ULTIMATE KCET

CRASH COURSE 2026

Relations And Functions → continue

Mathematics | Lecture : 01

Trigonometry and ITF for Differentiation and Integration

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Recap *of previous lecture*

1. Functions
2.
3.
4.

Topics *to be covered*

1. Functions – continue
2. Relation
3. Trigonometry & ITF
   for Differentiation & Integration
4.

Types of Relations

First check

if $R \subset A \times A$

i.e., R is a relation of A

Then what is A

Finite set

N

Z

R

W

Start with

Trial & error method

Ex: if A = Real No (R)

Irrational

0

+ve integer

-ve integer

Rational

Reflexive :-

$(a, a) \in R$ for each $a \in A$

$a \le b^3$

$R \subset \underline{R \times R}$

$\Downarrow$

$A = R$

0   +ve   −ve

① <u>Reflexive</u> :

→ $0 \le 0^3$ true

→ $2 \le 2^3$ true

→ $-3 \le (-3)^3$

$-3 \le -27$ False

$\therefore (-3,-3) \notin R$

$\therefore$ not reflexive.

(2) Symmetric :-

$a \le b^3$

if $(a, b) \in R \Rightarrow$ we need to first search the pair which satisfies the given condition (a, b)

if $(b, a) \in R$

$\Downarrow$

R is symmetric

if $(b, a) \notin R$

R is not symmetric

$a \le b^3$

WKT

$2 \le 9^3$

$(2,9) \in R$

but

$9 \nleq 2^3$

$(9,2) \notin R$

$\therefore$ R is not symmetric

usually we take

$a \,\&\, b$

0 & +ve

0 & -ve

+ve & -ve

0 & Rational

integer & Rational no.

Transitive:-

if $(a,b) \in R$

&

$(b,c) \in R$

if $(a,c) \in R$

R is transitive

if $(a,c) \notin R$

R is not transitive

$\Rightarrow$ First we need to search (or get) 2 ordered pairs which satisfies the given condition, provided the second (b) element in the 1st Pair should be the 1st element (b) in the second Pair.

Equivalence

1. Reflexive
2. symmetric
3. Transitive

QUESTION

Let S be the set of all real numbers and let R be a relation on S defined by $ab \Leftrightarrow a^2 + b^2 = 1$. Then, R is

A symmetric but neither reflexive nor transitive

B reflexive but neither symmetric nor transitive

C transitive but neither reflexive nor symmetric

D none of these

① $2^2 + 2^2 = 8 \neq 1$

$\therefore (2, 2) \notin R$

$\therefore R$ is not reflexive

② R is symmetric

if $(0,1) \in R$

$0^2 + 1^2 = 1$

$1^2 + 0^2 = 1$

$\therefore (1,0) \in R$

③ $(1,0) \in R$ | $(0,1) \in R$

$1^2 + 0^2 = 1$ | $0^2 + 1^2 = 1$

but $1^2 + 1^2 = 2 \neq 1$

$\therefore (1,1) \notin R \rightarrow$ Not transitive

$a^2+b^2=1$

$(a,b)=(2,4) \Rightarrow 2^2+4^2=18 \neq 1$

$\Downarrow$

$(2,4) \notin R$

$A=\{1,2,3\}$

Then there is no need to search for $(4,2)$

$R=\{(1,2),(2,1)\}$

$\hookrightarrow$ Here $(2,3) \notin R$

$\therefore$ there is no need to expect the existence of $(3,2)$

QUESTION

For the set $A = \{1,2,3\}$, define a relation R on the set A as follows :
$R = \{(1,1), (2,2), (3,3), (1,3)\}$ How many ordered pairs to be added to R to make it the smallest equivalence relation?

(A) 1 $\rightarrow$ (3,1)

(B) 2

(C) 3

(D) 4

QUESTION

Let R be a relation on N defined by $R = \{(1+x, 1+x^2) : x \leq 5, x \in N\}$. Which of the following is True ?

(A) R is reflexive

(B) R is symmetric

(C) R is transitive

(D) None of these

x	$1+x$	$1+x^2$
1	2	2
2	3	5
3	4	10
4	5	17
5	6	26

$R = \{(2,2), (3,5), (4,10), (5,17), (6,26)\}$

① Not reflexive :- $(3,3) \notin R$

② Not symmetric :- $(3,10) \in R$ but $(10,3) \notin R$

③ Not transitive :- Here $(3,5) \in R$ & $(5,17) \in R$ but $(3,17) \notin R$.

Transitive

Definition

$\Downarrow$

if $(a,b) \in R$ &

$(b,c) \in R$

Then $(a,c) \in R$

Contradictory method

$\Downarrow$

① if $(a,b) \in R$ & $(b,c) \notin R$

Then there is no need to expect the existence of (a,c) in R

② if $(a,b) \notin R$ & $(b,c) \notin R$

Transitive

Ex: Empty relation

Empty relation

is both symmetric

&

Transitive

QUESTION

$R \subset N \times N \mid A = N$

If the relation R on the set N of all natural numbers defined as $R = \{(x, y): y = x + 5 \text{ and } x < 4\}$, then R is

$R = \{(1,6), (2,7), (3,8)\}$

(A) reflexive

(B) symmetric

(C) transitive by contradictory method

(D) equivalence

QUESTION

$x, y \in$ Integers

If $R = \{(x, y) : x, y \in I, x^2 + y^2 \leq 4\}$ is a relation in I, then R is

Not transitive since $(2,2) \notin R$

$R = \{(2,0), (0,2)\}$

(1) $3^2 + 3^2 = 18 \nleq 4$

$\therefore (3,3) \notin R$

R is not reflexive

(3) $(x,y) = (2,0)$ & $(y,z) = (0,2)$
$\in R$ $\in R$

$x^2 + z^2 = 2^2 + 2^2 = 8 \nleq 4$

$(2,2) \notin R$

$\therefore$ R is not transitive.

