ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture - 02

Matrices and Determinants

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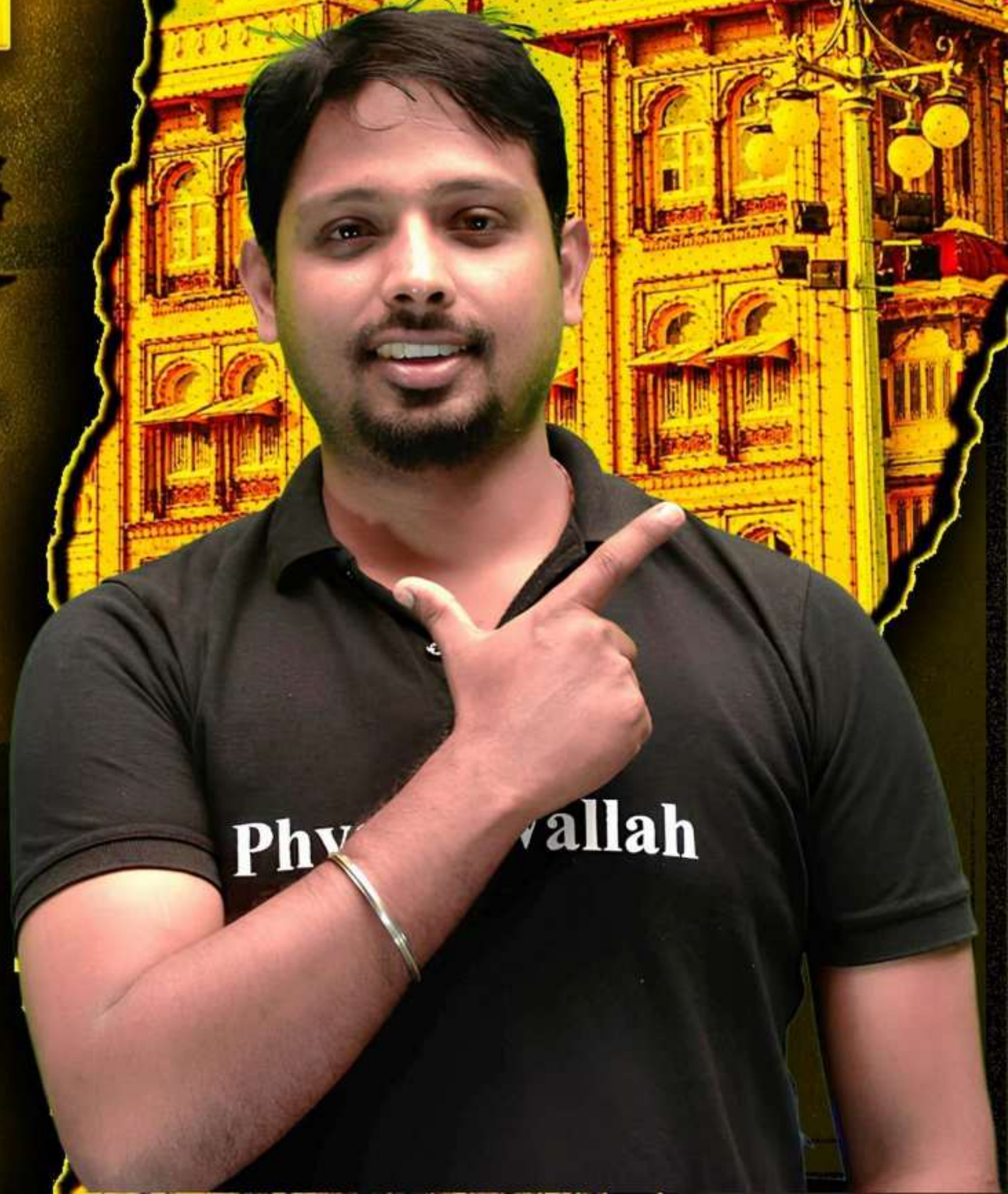
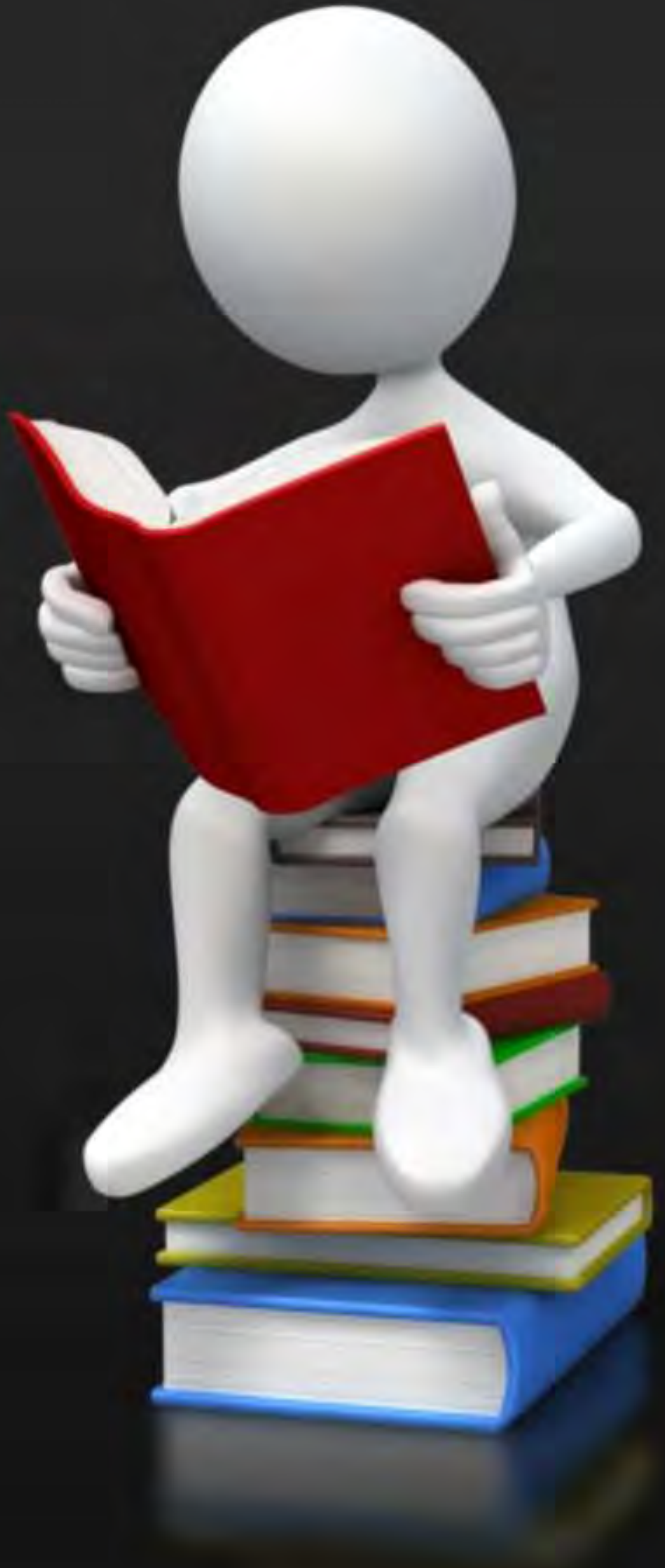


