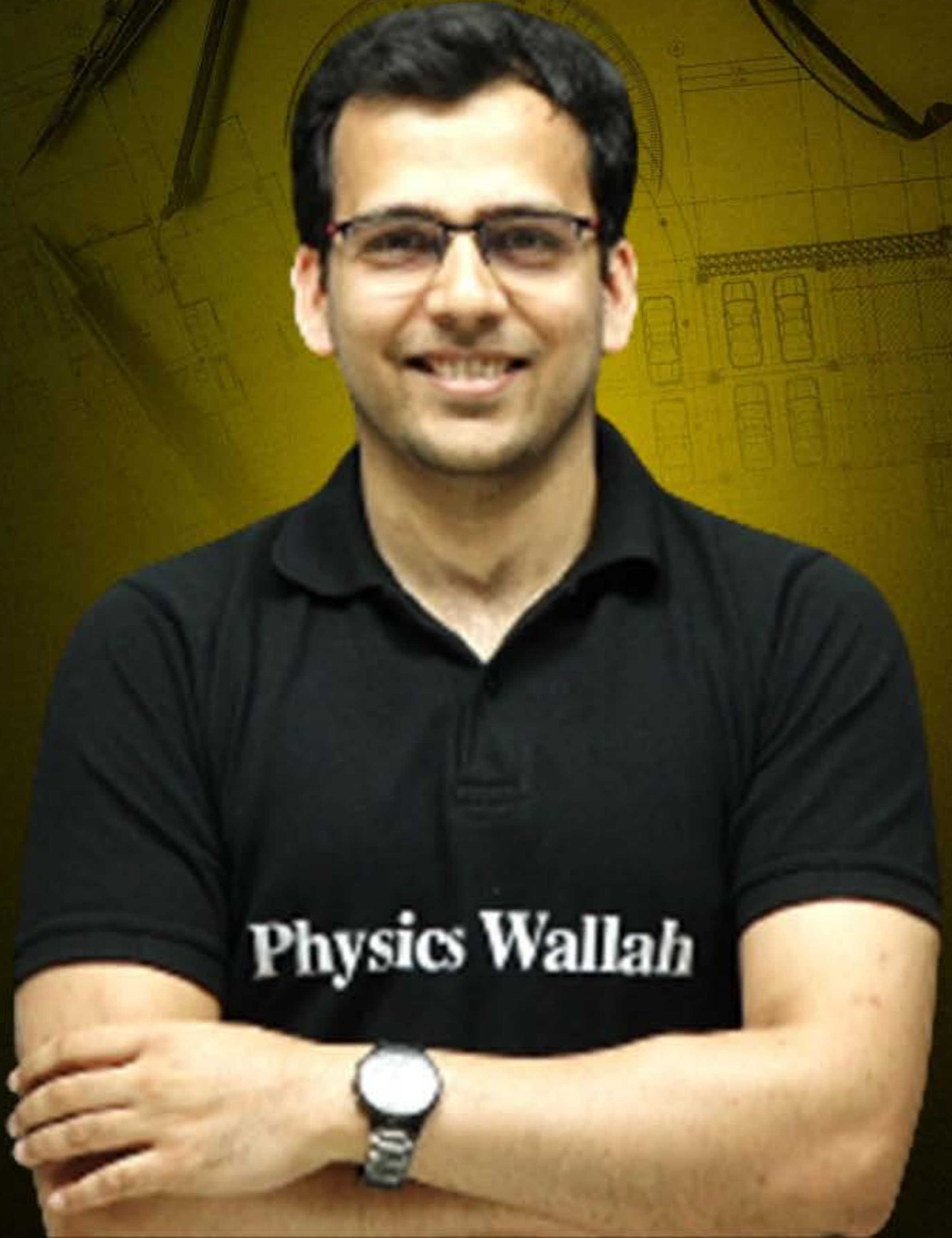
VARUN

ADVANCE 2026

Mathematics | Lecture - 01

Integration, D.E. & Area



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Topics *To be covered*

1. Indefinite Integration
2. Important Brain Teasers

Indefinite Integration

Brain Teaser-01

[Ans. A, D]

Let $\int \frac{(x^2+1)dx}{x\sqrt{x^2+2x-1}\sqrt{1-x^2-x}} = 2\sin^{-1}\left(\sqrt{x-\frac{1}{\alpha x}+\beta}\right)+c$, then

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1} x/a$

(A) $\alpha+\beta=3$ ✓

(B) $\alpha+\beta=4$

(C) $\alpha-\beta=2$

(D) $\alpha-\beta=-1$ ✓

$\frac{(x^2+1)\,dx}{x\sqrt{x}\sqrt{x+2-1/x}\,\sqrt{x}\sqrt{1/x-x-1}}$

$\int \frac{(1+1/x^2)\,dx}{\sqrt{x-1/x+2}\sqrt{1/x-x-1}} = \int \frac{2\cancel{t}\,dt}{\cancel{t}\sqrt{2-t^2-1}}$