A. Reflexive

B. Symmetric

C. Transitive

D. None of these

(2) $R = \{(0,0), (1,1), (0,1), (1,0), (0,2), (2,0), (-1,-1), (0,-1), (-1,0), (0,-2), (-2,0)\}$

Symmetric

$\boxed{\text{counter example}}$

$\Downarrow$

To disprove something (some rules, Person, Property)
we give example
which is called as
counter example.

Degree ✗

skills ✓ ✓ ✓

QUESTION

$S \subset R \times R$ | $A = R$

Let S be the set of all real numbers. Then the relation $R = \{(a, b): 1 + ab > 0\}$ on S is

(A) reflexive and symmetric but not transitive

(B) reflexive and transitive but not symmetric

(C) symmetric and transitive but not reflexive

(D) reflexive, transitive and symmetric

(1) Reflexive:-

WKT

$a \cdot a = a^2$

$\Downarrow$

$a^2 \geq 0$

$1 + a^2 \geq 1$

$1 + a^2 > 0$

$(a, a) \in R$ for each $a \in R$

Reflexive

(2) WKT

$ab = ba$

$\therefore 1 + ab = 1 + ba$

$\therefore$ if $(a, b) \in R$

$\Rightarrow 1 + ab > 0$

$\Rightarrow 1 + ba > 0$

$\Downarrow$

$(b, a) \in R$

$\therefore R$ is symmetric

(3) $a = -2,$ $b = 0.3$

$1 + ab$

$= 1 - 0.6$

$= 0.4 > 0$

$1 + ab > 0$

$(a, b) \in R$

$b = 0.3$

$c = 6$

$1 + bc$

$= 1 + 1.8$

$= 2.8 > 0$

$1 + bc > 0$

$(b, c) \in R$

$\therefore a = -2$ & $c = 6$

$1 + (-2)(6)$

$= 1 - 12 = -11 \not> 0$

$\therefore (a, c) \notin R$

$\therefore R$ is not transitive

QUESTION

Let S be the set of all real numbers. Then the relation $R = \{(a, b): 1 + ab < 0\}$ on S is

only Symmetric

$(0,1) \notin R$

$ab = 0$

$1 + ab$

$= 1 + 0 = 1 \nless 0$

$a = -3 \quad b = 2$

$1 + ab$

$1 - 6 = -5 < 0$

$\Downarrow$

$(a, b) \in R$

$b = 2 \quad c = -4$

$1 + bc$

$= 1 - 8 = -7 < 0$

$\Downarrow$

$(b, c) \in R$

$a = -3 \ \& \ c = -4$

$1 + ac$

$= 1 + 12 = 13 \nless 0$

$\Downarrow$

$(a, c) \notin R$

Symmetric

↓

First (a,b) should be in 'R'

$\Downarrow$

$(a,b) \in R$

only then we need to expect (b,a) should be in R

QUESTION

Consider the non-empty set consisting of children in a family and a relation R defined as aRb, if a is brother of b. Then, R is

→ Both Male & Female

A symmetric but not transitive

B transitive but not symmetric

C neither symmetric nor transitive

D both symmetric and transitive

(1) Not Reflexive

a cannot be brother of a

$(a,a) \notin R$

(2) a → Male b → Female

$(a,b) \in R$

but $(b,a) \notin R$

counter example

⇒ Not symmetric

$M \to$ Male & $F \to$ Female

③ **Transitive:-**

Case 1:- $a \to M$, $b \to M$, $c \to M$

Case 2:- $a \to M$, $b \to F$, $c \to M|F$

$(a,b) \in R$ & $(b,c) \notin R$ → contradictory method

Case 3:- $a \to M$ $b \to M$ $c \to M|F$

Case 4:- $a \to F$ $b \to M|F$ $c \to M|F$

$(a,b) \notin R$

empty Relation

QUESTION

Let us define a relation R in R as aRb if $a \geq b$. Then R is

(A) an equivalence relation

(B) reflexive, transitive but not symmetric

(C) symmetric, transitive but not reflexive

(D) neither transitive nor reflexive but symmetric

	Reflexive	Symmetric	Transitive
$a \geq b$	✓	✗	✓
$a \leq b$	✓	✗	✓
$a > b$	✗	✗	✓
$a < b$	✗	✗	✓

	A	B	$A \cap B$	$A \cup B$
Reflexive	✓	✓	✓	✓
Symmetric	✓	✓	✓	✓
Transitive	✓	✓	✓	✗
Equivalence	✓	✓	✓	✓

$X = \{1, 2, 3\}$

$A = \{(1,2)\}$

$B = \{(2,3)\}$

$A \cup B = \{(1,2), (2,3)\}$

$X = \{1, 2, 3\}$

$A = \{(1,1), (2,2), (3,3)\}$ Reflexive

$B = \{(1,1), (2,2), (3,3), (1,2)\}$ Reflexive

$A \cap B = \{(1,1), (2,2), (3,3)\}$ Reflexive

$A \cup B = \{(1,1), (2,2), (3,3), (1,2)\}$ Reflexive

QUESTION

Assume R and S (non-empty) relations in a set A. Which of the relations given below is false?

