VARUN

ADVANCE 2026

Physics

Lecture - 02

Center of Mass and Collision

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Topics to be covered

1. Center of mass
2.
3.
4.
5.

Question

System is release from rest, all surfaces are smooth. find speed of wedge, ball, Normal force on ball, wedge, V_{cm}, a_{cm} when line joining ball and center make an angle θ with horizontal

Question

Solution :→

→ $F_{ext.}$ on System in x-direction is zero (ball + wedge) $\left(\overset{\uparrow}{\underset{mg}{\downarrow}} \quad \underset{Mg}{\downarrow} \quad \uparrow N_2 \right)$

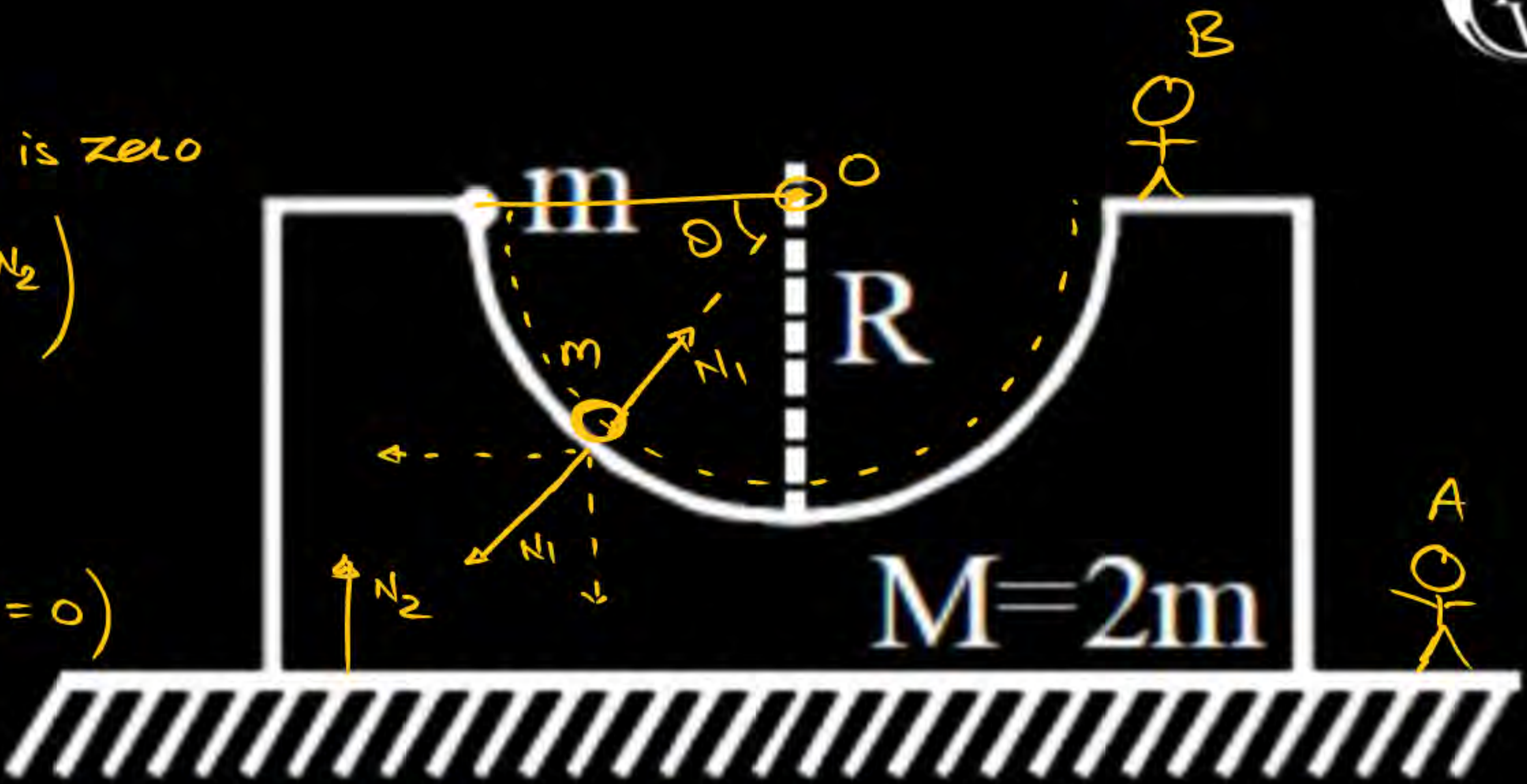
$$\vec{P}_i = \vec{P}_f \text{ in x-dir}^n$$

→ Energy Conservation

$\left(W_{N_1}(\text{ball + wedge}) + W_{N_2}(\text{wedge}) = 0 \right)$

→ $\vec{F} = m\vec{a}$

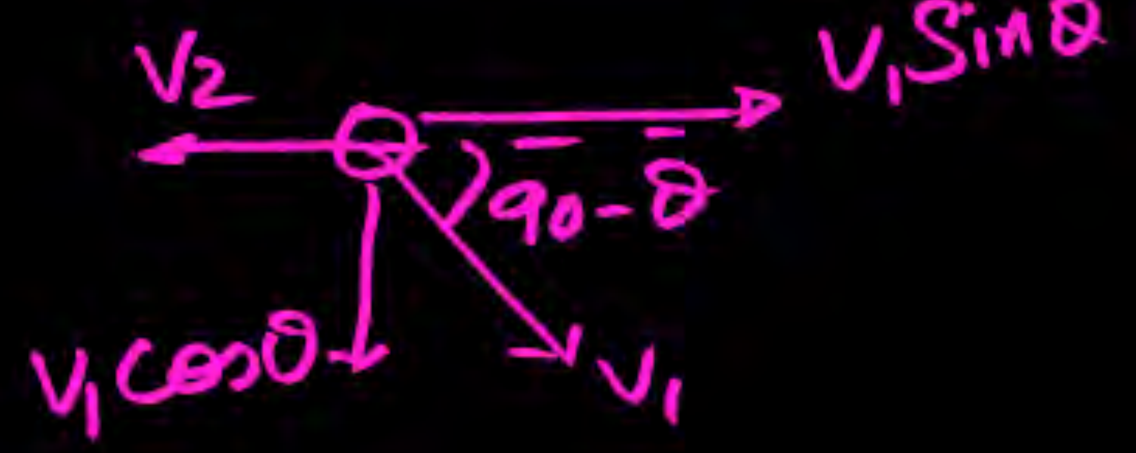
→



W.r.t wedge (observer B) → Path of ball is circular but it is not inertial frame

W.r.t Ground (observer A) → Path of ball is not circular but it is inertial

Question



$\rightarrow \vec{P_i} = \vec{P_f}$

$$0 = m(V_1\sin\theta - V_2) - 2mV_2 = 0$$

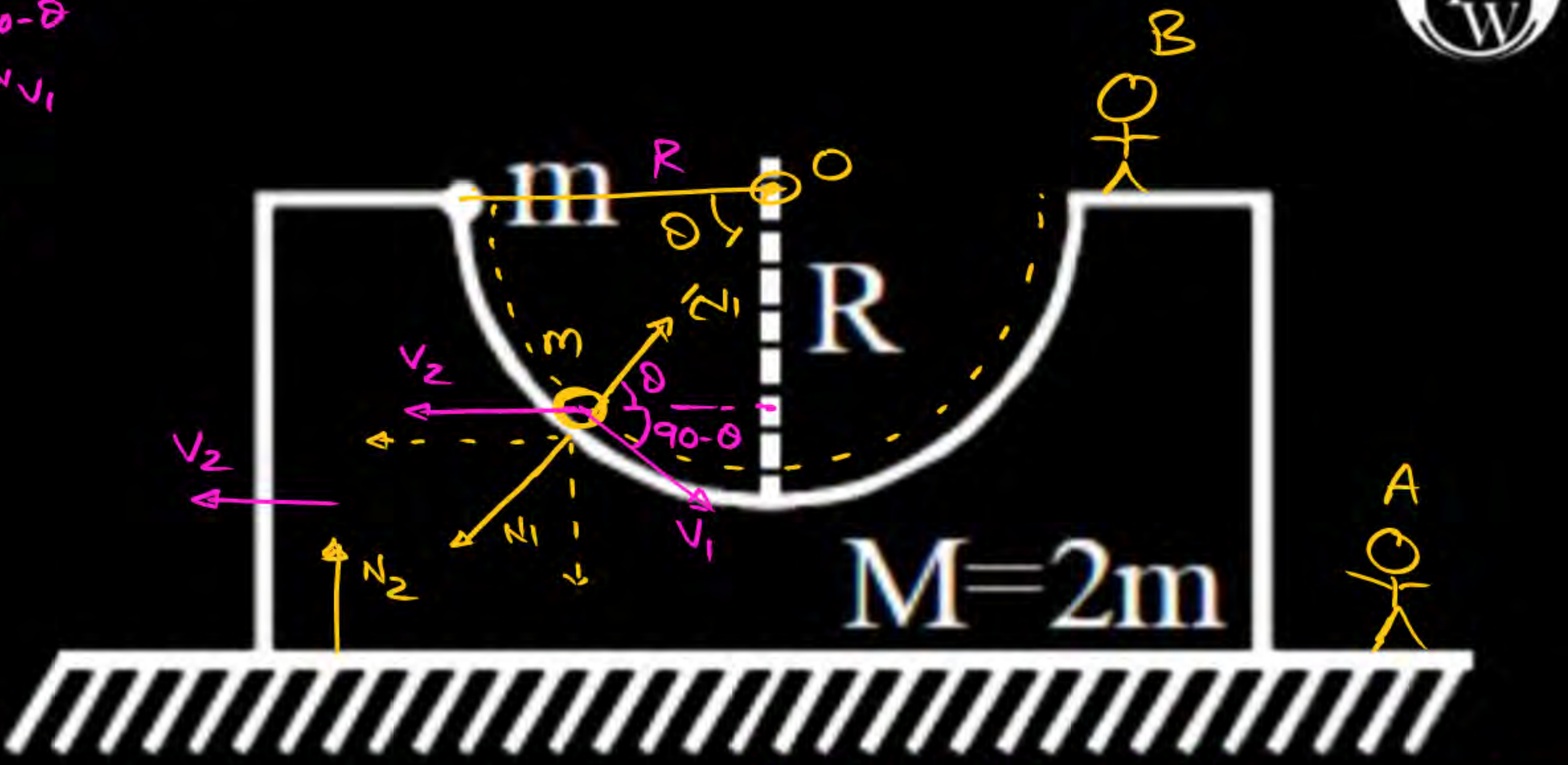
$$V_1\sin\theta = 3V_2 \quad -(i)$$

$\rightarrow K_1 + U_1 = K_2 + U_2$

$$0+0 = -mgR\sin\theta + \frac{1}{2}m\left((V_1\sin\theta - V_2)^2 + (V_1\cos\theta)^2\right) + \frac{1}{2}2mV_2^2 = 0$$

$$= 2gR\sin\theta = V_1^2 + V_2^2 - 2V_1V_2\sin\theta + 2V_2^2$$

$$= V_1^2 + 3V_2^2 - 2V_1V_2\sin\theta$$



$V_1 \rightarrow$ Velocity of ball w.r.t wedge

$$2gR\sin\theta = V_1^2 + 3\left(\frac{V_1\sin\theta}{3}\right)^2 - 2V_1\left(\frac{V_1\sin\theta}{3}\right)\sin\theta = V_1^2\left(1 + \frac{\sin^2\theta}{3} - \frac{2}{3}\sin^2\theta\right)$$

$$V_1 = \sqrt{\frac{3gR\sin\theta}{3-\sin^2\theta}}, \quad V_2 = \frac{V_1\sin\theta}{3}$$

Ans.

Speed of wedge v_2

Speed of ball $= \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \sin\theta}$