$= 2\int \frac{dt}{\sqrt{1-t^2}}$

(x^2-1) $\quad x^2+1$

Let $x-1/x+2=t^2$

$(1+1/x^2)\,dx = 2t\,dt$

$1/x-x = 2-t^2$

$2\sin^{-1}(t)+C = 2\sin^{-1}\sqrt{x-1/x+2}+c$

$\alpha=1,\ \beta=2$

Brain Teaser-02

For $x \in \left(0, \frac{\pi}{2}\right)$, If $\int \frac{\tan x\sqrt{\sec x}(1+\cos^2 x)dx}{\sqrt{\cos x + \sin^2 x}} = 2\sqrt{f(x)+1} + c$, where, $f\left(\frac{\pi}{3}\right) = \frac{3}{2}$, then $\left[f\left(\frac{\pi}{4}\right)\right] =$ ....

(where [.] denotes the greatest integer function)

$$\int \frac{\frac{\sin x}{\cos x(\cos x)^{1/2}}(1+\cos^2 x)\,dx}{\sqrt{\cos x + 1 - \cos^2 x}}$$

Let $\cos x = t$

$$-\int \frac{(1+t^2)\,dt}{(t)^{3/2}\sqrt{t+1-t^2}} = -\int \frac{(1+t^2)\,dt}{t^2\sqrt{1+\frac{1}{t}-t}}$$

$$= -\int \frac{\left(1+\frac{1}{t^2}\right)dt}{\sqrt{1+\frac{1}{t}-t}}$$

[Ans. 0]

$$1+\frac{1}{t}-t = z^2$$

$$\left(-\frac{1}{t^2}-1\right)dt = 2z\,dz$$

$$-\left(1+\frac{1}{t^2}\right)dt = 2z\,dz$$

$$\int \frac{2\cancel{z}\,dz}{\cancel{z}} = 2z + c$$

$$2\sqrt{1+\frac{1}{t}-t} + c$$

$$2\sqrt{1+\sec x - \cos x} + c$$

$$f(x) = \sec x - \cos x$$

$$f(\pi/4) = \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$[f(\pi/4)] = 0$$

Brain Teaser-03

[Ans. 3]

$f(x)$ be a polynomial of degree at most 7 which leaves remainders −1 and 1 upon division by $(x-1)^4$ and $(x+1)^4$ respectively. If the sum of pairwise product of all roots of $f(x)=0$ is n, then the value of $(5n+24)$ is ______.

$$\boxed{f(x) = (x-1)^4 Q(x) - 1} \quad \& \quad f(x) = (x+1)^4 P(x) + 1$$

$$f'(x) = (x-1)^4 Q'(x) + Q(x) \cdot 4(x-1)^3$$

$$f'(x) = (x-1)^3 \left[(x-1)Q'(x) + 4Q(x)\right]$$

$$f'(x) = 4(x+1)^3 P(x) + (x+1)^4 P'(x)$$

$$f'(x) = (x+1)^3 \left[4P(x) + (x+1)P'(x)\right]$$

$$\Rightarrow f'(x) = K(x-1)^3(x+1)^3$$

$$f'(x) = K(x^2-1)^3 = K\left[x^6 - 1 - 3x^2(x^2-1)\right]$$

$$f'(x) = K\left[x^6 - 1 - 3x^4 + 3x^2\right]$$

$$f(x) = K\left[\frac{x^7}{7} - x - \frac{3x^5}{5} + x^3\right] + C$$

$$S_2 = \alpha\beta + \beta\gamma + \gamma\delta + \ldots = +C/a$$

$$= \frac{-3\cancel{K}/5}{\cancel{K}/7}$$

$$S_2 = -\frac{21}{5}$$

$$n = -\frac{21}{5}$$

$$5n = -21$$

$$\text{Ans} = (-21+24) = 3$$

How to Find K & C

$$f(1) = -1 \quad \& \quad f(-1) = 1$$

$$K/7 - K - 3K/5 + K + C = -1$$

$$-K/7 + K + \tfrac{3}{5}K - K + C = 1$$

Add

$$\Rightarrow \boxed{C = 0}$$

$K =$ ✓

Brain Teaser-04

[Ans 5]

$\int \underbrace{(x^6 + x^3)} (x^3 + 2)^{\frac{1}{3}} dx = \lambda(x^6 + 2x^3)^{\frac{\alpha}{3}} + c$, then $8\lambda+\alpha$ is equal to $\underline{\quad 5 \quad}$.