Recap of previous lecture

1. Matrices
2.
3.
4.

Topics *to be covered*

1. Determinants
2.
3.
4.



① $|AB| = |A|\,|B|$

Ex:- if $|A| = 4$, $|B| = -5$ & $|C| = 3$

$\therefore |ABC| = |A|\,|B|\,|C|$

$= 4(-5)(3)$

$= -60$

② $|kA| = k^n |A|$, where $n \to$ order of matrix

Ex:- if $n = 3$

$|A| = 4$ & $|B| = -2$

Find $|2AB|$

$|2AB| = 2^n |AB|$

$= 2^3 |A|\,|B|$

$= 8(4)(-2)$

$= \underline{-64}$

$\circledast$ ① $A(\text{Adj}A) = |A|I$ ② $|A\cdot \text{Adj}A| = |A|^n$ $n \to$ order

(or)

$(\text{Adj}A)\cdot A = |A|I$ ③ $|\text{Adj}A| = |A|^{n-1}$

Ex:- if $A = \begin{bmatrix} 3 & 0 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix}_{3\times 3} \to n=3$

find ① $A\cdot \text{Adj}A$. ② $|\text{Adj}A|$

Soln:

① $|A| = 3(5)(-1) = -15$

WKT

$A\cdot \text{Adj}A = |A|I = -15I = -15\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} -15 & 0 & 0 \\ 0 & -15 & 0 \\ 0 & 0 & -15 \end{bmatrix}$

②

$|\text{Adj}A| = |A|^{n-1}$

$n=3$; $|A| = -15$

$|\text{Adj}A| = (-15)^{3-1}$

$= 225$

③ WKT $|\text{Adj}A| = |A|^{(n-1)^1}$

NOW

① $|\text{Adj}(\text{Adj}A)| = |A|^{(n-1)^2}$

② $|\text{Adj}(\text{Adj}(\text{Adj}A))| = |A|^{(n-1)^3}$

$A \cdot \text{Adj}A = |A| I$

Premultiply by A^{-1} on B.S

$A^{-1}A(\text{Adj}A) = A^{-1}|A| I$

$\downarrow$

$I(\text{Adj}A) = |A| A^{-1}$

$\therefore \text{Adj}A = |A| A^{-1}$

④ WKT ① $A(\text{Adj}A) = |A| I$

② $\text{Adj}A = |A| \cdot A^{-1}$

③ $\text{Adj}(\text{Adj}A) = |A|^{n-2} A$

$\Downarrow$

Proof is similar to

$A(\text{Adj}A) = |A| I$

QUESTION

k is a scalar, and A is an n-square matrix, then $|kA| =$

A) $k|A|^n$

B) $k|A|$

C) $k^n|A|^n$

D) $k^n|A|$

QUESTION

$\rightarrow |A| = 1000 - 1200 = -200$

$\rightarrow |B| = 1100 - 1000$
$= 100$

Let $A = \begin{pmatrix} 100 & 60 \\ 20 & 10 \end{pmatrix}, B = \begin{pmatrix} 110 & 100 \\ 10 & 10 \end{pmatrix}$; then the value of the determinant of the product matrix AB is

$|AB| = |A|\,|B|$
$= -20{,}000$

(A) 10000

(B) 20000

(C) -10000

(D) -20000

QUESTION

The roots of the equation $\begin{vmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{vmatrix} = 0$ are

Rule of sign

$\Downarrow$

$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$

Expand w.r.t 1st column

$-x\left(x^2-16(9)\right)=0$

$-x(x^2-144)=0$

$x=\underline{0} \mid x^2-144=0$

$x=\underline{\pm 12}$

(A) (0, 9, 16)

(B) (0, 12, 12)

(C) (0, 12, −12)

(D) (0, 12, 16)

QUESTION

If A is a square matrix of order 3 $\rightarrow n=3$ and $|A|=3$, then $|\operatorname{adj} A|$ is

$\Downarrow$

$|A|^{n-1}$

3^{3-1}

$=3^{2}$

$=9$

A 3

B 9

C (1/3)

D 0

QUESTION

$n=3$

If A and B are square matrices of order 3 and $|A| = -1, |B| = 3$, the determinant of $3AB$ is equal to

(A) -9

(B) 9

(C) -81

(D) 81

$|3AB|$

$= 3^n |A|\,|B|$

$= 3^3 (-1)(3)$

$= \underline{-81}$

QUESTION

If the elements of a row of a matrix are multiplied by the cofactors of the corresponding elements of any other row and added then their sum is

Property

(A) A

(B) 0

(C) $2A$

(D) $-A$

QUESTION

$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = |A|$

The sum of the product of the elements of the first row with the corresponding cofactors of $\begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$ is

$\Downarrow$

$|A|$

$|A| = 3(-3) - 2(-13) + 1(-9)$

$= -9 + 26 - 9$

$= 8$

(A) 0

(B) 8

(C) −8

(D) 4

QUESTION

If $A = \begin{bmatrix} 2 & -4 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$ then $\operatorname{adj}(AB)$ is

A $\begin{bmatrix} 8 & 24 \\ 5 & 14 \end{bmatrix}$

B $\begin{bmatrix} 22 & 8 \\ 1 & 0 \end{bmatrix}$

C $\begin{bmatrix} 0 & 8 \\ 1 & 22 \end{bmatrix}$

D $\begin{bmatrix} 22 & 1 \\ 8 & 0 \end{bmatrix}$

Consistent

↓

(1) unique solution (or) (2) Infinitely many soln

QUESTION

If the system of equations $\lambda x + y - z = 0, x + 2y - 3z = 0, 3x + 4y - 7z = 0$ is consistent, then the value of λ is not equal to

A -1

B $-1/2$

C 0

D 2

$$\begin{vmatrix} \lambda & 1 & -1 \\ 1 & 2 & -3 \\ 3 & 4 & -7 \end{vmatrix} \neq 0$$

$$\lambda(-14+12) - 1(-7+9) - 1(4-6) \neq 0$$

$$-2\lambda - 2 + 2 \neq 0$$

$$-2\lambda + 0 \neq 0$$

$$\lambda \neq 0$$

unique soln

⇓

$|A| \neq 0$

① if $|A| \neq 0$

① system is consistent

② unique soln

② if $|A| = 0$

if $(\text{Adj}\,A)B = 0$

$\Downarrow$

Infinitely many solution (consistent)

or

no soln (Inconsistent)

if $(\text{Adj}\,A)B \neq 0$

$\Downarrow$

No soln

(Inconsistent

QUESTION

The system of equations $5x + 3y = 8, 10x + 6y = 16$ has

(A) Infinitely many solutions

(B) Finitely many solutions

(C) Unique solution

(D) No solution

$A = \begin{bmatrix} 5 & 3 \\ 10 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ 16 \end{bmatrix}$

$|A| = 0$

$\text{Adj}A = \begin{bmatrix} 6 & -3 \\ -10 & 5 \end{bmatrix}$

Consider $(\text{Adj}A)B = \begin{bmatrix} 6 & -3 \\ -10 & 5 \end{bmatrix}\begin{bmatrix} 8 \\ 16 \end{bmatrix} = \begin{bmatrix} 48-48 \\ -80+80 \end{bmatrix} = 0$