$\underline{V_{cm}}$: $\longrightarrow$ $V_{cm})_x = U_{cm})_x = 0$

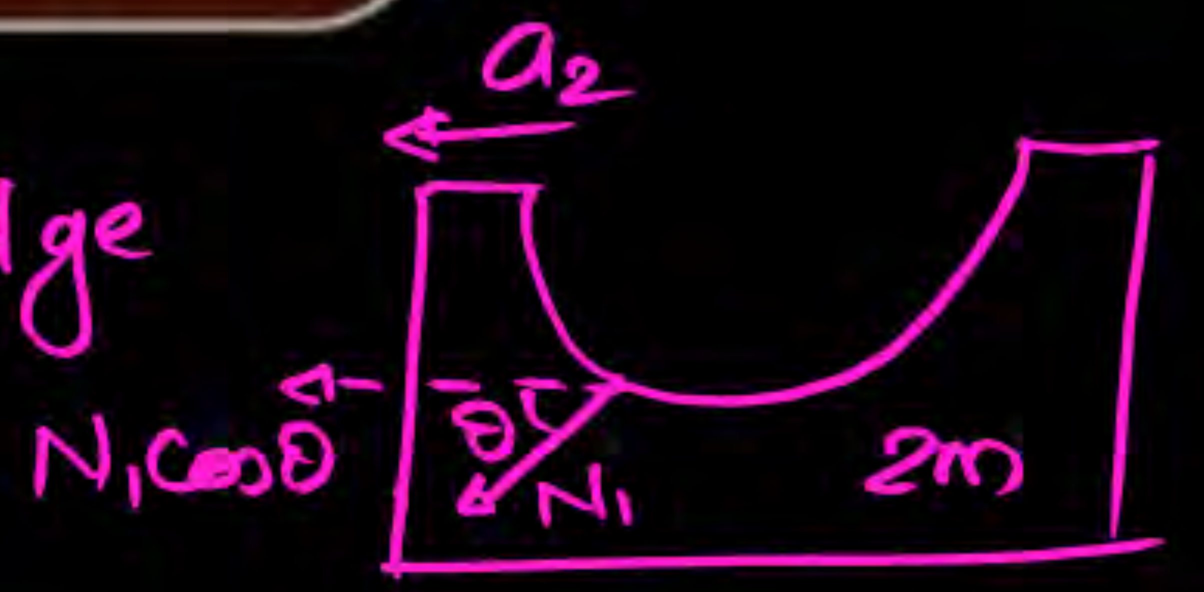
$\left(F_{ext.})_x = 0 \right)$

$$V_{cm})_y = \frac{m v_1 \cos\theta(-\hat{j}) + 2m(0)}{3m}$$

$$V_{cm})_y = \frac{v_1 \cos\theta}{3}(-\hat{j})$$

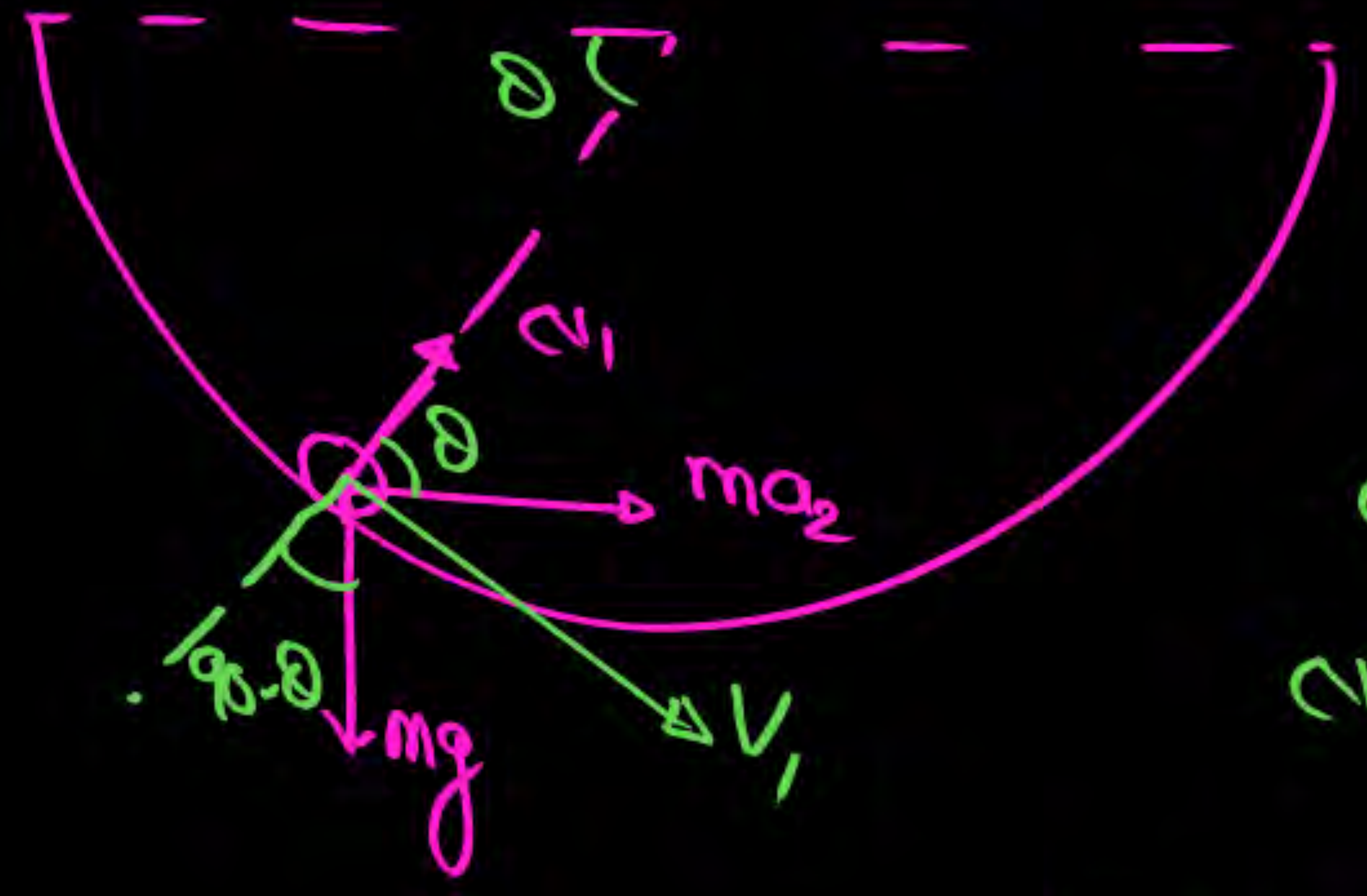
Question

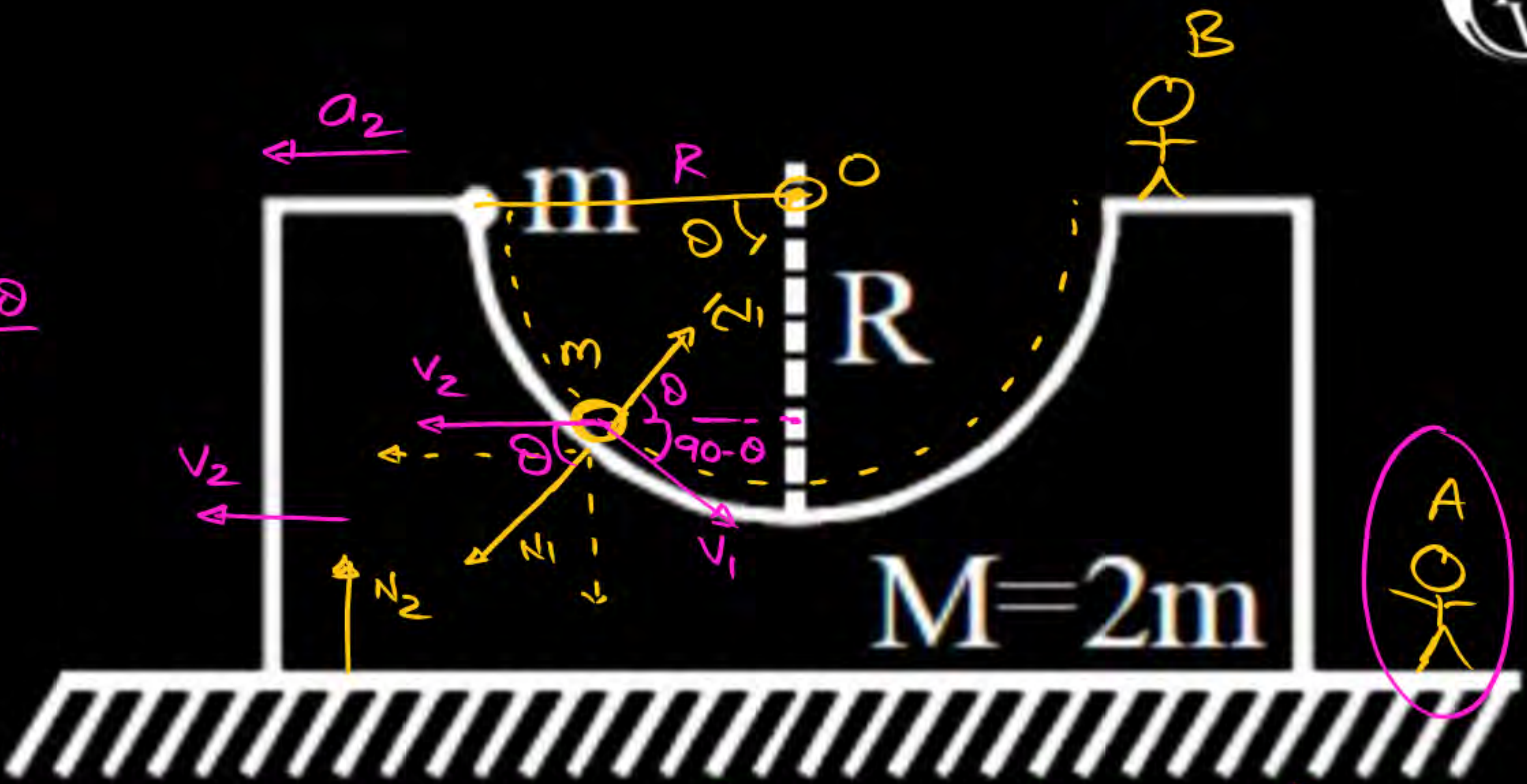
Inledge



$N_1\cos\theta = 2ma_2 \Rightarrow a_2 = \frac{N_1\cos\theta}{2m}$

W.r.t wedge, ball → Circular





$N_1 + ma_2\cos\theta - mg\sin\theta = \frac{mv_1^2}{R}$

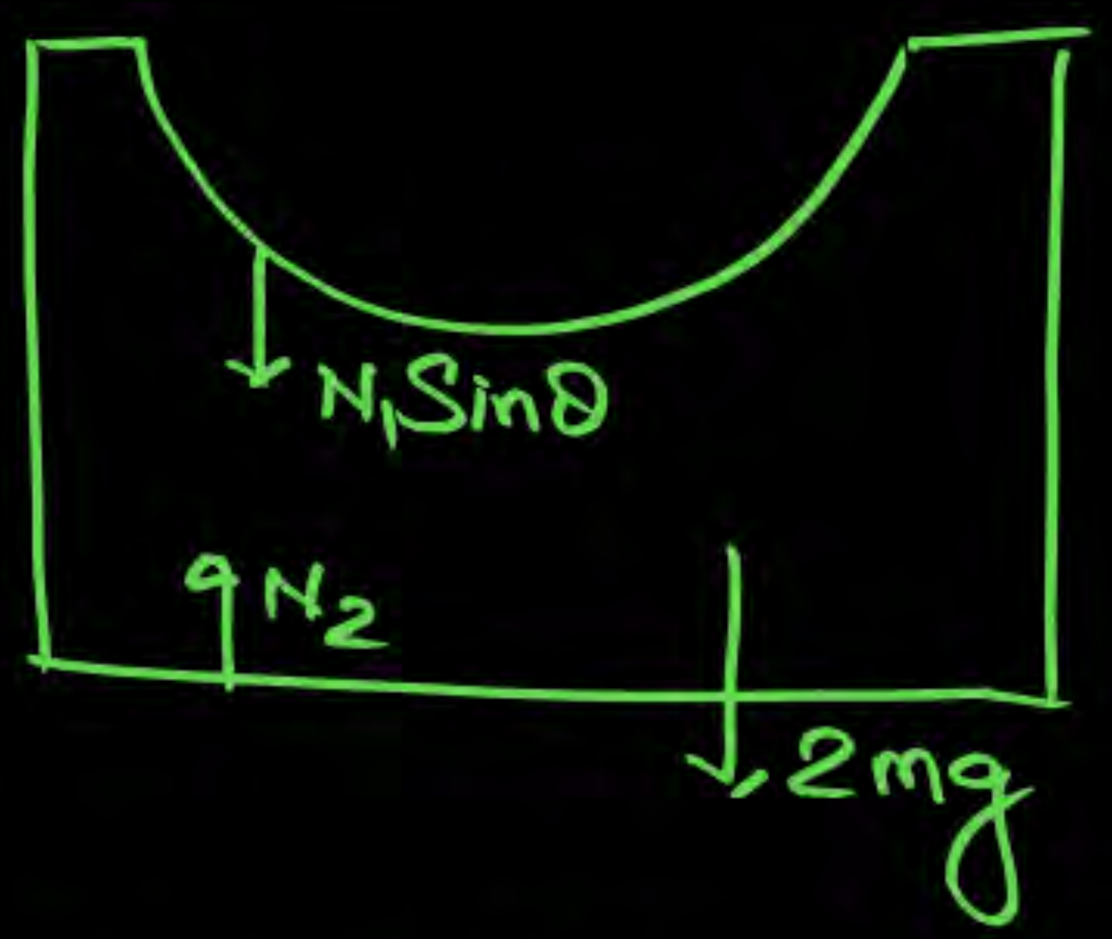
$N_1 + \frac{N_1\cos^2\theta}{2} = mg\sin\theta + m\cdot\left(\frac{3g\sin\theta}{3-\sin^2\theta}\right)$

$\frac{N_1(2+\cos^2\theta)}{2} = \frac{mg\sin\theta}{3-\sin^2\theta}\left(3-\sin^2\theta+3\right)$

$N_1 = \frac{2mg\sin\theta(6-\sin^2\theta)}{(3-\sin^2\theta)(2+\cos^2\theta)}$

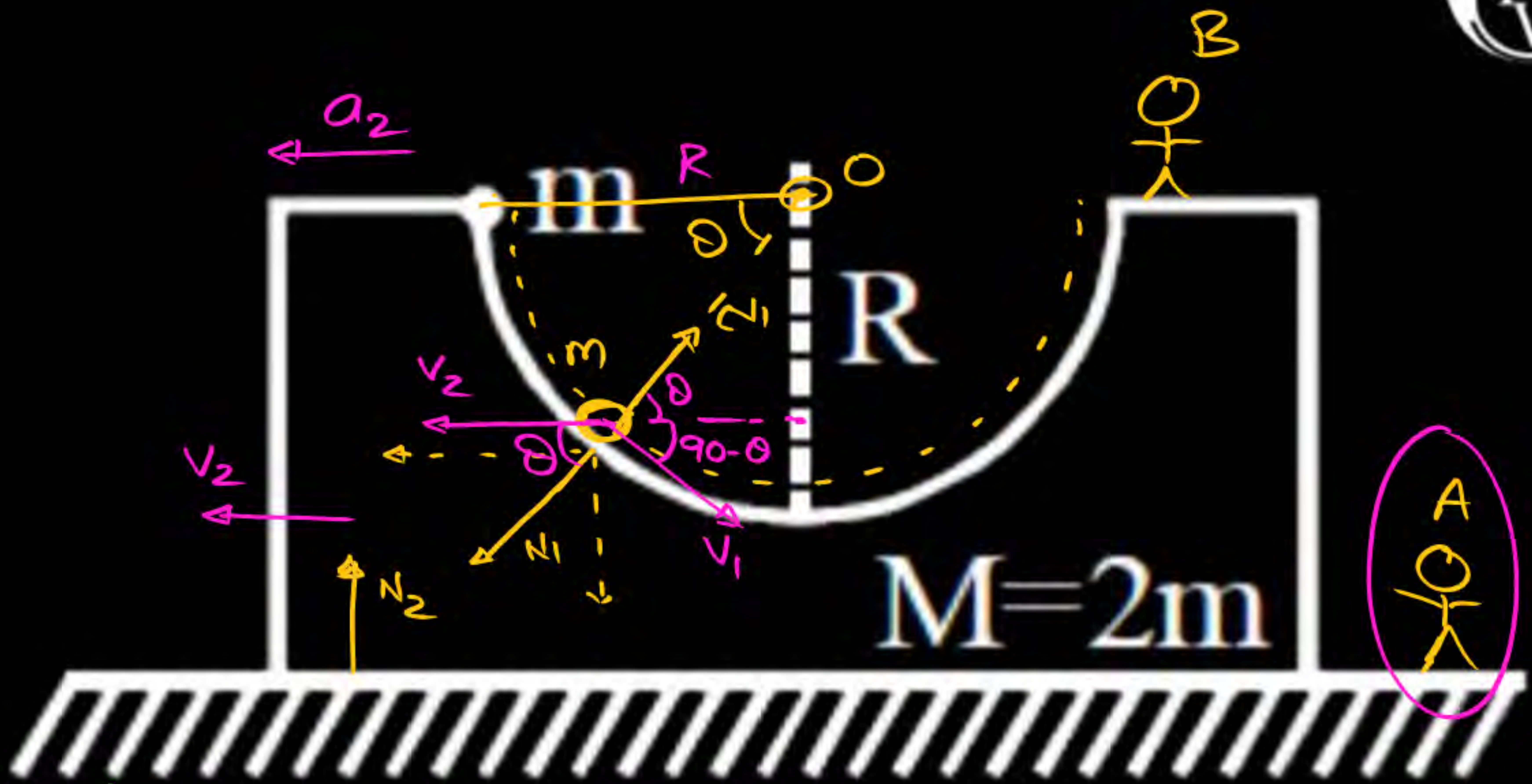
Question

N_2 :→ Wedge



$$N_2 = N_1 \sin\theta + 2mg$$

$$N_2 = 2mg + \frac{2mg \sin^2\theta\,(6 - \sin^2\theta)}{(3 - \sin^2\theta)(2 + \cos^2\theta)}$$