$$\int x(x^5 + x^2)(x^3+2)^{1/3}\,dx$$

$$\int (x^5 + x^2)\left(\underbrace{x^6 + 2x^3}\right)^{1/3} dx$$

$$\int (t^3)^{1/3} \frac{t^2\,dt}{2}$$

$$\int \frac{t^3\,dt}{2} = \frac{t^4}{8} + c$$

$$= \frac{(x^6 + 2x^3)^{4/3}}{8} + c$$

$x^6 + 2x^3 = t^3$

$(6x^5 + 6x^2)\,dx = 3t^2\,dt$

$(x^5 + x^2)\,dx = \frac{3t^2\,dt}{6}$

$= \frac{t^2\,dt}{2}$

$\lambda = \frac{1}{8} \Rightarrow 8\lambda = 1$

$\alpha = 4$

Brain Teaser-05

$(x^4+x^2+1)=(x^2+x+1)(x^2-x+1)$ [Ans. 0.25

If $\int \frac{1+x^2}{\boxed{3-4\sqrt{2}x+4x^4}}dx = \frac{a}{x+b} + c\tan^{-1}(x+d)+e$, then $2024a+2025b+2026c+2025d$ is equal to ______.

$$\frac{1}{4}\int \frac{1+x^2}{\left(x^2-\sqrt{2}x+\frac{1}{2}\right)\left(x^2+\sqrt{2}x+3/2\right)}dx$$

$$\frac{1}{8}\int \frac{\left(x^2+\sqrt{2}x+3/2\right)+\left(x^2-\sqrt{2}x+1/2\right)}{\left(x^2-\sqrt{2}x+1/2\right)\left(x^2+\sqrt{2}x+3/2\right)}dx$$

$$\frac{1}{8}\int \frac{dx}{\left(x^2-\sqrt{2}x+1/2\right)} + \frac{1}{8}\int \frac{dx}{x^2+\sqrt{2}x+3/2}$$

$$\frac{1}{8}\int \frac{dx}{\left(x-\frac{1}{\sqrt{2}}\right)^2} + \frac{1}{8}\int \frac{dx}{\left(x+\frac{1}{\sqrt{2}}\right)^2+1}$$

$$4x^4-4\sqrt{2}x+3$$

$$4\left[x^4-\sqrt{2}x+3/4\right]$$

$$\overbrace{x^4-x^2+x^2-\sqrt{2}x+\tfrac{1}{2}+\tfrac{1}{4}}$$

$$4\left[\left(x^2-1/2\right)^2+\left(x-\frac{1}{\sqrt{2}}\right)^2\right]$$

$$4\left[\left(x-\frac{1}{\sqrt{2}}\right)\left(x+\frac{1}{\sqrt{2}}\right)\right]^2+\left(x-\frac{1}{\sqrt{2}}\right)^2$$

$$4\left(x-\frac{1}{\sqrt{2}}\right)^2\left[\left(x+\frac{1}{\sqrt{2}}\right)^2+1\right]$$

$$4\left(x^2-\sqrt{2}x+1/2\right)\left(x^2+\sqrt{2}x+3/2\right)$$

$$-\frac{1}{8}\,\frac{1}{\left(x-\frac{1}{\sqrt{2}}\right)}+\frac{1}{8}\tan^{-1}\left(x+\frac{1}{\sqrt{2}}\right)+c$$

$b=-\frac{1}{\sqrt{2}}$ $\qquad$ $c=\frac{1}{8}$, $d=\frac{1}{\sqrt{2}}$

$a=-\frac{1}{8}$

$$2024\left(-\frac{1}{8}\right)+\cancel{2025}\left(\frac{-1}{\sqrt{2}}\right)+2026\times\frac{1}{8}+\cancel{2025}\left(\frac{1}{\sqrt{2}}\right)$$

$$(2026-2024)\times\frac{1}{8}$$

$$=\frac{2}{8}=\frac{1}{4}=0.25$$

CLASS ILLUSTRATION

$\boxed{\text{M-1}}$ ✓ Forcing by Parts

[Ans. C]

$$\int \frac{x^2}{(x\sin x+\cos x)^2}\,dx =$$

(A) $\dfrac{\sin x+\cos x}{x\sin x+\cos x}$

(B) $\dfrac{x\sin x-\cos x}{x\sin x+\cos x}$

(C) $\dfrac{\sin x-x\cos x}{x\sin x+\cos x}$ ✓

(D) None of these

$$\int \underbrace{\frac{x}{\cos x}}_{\text{I}}\;\underbrace{\frac{x\cos x}{(x\sin x+\cos x)^2}}_{\text{II}}\,dx$$

$$\frac{d}{dx}(x\sin x+\cos x)$$

$$x\cos x+\cancel{\sin x}-\cancel{\sin x}$$

$$x\cos x$$

$$-\frac{x}{\cos x}\,\frac{1}{(x\sin x+\cos x)}+\int\frac{(\cos x+x\sin x)}{\cos^2 x}\,\frac{1}{(x\sin x+\cos x)}\,dx$$