$(\text{Adj}A)B = 0$

infinitely many solu

QUESTION

The equations $2x - 3y + 6z = 4$, $5x + 7y - 14z = 1$, $3x + 2y - 4zx = 0$, have

(A) unique solution

(B) no solution

(C) infinitely many solutions

(D) two solutions

QUESTION

The value of $\begin{vmatrix} 4\sin^2\theta & \cos 2\theta \\ -\cos 2\theta & \cos\theta \end{vmatrix}$

(A) $8\sin^2\theta\cos^2\theta$

(B) 1

(C) $4\sin^2 2\theta\cos\theta$

(D) $4\cos^3\theta - 3\cos\theta$

QUESTION

The equation $\begin{vmatrix} x-2 & 3 & 1 \\ 4x-2 & 10 & 4 \\ 2x-1 & 5 & 1 \end{vmatrix} = 0$ is satisfied by

A $x = -2$

B $x = -5$

C $x = -7$

D $x = -9$

QUESTION

The factors of $\begin{vmatrix} 1 & 1 & 1 \\ x & y & 1 \\ x^2 & y^2 & 1 \end{vmatrix}$ are

(A) $x - 1, y - 1$ and $y - x$

(B) $x - 1, y - 1$ and $x + y$

(C) x, y and $x - y$

(D) $x - 1, y + 1$ and $x + y$

QUESTION

The value of $\begin{vmatrix} \sec x & \sin x & \tan x \\ 0 & 1 & 0 \\ \tan x & \cot x & \sec x \end{vmatrix}$ is

A. 0

B. 1

C. $\sin x$

D. $\tan x$

QUESTION

$\rightarrow n=3$ $\Rightarrow |A|=11$

If the value of a third order determinant is 11 , then the value of the square of the determinant formed by its cofactors will be

$|\text{cofactor matrix}|^2 = |\text{Transpose of Adj}A|^2$

$= |(\text{Adj}A)^T|^2$

$|\text{Adj}A| = |A|^{n-1}$

$= |\text{Adj}A|^2$

$= (|A|^{n-1})^2$

$= (11^{3-1})^2 = (11^2)^2 = 11^4 = 121 \times 121$

$= 14641$

A 11

B 121

C 1331

D 14641 ✓

QUESTION

The solution se of the equation $\begin{vmatrix} 2 & 3 & x \\ 2 & 1 & x^2 \\ 6 & 7 & 3 \end{vmatrix} = 0$ is

(A) ϕ

(B) $\{0, 1\}$

(C) $\{1, -1\}$

(D) $\{1, -3\}$

QUESTION

The points $P(x_1, y_1), Q(x_2, y_2), R(x_3, y_3)$ are collinear if and only if

A $\begin{vmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{vmatrix} = 1$

B $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

C $\begin{vmatrix} x_1 & 1 & y_1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

D $\begin{vmatrix} x_1 & y_1 & x_1y_1 \\ x_2 & y_2 & x_2y_2 \\ x_3 & y_3 & x_3y_3 \end{vmatrix} = 0$

QUESTION

If the matrix $\begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is singular, then λ is

A -2

B 4

C 2

D -4

QUESTION

If the matrix $\begin{bmatrix} 1 & 3 & \lambda+2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$ is singular, then λ is

(A) -2

(B) 4

(C) 2

(D) -4

QUESTION

If A is a matrix of order 3, and $|A| = 8$, then $|\text{adj}A|$ is

$n=3$

(A) 8

(B) 8^2

(C) 8^3

(D) $1/8$

$|A|^{n-1}$

8^{3-1}

$= 8^2$

QUESTION

The matrix having multiplicative inverse is

A $\begin{bmatrix} 4 & 12 \\ 3 & 9 \end{bmatrix}$

B $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

C $\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$

D $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

QUESTION

The inverse of $\begin{bmatrix} \sec\theta & -\tan\theta \\ -\tan\theta & \sec\theta \end{bmatrix}$ is

(A) $\begin{bmatrix} 1 & -\sin\theta \\ -\sin\theta & 1 \end{bmatrix}$

(B) $\begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(C) $\begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix}$

(D) $\begin{bmatrix} \sec\theta & \tan\theta \\ -\tan\theta & \sec\theta \end{bmatrix}$

QUESTION

The inverse of $\begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1/2 & 0 \\ 1/3 & 0 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$

QUESTION

The inverse of the matrix $\begin{bmatrix} 1 & 3 & k \\ 2 & 3 & 4 \\ 1 & 0 & -1 \end{bmatrix}$ does not exist when k is equal to

(A) 4

(B) -1

(C) 3

(D) 5

QUESTION

The multiplicative inverse of $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is

(A) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(B) $\begin{bmatrix} \cos\theta & -\sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(C) $\begin{bmatrix} \sin\theta & \cos\theta \\ -\cos\theta & \sin\theta \end{bmatrix}$

(D) $\begin{bmatrix} \sin\theta & -\sin\theta \\ \cos\theta & \cos\theta \end{bmatrix}$

QUESTION

If A is a matrix of order 3×3 and $detA = 4$ then $det(adjA)$ is equal to

A 4

B $1/4$

C 16

D 64

QUESTION

The inverse of $A(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is

A itself

B $A(-\theta)$

C I

D $A(2\theta)$

QUESTION

If $x, y \in R$ then the determinant $\Delta = \begin{vmatrix} \cos x & -\sin x & 1 \\ \sin x & \cos x & 1 \\ \cos(x+y) & -\sin(x+y) & 0 \end{vmatrix}$ lies in the interval

A $[-\sqrt{2}, \sqrt{2}]$

B $[-1, 1]$

C $[-\sqrt{2}, 1]$

D $[-1, -\sqrt{2}]$

QUESTION

If $A = \begin{bmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{bmatrix}$ is singular, then the possible values of x are

A) $0, 4, -4$

B) $0, 5, -5$

C) $0, +12, -12$

D) $0, 1, -1$

QUESTION

$|A| = 1(8-6) - 0(\ \ ) + 3(6-4) = 2 + 3(2) = 8$

$|A|I = 8I = 8\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$= \begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}$

If $A = \begin{bmatrix}1 & -2 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}$, then $A \cdot \text{adj}(A)$ is equal to

(A) $\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}$

(B) $\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

(C) $\begin{bmatrix}5 & 1 & 1\\ 1 & 5 & 1\\ 1 & 1 & 5\end{bmatrix}$

(D) $\begin{bmatrix}5 & 0 & 0\\ 0 & 5 & 0\\ 0 & 0 & 5\end{bmatrix}$

QUESTION

$|A| = 2(4) - 1(-1)$

$= 9$

If $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$ then $|\mathrm{adj}A| =$

$\hookrightarrow |A|^{n-1}$

$3 \times 3 \Downarrow n = 3$

$= 9^{3-1}$

$= 9^2$

$= 81$

A 81

B 0

C 9

D 1/9

QUESTION

The constant term of the polynomial $\begin{vmatrix} x+3 & x & x+2 \\ x & x+1 & x-1 \\ x+2 & 2x & 3x+1 \end{vmatrix}$ is