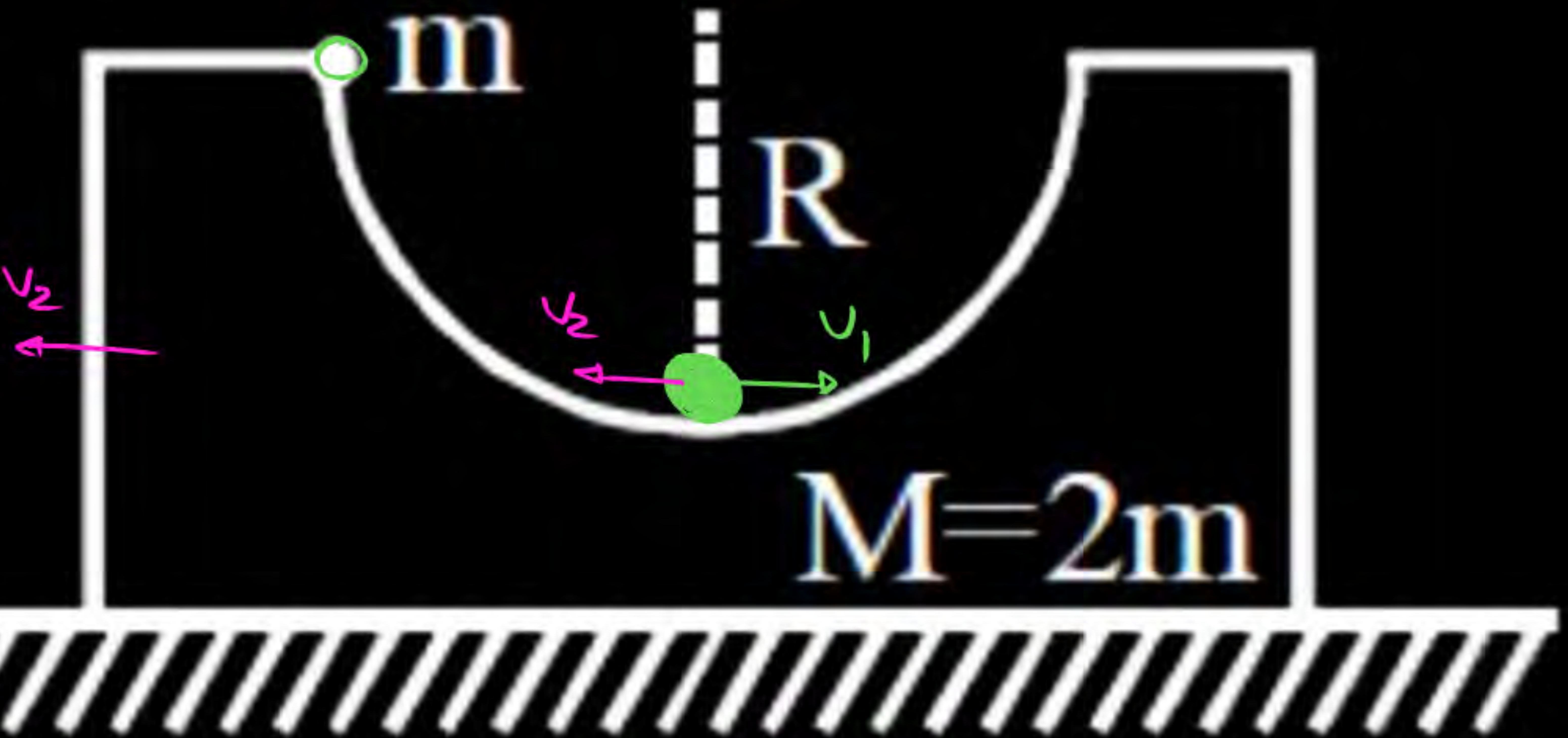
$$a_{cm} = \frac{F_{ext.})_y}{3m} \quad \text{on System}$$

$$a_{cm} = \frac{(N_2 - 3mg)\,\hat{j}}{3m}$$

Question

Wedge of mass $M = 2$ m has hemispherical groove of radius as $R = 9$ cm as shown in the figure. A particle of mass m kept at upper extreme of groove and released from rest. Find the radius of curvature in cm of particle, when it reaches its lowermost point.

$v_1 \rightarrow$ w.r.t wedge



Sol^n $\rightarrow$ $P_i = P_f$ in x-dir^n

$$0 = m(v_1 - v_2) - 2mv_2 = 0$$

$$v_2 = v_1/3$$

$\rightarrow$ $k_1 + U_1 = k_2 + U_2$

$$0 + 0 = \frac{1}{2}m(v_1 - v_2)^2 + \frac{1}{2}2mv_2^2 - mgR$$

$$2gR = \frac{4v_1^2}{9} + 2 \cdot \frac{v_1^2}{9} = \frac{2v_1^2}{3}$$

$$v_1 = \sqrt{3gR}, \quad v_2 = \sqrt{\frac{gR}{3}}$$

$$\left(\begin{array}{l} k_1 + U_1 = k_2 + U_2 \\ 0 + 0 = \frac{1}{2}\left(\frac{2m \times m}{2m + m}\right)v_1^2 - mgR = 0 \quad \Rightarrow v_1 = \sqrt{3gR} \end{array} \right)$$

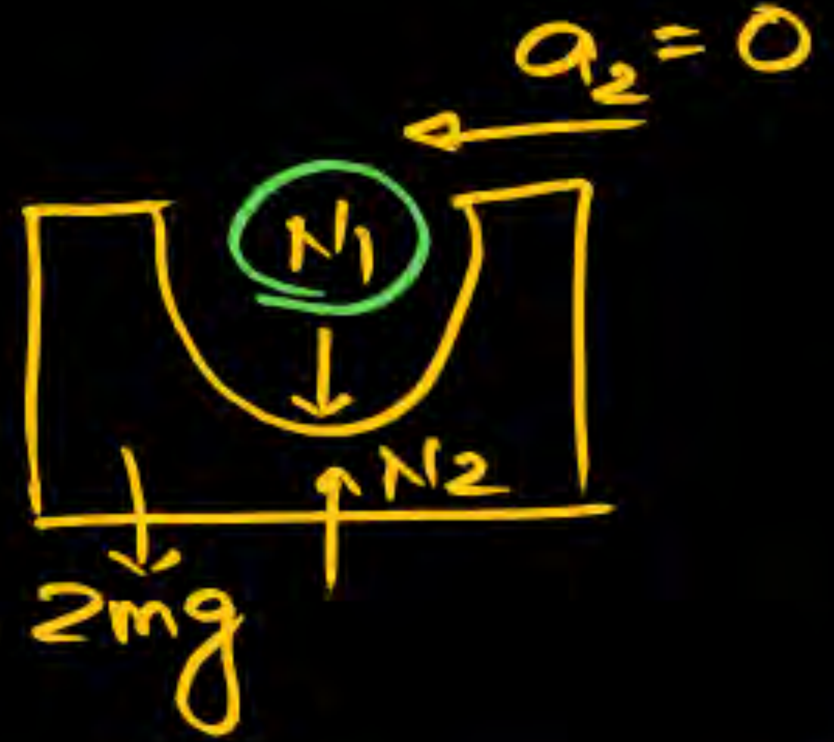
Ans. 4

V_{cm} :→ $V_{cm})_x = u_{cm})_x = 0$

$$V_{cm})_y = \frac{m(0) + 2m(0)}{3m} = 0$$

$$V_{cm} = 0$$

a_{cm} :→ w.r.t wedge ($a_2 = 0$)