$$\frac{-x}{\cos x[x\sin x+\cos x]}+\tan x+C$$

$$\frac{-x}{\cos x[x\sin x+\cos x]}+\frac{\sin x}{\cos x}+C$$

$$\frac{-x+\sin x(x\sin x+\cos x)}{\cos x(x\sin x+\cos x)}+C$$

$$-\frac{\underbrace{x + x\sin^2 x} + \sin x\cos x}{\cos x\,(x\sin x + \cos x)} + C$$

$$\frac{-x\cos^{\cancel{2}} x + \sin x\,\cancel{\cos x}}{\cancel{\cos} x\,(x\sin x + \cos x)} + C$$

$$\frac{-x\cos x + \sin x}{x\sin x + \cos x} + C$$

$\boxed{M-2}$ By Substitution

$x = \tan\theta$

$$\int \frac{x^2\,dx}{(x\sin x + \cos x)^2}$$

$$\int \frac{\tan^2\theta\,\sec^2\theta\,d\theta}{\left[\frac{\sin\theta}{\cos\theta}\sin(\tan\theta) + \cos(\tan\theta)\right]^2}$$

$$\int \frac{\tan^2\theta\,\sec^{\cancel{2}}\theta\,\cancel{\cos^2}\theta\,d\theta}{\left[\sin\theta\sin(\tan\theta) + \cos\theta\cos(\tan\theta)\right]^2}$$

$$\int \frac{\tan^2\theta\,d\theta}{\cos^2(\tan\theta - \theta)}$$

$\tan\theta - \theta = z$

$(\sec^2\theta - 1)\,d\theta = dz$

$\tan^2\theta\,d\theta = dz$

$$\int \frac{dz}{\cos^2 z} = \tan z + C$$

$$\tan(\tan\theta - \theta) + C$$

$$\tan(\tan\theta - \theta) + C$$

$$\frac{\tan(\tan\theta) - \tan\theta}{1 + \tan(\tan\theta)\tan\theta} + C$$

$$\frac{\tan x - x}{1 + x\tan x} + C$$

$$\frac{\sin x - x\cos x}{\cos x + x\sin x} + C$$

Brain Teaser-06

$\int \frac{(1+x^2)(2+x^2)}{(x\cos x+\sin x)^4}dx$ equals

[Ans. B, C]

(A) $\tan(x+\cot^{-1}x)+\frac{\tan^3(x+\cot^{-1}x)}{3}+c$

(B) $\tan(x-\cot^{-1}x)+\frac{\tan^3(x-\cot^{-1}x)}{3}+c$

(C) $-\frac{1}{3}\left(\frac{1-x\tan x}{x+\tan x}\right)^3-\left(\frac{1-x\tan x}{x+\tan x}\right)+c$

(D) $-\frac{1}{3}\left(\frac{1+x\tan x}{x-\tan x}\right)^3-\left(\frac{1+x\tan x}{x-\tan x}\right)+c$

Let $x=\cot\theta$

$-\int \frac{(1+\cot^2\theta)(2+\cot^2\theta)\cdot \operatorname{cosec}^2\theta\, d\theta}{\left[\frac{\cos\theta}{\sin\theta}\cos(\cot\theta)+\sin(\cot\theta)\right]^4}$

$-\int \frac{\cancel{\operatorname{cosec}^4\theta}\,(2+\cot^2\theta)\,\cancel{\sin^4\theta}\, d\theta}{[\cos\theta\cos(\cot\theta)+\sin\theta\sin(\cot\theta)]^4}$

$-\int \frac{(1+\operatorname{cosec}^2\theta)\,d\theta}{[\cos(\theta-\cot\theta)]^4}$

$\theta-\cot\theta=z$

$(1+\operatorname{cosec}^2\theta)d\theta=dz$

$-\int\frac{dz}{\cos^4 z}=-\int\sec^2 z\sec^2 z\,dz$

$-\int(1+\tan^2 z)\sec^2 z\,dz$

$\tan z=\alpha$

$-\int(1+\alpha^2)d\alpha$

$=-\alpha-\alpha^3/3+C$

$$-\alpha-\alpha^3/3+c$$

$$-\tan z-\frac{\tan^3 z}{3}+c$$

$$\tan(\underline{\cot}\,\theta-\theta)+\frac{\tan^3(\cot\theta-\theta)}{3}+c$$

$$\tan(x-\cot^{-1}x)+\frac{\tan^3(x-\cot^{-1}x)}{3}+c$$

Brain Teaser-07

If $I = \int \frac{x^2+20}{(x\sin x + 5\cos x)^2}\,dx$, then I equals:

[Ans. A]

(A) $-\frac{x}{\cos x(x\sin x + 5\cos x)} + \tan x + c$ ✓

(B) $\frac{x}{\sin x(x\sin x + 5\cos x)} + \cot x + c$

(C) $(x\sin x - 5\cos x)^{-1}\sin x + 7x + c$

(D) None of these

Forcing by Parts

$\boxed{\text{M-1}}$ $\int \frac{(x^2+20)x^8}{(x^5\sin x + 5x^4\cos x)^2}\,dx$

$\int \underbrace{\frac{x^5}{\cos x}}_{\text{I}} \underbrace{\frac{x^3(x^2+20)\cos x}{(x^5\sin x + 5x^4\cos x)^2}}_{\text{II}}\,dx$

$-\frac{x^5}{\cos x}\frac{1}{(x^5\sin x + 5x^4\cos x)} + \int \frac{(\cos x \cdot 5x^4 + x^5\sin x)}{\cos^2 x}\frac{1}{(x^5\sin x + 5x^4\cos x)}\,dx$

$-\frac{x}{\cos x[x\sin x + 5\cos x]} + \tan x + c$

$$\frac{d}{dx}\left(x^5 \sin x + 5x^4 \cos x\right)$$

$$x^5 \cos x + \cancel{5x^4 \sin x} - \cancel{5x^4 \sin x} + 20x^3 \cos x$$

$$x^3 \cos x\left(x^2 + 20\right)$$

$\boxed{M-2}$ $x = 5\tan\theta$

$$Dr \rightarrow \left[5\tan\theta \sin(5\tan\theta) + 5\cos(5\tan\theta)\right]^2$$

$$= 5^2\left[\frac{\sin\theta}{\cos\theta}\sin(5\tan\theta) + \cos(5\tan\theta)\right]^2$$

$$D^r \rightarrow \left(5\,\frac{\cos(5\tan\theta - \theta)}{\cos\theta}\right)^2$$

Brain Teaser-08

[Ans. 36]

If $\int x^{26} \cdot (x-1)^{17} \cdot (\underbrace{5x-}3) dx = \frac{x^{m} \cdot (x-1)^{18}}{k} + C$ where

C is a constant of integration, then the value of $m + k$ is equal to ______.

$$\int x^{26} \overbrace{(x-1)^{17} ((3x-3)}+2x)\, dx$$

$$\int 3 \underbrace{x^{26}}_{\text{II}} \underbrace{(x-1)^{18}}_{\text{I}} + \int 2x^{27}(x-1)^{17}\, dx$$

$$3(x-1)^{18}\frac{x^{27}}{27} - \int \cancel{18}^{2}(x-1)^{17}\cdot\frac{\cancel{3}x^{27}}{\cancel{27}_{\cancel{9}}}\,dx + \int 2x^{27}(x-1)^{17}\,dx$$

$$\frac{(x-1)^{18}x^{27}}{9} - \cancel{\int 2(x-1)^{17}x^{27}\,dx} + \cancel{\int 2x^{27}(x-1)^{17}\,dx}$$

$m = 27$

$K = 9$

$m + K = 36.$

Brain Teaser-09

Find the value of $\int \frac{x^n}{1+\frac{x}{1!}+\frac{x^2}{2!}+\cdots+\frac{x^n}{n!}}dx$

Ans.

Brain Teaser-10

[Ans. 2]

Let $A(\theta) = \int \sin(\theta + \sin\theta)e^{\cos\theta}\,d\theta$ & $B(\theta) = \int \cos(\theta + \sin\theta)e^{\cos\theta}\,d\theta$.

where $A(0) = -1$ & $B(0) = 0$. Find the value of $\frac{\ln\left((A(\theta))^2 + (B(\theta))^2\right)}{\cos\theta}$.

$$A(\theta) = \int \left(\sin\theta\cos(\sin\theta) + \cos\theta\sin(\sin\theta)\right)e^{\cos\theta}\,d\theta$$

$$\int \underbrace{e^{\cos\theta}\sin\theta}_{\mathrm{II}}\ \underbrace{\cos(\sin\theta)}_{\mathrm{I}}\,d\theta + \int \cos\theta\, e^{\cos\theta}\sin(\sin\theta)\,d\theta$$

$$-\cos(\sin\theta)\cdot e^{\cos\theta} - \int \sin(\sin\theta)\cos\theta\cdot e^{\cos\theta}\,d\theta$$

$$A(\theta) = -\cos(\sin\theta)e^{\cos\theta}$$

$$B(\theta) = \int \underbrace{\cos\theta\cos(\sin\theta)}_{\mathrm{II}}\ \underbrace{e^{\cos\theta}}_{\mathrm{I}} - \int \sin\theta\sin(\sin\theta)e^{\cos\theta}\,d\theta$$