(A) -1

(B) 1

(C) 0

(D) 2

QUESTION

$|kA| = k^n |A|$

$\rightarrow n = 3$

$|A^{-1}| = \frac{1}{|A|}$

If A is a 3 × 3 non-singular matrix and if $|A| = 3$ then, $|(2\,A)^{-1}| =$

A) 1/3

B) 1/24

C) 24

D) 3

$|2^{-1} A^{-1}|$

$= (2^{-1})^n |A^{-1}|$

$= (2^{-1})^3 \frac{1}{|A|}$

$= 2^{-3} \left(\frac{1}{3}\right)$

$= \frac{1}{8} \frac{1}{3} = \frac{1}{24}$

QUESTION

If $\begin{bmatrix} 1 & 2 & -1 \\ 1 & x-2 & 1 \\ x & 1 & 1 \end{bmatrix}$ is singular, then the value of x is

A 0

B 1

C 3

D 2

QUESTION

The sum of the product of the elements of the first row with the corresponding cofactors of the elements of the same row of the matrix $\begin{bmatrix} 3 & 2 & 1 \\ 1 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$ is

A) 0

B) 8

C) -8

D) 4

QUESTION

If $A=\begin{bmatrix}0 & x & 16\\ x & 5 & 7\\ 0 & 9 & x\end{bmatrix}$ is singular, then the possible values of x are

(A) 0,4,-4

(B) 0,5,-5

(C) 0,+12,-12

(D) 0,1,-1

QUESTION

If A is a matrix of order 3×3 and $|A| = 4$, then $|5A| =$

$5^n|A|$

$5^3(4)$

125×4

$= 500$

(A) 400

(B) 20

(C) 600

(D) 500

QUESTION

If a matrix of order 4×4 and $|A| = 3$ then $|6A|^{-1} =$

$n = 4$

$\frac{\overset{3}{216} \times 6}{1296}$

(A) 1/6480

(B) 1/18

(C) 6480

(D) 1/3888

① $|A^n| = |A|^n$

② $|A^{-1}| = |A|^{-1}$

③ $|A^T| = |A|$

$$|6^{-1} A^{-1}|$$

$$= (6^{-1})^4 \, |A^{-1}|$$

$$= \frac{1}{6^4} \, \frac{1}{|A|}$$

$$= \frac{1}{1296} \, \frac{1}{3}$$

$$= \frac{1}{3888}$$

QUESTION

If $|A| = 3, |B| = -6$ where A and B are of order 3×3, Then $|4\,AB| =$

$\rightarrow n = 3$

$4^n |A| |B|$

$4^3 (3)(-6)$

$64(-18)$

$= \underline{-1152}$

(A) –1152

(B) –72

(C) 1/75

(D) 48

if $f(x) = x^3 + 4x^2 + 3x + 7$

$\Downarrow$

constant term = 7

To get only constant

we need to substitute $x = 0$

Note:-

If given a polynomial eq^n in x

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_0$

$\Downarrow$

To get only constant 'a_0'

Put $x = 0$

$f(0) = a_0$

QUESTION

If $\underbrace{ax + bx^3 + cx^2 + dx}$ $+ e = \begin{vmatrix} x^3+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix}$ then $e =$

↳ constant term

Put $x = 0$

$$0 + e = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix}$$

↳ skew symmetric matrix of odd order

$e = 0$

(A) -1

(B) 1

(C) 0

(D) 2

QUESTION

The constant form of the Polynomial $\begin{vmatrix} x^2+4 & 2x+2 & 4 \\ -4x+3 & 3x+1 & -6x+1 \\ 7x+2 & x^2-3 & x+1 \end{vmatrix}$ is

Put $x=0$

$$\begin{vmatrix} 4 & 2 & 4 \\ 3 & 1 & 1 \\ 2 & -3 & 1 \end{vmatrix}$$

$4(1+3)-2(3-2)+4(-9-2)$

$=16-2-44$

$=14-44=-30$

(A) 0

(B) –30

(C) 60

(D) –10

QUESTION

The constant term of the polynomial The $\begin{vmatrix} x^2+3x & 4x-3 & -2x+4 \\ 3x+3 & 2x & 6x-2 \\ 3x-4 & 2x+2 & 3x^2+2x \end{vmatrix}$ is

Put $x=0$

$$\begin{vmatrix} 0 & -3 & 4 \\ 3 & 0 & -2 \\ -4 & 2 & 0 \end{vmatrix}$$

$$=0$$

(A) 3

(B) 108

(C) 0

(D) –7

if $|A| = 3$, $|B| = 5$

Find $|AB^T|$

Soln:

$$|AB^T| = |A|\,|B^T|$$

$$= |A|\,|B|$$

$$= 3(5)$$

$$= 15$$

QUESTION

$\rightarrow |A| = 2(-2) - 1(0) - 2(2)$

$= -8$

If $A = \begin{bmatrix} 2 & 1 & -2 \\ 3 & 4 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ then $|\text{Adj } A| =$

3×3

$\Downarrow$

$n=3$

$|A|^{n-1}$

$= (-8)^{3-1}$

$= (-8)^2$

$= 64$

A) -64

B) 16

C) -16

D) 64

QUESTION

$|A^n| = |A|^n$

$\rightarrow |A| = \alpha^2 - 9$

If $A = \begin{bmatrix} \alpha & 3 \\ 3 & \alpha \end{bmatrix}$ and $|A^3| = 216$ then $\alpha =$

$\Downarrow$

$|A|^3 = 6^3$

$|A| = 6$

$\Downarrow$

$\alpha^2 - 9 = 6$

$\alpha^2 = 15$

$\alpha = \underline{\pm}\sqrt{15}$

A) ± 5

B) $\pm\sqrt{15}$

C) $\pm\sqrt{17}$

D) ± 6

QUESTION

If the determinant of the adjoint of (real) matrix of order 3 is 16 , Then determinant of the inverse of the matrix is

(A) ± 0.25

(B) ± 0.2

(C) ± 5

(D) ± 4

$|Adj A| = 16$ where $n = 3$

$|A|^{n-1} = 16$

$|A|^{3-1} = 16$

$|A|^2 = 16$

$|A| = \pm 4$

$|A^{-1}| = \frac{1}{|A|} = \frac{1}{\pm 4}$

$= \underline{\pm 0.25}$

QUESTION

If 'A' is a matrix of order 4 such that $A(\text{adj}A) = 9I$ Then $|\text{Adj } A| =$

$\rightarrow n = 4$

(A) 81

(B) 729

(C) 1/9

(D) 1/81

$A(\text{Adj}A) = |A|I$

$\Rightarrow |A| = 9$

$\hookrightarrow |A|^{n-1}$

$= 9^{4-1}$

$= 9^3$

$= 729$

QUESTION

$\rightarrow$ 2023 PYQ

If $f(x) = \begin{vmatrix} x-3 & 2x^2-18 & 3x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$, then $f(1)\cdot f(3) + f(3)\cdot f(5) + f(5)\cdot f(1)$ is

(A) -1

(B) 0

(C) 1

(D) 2

$$f(x) = \begin{vmatrix} x-3 & 2(x^2-3^2) & 3(x^3-3^3) \\ x-5 & 2(x^2-5^2) & 4(x^3-5^3) \\ 1 & 2 & 3 \end{vmatrix}$$

$$f(3) = \begin{vmatrix} 0 & 0 & 0 \\ -2 & 2(9-25) & 4(27-5^3) \\ 1 & 2 & 3 \end{vmatrix}$$