False

A. If R and S are transitive, then $R \cup S$ is transitive

B. If R and S are transitive, then $R \cap S$ is transitive

C. If R and S are symmetric, then $R \cup S$ is symmetric

D. If R and S are reflexive, then $R \cap S$ is reflexive

QUESTION

If R and R' are symmetric relations (not disjoint) on a set A, then the relation $R \cap R'$ is

(A) Reflexive

(B) Symmetric

(C) Transitive

(D) None of these

QUESTION

Let X be a non-empty set and $P(X)$ be the set of all subsets of X. For $A, B \in P(X)$, ARB if and only if $\boxed{A \cap B = \phi}$ then the relation

A R is reflexive

B R is not symmetric

C R is not transitive

D R is an equivalence relation

① Not Reflexive :-

$A \cap A = A$

$\neq \phi$

$\therefore (A, A) \notin R$

② Symmetric :-

WKT $A \cap B = B \cap A$

if $A \cap B = \phi \Rightarrow (A, B) \in R$

Then $B \cap A = \phi \Rightarrow (B, A) \in R$

R is symmetric

③ Transitive:-

if $A = \{1,2,3\}$

$B = \{4,5,6\}$

$A \cap B = \phi$

$(A,B) \in R$

$B = \{4,5,6\}$

$C = \{1,2,3\}$

$B \cap C = \phi$

$(B,C) \in R$

But $A \cap C = \{1,2,3\} \neq \phi$

$\therefore (A,C) \notin R$

Not transitive

$f(x) = \sin x$

one-one

$\Downarrow$

$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Downarrow$

Range of $\sin^{-1}x$

onto

$\Downarrow$

$[-1, 1]$

$\downarrow$

Domain of $\sin^{-1}x$

$\sin x : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$

$\Downarrow$

$\sin x$ is both one-one & onto

$\sin^{-1}x : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Downarrow$

$\sin^{-1}x$ is both one-one & onto

$f(x) = \cos x$

one-one

$\Downarrow$

$[0, \pi]$

$\downarrow$

Range of $\cos^{-1}x$

onto

$\Downarrow$

$[-1, 1]$

$\downarrow$

Domain of $\cos^{-1}x$

$\cos x : [0, \pi] \to [-1, 1]$

$\Downarrow$

$\cos x$ is both 1-1 & onto

$\cos^{-1}x : [-1, 1] \to [0, \pi]$

$\Downarrow$

$\cos^{-1}x$ is both 1-1 & onto

$$f(x) = A\sin x \pm B\cos x$$

or

$$= A\cos x \pm B\sin x$$

$\Downarrow$

$x'y$ & divide by

$$\sqrt{A^2+B^2}$$

① Find the interval in which $f(x) = \sin x + \cos x$ is both one-one & onto

Soln:-

$f(x) = (1)\sin x + (1)\cos x$

$A = 1 \qquad B = 1$

$\sqrt{A^2+B^2} = \sqrt{2}$

x'y & ÷ by $\sqrt{2}$

$f(x) = \sqrt{2}\left[\sin x \frac{1}{\sqrt{2}} + \cos x \frac{1}{\sqrt{2}}\right]$

$\frac{1}{\sqrt{2}} \to \cos\frac{\pi}{4}$, $\frac{1}{\sqrt{2}} \to \sin\frac{\pi}{4}$

$f(x) = \sqrt{2}\left[\sin\left(x+\frac{\pi}{4}\right)\right]$

one-one:-

f(x) is 1-1 if

$-\frac{\pi}{2} \le x + \frac{\pi}{4} \le \frac{\pi}{2}$

Add $-\frac{\pi}{4}$

$-\frac{3\pi}{4} \le x \le \frac{\pi}{4}$

$x \in \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$ → This is the interval in which f(x) is 1-1 onto:

$$f(x) = \sqrt{2}\left[\underbrace{\sin\left(x+\frac{\pi}{4}\right)}\right]$$

WKT

$$-1 \le \sin\left(x+\frac{\pi}{4}\right) \le 1$$

x^{ly} by $\sqrt{2}$

$$-\sqrt{2} \le \sqrt{2}\sin\left(x+\frac{\pi}{4}\right) \le \sqrt{2}$$

$$f(x) \in [-\sqrt{2}, \sqrt{2}]$$

This is interval in which $f(x)$ is onto.

method ①

② if $f(x) = \sqrt{3}\cos x - \sin x$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

find the interval in which $f(x)$ is one-one & onto

Soln:

$f(x) = 2\left[\cos x \frac{\sqrt{3}}{2} - \sin x \frac{1}{2}\right]$

$= 2\left[\cos x \cos\frac{\pi}{6} - \sin x \sin\frac{\pi}{6}\right]$

$f(x) = 2\left[\cos\left(x + \frac{\pi}{6}\right)\right]$

onto:

Range $= [-2, 2]$

one-one:

$0 \le x + \frac{\pi}{6} \le \pi$

$-\frac{\pi}{6} \le x \le \frac{5\pi}{6}$

$x \in \left[-\frac{\pi}{6}, \frac{5\pi}{6}\right]$

method ②

$f(x) = \sqrt{3}\cos x - \sin x$

$f(x) = 2\left[\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x\right]$

$= 2\left[\sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\sin x\right]$

$f(x) = 2\left[\sin\left(\frac{\pi}{3} - x\right)\right]$

$\sin(A-B)$
$= \sin A\cos B - \cos A\sin B$

one-one :-

$-\frac{\pi}{2} \le \frac{\pi}{3} - x \le \frac{\pi}{2}$

Add $-\frac{\pi}{3}$

$-\frac{5\pi}{6} \le -x \le \frac{\pi}{6}$

x by -1

$\frac{5\pi}{6} \geq x \geq -\frac{\pi}{6}$

$\Downarrow$

$-\frac{\pi}{6} \le x \le \frac{5\pi}{6}$

$x \in \left[-\frac{\pi}{6}, \frac{5\pi}{6}\right]$

QUESTION

P-70

The function $f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$ is one-one in the interval.