$$B(\theta) = e^{\cos\theta} \cdot \sin(\sin\theta) + \int e^{\cos\theta}(\sin\theta) \cdot \sin(\sin\theta)$$

$$B(\theta) = e^{\cos\theta} \sin(\sin\theta)$$

$$(A(\theta))^2 + (B(\theta))^2 = e^{2\cos\theta}\left[\sin^2(\sin\theta) + \cos^2(\sin\theta)\right]$$

$$= e^{2\cos\theta}$$

$$\ln\left((A(\theta))^2 + (B(\theta))^2\right) = 2\cos\theta$$

Brain Teaser-10

$\cos\alpha + i\sin\alpha = e^{i\alpha}$

[Ans. 2]

Let $A(\theta) = \int \sin(\theta + \sin\theta)e^{\cos\theta}\,d\theta$ & $B(\theta) = \int \cos(\theta + \sin\theta)e^{\cos\theta}\,d\theta$.

where $A(0) = -1$ & $B(0) = 0$. Find the value of $\frac{\ln\left((A(\theta))^2+(B(\theta))^2\right)}{\cos\theta}$.

Consider

$$B(\theta) + iA(\theta) = \int\left(\cos(\theta+\sin\theta)e^{\cos\theta} + i\sin(\theta+\sin\theta)e^{\cos\theta}\right)d\theta$$

$$\int\left(\underbrace{\cos(\theta+\sin\theta) + i\sin(\theta+\sin\theta)}\right)e^{\cos\theta}\,d\theta$$

$$\int e^{i(\theta+\sin\theta)} \cdot e^{\cos\theta}\,d\theta$$

$$\int e^{i\theta} \cdot e^{i\sin\theta} e^{\cos\theta}\,d\theta$$

$$= \int e^{i\theta} e^{\cos\theta + i\sin\theta}\,d\theta$$

$$B(\theta) + iA(\theta) = \int \underbrace{e^{i\theta}\, e^{e^{i\theta}}\, d\theta}$$

$$e^{i\theta} = \alpha$$

$$e^{i\theta} \cdot i\, d\theta = d\alpha$$

$$e^{i\theta} d\theta = \frac{d\alpha}{i}$$

$$\frac{1}{i} \int e^{\alpha}\, d\alpha$$

$$\frac{1}{i} e^{\alpha} + c$$

$$\frac{1}{i} e^{(\cos\theta + i\sin\theta)} + c$$

$$e^{\cos\theta} \cdot \frac{1}{i} \cdot e^{i\sin\theta} + c$$

$$-i\, e^{\cos\theta}\left[\cos(\sin\theta) + i\sin(\sin\theta)\right] + c$$

$$B(\theta) + iA(\theta) = e^{\cos\theta}\left[-i\cos(\sin\theta) + \sin(\sin\theta)\right] + c$$

$$B(\theta) = e^{\cos\theta} \sin(\sin\theta)$$

$$A(\theta) = -e^{\cos\theta} \cos(\sin\theta)$$

direct mod

$$\sqrt{B^2(\theta) + A^2(\theta)} = e^{\cos\theta}$$

$$B^2(\theta) + A^2(\theta) = e^{2\cos\theta}$$

Brain Teaser-11

[Ans. C]

If $y = \cfrac{x}{x^2 + \cfrac{x}{x^2 + \cfrac{x}{x^2 + \dots\infty}}}$ and $\int \frac{(y-x^2)dx}{(x^2+y)(x+y^2)} = \underbrace{f(y)}_{} + c$, then $\frac{d}{dy}(f(y))$ for $x = 1$ is equal to:

(A) 1

(B) $\frac{\sqrt{5}-1}{2}$

(C) $\frac{\sqrt{5}+1}{2}$ ✓

(D) $\frac{2-\sqrt{5}}{2}$

$y = \frac{x}{x^2+y}$

$x^2y + y^2 = x$

$y^2 = x - x^2y$

$\frac{(y-x^2)}{(x^2+y)(x+x-x^2y)}$

$\frac{y-x^2}{x(x^2+y)(2-xy)}$

$\frac{(y-x^2)}{x[2x^2+2y-x^3y-xy^2]}$

$\frac{y-x^2}{x[2x^2+2y-x^3y-x(x-x^2y)]}$

$\frac{y-x^2}{x[x^2+2y-\cancel{x^3}y+\cancel{x^3}y]}$

$= \int \frac{(y-x^2)}{x(x^2+2y)}dx$

$y^2 = x - x^2y$

$2y\,dy = dx - x^2dy - 2xy\,dx$

$(2y+x^2)\,dy = dx(1-2xy)$

$dx = \frac{(2y+x^2)dy}{1-2xy}$

$$\int \frac{(y-x^2)\,dx}{x(x^2+2y)}$$

$$dx = \frac{(2y+x^2)}{1-2xy}\,dy$$

$$\frac{dx}{2y+x^2} = \frac{dy}{1-2xy}$$

$$\int \frac{(y-x^2)}{x(1-2xy)}\,dy$$

$$\frac{(y-x^2)\,dy}{x-2x^2y}$$

$$\int \frac{(y-x^2)\,dy}{x-2(x-y^2)} = \int \frac{y-x^2}{2y^2-x}\,dy = \int \frac{y-x^2}{2y^2-(y^2+x^2y)}\,dy = \int \frac{\cancel{y-x^2}}{y\left[\cancel{y-x^2}\right]}\,dy$$

$$= \int \frac{dy}{y} = \ln y + c$$

$$f(y) = \ln y$$

$$f'(y) = 1/y$$

$$= \frac{2}{\sqrt{5}-1} \cdot \frac{(\sqrt{5}+1)}{\sqrt{5}+1}$$

$$\frac{2(\sqrt{5}+1)}{4} = \frac{\sqrt{5}+1}{2}$$

$$x = 1$$

$$\Rightarrow y^2 + y = 1$$

$$y^2 + y - 1 = 0$$

$$y = \frac{-1 \pm \sqrt{5}}{2}$$

$$y = \frac{-1+\sqrt{5}}{2}$$

Brain Teaser-11

M-2

[Ans. C]

If $y = \frac{x}{x^2 + \frac{x}{x^2 + \frac{x}{x^2 + \ldots\infty}}}$ and $\int \frac{(y - x^2)dx}{(x^2+y)(x+y^2)} = f(y) + c$, then $\frac{d}{dy}(f(y))$ for $x = 1$ is equal to:

(A) 1

(B) $\frac{\sqrt{5}-1}{2}$

(C) $\frac{\sqrt{5}+1}{2}$ ✓

(D) $\frac{2-\sqrt{5}}{2}$

$y = \frac{x}{x^2+y}$

$x^2y + y^2 = x$

$y^2 = x - x^2y$

$\frac{y-x^2}{(x^2+y)(x+y^2)} = \frac{d}{dx}f(y)$

$\frac{d}{dy}f(y) \cdot \frac{dy}{dx}$

$\frac{y-x^2}{(x^2+y)(x+y^2)} = f'(y) \cdot \frac{1-2xy}{(2y+x^2)}$

$1 - 2xy$

$x = 1$

$\frac{y-1}{(1+y)(y^2+1)} = f'(y)\frac{(1-2y)}{(2y+1)}$

get $f'(y) =$ ✓

Brain Teaser-12

[Ans. D]

$I = \int \frac{x^2 dx}{(x^2-4)\sin x + 4x\cos x}$ and $y = 2\cot^{-1}(x/2) + x$, then $I =$

(A) $\ln(\operatorname{cosec} 2y + \cot y) + c$

(B) $\ln(\operatorname{cosec} 2y - \cot 2y) + c$

(C) $\ln(\operatorname{cosec} y + \cot y) + c$

(D) $\ln(\operatorname{cosec} y - \cot y) + c$ ✓

$dy = \left(-\cancel{2}\frac{1}{1+x^2/4}\cdot\frac{1}{\cancel{2}} + 1\right)dx$

$dy = \left(-\frac{4}{x^2+4} + 1\right)dx$

$dy = \frac{(-\cancel{4}+x^2+\cancel{4})\,dx}{x^2+4}$

$dy = \frac{x^2 dx}{(x^2+4)}$

$\int \frac{(x^2+4)\,dy}{(x^2-4)\sin x + 4x\cos x}$

$y - x = 2\cot^{-1}\left(\frac{x}{2}\right)$

$y - x = 2\left(\pi/2 - \tan^{-1} x/2\right)$

$y - x = \pi - 2\tan^{-1} x/2$

$y - x = \pi - \tan^{-1}\left(\frac{2x/2}{1 - x^2/4}\right)$

$y - x = \pi - \tan^{-1}\left(\frac{4x}{4-x^2}\right)$

$\tan(y-x) = -\tan\tan^{-1}\left(\frac{4x}{4-x^2}\right)$

$\tan(y-x) = \frac{4x}{x^2-4}$

$\frac{\tan y - \tan x}{1 + \tan y \tan x} = \frac{4x}{x^2-4}$

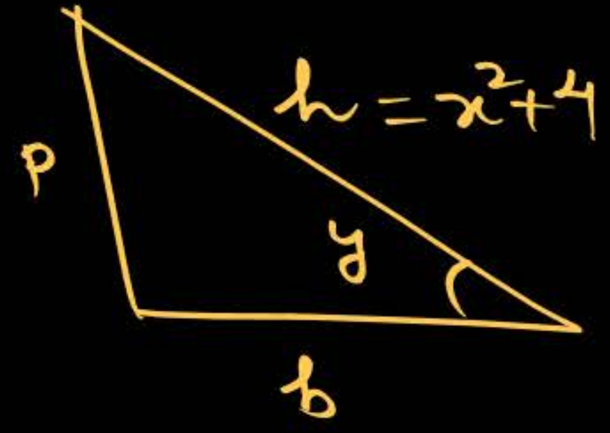
$$(\tan y - \tan x)(x^2-4) = 4x(1+\tan x \tan y)$$