$= 0$

$$f(5) = \begin{vmatrix} 2 & 2(25-9) & 3(5^3-3^3) \\ 0 & 0 & 0 \\ 1 & 2 & 3 \end{vmatrix}$$

$= 0$

$\therefore f(1)\cdot f(3) + f(3) f(5) + f(5) f(1)$

$f(1)(0) + 0(0) + 0 f(1)$

$= \underline{0}$

QUESTION

Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$. Then $\lim_{x\to 0} \frac{f(x)}{x^2} =$

A) -1

B) 0

C) 3

D) 2

QUESTION

$$|P| = 1(0) - \alpha(-2) + 3(-2)$$
$$= 2\alpha - 6$$

$n = 3$

If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

(A) 4

(B) 5

(C) 11

(D) 0

$P = \text{Adj } A$

$|P| = |\text{Adj } A|$

$|P| = |A|^{n-1}$

$|P| = 4^{3-1} = 4^2 = 16$

$2\alpha - 6 = 16$

$2\alpha = 22$

$\alpha = 11$

$A = \begin{bmatrix} \; \end{bmatrix}$

$\Downarrow$

Adj A

$\Downarrow$

compare

QUESTION

If $A = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$ and $B = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$, then $\frac{dB}{dx}$ is

A. 3A

B. –3B

C. 3B+1

D. 1–3A

QUESTION

The constant term of the polynomial $\begin{vmatrix} x+3 & x & x+2 \\ x & x+1 & x-1 \\ x+2 & 2x & 3x+1 \end{vmatrix}$ is

(A) 2

(B) 0

(C) 1

(D) –1

QUESTION

If $\begin{bmatrix} 1 & 2 & -1 \\ 1 & x-2 & 1 \\ x & 1 & 1 \end{bmatrix}$ is singular, then the value of x is

A. 2

B. 3

C. 1

D. 0

QUESTION

If ω is an imaginary cube root of unity, then the value of $\begin{bmatrix} 1 & \omega^2 & 1-\omega^4 \\ \omega & 1 & 1+\omega^5 \\ 1 & \omega & \omega^2 \end{bmatrix}$ is

A -4

B ω^2-4

C ω^2

D 4

QUESTION

If $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & x \end{vmatrix}$, then $\mathrm{Lt}_{x\to 0} \dfrac{f(x)}{x^2} =$

A. 1

B. 0

C. 3

D. 2

Sandwich Theorem

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$\lim_{x\to 0} \frac{\tan x}{x} = 1$

$f(x) = \sin x(x^3 - x) - \cos x(x^4 - 2x^2) + \tan x(x^3 - 2x^3)$

$= x^3 \sin x - x\sin x - x^4\cos x + 2x^2\cos x - x^3\tan x$

$\div$ by x^2

$\frac{f(x)}{x^2} = x\sin x - \frac{\sin x}{x} - x^2\cos x + 2\cos x - x\tan x$

As $x \to 0$

$= 0 - 1 - 0 + 2(1) - 0 = -1 + 2 = 1$

QUESTION

If $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 0 & 2\cos x & 3 \\ 0 & 1 & 2\cos x \end{vmatrix}$, then $\lim_{x\to\pi} f(x) =$

(A) -1 ✓

(B) 1

(C) 0

(D) 3

$f(x) = \cos x\,[4\cos^2 x - 3]$

As $x \to \pi$

$= \cos\pi\,[4\cos^2\pi - 3]$

$= (-1)[4(-1)^2 - 3]$

$= (-1)[4-3] = \underline{-1}$

QUESTION

$n = 3$

If the determinant of the adjoint of a (real) matrix of order 3 is 25 , then the determinant of the inverse of the matrix is

(A) $+0.2$

(B) ± 5

(C) $\frac{1}{\sqrt[5]{625}}$

(D) ± 0.2

$|Adj A| = 25$

$|A|^{3-1} = 5^2$

$|A|^2 = 5^2$

$|A| = \pm 5$

$|A^{-1}| = \frac{1}{|A|}$

$= \pm \frac{1}{5} = \pm 0.2$

if $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \longrightarrow$ Diagonal matrix

Then

$$A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$$

QUESTION

The inverse of the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

(A) $\frac{1}{24}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

(C) $\frac{1}{24}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$

QUESTION

The system of linear equations $x + y + z = 6, x + 2y + 3z = 10$ and $x + 2y + az = b$ has no solution when ______

A $a = 3, b \neq 10$

B $b = 3, a \neq 10$

C $a = 2, b \neq 3$

D $b = 2, a = 3$

QUESTION

$\rightarrow |A| = \alpha^2 - 4$ $\qquad |A^n| = |A|^n$

If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 27$, then $\alpha =$

$|A|^3 = 3^3$

(A) ± 2

$|A| = 3$

(B) $\pm\sqrt{5}$

$\alpha^2 - 4 = 3$

(C) ± 1

$\alpha^2 = 7$

(D) $\pm\sqrt{7}$

$\alpha = \underline{\pm}\sqrt{7}$

QUESTION

If $\begin{vmatrix} 2a & x_1 & y_1 \\ 2b & x_2 & y_2 \\ 2c & x_3 & y_3 \end{vmatrix} = \frac{abc}{2} \neq 0$, then the area of the triangle whose vertices are $\left(\frac{x_1}{a}, \frac{y_1}{a}\right), \left(\frac{x_2}{b}, \frac{y_2}{b}\right)$ and $\left(\frac{x_3}{c}, \frac{y_3}{c}\right)$ is

A $\frac{1}{8}abc$

B $\frac{1}{8}$

C $\frac{1}{4}abc$

D $\frac{1}{4}$

QUESTION

Evaluate $\begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

(A) 0

(B) 3

(C) 1

(D) 2

QUESTION

If A is any square matrix of order 3×3 then $|3A|$ is equal to

A $3|A|$

B $\frac{1}{3}|A|$

C $27|A|$

D $9|A|$

QUESTION

If $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$ then x is equal to

A) 8

B) 4

C) $\pm 2\sqrt{2}$

D) 2

QUESTION

If A is a square matrix of order 3×3, then $|KA|$ is equal to

A. $K|A|$

B. $K^2|A|$

C. $3K|A|$

D. $K^3|A|$

QUESTION

If $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) are the vertices of a triangle whose area is k square units, then $\begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2$ is

(A) $32k^2$

(B) $16k^2$

(C) $64k^2$

(D) $48k^2$

QUESTION

Let A be a square matrix of order 3×3, then $|5A| =$

(A) $5|A|$

(B) $125|A|$

(C) $25|A|$

(D) $15|A|$

QUESTION

$\rightarrow |A| = 2 - 12 = -10$

$\rightarrow |B| = 4 + 1 = 5$

If $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$, then $|ABB^T| =$

$\Downarrow$

$|A|\,|B|\,|B^T|$

$|A|\,|B|\,|B|$

$= (-10)(5)(5)$

$= \underline{-250}$

(A) 50

(B) 100

(C) -250

(D) 250

QUESTION

$\rightarrow n=3$ $\quad |A|=16$

If the value of a third order determinant is 16 , then the value of the determinant formed by replacing each of its elements by its cofactor is

(A) 96

(B) 256

(C) 48

(D) 1

Minor $\rightarrow M_{ij}$

Cofactors $\Rightarrow A_{ij}$

$$\det\begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} \Rightarrow \text{Cofactor matrix}$$