(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(B) $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

(C) $\left[-\frac{\pi}{6}, \frac{\pi}{3}\right]$ ✓

(D) $[-\pi, \pi]$

$f(x) = 2\left[\sin 2x \frac{\sqrt{3}}{2} - \cos 2x \frac{1}{2}\right] + 4$

$= 2\left[\sin 2x \cos\frac{\pi}{6} - \cos 2x \sin\frac{\pi}{6}\right] + 4$

$= 2\left[\sin\left(2x - \frac{\pi}{6}\right)\right] + 4$

$-\frac{\pi}{2} \le 2x - \frac{\pi}{6} \le \frac{\pi}{2}$

Add $\frac{\pi}{6}$

$-\frac{\pi}{3} \le 2x \le \frac{2\pi}{3}$

$\div$ by 2

$-\frac{\pi}{6} \le x \le \frac{\pi}{3}$

Range :- Here

$-1 \le \sin\left(2x - \frac{\pi}{6}\right) \le 1$

$-2 \le 2\sin\left(2x - \frac{\pi}{6}\right) \le 2$

Add 4

$2 \le 2\sin\left(2x - \frac{\pi}{6}\right) + 4 \le 6$

Range $= [2, 6]$

method 2

$$f(x) = \sqrt{3}\sin 2x - \cos 2x + 4$$

$$= -[\cos 2x - \sqrt{3}\sin 2x] + 4$$

$$= -2\left[\cos 2x \tfrac{1}{2} - \sin 2x \tfrac{\sqrt{3}}{2}\right] + 4$$

$$= -2\left[\cos 2x \cos\tfrac{\pi}{3} - \sin 2x \sin\tfrac{\pi}{3}\right] + 4$$

$$= -2\left[\cos\left(2x + \tfrac{\pi}{3}\right)\right] + 4$$

me-one:-

$$0 \le 2x + \frac{\pi}{3} \le \pi$$

$$-\frac{\pi}{3} \le 2x \le \frac{2\pi}{3}$$

$$-\frac{\pi}{6} \le x \le \frac{\pi}{3}$$

$f: R \to R$

$f(x) = 2x + 3$

one-one ✓

onto ✓

$f: Z \to Z$

$f(x) = 2x + 3$

one-one ✓

$\boxed{\text{onto } \times}$

↓

Consider

$f(x) = y$

$2x + 3 = y$

$x = \frac{y-3}{2}$

$y = 0 \Rightarrow x = -\frac{3}{2} \notin Z$ (Domain)

$y = 0$ Doesnot Have Preimage

$f: N \to N$

$f(x) = 2x + 3$

one-one ✓

$\boxed{\text{onto } \times}$

$\hookrightarrow f(x) = y$

$2x + 3 = y$

$x = \frac{y-3}{2}$

$y = 1$

$x = -\frac{2}{2} = -1 \notin N$

$\therefore y = 1$ doesnot have preimage.

$N = \{1, 2, 3, 4, 5, \ldots\}$

QUESTION

The mapping $f: N \to N$ given by $f(n) = 2 + n^2$, $n \in N$, where N is the set of natural numbers is

$f(n) = 2 + n^2$

$f(1) = 3$

$f(2) = 6$

$f(3) = 11$

one-one

onto:

Range $= \{3, 6, 11, 18, 27, \ldots\}$

$\neq N$ (co-domain)

Not onto

(A) one-one and onto

(B) onto but not one-one

(C) one-one but not onto

(D) neither one-one nor onto

QUESTION

Let $f : N \to N$ defined by $f(x) = x^2 + x + 1, x \in N$, then f is

(A) one-one onto

(B) many-one onto

(C) one-one but not onto

(D) none of these

$f(x) = x^2 + x + 1$

$f(1) = 3$

$f(2) = 7$

$f(3) = 13$

$f(4) = 21$

$\vdots$

one-one

onto :-

Range $= \{3, 7, 13, 21, \ldots\}$

$\neq N$ (codomain)