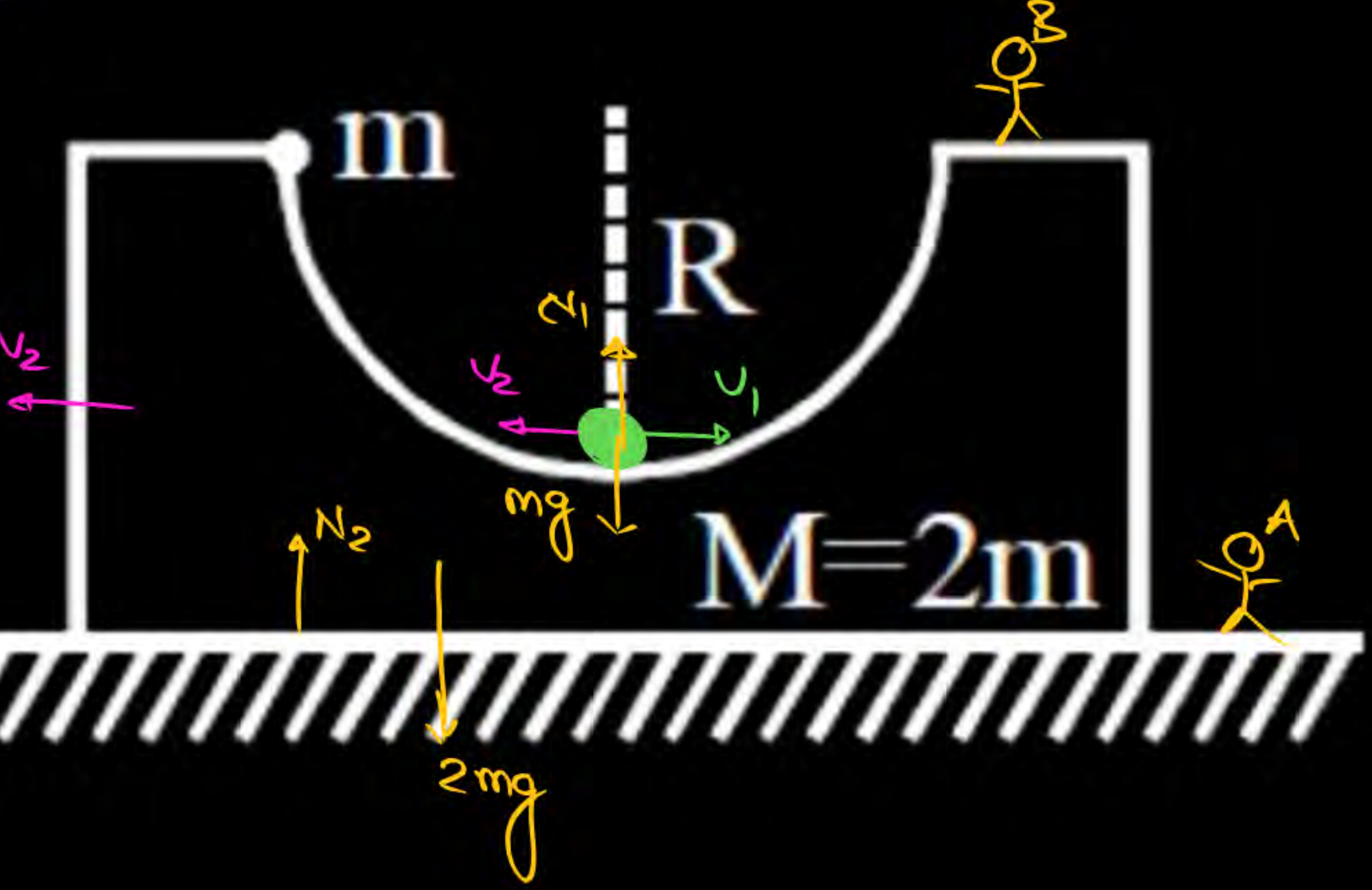


$$N_1 - mg = \frac{mv_1^2}{R} = 3mg$$

$$N_1 = 4mg$$

$$N_1 + 2mg = N_2 \Rightarrow N_2 = 6mg$$

$$a_{cm})_y = \frac{N_2 - 3mg}{3m} = g \uparrow$$



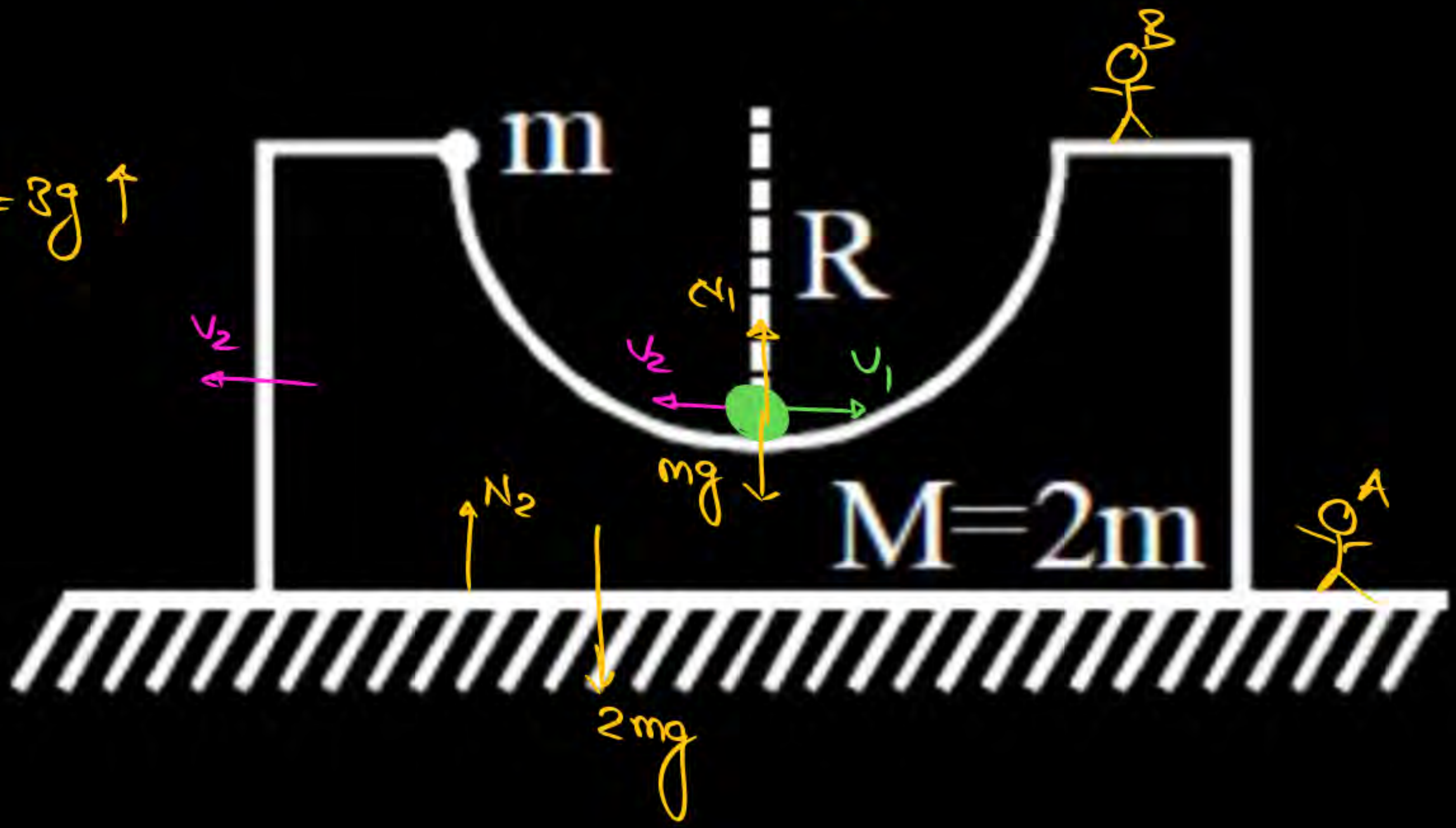
Ans. 4

ROC w.r.t Ground

$$ROC = \frac{v^2}{a_\perp} = \frac{(v_1 - v_2)^2}{3g} = \frac{\left(v_1 - \frac{v_1}{3}\right)^2}{3g} = \frac{4v_1^2}{9(3g)} = \frac{4(3gR)}{27g}$$

$$= \frac{4R}{9}$$

$$a_\perp = \frac{N_1 - mg}{m} = \frac{4mg - mg}{m} = 3g \uparrow$$



Ans. 4

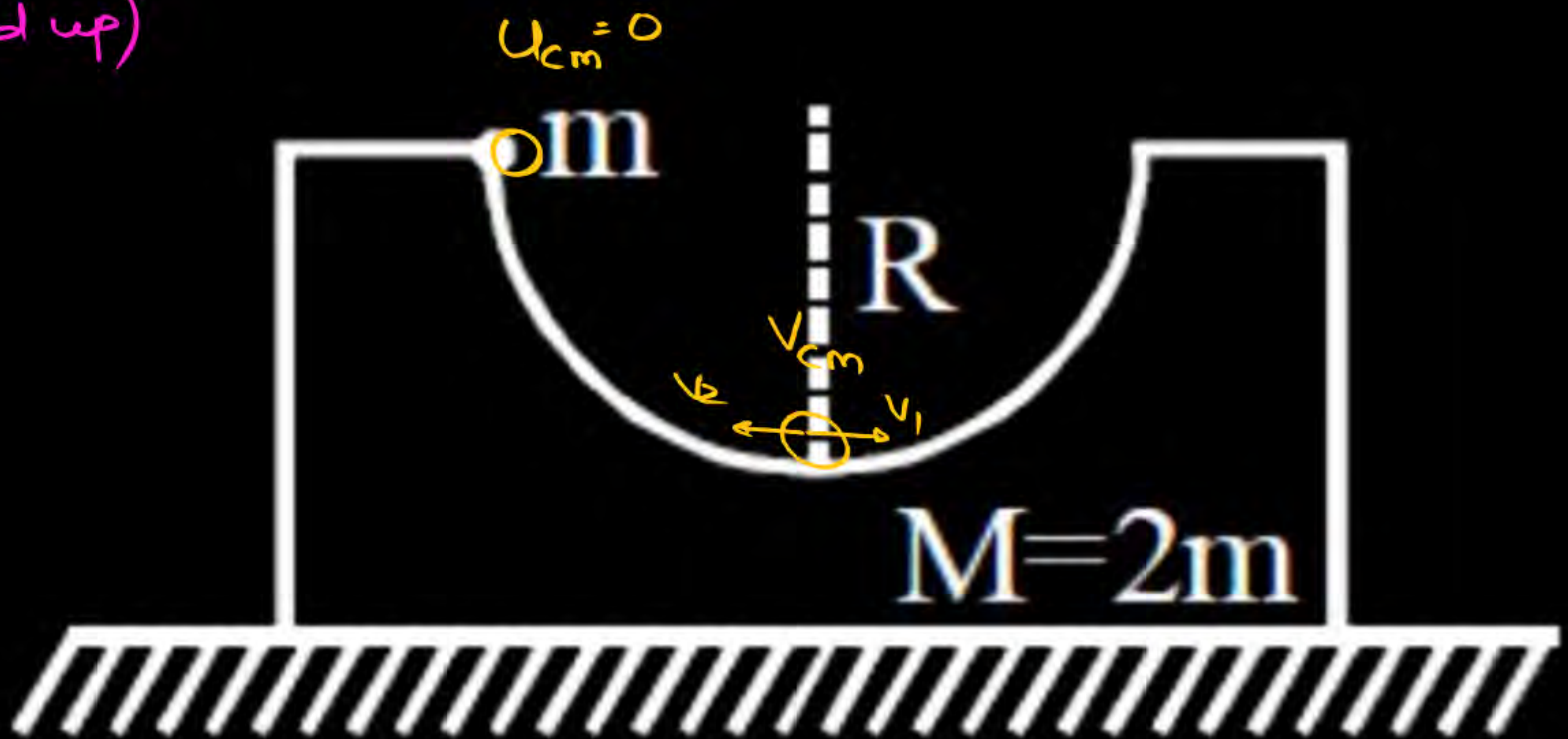
COM path → St. line

$u_{cm} = 0$

→ $a_{cm} \downarrow$ (Speed up)

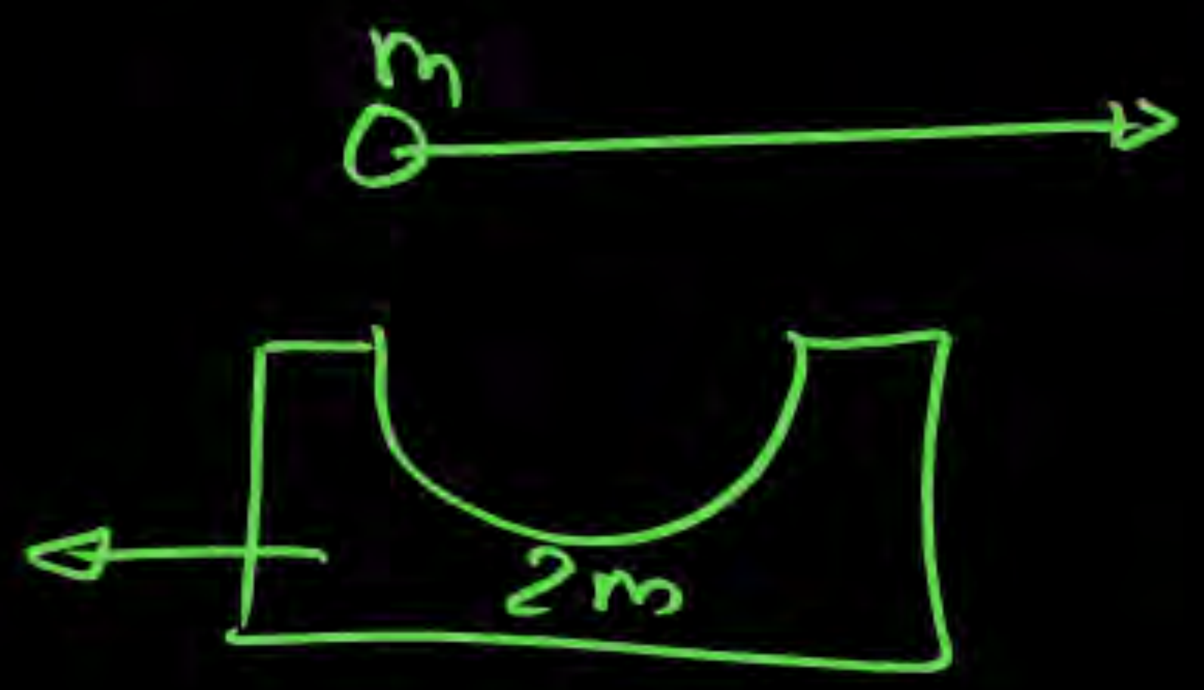
→ $a_{cm} \uparrow$ (Speed up)