$= |\text{Transpose of Adj } A|$

$= |(\text{Adj } A)^T|$

$= |\text{Adj } A|$ $\quad$ WKT $|X| = |X^T|$

$= |A|^{n-1} = 16^{3-1} = 256$

Cofactor matrix A = Transpose of Adj A

if $A = \begin{bmatrix} 3 & 5 & 6 \\ 1 & -2 & -4 \\ -1 & -5 & 4 \end{bmatrix}$ $\to a_{12}$ $\to a_{23}$

$a_{ij} \to$ element

Find $a_{13}A_{23} + M_{12}A_{22}$

$M_{ij} \to$ minor of a_{ij}

$A_{ij} \to$ cofactor of a_{ij}
$= (-1)^{i+j} M_{ij}$

Soln:

$a_{13} = 6$

$A_{23} = (-1)^{2+3} M_{23} = -1 \begin{vmatrix} 3 & 5 \\ -1 & -5 \end{vmatrix} = -(-15+5) = -(-10) = 10$

$M_{12} = \begin{vmatrix} 1 & -4 \\ -1 & 4 \end{vmatrix} = 4-4 = 0$

$A_{22} = (-1)^{2+2} M_{22} = +\begin{vmatrix} 3 & 6 \\ -1 & 4 \end{vmatrix} = 12+6 = 18$

$a_{13}A_{23} + M_{12}A_{22}$

$= 6(10) + 0(18)$

$= \underline{60}$

QUESTION

The constant term in the expansion of $\begin{vmatrix} 3x+1 & 2x-1 & x+2 \\ 5x-1 & 3x+2 & x+1 \\ 7x-2 & 3x+1 & 4x-1 \end{vmatrix}$ is

Put $x=0$

$$\begin{vmatrix} 1 & -1 & 2 \\ -1 & 2 & 1 \\ -2 & 1 & -1 \end{vmatrix}$$

$= 1(-3)+1(3)+2(3)$

$= -3+3+6$

$= 6$

A) 0

B) –10

C) 2

D) 6

QUESTION

If $f(x) = \begin{vmatrix} x^3 - x & a + x & b + x \\ x - a & x^2 - x & c + x \\ x - b & x - c & 0 \end{vmatrix}$, then

(A) $f(2)=0$

(B) $f(0)=0$

(C) $f(-1)=0$

(D) $f(1)=0$

QUESTION

If A is a square matrix of order 3 and $|A| = 5$, then $|A \operatorname{adj} A|$ is

A 125

B 25

C 625

D 5

QUESTION

If A and B are matrices of order 3 and $|A| = 5, |B| = 3$, then $|3AB|$ is

A. 425

B. 405

C. 565

D. 585

QUESTION

If A and B are invertible matrices, then which of the following is not correct?

A. $\operatorname{adj} A = |A| A^{-1}$

B. $\det(A^{-1}) = [\det(A)]^{-1}$

C. $(AB)^{-1} = B^{-1} A^{-1}$

D. $(A + B)^{-1} = B^{-1} + A^{-1}$

QUESTION

If $x^3 - 2x^2 - 9x + 18 = 0$ and $A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & x & 6 \\ 7 & 8 & 9 \end{vmatrix}$ then the maximum value of A is

A. 96

B. 36

C. 24

D. 120

QUESTION

If A is a 3×3 matrix such that $|5 \cdot \text{adj } A| = 5$ then $|A|$ is equal to

A ± 1

B $\pm 1/5$

C $\pm 1/25$

D ± 5

QUESTION

If there are two values of ' a ' which makes determinant $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$

Then the sum of these numbers is

(A) -4

(B) 4

(C) 9

(D) 5

QUESTION

If A is a matrix of order 3 , such that $A(\operatorname{adj} A)=10 I$, then $|\operatorname{adj} A|=$

(A) 1

(B) 10

(C) 100

(D) $10 I$

① $\text{Adj}(AB) = (\text{Adj}\,B)\cdot(\text{Adj}\,A)$

② $\text{Adj}(A') = (\text{Adj}\,A)'$

QUESTION

If M is any square matrix of order 3 over R and if M' be the transpose of M , then adj (M') – (adj M)' is equal to

(A) M

(B) M'

(C) Null matrix

(D) Identity matrix

$\text{Adj}(M') - (\text{Adj}\, M)'$

$= (\text{Adj}\, M)' - (\text{Adj}\, M)'$

$= 0$

Note

$$\operatorname{Adj}(KA) = K^{n-1}(\operatorname{adj} A)$$

QUESTION

$n=3$

If X is a square matrix of order 3×3 and λ is a scalar, then $\text{adj}(\lambda X)$ is equal to

$= \lambda^{n-1} \text{Adj}(x)$

$= \lambda^{3-1} \text{Adj}\, x$

$= \lambda^{2} \text{Adj}\, X$

A $\lambda\, \text{adj}\, X$

B $\lambda^3 \cdot \text{adj} X$

C $\lambda^2\, \text{adj}\, X$

D $\lambda^4 \text{adj} X$

If $\text{Adj}A = \begin{bmatrix} 3 & 4 & -2 \\ 4 & -5 & 6 \\ 7 & 2 & 8 \end{bmatrix}$ Then $\text{Adj}\,2A$

(A) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

(B) $\begin{bmatrix} 6 & 8 & -4 \\ 8 & -10 & 12 \\ 14 & 4 & 16 \end{bmatrix}$

(C) $\begin{bmatrix} 12 & 16 & -8 \\ 16 & -20 & 24 \\ 28 & 8 & 32 \end{bmatrix}$ ✓

(D) None

Soln: order $= n = 3$

$$\text{Adj}\,2A = 2^{3-1}\,\text{Adj}A$$

$$= 4\begin{bmatrix} 3 & 4 & -2 \\ 4 & -5 & 6 \\ 7 & 2 & 8 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 16 & -8 \\ 16 & -20 & 24 \\ 28 & 8 & 32 \end{bmatrix}$$

QUESTION

$|A| = 0$

If A is a singular matrix, then adjA is

A singular

B non-singular

C symmetric

D not defined

$$|\text{Adj}A| = |A|^{n-1}$$
$$= 0^{n-1}$$

$$|\text{Adj}A| = 0$$

$\Rightarrow$ Adj A is singular

QUESTION

If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible values of the determinant of P are

A. ± 2

B. ± 1

C. ± 3

D. ± 4

$$\text{Adj } P = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$$

$$|\text{Adj } P| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$$

$$|P|^{3-1} = 1(-4) - 4(-1) + 4(1)$$

$$|P|^2 = -4 + 4 + 4 = 4$$

$$|P|^2 = 4$$

$$|P| = \pm 2$$

QUESTION

$|A| = 0$

If A is a singular matrix of order n , then A.(adj A) is

$$A \cdot (\text{adj}\, A) = |A| I$$
$$= 0 (I)$$
$$A \cdot (\text{Adj}\, A) = 0$$

A zero matrix

B row matrix

C unit matrix

D column matrix

QUESTION

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\text{adj}(\text{adj}A)$ is equal to