f is not onto

(1) if $f(x) = x^2 + x + 1$, The interval in which $f(x)$ is one-one is

(A) R

(B) $\left(\frac{3}{2}, 5\right)$

(C) $\left(-\frac{1}{2}, \infty\right)$

(D) $(-\infty, -1)$

(E) $\left(-\infty, -\frac{1}{2}\right)$

$f'(x) = 2x + 1$

Consider $f'(x) = 0$

$2x + 1 = 0$

$x = -\frac{1}{2}$

$-$ $\quad$ $+$

$-\infty \quad -\frac{1}{2} \quad \infty$

$\Downarrow \quad \Downarrow$

$f'(x) < 0 \quad f'(x) > 0$

In $\left(-\frac{1}{2}, \infty\right)$

$f'(x) > 0$

$\therefore f(x)$ is strictly increasing

$\therefore f$ is one-one in $\left(-\frac{1}{2}, \infty\right)$

$$f(x) = x^2 + x + 1$$

$$= x^2 + x + \frac{1}{4} - \frac{1}{4} + 1$$

Range $= \left[\frac{3}{4}, \infty\right)$ $\Longleftarrow$

$$= \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

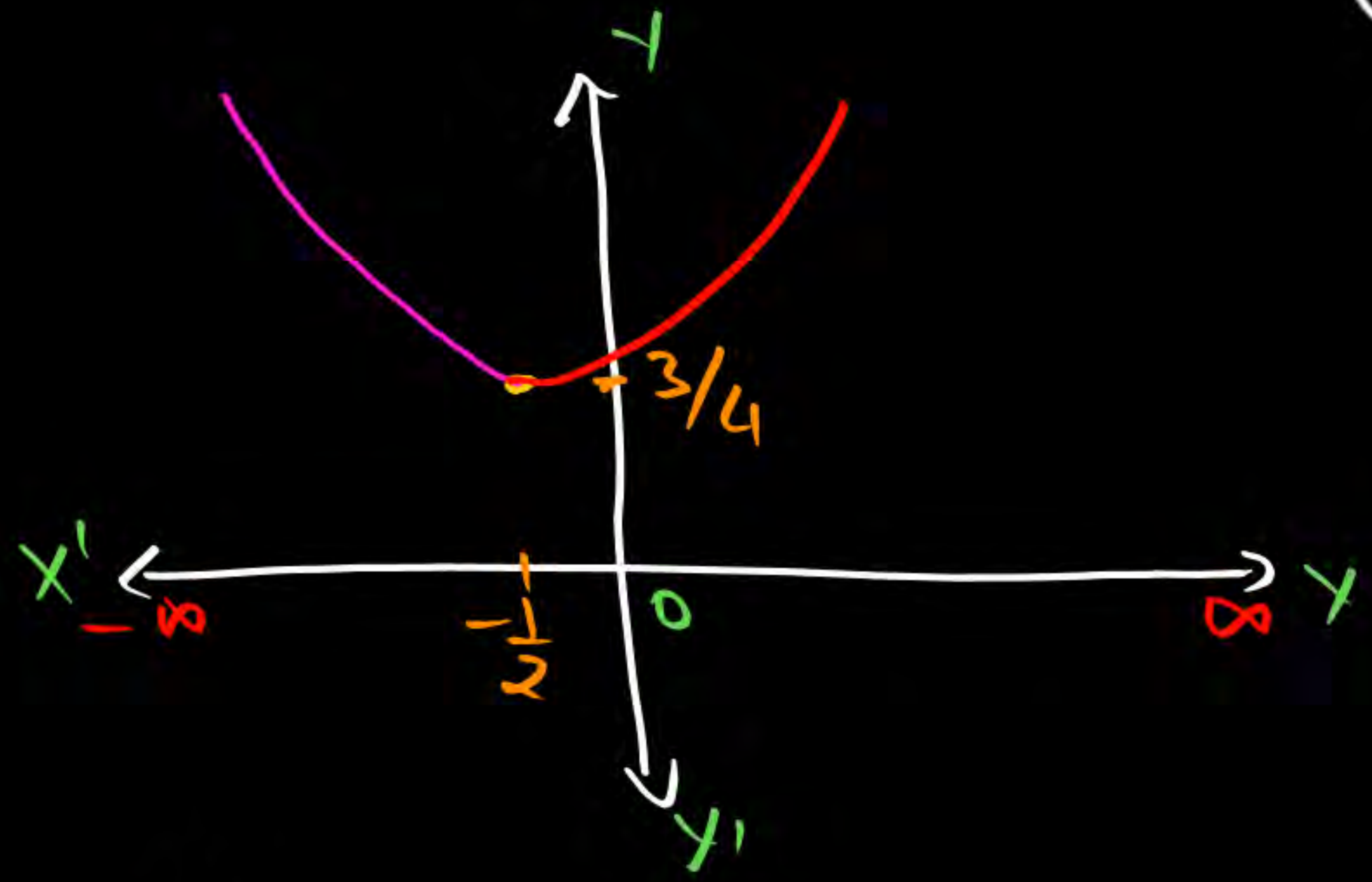
$$\Downarrow$$

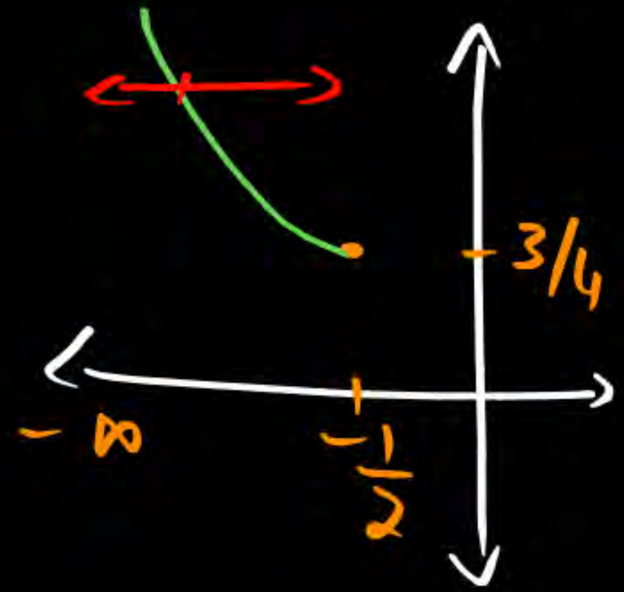
$$x = -\frac{1}{2}$$

$$f(x) = \frac{3}{4}$$

$$\Downarrow$$

$$\left(-\frac{1}{2}, \frac{3}{4}\right)$$





QUESTION

$f: R \to R$

Which one of the following is a bijective function on the set of real numbers?

Option	Function	one-one	onto	Range
A	$2x - 5$	✓	✓	R
B	$\lvert x \rvert$	✗	✗	$[0, \infty)$
C	x^2	✗	✗	$[0, \infty)$
D	$x^2 + 1$	✗	✗	$[1, \infty)$

QUESTION

Which of the following functions is not one-one?

(A) $f: R \to R$ defined by $f(x) = 6x - 1$

(B) $f: R \to R$ defined by $f(x) = x^2 + 1$

(C) $f: R \to R$ defined by $f(x) = x^3$

(D) $f: R - \{7\} \to R$ defined by $f(x) = \frac{2x+1}{x-7}$

one-one	onto	Range
✓	✓	R
X	X	$[1, \infty)$
✓	✓	R
✓	X	$R - \{2\}$

$f(x) = y$

$\frac{2x+1}{x-7} = y$

$2x+1 = xy - 7y$

$x(2-y) = -1 - 7y$

$x = \frac{-1-7y}{2-y}$

$2 - y \neq 0$

$y \neq 2$

Range $= R - \{2\}$

QUESTION

If $f: A \to B$ is a constant function which is onto then B is

(A) a singleton set

(B) a null set

(C) an infinite set

(D) a finite set

Ex: $f: A \to B$

$f(x) = 2$

$f(1) = 2$

$f(2) = 2$

$f(3) = 2$

$\vdots$

$f(100) = 2$

Range = $\{2\}$

↳ singleton set

QUESTION

Which of the following functions is not invertible?