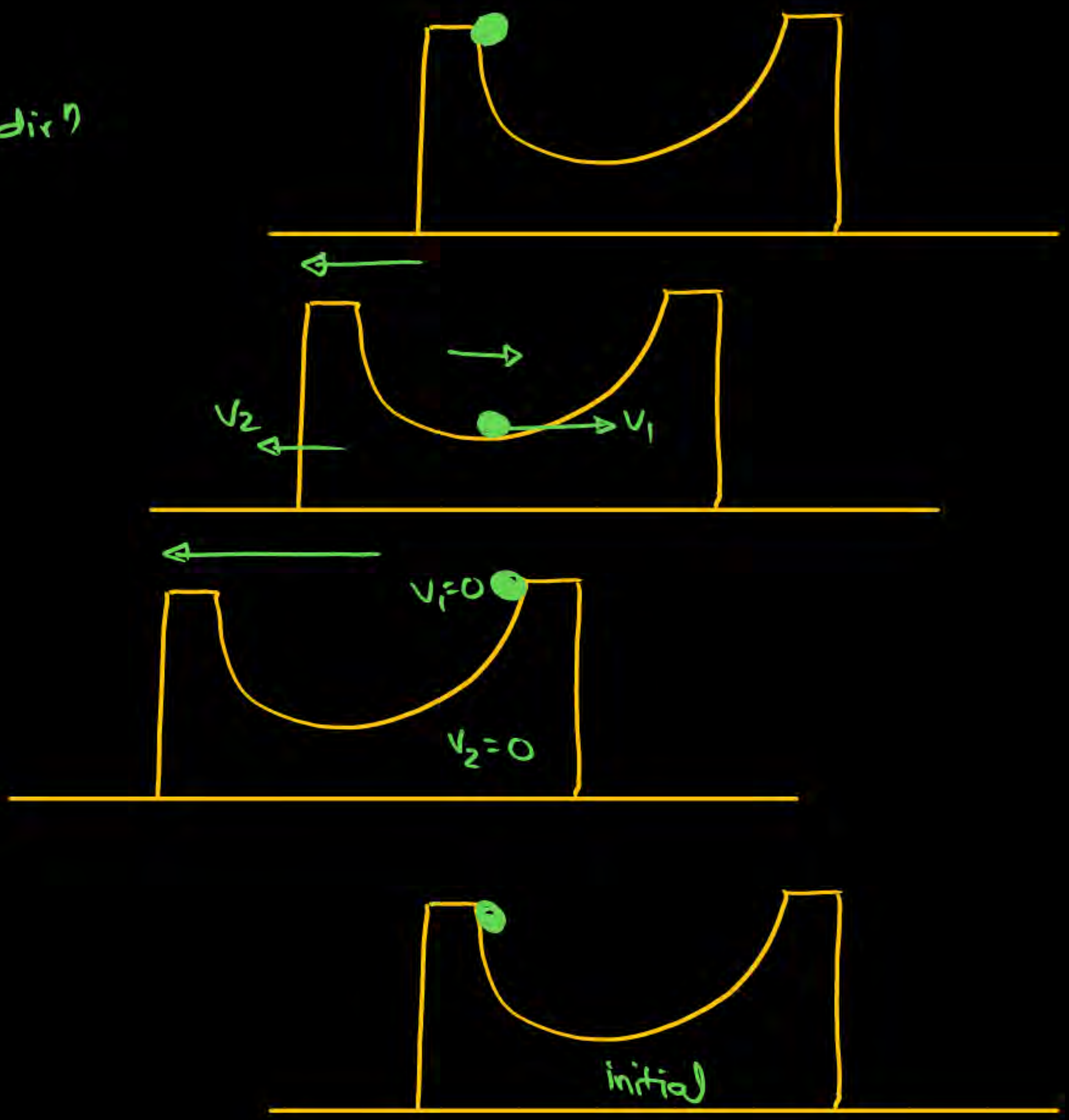
$v_{cm} = 0$

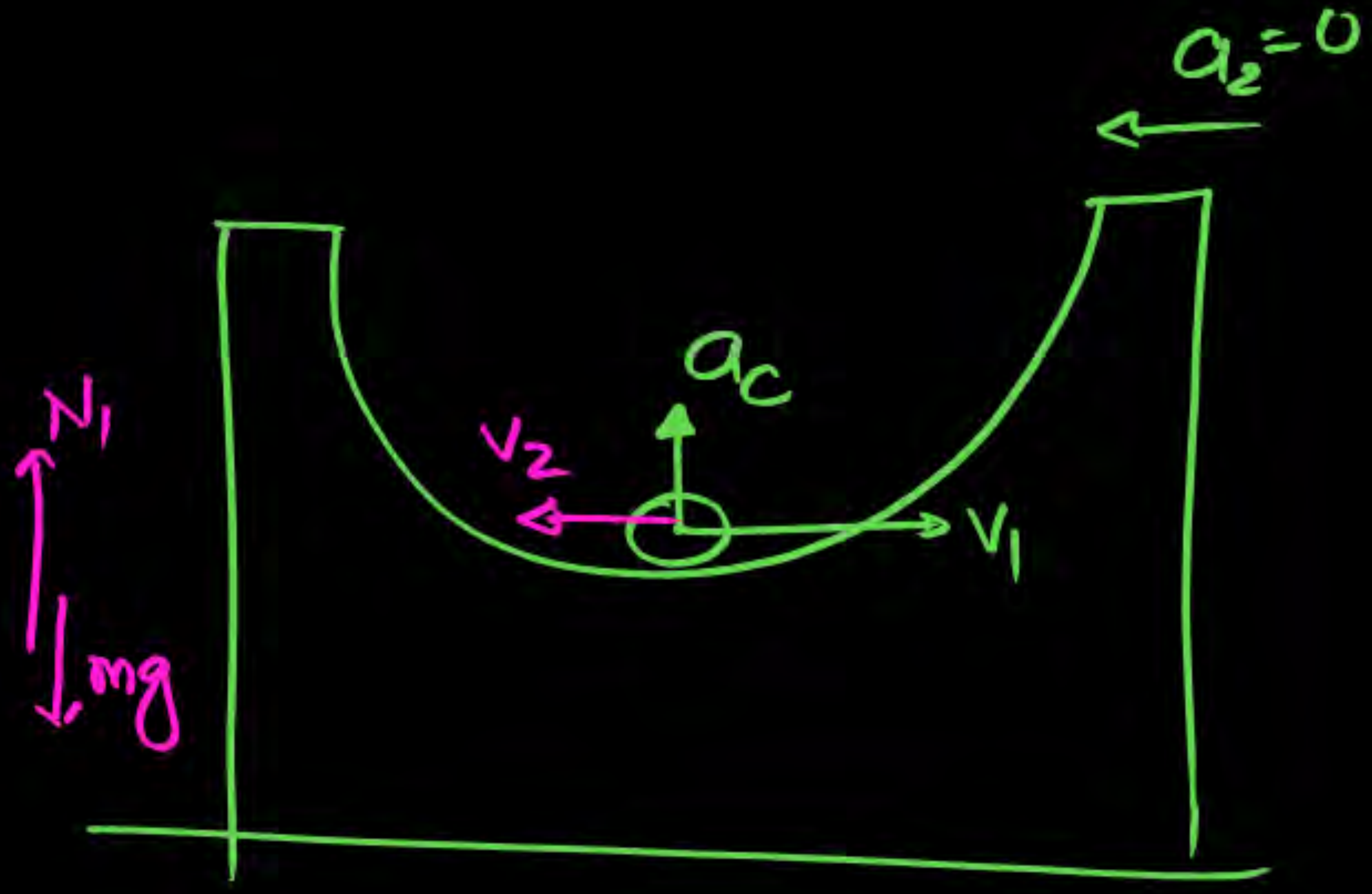


Ans. 4

COM is stationary in x-dir^n







$$a_{com} = \frac{m_1 a_1 + m_2 a_2 + \cdots}{M}$$

$$a_{cm} = \frac{F_{ext.}}{M} \rightarrow \frac{N_2 - 3mg}{3m}$$

$$a_{cm} = \frac{m a_c + 2m(0)}{3m} = \frac{a_c}{3} = \frac{1}{3}\frac{V_1^2}{R} = \frac{(V_1 - V_2)^2}{3(ROC)}$$

w.r.t wedge (because $a_2 = 0$)

w.r.t Ground

Displacement

$$\vec{S}_{ball/G} = \vec{S}_{ball/w} + \vec{S}_{wedge}$$

$$m_1\vec{S}_1 + m_2\vec{S}_2 + \dots = 0$$

(in x-dir^n + from Ground frame)

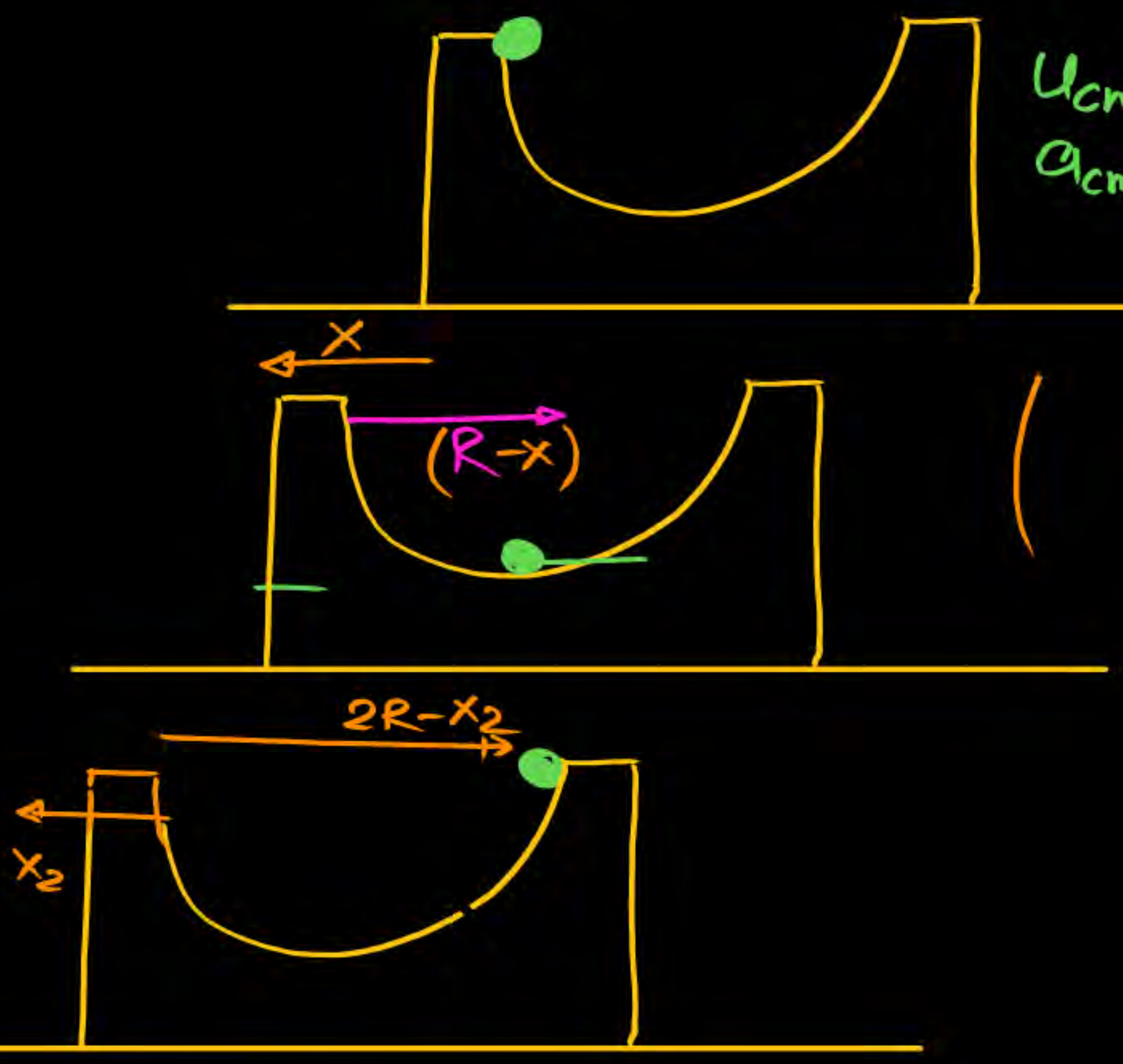
$$\left((F_{ext.})_x = 0 \text{ and } (u_{cm})_x = 0 \right)$$

$$m(R-x) + 2m(-x) = 0$$

$$x = R/3$$

$$S_{b/G} = R - R/3 = 2R/3 \; (\rightarrow)$$

$$S_{w/G} = R/3 \; (\leftarrow)$$

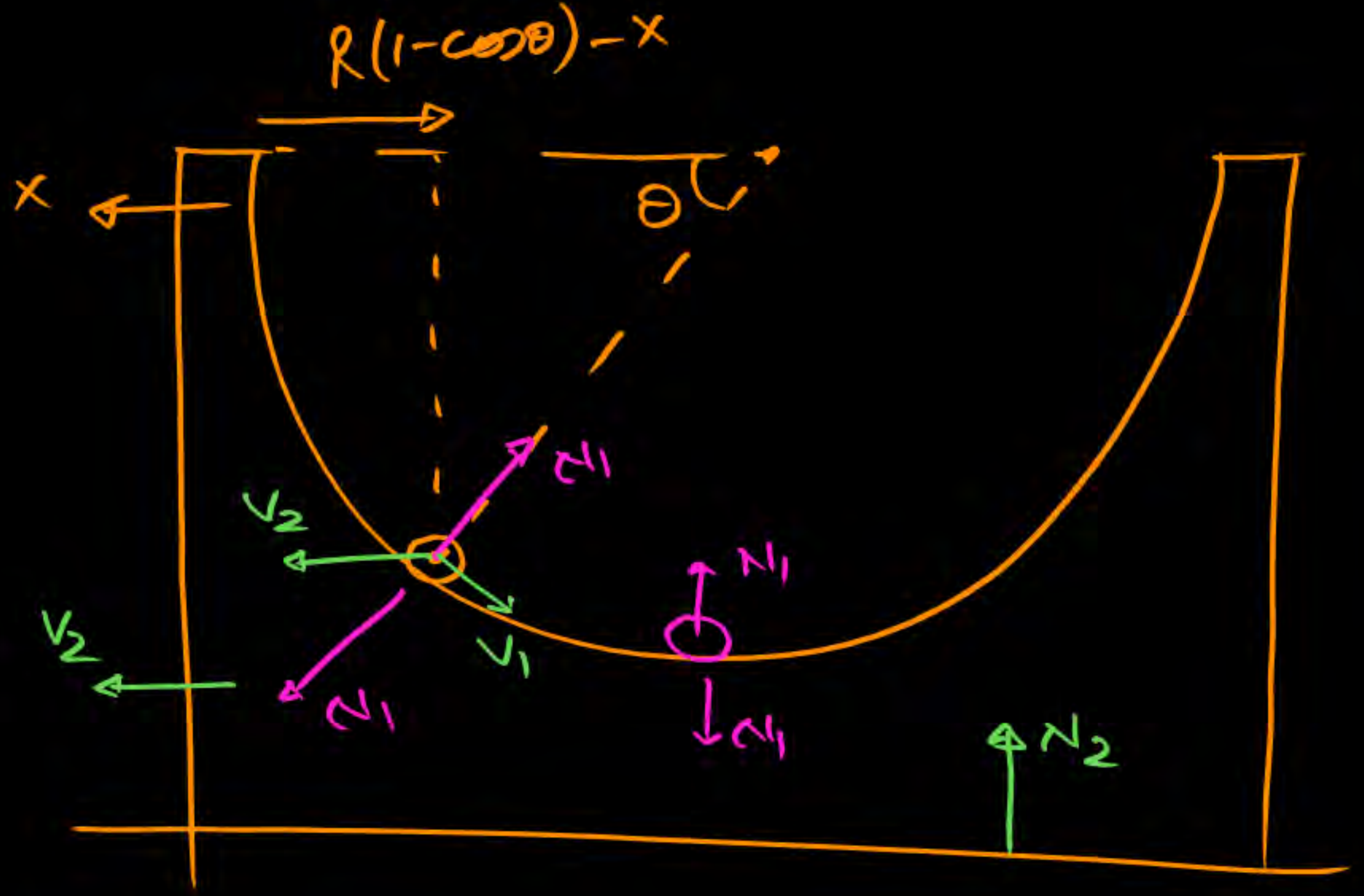


$$\left.\begin{aligned} u_{cm} &= 0 \\ a_{cm} &= 0 \end{aligned}\right\} \rightarrow \text{in x-dir}^n$$