2×2

$n=2$

$|A|^{n-2} A$

$= |A|^{2-2} A$

$= |A|^{0} A$

$= A$

A adj A

B A

C A^T

D $-A$

QUESTION

WKT $adj(adjA) = |A|^{n-2} A$

$n=2$

$= |A|^{2-2} A$

$= (1) A = A$

Let A be a 2×2 matrix

Statement-1 : $\text{adj}(\text{adj}A) = A$ → True

Statement-2 : $|\text{adj}A| = |A|$ True

(A) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(B) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1

(C) Statement-1 is true, Statement-2 is false

(D) Statement-1 is false, Statement-2 is true

QUESTION

If A is a square matrix such that $A(\text{adj}A) = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then $\dfrac{|\text{adj}(\text{adj}A)|}{|\text{adj}A|}$ is equal to

$A(\text{adj}A) \rightarrow |A|I$

$|\text{adj}(\text{adj}A)| \rightarrow |A|^{(n-1)^2}$

$|\text{adj}A| \rightarrow |A|^{n-1}$

$3\times3 \rightarrow n=3$

$= 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow |A| = 4$

$= \dfrac{4^{(3-1)^2}}{4^{3-1}} = \dfrac{(4^2)^2}{4^2}$

$= \dfrac{16^2}{16} = 16$

(A) 256

(B) 16 ✓

(C) 32

(D) 64

QUESTION

If $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = \begin{cases} i+j, & \text{if } i \neq j \\ i^2 - 2j, & \text{if } i = j \end{cases}$ then A^{-1} is equal to

(A) $\frac{1}{9}\begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$

(B) $\frac{1}{9}\begin{bmatrix} 0 & -3 \\ -3 & -1 \end{bmatrix}$

(C) $\frac{1}{9}\begin{bmatrix} 0 & 3 \\ 3 & 1 \end{bmatrix}$

(D) None of these

QUESTION

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

if $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

If $A = \begin{bmatrix} \frac{k}{2} & 0 & 0 \\ 0 & \frac{l}{3} & 0 \\ 0 & 0 & \frac{m}{4} \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{4} \end{bmatrix}$ then $k + l + m =$

$$A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$$

(A) 1

(B) 9

(C) 14

(D) 29

$\frac{k}{2} = 2$, $k = 4$ | $\frac{l}{3} = 3$, $l = 9$ | $\frac{m}{4} = 4$, $m = 16$

$$k + l + m = 4 + 9 + 16$$
$$= 29$$

QUESTION

From the matrix equation $AB = AC$ we can conclude that $B = C$, provided

Premultiply by A^{-1}

is possible only if A^{-1} exists

$\Downarrow$

$A \rightarrow$ non singular

A A is singular

B A is non-singular

C A is symmetric

D A is square

QUESTION

If $A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix}$, then the value of x is

A 2

B 3

C -4

D 4

$A \cdot A^{-1} = I$

$$\begin{bmatrix} \frac{7x}{34} + \frac{6}{34} & \_\_ \\ \_\_ & \_\_ \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\frac{7x+6}{34} = 1$

$7x + 6 = 34$

$7x = 28$

$x = 4$

QUESTION

If A and B are two square matrices such that $B = -A^{-1}BA$, then $(A + B)^2 =$

Premultiply by A on B.S

(A) 0

$$AB = -BA$$

(B) $A^2 + B^2$

$$\boxed{AB + BA = 0}$$

(C) $A^2 + 2AB + B^2$

Consider $(A+B)^2$

$$= A^2 + B^2 + AB + BA$$

(D) $A + B$

$$= A^2 + B^2 + 0$$

$$= \underline{A^2 + B^2}$$

QUESTION

$A^2 = I$

$\Downarrow$

$A \cdot A = I \Rightarrow A^{-1} = A$

If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and $A^2 = I$, then A^{-1} is equal to

$\Downarrow$

$$\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x^2+1 & \_\_ \\ \_\_ & \_\_ \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$x^2 + 1 = 1$

$x^2 = 0$

$x = 0$

$$A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A$$

$$= \frac{1}{-1}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

A. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

QUESTION

If A and B are square matrices of the same order and $AB = 3I$, then A^{-1} is equal to

(A) $3B$

(B) $1/3\ B$

(C) $3\ B^{-1}$

(D) $1/3\ B^{-1}$

$AB = 3I$

Premultiply by A^{-1}

$B = 3A^{-1}$

$A^{-1} = \frac{1}{3} B$

QUESTION

If Matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ such that $AX = I$, then X = ___

$$\Downarrow$$

$$X = A^{-1}$$

$$= \frac{1}{|A|} \text{Adj} A$$

$$= \frac{1}{-5} \begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix}$$

$$= +\frac{1}{5} \begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}$$

(A) $\frac{1}{5}\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

(B) $\frac{1}{5}\begin{bmatrix} 4 & 2 \\ 4 & -1 \end{bmatrix}$

(C) $\frac{1}{5}\begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}$

(D) $\frac{1}{5}\begin{bmatrix} -1 & 2 \\ -1 & 4 \end{bmatrix}$

QUESTION

If the inverse of the matrix $\begin{bmatrix} \alpha & 14 & -1 \\ 2 & 3 & 1 \\ 6 & 2 & 3 \end{bmatrix}$ does not exist, then the value of α is

$\rightarrow |A| = 0$

A 1

B -1

C 0

D -2

$\alpha(7) - 14(0) - 1(-14) = 0$

$7\alpha + 14 = 0$

$\alpha = -2$

QUESTION

$|A| = -3$

The inverse of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$ is

$$\begin{array}{ccccc} 1 & 0 & 0 & 1 & 0 \\ 3 & 3 & 0 & 3 & 3 \\ 5 & 2 & -1 & 5 & 2 \\ 1 & 0 & 0 & 1 & 0 \\ 3 & 3 & 0 & 3 & 3 \end{array}$$

(A) $-\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 2 & -3 \end{bmatrix}$

(B) $-\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

$\text{Adj}A = \begin{bmatrix} -3 \\ 3 \\ -9 \end{bmatrix}$

(C) $-\frac{1}{3}\begin{bmatrix} 3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

(D) $-\frac{1}{3}\begin{bmatrix} -3 & 0 & 0 \\ -3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$

QUESTION

$n=3$

If A and B are square matrices of order 3 such that $|A|=2, |B|=4$, then $|A(\operatorname{adj}B)|=$

$|A|\,|\operatorname{Adj} B|$

$(2)\ |B|^{n-1}$

$(2)\ (4)^{3-1}$

$(2)\ 4^2$

$=2(16)$

$=32$

A 32

B 64

C 8

D 16

QUESTION

$A = \text{diag}(a, b, c)$

$A^n = \text{diag}(a^n, b^n, c^n)$

If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, then $\underline{A^4\ A^{-1}} =$

A^3

$= \text{diag}(2^3, (-2)^3, (-1)^3)$

$= \text{diag}(8, -8, -1)$

A $\begin{bmatrix} -4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

B $\begin{bmatrix} 3 & 0 & 0 \\ 0 & \frac{-1}{2} & 0 \\ 0 & 0 & -1 \end{bmatrix}$

C $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

D $\begin{bmatrix} 8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

1. $A(\text{Adj}A) = |A| I$
2. $\text{Adj}A = |A| \cdot A^{-1}$
3. $\text{Adj}(\text{Adj}A) = |A|^{n-2} A$