→ not one-one (or) not onto (or) both

$f: R^+ \to R^+$

$f(x) = \frac{1}{x^3}$

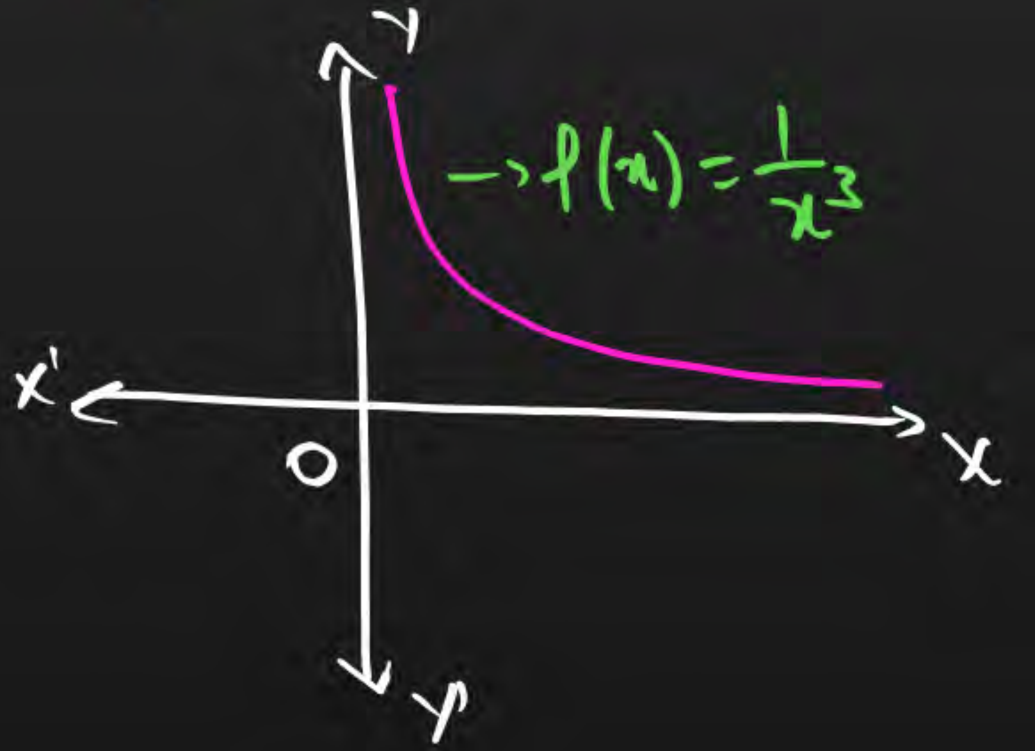
(A) $f: R \to R, f(x) = 3x + 1$

(B) $f: R \to [0, \infty), f(x) = x^2$

(C) $f: R^+ \to R^+, f(x) = \frac{1}{x^3}$

(D) None of these

1-1	onto	Range
✓	✓	R
×	✓	$[0, \infty)$
✓	✓	R^+ $(0, \infty)$



f is invertible

$\Downarrow$

f is both one-one

&

onto

QUESTION

The function $f : R \to R$ given by $f(x) = x^3 - 1$ is

(A) One-one but not onto

(B) Onto but not one-one

(C) A bijection

(D) Neither one-one nor onto

QUESTION

If R is the set of all real numbers and if $f : R - \{2\} \to R$ is defined by $f(x) = \frac{2+x}{2-x}$ for $x \in R - \{2\}$, then the range of f is

(A) R

(B) $R - \{1\}$

(C) $R - \{-1\}$

(D) $R - \{-2\}$

$f(x) = y$

$\frac{2+x}{2-x} = y$

$2 + x = 2y - xy$

$x + xy = 2y - 2$

$x(1+y) = 2y - 2$

$x = \frac{2y-2}{1+y}$

$D^r \neq 0$

$1 + y \neq 0$

$y \neq -1$

Range $= R - \{-1\}$

QUESTION

The function $f: R \to R$ defined by $f(x) = (x - 1)(x - 2)(x - 3)$ is

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

$f(1) = 0$

$f(2) = 0$

Not one-one

onto :-

The above func is a cubic expression whose domain is R

$\therefore$ Range is equal to R

$\therefore$ Range = Codomain

$\Downarrow$

onto

QUESTION

input = odd ⇓ output = 0 | input = +ve even ⇓ output = {1, 2, 3, ...} | input = -ve even ⇓ output = {-1, -2, -3, ...}

On the set of integers Z, define $f: Z \to Z$ as $f(n) = \begin{cases} \frac{n}{2}, & n \text{ is even} \\ 0, & n \text{ is odd} \end{cases}$ then f is

(A) surjective but not injective

(B) bijective

(C) injective but not surjective

(D) neither injective nor surjective

$f(1) = 0$	$f(2) = \frac{2}{2} = 1$	$f(-2) = -1$
$f(3) = 0$	$f(4) = 2$	$f(-4) = -2$
$f(5) = 0$	$f(6) = 3$	$f(-6) = -3$
not one-one	$f(8) = 4$	$f(-8) = -4$
	$\vdots$	$\vdots$

Range $= \{0, 1, 2, 3, \ldots, -1, -2, -3, \ldots\} = Z$ = codomain

f is onto

QUESTION

$f: R - 2^{+} \to R$ $\quad f(2/3) = \frac{4/3}{-1/3} = -4$

Let $A = \{x : x \in R ; x \text{ is not a positive integer}\}$ Define $f : A \to R$ as $f(x) = \frac{2x}{x-1}$, then f is

- (A) injective but not surjective
- (B) surjective but not injective
- (C) bijective
- (D) neither injective nor surjective

$f(0) = 0$

$f(-1) = \frac{-2}{-2} = +1$

$f(-2) = \frac{-4}{-3} = \frac{4}{3}$

$f(-1/2) = \frac{-1}{-3/2} = \frac{2}{3}$

one-one

onto :-

Range :-

$f(x) = y$

$\frac{2x}{x-1} = y$

$2x = xy - y$

$y = x(y-2)$

$x = \frac{y}{y-2}$

Range $= R - \{2\}$

$\neq$ codomain (R)

f is not onto

PCM

$\downarrow$

$$\frac{\text{Marks} \geq 296}{300}$$

290

$\downarrow$

1500 to 2000 Rank gone

Samarth & Sujan

$\Downarrow$ $\Downarrow$

854 1100 something

$\Downarrow$ $\Downarrow$

$\frac{150}{180}$ $\frac{144}{180}$

Boards $\rightarrow$ 297

Conversion of Algebraic Expressions into Trigonometric Expressions

Algebraic Expression	Substitution	Trigonometric Expression
(1) $2x\sqrt{1-x^2}$	Put $x = \sin\theta$ or $x = \cos\theta$	$2\sin\theta\cos\theta$
(2) $\frac{2x}{1+x^2}$	Put $x = \tan\theta$	$\frac{2\tan\theta}{1+\tan^2\theta} = \sin 2\theta$
(3) $\frac{2x}{1-x^2}$	Put $x = \tan\theta$	$\frac{2\tan\theta}{1-\tan^2\theta} = \tan 2\theta$
(4) $\frac{1-x^2}{1+x^2}$	Put $x = \tan\theta$	$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$