$$\left(0 \xrightarrow{\qquad} R \quad \xleftarrow{x} \right)$$

$$m(2R - x_2) + 2m(-x_2) = 0$$

$$x_2 = \frac{2R}{3}$$



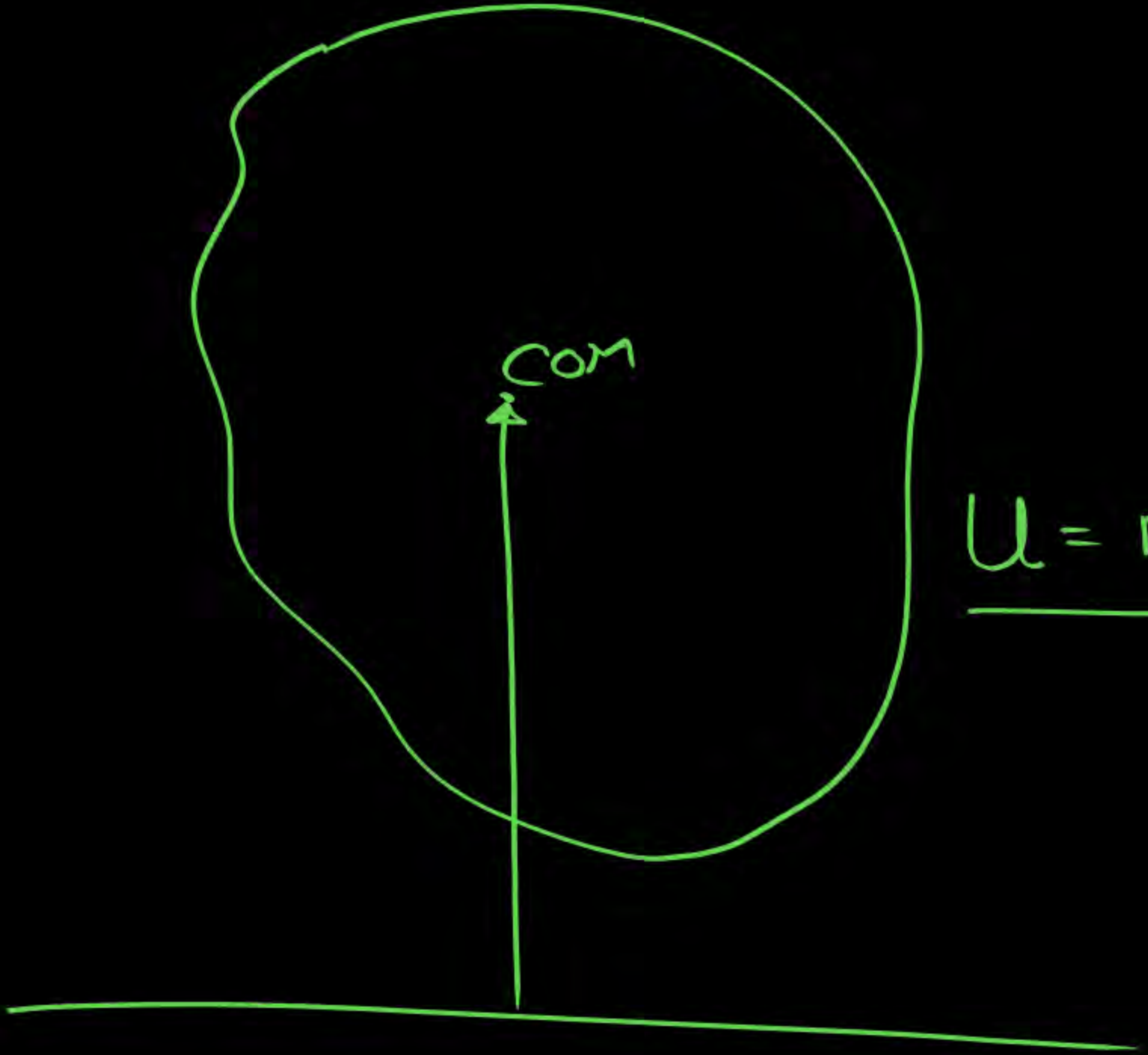
$$m\left(R(1-\cos\theta)-x\right)+2m(-x)=0$$

$$x=\frac{R(1-\cos\theta)}{3}$$

$$U = m_1 g h_1 + m_2 g h_2 + \cdots$$

$$U = (m_1 h_1 + m_2 h_2 + \cdots) g$$

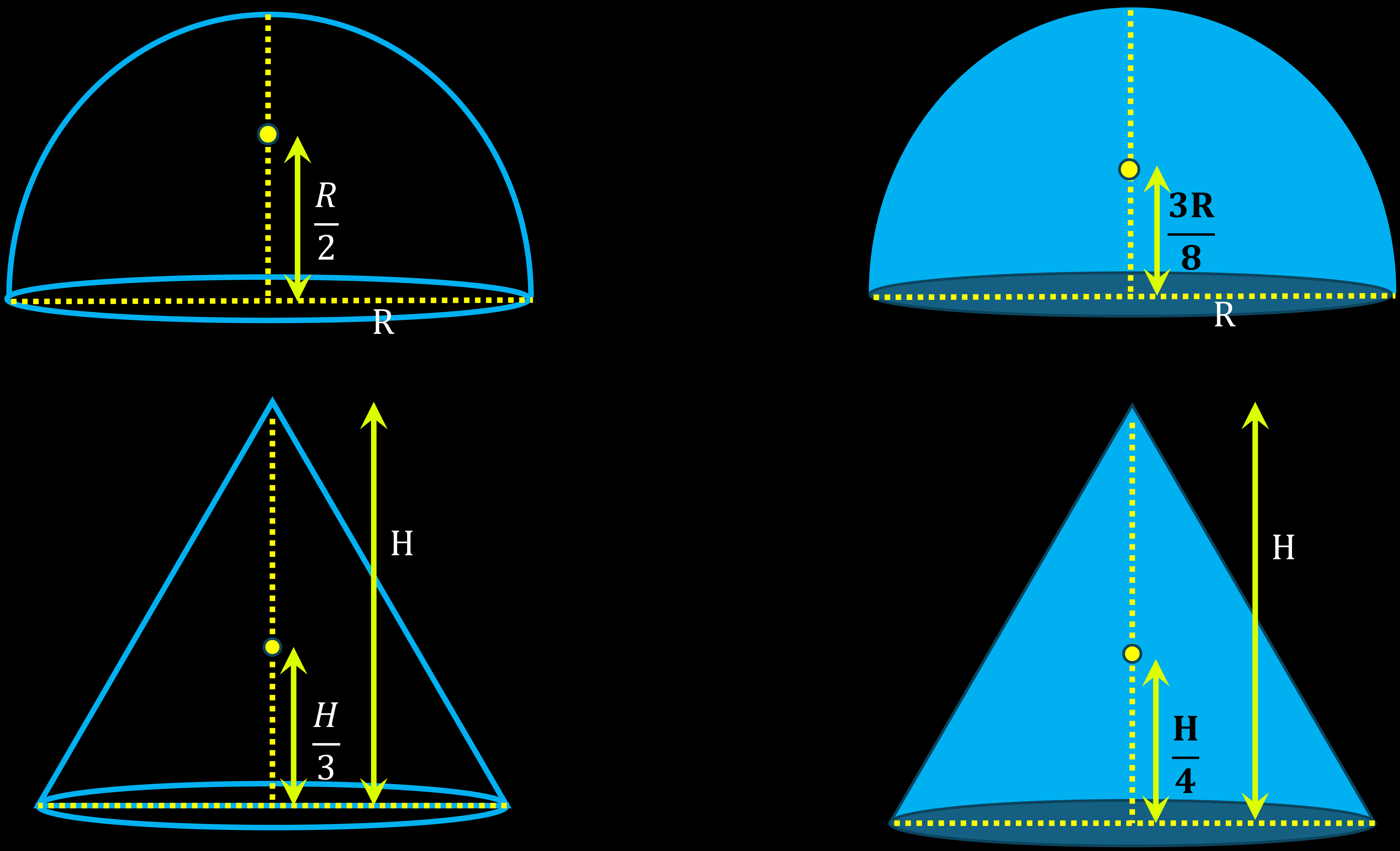
$$U = (M h_{cm}) g$$



$$U = M g h_{cm}$$

Location of com :→

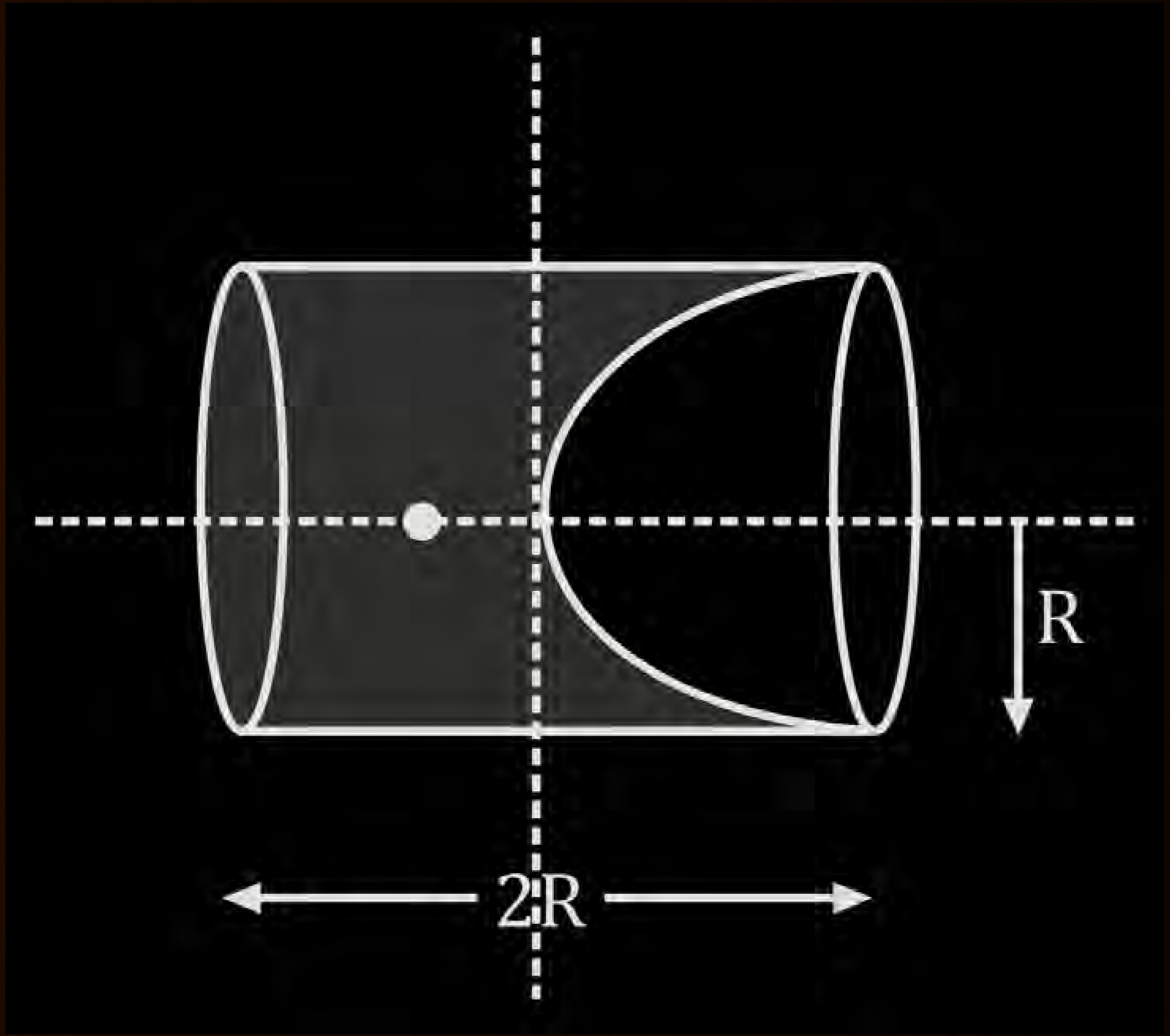
R
$\frac{2R}{\phi} \sin\frac{\phi}{2}$
R
$\frac{2R}{\pi}$
R
$\frac{2\sqrt{2}R}{\pi}$
R
$\frac{4R}{3\phi} \sin\frac{\phi}{2}$
R
$\frac{4R}{3\pi}$
R
$\frac{4\sqrt{2}R}{3\pi}$



R/2
R
3R/8
R
H
H/3
H
H/4

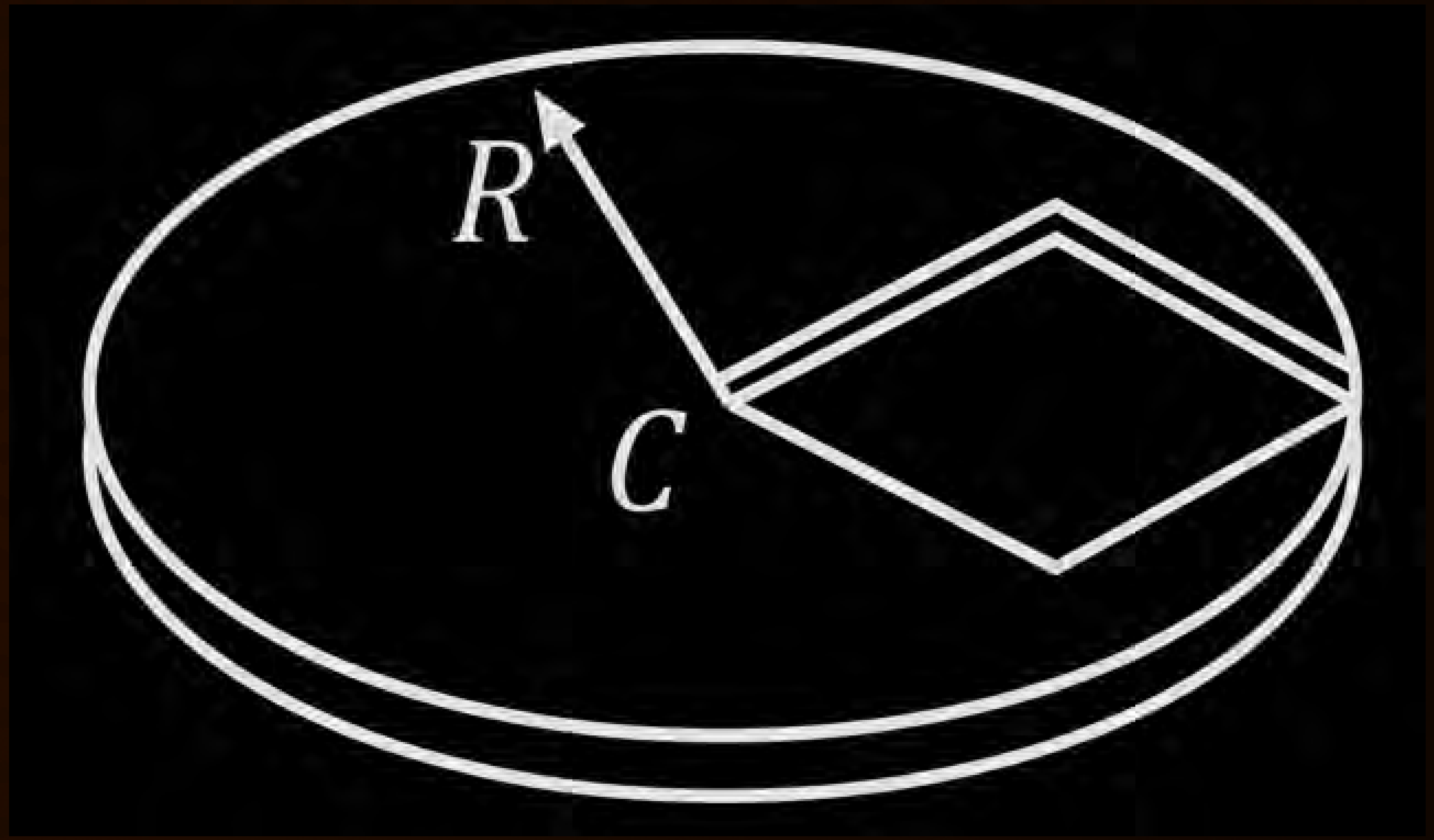
Question

A spherical part is removed from a cylinder. Find Distance of com of remaining part from center of cylinder. Assume uniform density throughout. (R = 32cm)