$$(x^2-4)\tan y - (x^2-4)\tan x = 4x + 4x \tan x \tan y$$

$$(x^2-4-4x\tan x)\tan y = 4x + (x^2-4)\tan x$$

$$\left((x^2-4)\cos x - 4x\sin x\right)\tan y = 4x\cos x + (x^2-4)\sin x$$

$$\tan y = \frac{(x^2-4)\sin x + 4x\cos x}{(x^2-4)\cos x - 4x\sin x} = p/b$$



$$h^2 = \left((x^2-4)\sin x + 4x\cos x\right)^2 + \left((x^2-4)\cos x - 4x\sin x\right)^2$$

$$(x^2-4)^2 + 16x^2 = x^4+16-8x^2+16x^2$$

$$h^2 = x^4+16+8x^2 = (x^2+4)^2 \Rightarrow h = x^2+4$$

$$\int \frac{h}{p} \cdot dy$$

$$\int \operatorname{cosec} y \, dy$$

$$= \ln(\operatorname{cosec} y - \cot y) + c$$

Definite Integration

Brain Teaser-13

[Ans. A]

$$\sum_{k=1}^{\infty}\left(\frac{1}{3k-1}-\frac{1}{3k}\right)$$ is equal to:

(A) $\log\sqrt{3}-\frac{\pi}{6\sqrt{3}}$

(B) $\log\sqrt{3}-\frac{\pi}{6}$

(C) $\log 3-\frac{\pi}{6\sqrt{3}}$

(D) $\log\sqrt{3}-\frac{\pi}{\sqrt{3}}$

$$\int_0^1 x^{3K-2}\,dx$$

$$\left.\frac{x^{3K-1}}{3K-1}\right|_0^1$$

$$\frac{1}{3K-1}$$

$$\int_0^1 x^{3K-1}\,dx$$

$$\left.\frac{x^{3K}}{3K}\right|_0^1$$

$$= 1/3K$$

$$\sum_{K=1}^{\infty}\int_0^1\left(x^{3K-2}-x^{3K-1}\right)dx$$

$$\int_0^1\sum_{K=1}^{\infty}\left(x^{3K-2}-x^{3K-1}\right)dx$$

$$\int_0^1\left[\left(x^1+x^4+x^7+\cdots\infty\right)-\left(x^2+x^5+x^8+\cdots\infty\right)\right]dx$$

$$\int_0^1 \left(\frac{x}{1-x^3} - \frac{x^2}{1-x^3}\right)dx$$

$$\boxed{x = A\,[2x+1] + B}$$

$$\int_0^1 \frac{x\,(\cancel{1-x})}{(\cancel{1-x})(1+x^2+x)}\,dx$$

$$\int_0^1 \frac{x\,dx}{x^2+x+1}$$

Brain Teaser-14

HW

[Ans. A]

The value of $\int_0^\infty \frac{(x^2+4)\ln x dx}{(x^4+16)}$ is equal to:

A $\frac{\pi}{2\sqrt{2}}\ln 2$

B $\frac{\pi}{2}\ln 2$

C $\pi \ln 2$

D $\frac{\pi}{\sqrt{2}}\ln 2$

Brain Teaser-15

[Ans. B]

The value of $\int_0^2 \frac{x^4+1}{(x^4+2)^{3/4}} dx$, is:

A $\sqrt{2\sqrt{2}}$

B $\sqrt{3\sqrt{2}}$

C $\sqrt{2\sqrt{3}}$

D $\sqrt{3\sqrt{3}}$

$$\frac{x^3(x^4+1)}{x^3(x^4+2)^{3/4}}$$

$$\frac{x^3(x^4+1)}{\left[x(x^4+2)^{1/4}\right]^3}$$

$$\frac{x^3(x^4+1)}{\left[x^4(x^4+2)\right]^{3/4}}$$

$$\int_0^2 \frac{(x^7+x^3)\,dx}{\left[x^8+2x^4\right]^{3/4}}$$

$$x^8+2x^4=t^4$$

$$8(x^7+x^3)dx=4t^3dt$$

Homework

Redo class Ques

Attempt L-2 B.Ts

THANK

YOU