Conversion of Algebraic Expressions into Trigonometric Expressions

Algebraic Expression	Substitution	Trigonometric Expression
(5) $2x^2 - 1$	Put $x = \cos\theta$	$2\cos^2\theta - 1 = \cos 2\theta$
(6) $1 - 2x^2$	Put $x = \sin\theta$	$1 - 2\sin^2\theta = \cos 2\theta$
(7) $\frac{1+x}{1-x}$	Put $x = \tan\theta$ Put $x = \cos\theta$	$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta} = \tan\left(\frac{\pi}{4} + \theta\right)$ $\Rightarrow \frac{1+\cos\theta}{1-\cos\theta} = \frac{2\cos^2\theta/2}{2\sin^2\theta/2} = \cot^2\theta/2$
(8) $3x - 4x^3$	Put $x = \sin\theta$	$3\sin\theta - 4\sin^3\theta = \sin 3\theta$
(9) $4x^3 - 3x$	Put $x = \cos\theta$	$4\cos^3\theta - 3\cos\theta = \cos 3\theta$

Conversion of Algebraic Expressions into Trigonometric Expressions

Algebraic Expression	Substitution	Trigonometric Expression
(10) $\frac{3x-x^3}{1-3x^2}$	Put $x = \tan\theta$	$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \tan 3\theta$
(11) $1 + x^2$	Put $x = \tan\theta$ or $x = \cot\theta$	$\Rightarrow 1 + \tan^1\theta = \sec^2\theta$ or $\Rightarrow 1 + \cot^2\theta = \operatorname{cosec}^2\theta$
(12) $x^2 - 1$	Put $x = \sec\theta$ or $x = \operatorname{cosec}\theta$	$\Rightarrow \sec^2\theta - 1 = \tan^2\theta$ or $\Rightarrow \operatorname{cosec}^2\theta - 1 = \cot^2\theta$

Conversion of Algebraic Expressions into Trigonometric Expressions

Algebraic Expression	Substitution	Trigonometric Expression
(13) $1 - x$	Put $x = \cos\theta$ or $x = \cos\theta$	$\Rightarrow 1 - \cos\theta = 2\sin^2\theta/2$ $\Rightarrow 1 - \cos 2\theta = 2\sin^2\theta$
(14) $1 + x$	Put $x = \cos\theta$ or $x = \cos\theta$	$\Rightarrow 1 + \cos\theta = 2\cos^2\theta/2$ $\Rightarrow 1 + \cos 2\theta + 2\cos^2\theta$
(15) $\frac{1-x}{1+x}$	Put $x = \tan\theta$ or Put $x = \cos\theta$ or Put $x = \tan^2\theta$	$\Rightarrow \frac{1-\tan\theta}{1+\tan\theta} = \tan\left(\frac{\pi}{4} - \theta\right)$ $\Rightarrow \frac{1-\cos\theta}{1+\cos\theta} = \frac{2\sin^2\theta/2}{2\cos^2\theta/2} = \tan^2\frac{\theta}{2}$ $\Rightarrow \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$

QUESTION

$$y = \tan^{-1}\left(\frac{3x}{1 + 4x^2}\right)$$

$3x \Rightarrow A-B$

$4x^2 \Downarrow AB$

$4x^2 = 4x - x$

$\Rightarrow A = 4x$ & $B = x$

$$y = \tan^{-1}\left(\frac{4x - x}{1+4x^2}\right)$$

$$y = \tan^{-1}4x - \tan^{-1}x$$

$\tan^{-1}A - \tan^{-1}B$

$= \tan^{-1}\left(\frac{A-B}{1+AB}\right)$

Observation :-

① In D^r First term is one

② In D^r b/w 1 & AB There is +ve sign

③ ∴ In N^r we have $A - B$

$\tan^{-1}A + \tan^{-1}B$

$= \tan^{-1}\left(\frac{A+B}{1-AB}\right)$

Observation:-

① In D^r 1st term is one

② In D^r b/w 1 & AB There is −ve sign

if $y = \tan^{-1}\left(\frac{3x}{1+4x^2}\right)$ find $\frac{dy}{dx}$

Soln:

$$y = \tan^{-1}\left(\frac{4x - x}{1+(4x)(x)}\right]$$

$$y = \tan^{-1} 4x - \tan^{-1} x$$

$$\frac{dy}{dx} = \frac{4}{1+16x^2} - \frac{1}{1+x^2}$$

(2) If $y = \tan^{-1}x\left[\dfrac{5x}{1-6x^2}\right]$ find $\dfrac{dy}{dx}$

$5x \Rightarrow A+B$

$6x^2 \Rightarrow AB$

Soln:-

$$y = \tan^{-1}\left[\frac{3x+2x}{1-(3x)(2x)}\right]$$

$$y = \tan^{-1}3x + \tan^{-1}2x$$

$$\frac{dy}{dx} = \frac{3}{1+9x^2} + \frac{2}{1+4x^2}$$

QUESTION

$$y = \tan^{-1}\left(\frac{6x}{1-5x^2}\right) \Rightarrow A+B$$

$\Downarrow$ AB

Find $\frac{dy}{dx}$

$5x^2 = (5x)(x)$

$\downarrow$ A $\quad$ $\downarrow$ B

$\tan^{-1}\left(\frac{5x+x}{1-5x^2}\right)$

$y = \tan^{-1}5x + \tan^{-1}x$

$\frac{dy}{dx} = \frac{5}{1+25x^2} + \frac{1}{1+x^2}$

QUESTION

$$y = \tan^{-1}\left[\frac{2+3x}{2-3x}\right]$$

Find $\frac{dy}{dx}$

In this place there should be one

$\therefore \div$ both N^r & D^r by 2

$$y = \tan^{-1}\left[\frac{1+\frac{3}{2}x}{1-\frac{3}{2}x}\right] = \tan^{-1}\left[\frac{1+\frac{3}{2}x}{1-(1)\left(\frac{3}{2}x\right)}\right]$$