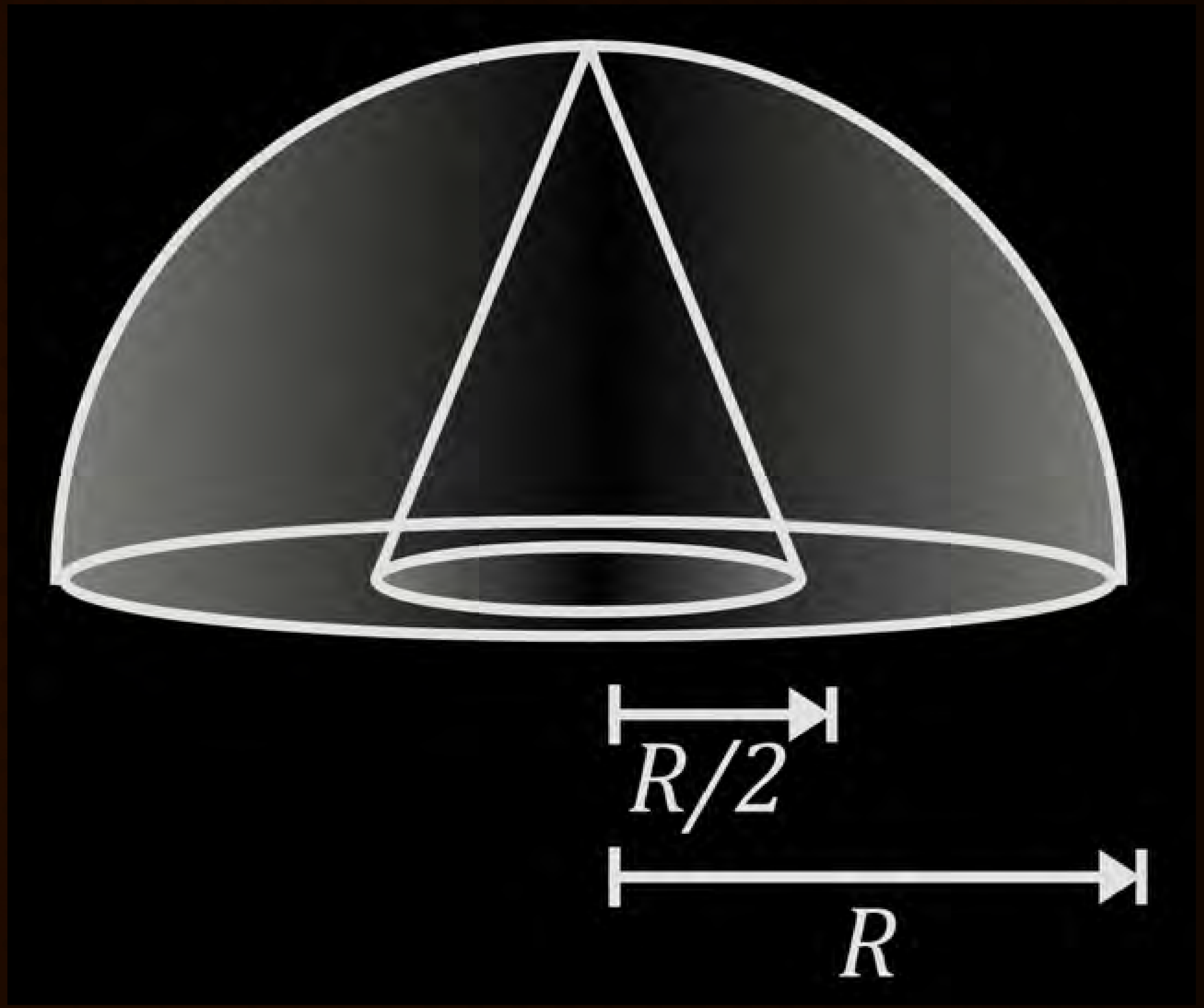
Question

Find the center of mass of the disc of radius R (R = 12cm) as shown in figure. A square is removed from the disc as shown. Distance of COM from point C is $\frac{N}{(2\pi-1)}$.find Value of N



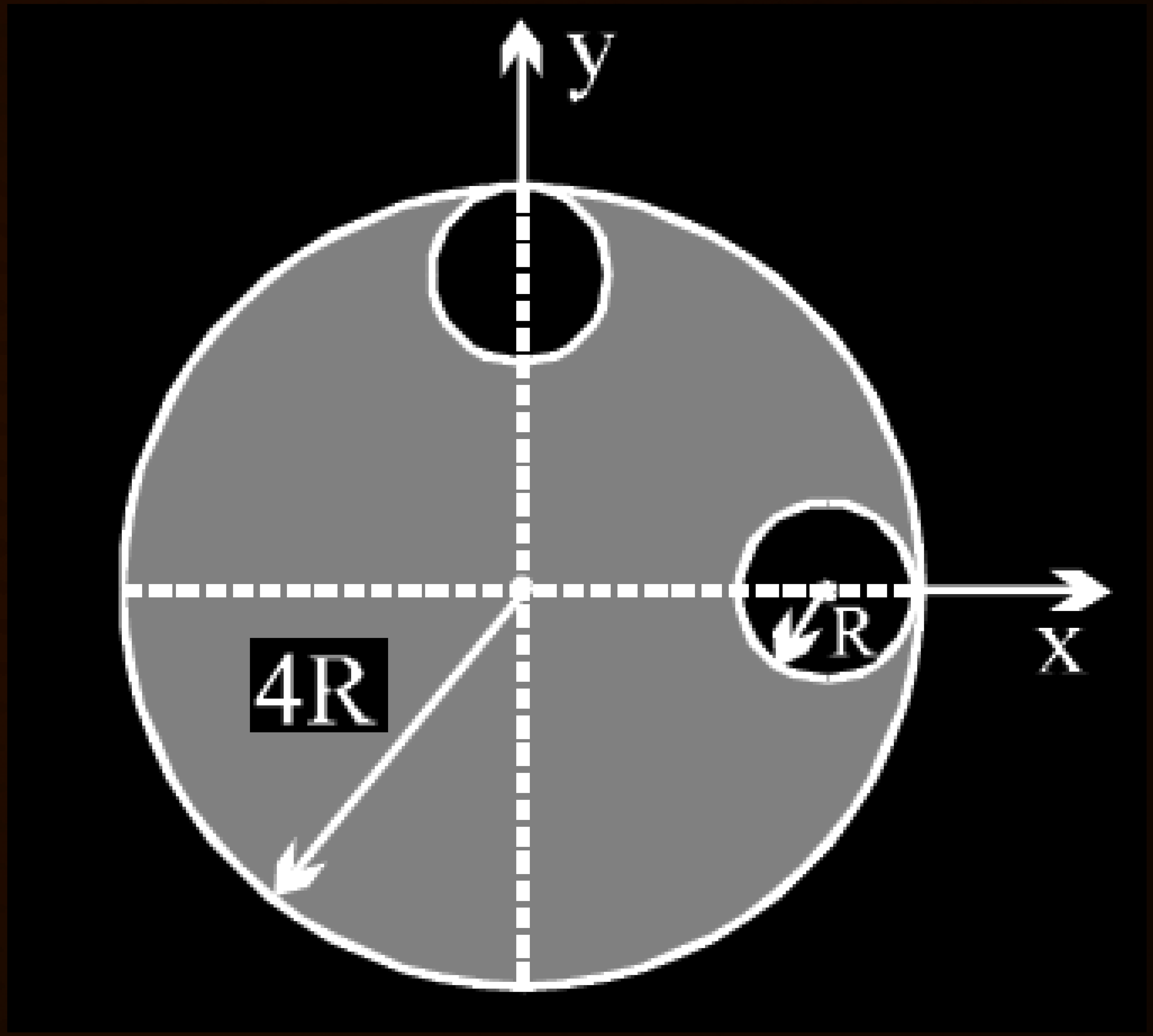
Question

From a hemisphere of radius R a cone of base radius $R/2$ and height R is cut as shown in figure. Find the height of center of mass of the remaining object. (R = 56cm)



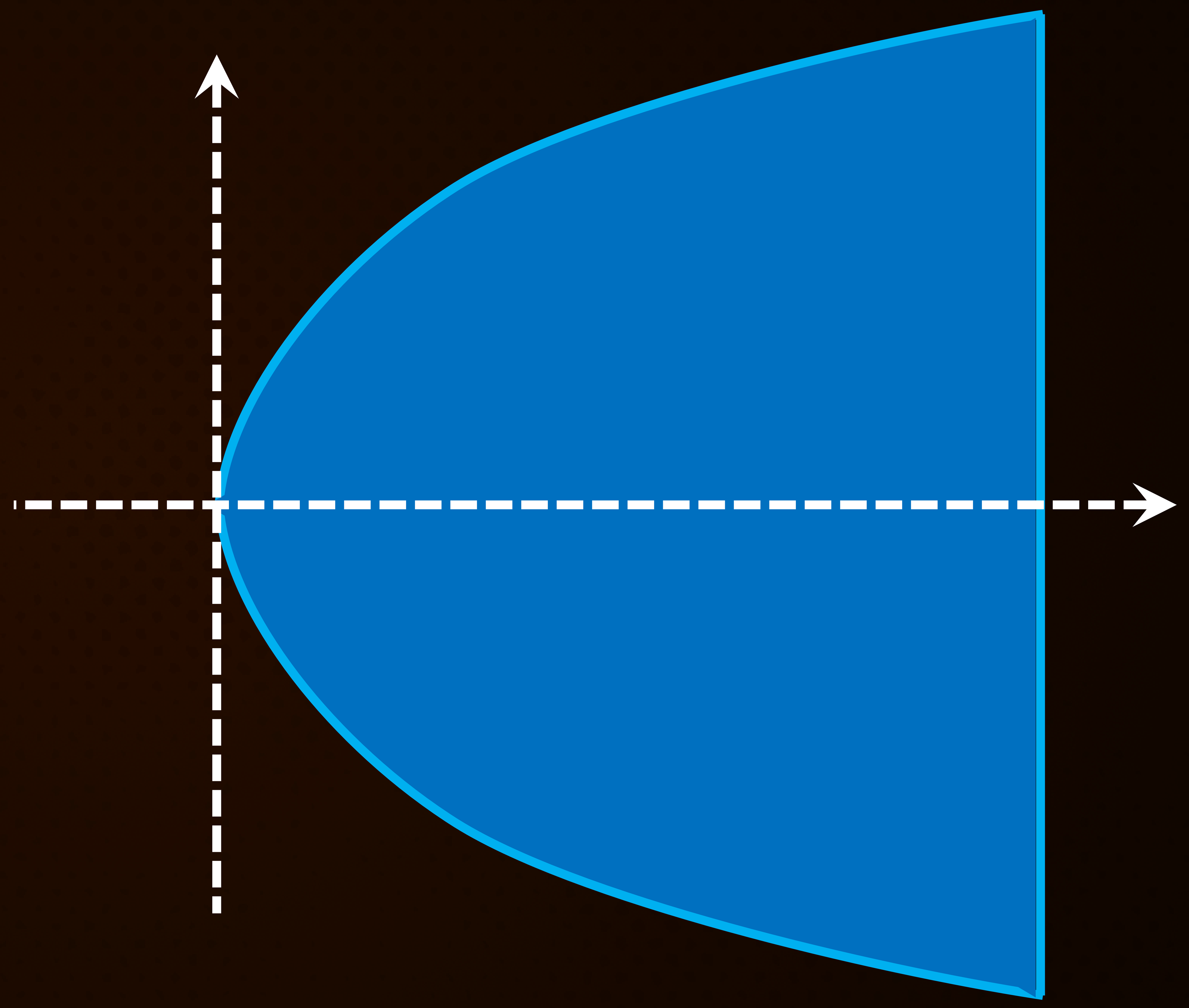
Question

From the circular disc of radius 4R two small disc of radius R are cut off. The center of mass of the new structure will be :



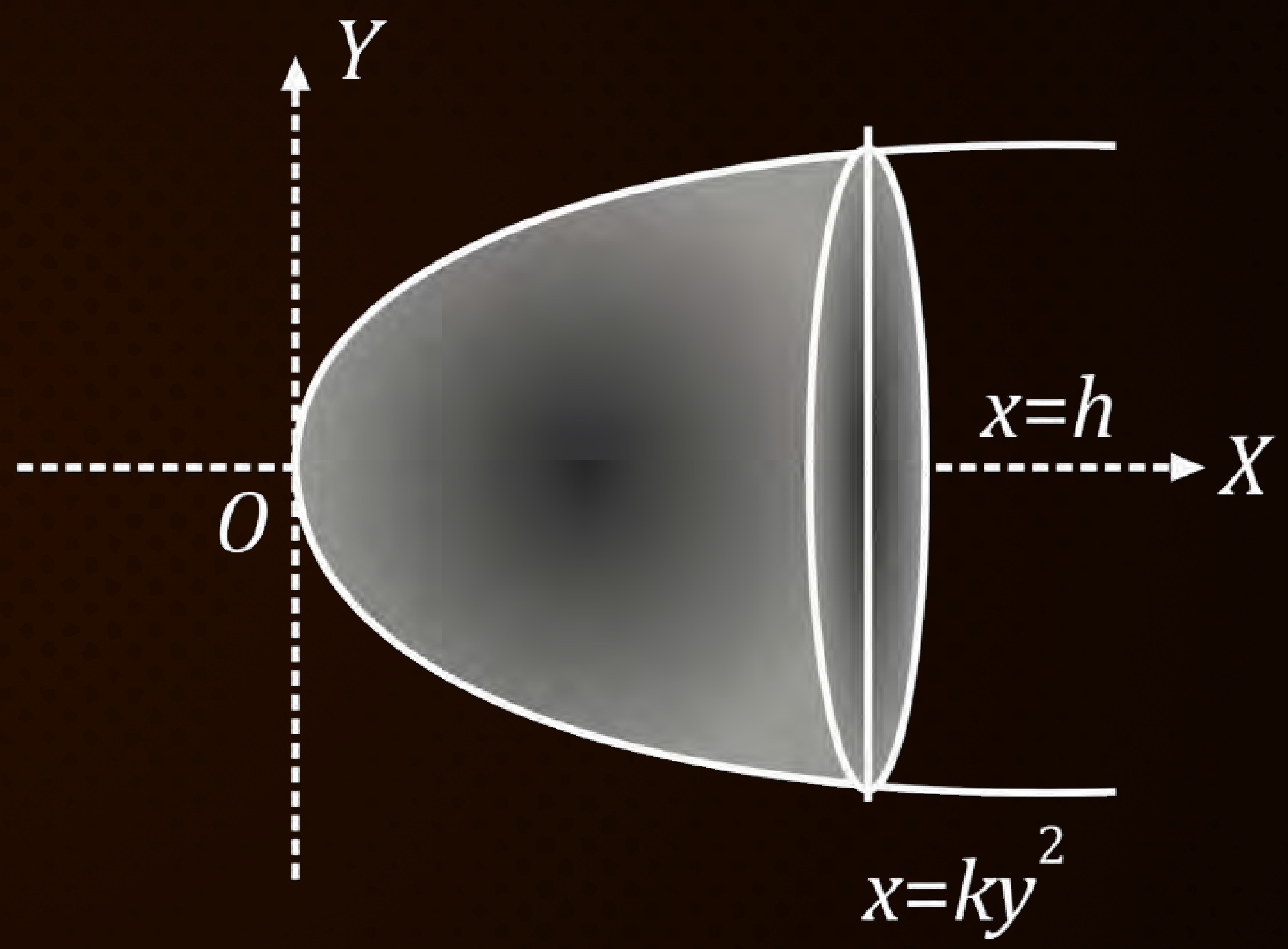
Question

Find center of mass of an object (Parabola) which is formed by a parabola $x = 4y^2$ as shown in figure (From x = 0 to x = h). Assume the object is of uniform density. (h = 15cm)