1. $|A \cdot \text{Adj}A| = |A|^n$
2. $|\text{Adj}A| = |A|^{n-1}$
3. $|\text{Adj}\cdot(\text{Adj}A)| = |A|^{(n-1)^2}$

QUESTION

$|A| = 1(-1) - 2(-1) + 1(0)$

$= -1 + 2 = 1$

If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, then $[\text{adj}(\text{adj}A)]^{-1} =$

3×3

$n = 3$

$\Downarrow$

$[|A|^{n-2} A]^{-1}$

$(|A| \cdot A)^{-1}$

$((1) A)^{-1}$

$= \underline{A^{-1}}$

A I

B 2A

C A^2

D A^{-1} ✓

QUESTION

$$A(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}, \text{ then } [A^2(\alpha)]^{-1} =$$

A) $A(\alpha)$

B) $A^2(\alpha)$

C) $A(-2\alpha)$

D) $A(2\alpha)$

QUESTION

The sum of the cofactors of the elements of second row of the matrix $\begin{bmatrix} 1 & 3 & 2 \\ -2 & 0 & 1 \\ 5 & 2 & 1 \end{bmatrix}$ is

(A) 23

(B) 5

(C) 3

(D) −23

$A_{21} + A_{22} + A_{23} = (-1)M_{21} + M_{22} - M_{23}$

$= -\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix}$

$= -(-1) + (-9) - (-13)$

$= 1 - 9 + 13 = 14 - 9 = 5$

QUESTION

If $A = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & -2 \\ 0 & 2 & 1 \end{bmatrix}$, adj $A = \begin{bmatrix} 5 & x & -2 \\ 1 & 1 & 0 \\ -2 & 2 & y \end{bmatrix}$, then value of $x + y$ is

cofactor of $a_{21} = A_{21} = -\begin{vmatrix} 0 & 2 \\ 2 & 1 \end{vmatrix} = -(-4) = 4$

cofactor of $a_{33} = A_{33} = \begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix} = 1$

$\therefore x + y = 4 + 1 = 5$

(A) 6

(B) 3

(C) 4

(D) 5

QUESTION

For a 3×3 matrix A, if $A(\text{adj}A) = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -10 \end{bmatrix}$, then the value of determinant of A is

Connection b/w A & A^{-1}

(1) $|A^{-1}| = \frac{1}{|A|}$

(2) $AA^{-1} = I$

(3) $A^{-1} = \frac{1}{|A|} \text{adj}A$

A 100

B -1000

C -10

D 20

$A(\text{Adj}A) = -10 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= -10\, I$

$\downarrow$

$|A| = -10$

$A(\text{Adj}A) = |A|\, I$

QUESTION

$\rightarrow |A| = 1(0) - 2(-6) + 3(-3)$

$= 12 - 9 = 3$

If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$, and $A(\text{adj}A) = kI$, then the value of $(k+1)^4$ is

$= |A| I$

$\Downarrow$

$|A| = K = 3$

$(3+1)^4$

$= 4^4$

$= 2^8$

$= \underline{256}$

A 256

B 81

C 16

D 625

QUESTION

$|A| = 2(2)(2)$
$= 8$

If $A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$, then adj(adj$A$) is equal to

3×3
$n=3$

$\hookrightarrow |A|^{n-2} A = (8^{3-2}) A = 8A = 8\begin{bmatrix} 2 & 0 & 0 \\ 2 & 2 & 0 \\ 2 & 2 & 2 \end{bmatrix}$

$= 8(2)\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

$= 16\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

(A) $4\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

(B) $8\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

(C) $16\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

(D) None of these

QUESTION

HW

If $A=\begin{bmatrix}0&1&2\\1&2&3\\3&a&1\end{bmatrix}$ and $A^{-1}=\frac{1}{2}\begin{bmatrix}1&-1&1\\-8&6&2c\\5&-3&1\end{bmatrix}$, Then

A $a=2, c=-\frac{1}{2}$

B $a=1, c=-1$

C $a=-1, c=1$

D $a=\frac{1}{2}, c=\frac{1}{2}$

QUESTION

If $A=\begin{bmatrix}0 & 1+2i & i-2\\ -1-2i & 0 & K\\ 2-i & 7 & 0\end{bmatrix}$ and A^{-1} does not exists, then $K=$ (where $i=\sqrt{-1}$)

(A) $1+2i$

(B) -7

(C) 7

(D) $1-2i$

QUESTION

$|A| = \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = 1(1-0) = 1$

The element of first row and third column of the inverse of the matrix $\begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ is

a_{13} in A^{-1}

$|A| = 1$

(A) -2

(B) 0

(C) 1

(D) 7

$A^{-1} = \frac{1}{|A|} \operatorname{Adj} A = \begin{bmatrix} & & \underline{A_{31}} \\ & & \\ & & \end{bmatrix}$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = 4 + 3 = \underline{7}$

QUESTION

$|A| = 1(-13) - a(-3) + 3(2) = -13 + 3a + 6 \Rightarrow 3a - 7$

If $A = \begin{bmatrix} 1 & a & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & b & 2 \\ -2 & 0 & 1 \end{bmatrix}$, then the values of a and b are respectively

(A) 1, 2

(B) 2, −1

(C) 2, 1

(D) −1, 2

WKT $AA^{-1} = I$

$$\begin{bmatrix} 1 & a & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}\begin{bmatrix} 13 & 2 & -7 \\ -3 & b & 2 \\ -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} & & -7+2a+3 \\ & 2+b & \\ & & \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$-7 + 2a + 3 = 0$

$2a - 4 = 0$

$a = 2$

$2 + b = 1$

$b = -1$

$$3a = 13(39a - 91) + 13$$

$$3a = 507a - 1183 + 13$$

$$3a = 507a - 1170$$

$$504a = 1170$$

$$a =$$

QUESTION

If $A=\begin{bmatrix}0&1&2\\1&2&3\\3&a&1\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1/2&-1/2&1/2\\-4&3&c\\5/2&-3/2&1/2\end{bmatrix}$, then

WKT $AA^{-1}=I$

$$\begin{bmatrix} & & 0+c+1\\ & & \\ \frac{3}{2}-4a+\frac{5}{2} & & \end{bmatrix}=\begin{bmatrix} & & 0\\ & & \\ 0 & & \end{bmatrix}$$

(A) $a=2, c=1/2$

(B) $a=1, c=-1$

(C) $a=-1, c=1$

(D) $a=1/2, c=1/2$

$\frac{3}{2}-4a+\frac{5}{2}=0$

$\frac{8}{2}-4a=0$

$4-4a=0$

$a=1$

$c+1=0$

$c=-1$

if $A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 6 & 7 \\ 4 & 2 & 3 \end{bmatrix}$

$$\begin{array}{ccccc} 2 & 3 & 5 & 2 & 3 \\ 1 & 6 & 7 & 1 & 6 \\ 4 & 2 & 3 & 4 & 2 \\ 2 & 3 & 5 & 2 & 3 \\ 1 & 6 & 7 & 1 & 6 \end{array}$$

$\operatorname{Adj} A = \begin{bmatrix} 4 & 1 & -9 \\ 25 & -14 & -9 \\ -22 & 8 & 9 \end{bmatrix}$

Thank

You