \frac{1}{1-n}\left(\frac{1}{x^n}+1\right)^{1-\frac{1}{n}} + c$$

QUESTION

#Q. The value of $\int \frac{(x^9-x)^{\frac{1}{9}}}{x^{10}} dx$ is

(A) $\left(1-\frac{1}{x^8}\right)^{\frac{19}{9}}+c$

(B) $\left(1-\frac{1}{x^8}\right)^{\frac{10}{9}}+c$

(C) $\frac{9}{80}\left(1-\frac{1}{x^8}\right)^{\frac{10}{9}}+c$

(D) $\frac{1}{80}\left(1-\frac{1}{x^8}\right)^{\frac{10}{9}}+c$

$$= \int \frac{\left\{x^9\left[1-\frac{x}{x^9}\right]\right\}^{1/9}}{x^{10}} dx$$

$$(x^9)^{1/9}$$

$$= \int \frac{x\,(1-x^{-8})^{1/9}}{x^{10}}$$

$$= \int \frac{(1-x^{-8})^{1/9}}{x^9} dx$$

Put $1-x^{-8}=t$

$$8x^{-9}\,dx = dt$$

$$\frac{dx}{x^9} = \frac{dt}{8}$$

$$= \frac{1}{8}\int t^{1/9}\,dt = \frac{1}{8}\,\frac{t^{10/9}}{10/9}+c$$

$$= \frac{9}{80}\left(1-x^{-8}\right)^{10/9} dt$$

QUESTION

#Q. The value of $\int \frac{dx}{x^{1/5}\left[1+x^{4/5}\right]^{1/2}}$ is

(A) $\sqrt{1+x^{4/5}}+c$

(B) $\frac{5}{2}\sqrt{1+x^{4/5}}+c$

(C) $x^{4/5}\left(1+x^{4/5}\right)+c$

(D) $\frac{2}{5}x^{4/5}\left(1+x^{4/5}\right)+c$

$1+x^{4/5}=t$

$\frac{4}{5}x^{-1/5}\,dx=dt$

$\frac{dx}{x^{1/5}}=\frac{5}{4}dt$

$I=\frac{5}{4}\int\frac{1}{t^{1/2}}dt$

$I=\frac{5}{2}\int\frac{1}{2\sqrt{t}}dt$

$=\frac{5}{2}\sqrt{t}+c$

$=\frac{5}{2}\sqrt{1+x^{4/5}}+c$

Thank

You