$$y = \tan^{-1}(1) + \tan^{-1}\left(\frac{3}{2}x\right)$$

$$y = \frac{\pi}{4} + \tan^{-1}\left(\frac{3}{2}x\right)$$

$$\frac{dy}{dx} = 0 + \frac{1}{1+\frac{9x^2}{4}}\left(\frac{3}{2}\right)$$

$$= \frac{4}{4+9x^2}\left(\frac{3}{2}\right)$$

$$= \frac{6}{4+9x^2}$$

QUESTION

$$y = \tan^{-1}\left[\frac{x+y}{x-y}\right]$$

There should be one

$\div$ N^r & D^r by x

$$y = \tan^{-1}\left[\frac{1+\frac{y}{x}}{1-(1)\left(\frac{y}{x}\right)}\right]$$

$\Rightarrow A+B$

$\Downarrow AB$

$$y = \tan^{-1}(1) + \tan^{-1}\frac{y}{x}$$

$$\frac{dy}{dx} = 0 + \frac{1}{1+\left(\frac{y}{x}\right)^2}\left[\frac{xy_1 - y}{x^2}\right]$$

$$y_1 = \frac{\cancel{x^2}}{x^2+y^2}\left[\frac{xy_1-y}{\cancel{x^2}}\right]$$

$$y_1 = \frac{x}{x^2+y^2}y_1 - \frac{y}{x^2+y^2}$$

$$\frac{y}{x^2+y^2} = y_1\left[\frac{x}{x^2+y^2} - 1\right]$$

$$y = y_1\left[x - x^2 - y^2\right]$$

$$y_1 = \frac{y}{x-x^2-y^2}$$

QUESTION

$$y = \tan^{-1}\sqrt{x^2 - 1}$$

Put $x = \sec\theta$

$\theta = \sec^{-1}x$

$y = \tan^{-1}\sqrt{\sec^2\theta - 1}$

$y = \tan^{-1}(\tan\theta)$

$y = \theta$

$y = \sec^{-1}x$

$$\frac{dy}{dx} = \frac{1}{x\sqrt{x^2-1}}$$

$$I = \int \tan^{-1}\sqrt{x^2-1}\, dx$$

Put $x = \sec\theta \Rightarrow \theta = \sec^{-1}x$

$dx = \sec\theta . \tan\theta \, d\theta$

$$\tan^{-1}\sqrt{x^2-1} = \tan^{-1}(\tan\theta)$$

$$= \theta$$

$$\int \sec\theta \tan\theta \, d\theta = \sec\theta + c$$

$$\int \sec\theta \, d\theta = \log|\sec\theta + \tan\theta| + c$$

$$I = \int \underset{u}{\theta}\, \underbrace{\sec\theta \tan\theta \, d\theta}_{v}$$

$$I = \theta(\sec\theta) - \int \sec\theta \, d\theta$$

$$I = \theta \sec\theta - \log|\sec\theta + \tan\theta| + c$$

$$I = \sec^{-1}x(x) - \log\left|x + \sqrt{x^2-1}\right| + c$$

$\sec\theta = x$

$\tan\theta = \sqrt{\sec^2\theta - 1} = \sqrt{x^2-1}$

QUESTION

$$y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

Put $x = \tan\theta$

$$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$y = \sin^{-1}(\sin 2\theta)$$

$$y = 2\theta$$

$$y = 2\tan^{-1}x$$

QUESTION

$$y = \cot^{-1}\left[\frac{\underset{\to\, 1-2\sin^2\theta}{1-2x^2}}{\underset{\to\, 2\sin\theta\cos\theta}{2x\sqrt{1-x^2}}}\right]$$

Find $\frac{dy}{dx}$

Put $x = \sin\theta$

$\theta = \sin^{-1}x$

$\sqrt{1-x^2} = \sqrt{1-\sin^2\theta} = \cos\theta$

$1-2x^2 = 1-2\sin^2\theta = \cos 2\theta$

$$y = \cot^{-1}\left[\frac{\cos 2\theta}{2\sin\theta\cos\theta}\right]$$

$$y = \cot^{-1}\left[\frac{\cos 2\theta}{\sin 2\theta}\right]$$

$$y = \cot^{-1}(\cot 2\theta)$$

$$y = 2\theta$$

$$y = 2\sin^{-1}x$$

$$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$$

QUESTION

$2\cos^2\theta - 1$

$$y = \tan^{-1}\left[\frac{2x^2 - 1}{2x\sqrt{1-x^2}}\right]$$

$2\cos\theta\sin\theta$

Find $\frac{dy}{dx}$

Put $x = \cos\theta$ | $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-x^2}$

$\theta = \cos^{-1}x$ | $2x^2 - 1 = 2\cos^2\theta - 1 = \cos 2\theta$

$y = \tan^{-1}\left[\frac{\cos 2\theta}{\sin 2\theta}\right]$

$y = \tan^{-1}(\cot 2\theta)$

$y = \frac{\pi}{2} - \cot^{-1}(\cot 2\theta)$

$y = \frac{\pi}{2} - 2\theta$

$y = \frac{\pi}{2} - 2\cos^{-1}x$

$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$

Thank

You