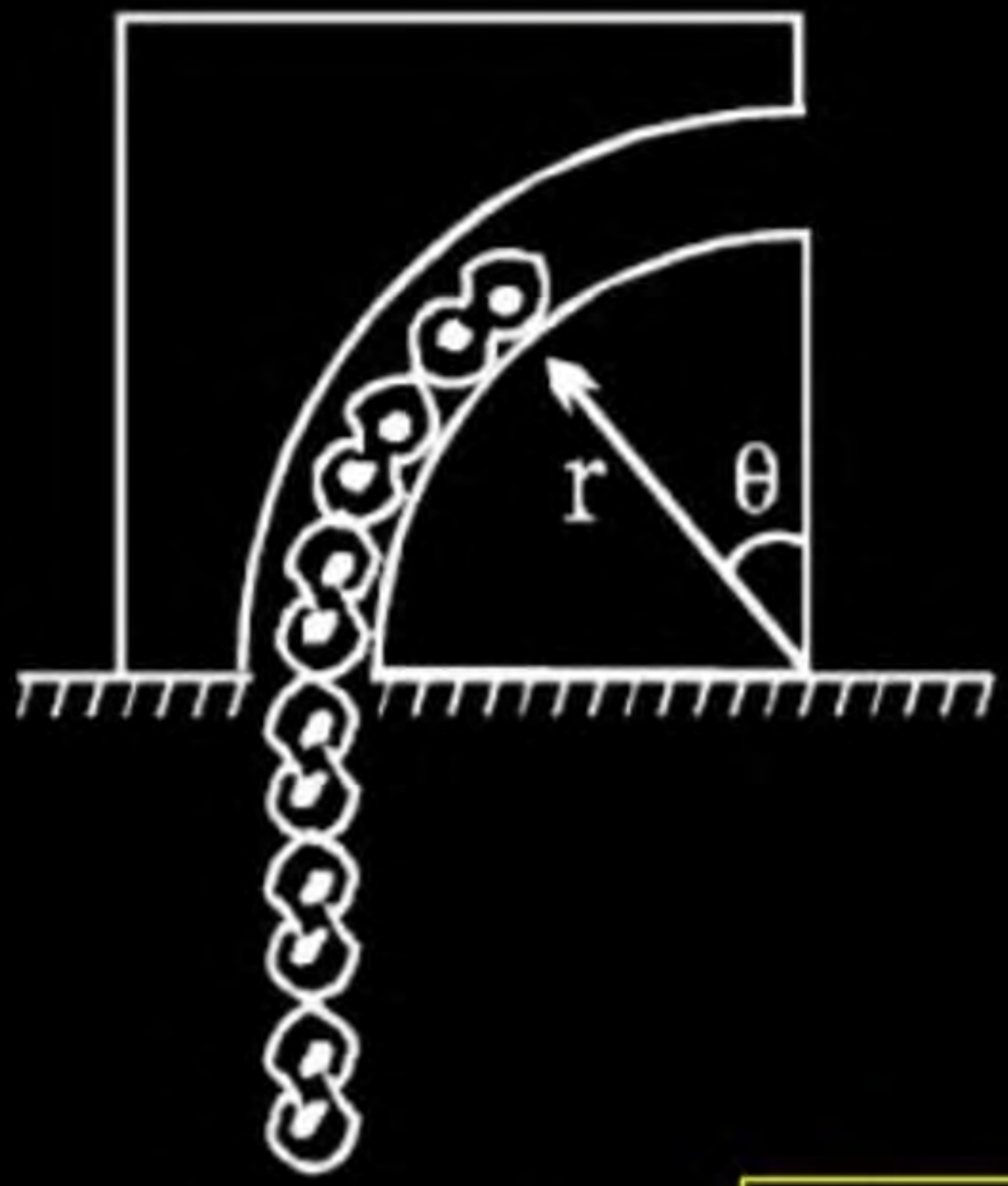
Question

Find center of mass of an object (Paraboloid) which is formed by rotating a parabola $x = 4y^2$ about x-axis as shown in figure (From x = 0 to x = h). Assume the object is of uniform density. (h = 15cm)



Question

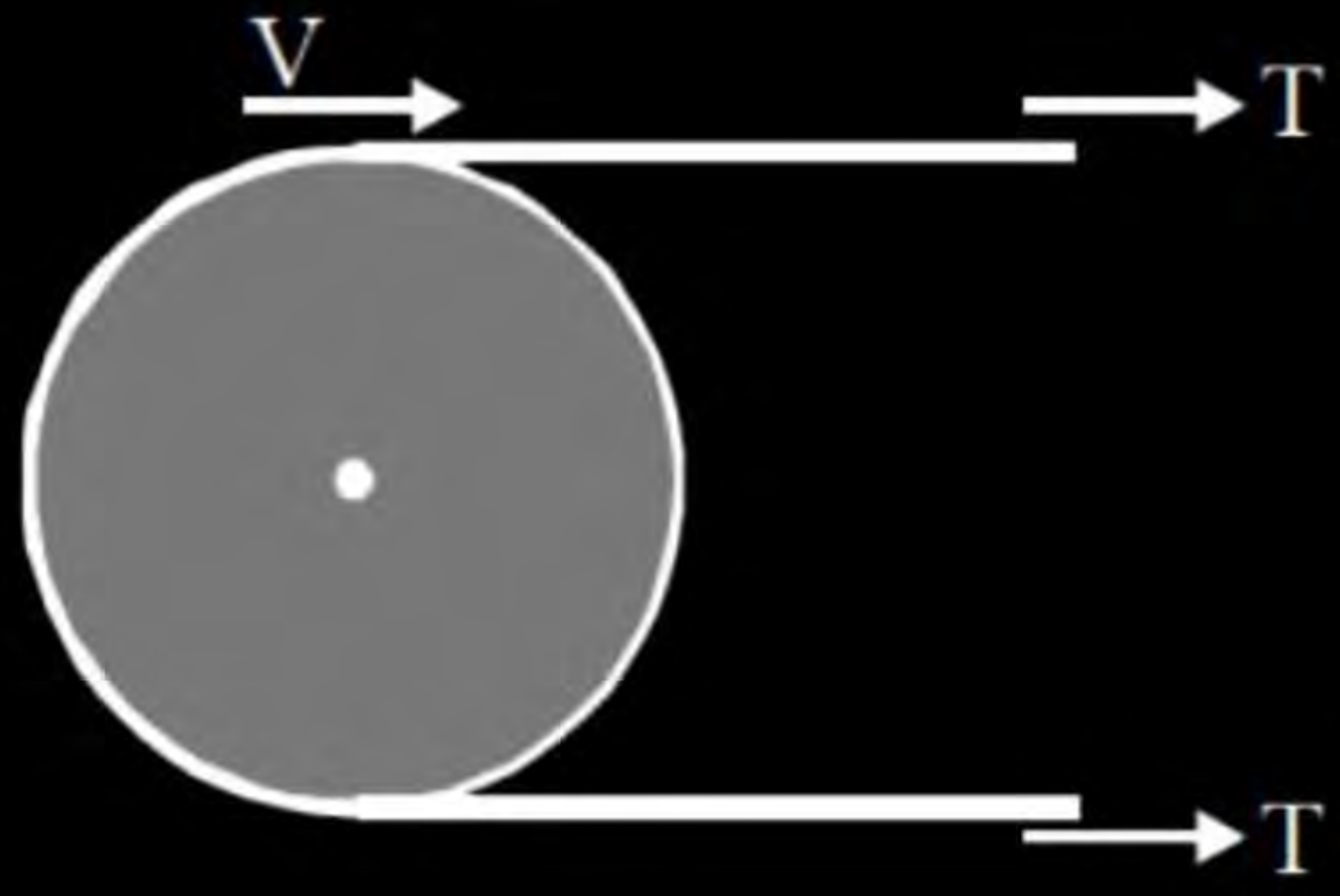
The flexible bicycle type chain of length $\frac{\pi r}{2}$ and mass per unit length r is released from rest with $\theta = 0°$ in the smooth circular channel and falls through the hole in the supporting surface. Determine the velocity v of the chain as the last link leaves the slot.



Ans. $\sqrt{gr\left(\frac{\pi}{2}+\frac{4}{\pi}\right)}$

Question

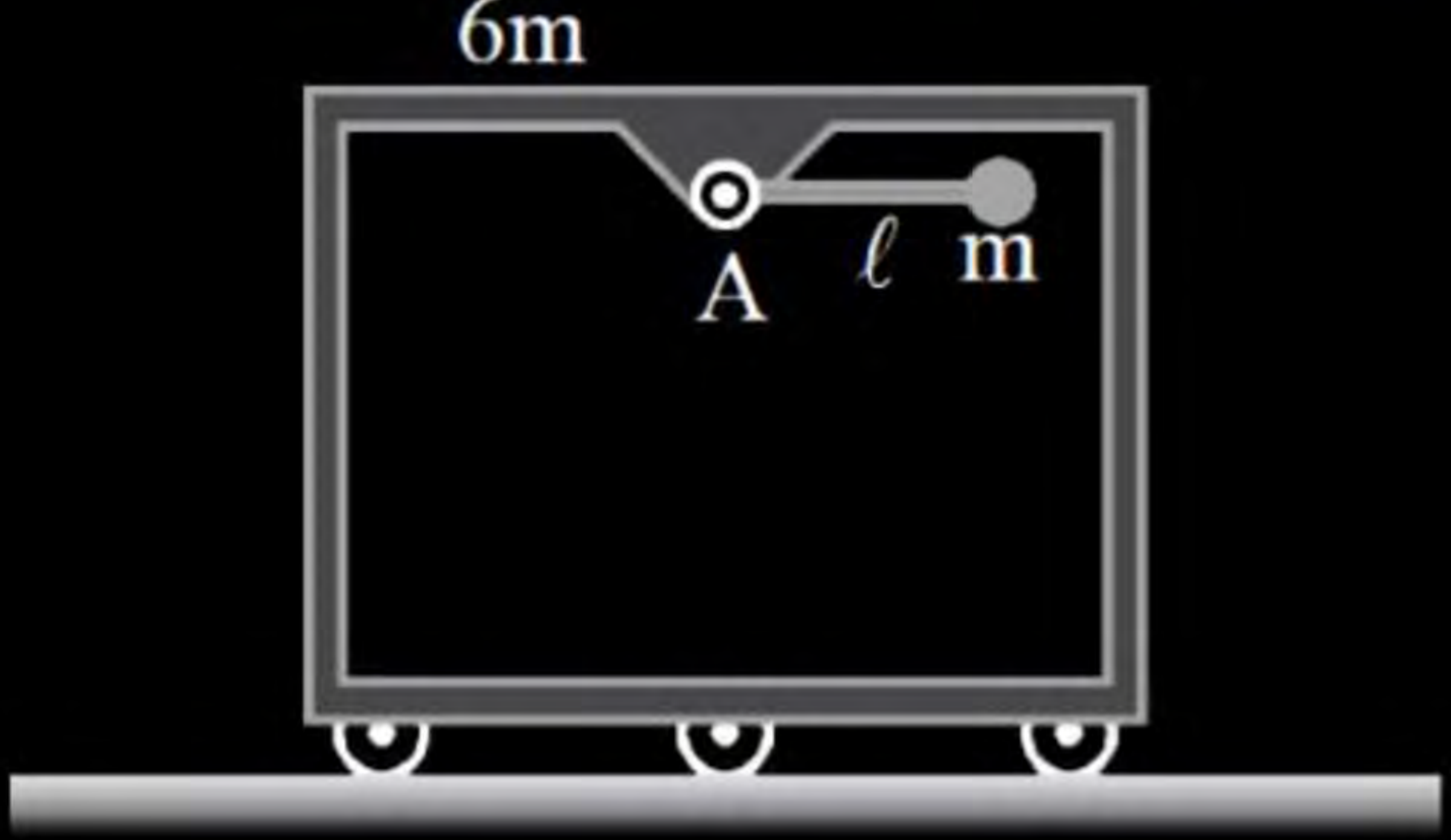
A flexible drive belt runs over a frictionless flywheel (see Figure). The mass per unit length of the drive belt is 1 kg/m, and the tension in the drive belt is 10 N. The speed of the drive belt is 2 m/s. The whole system is located on a horizontal plane. Find the normal force (in N) exerted by the belt on the flywheel.



Ans. 12

Question

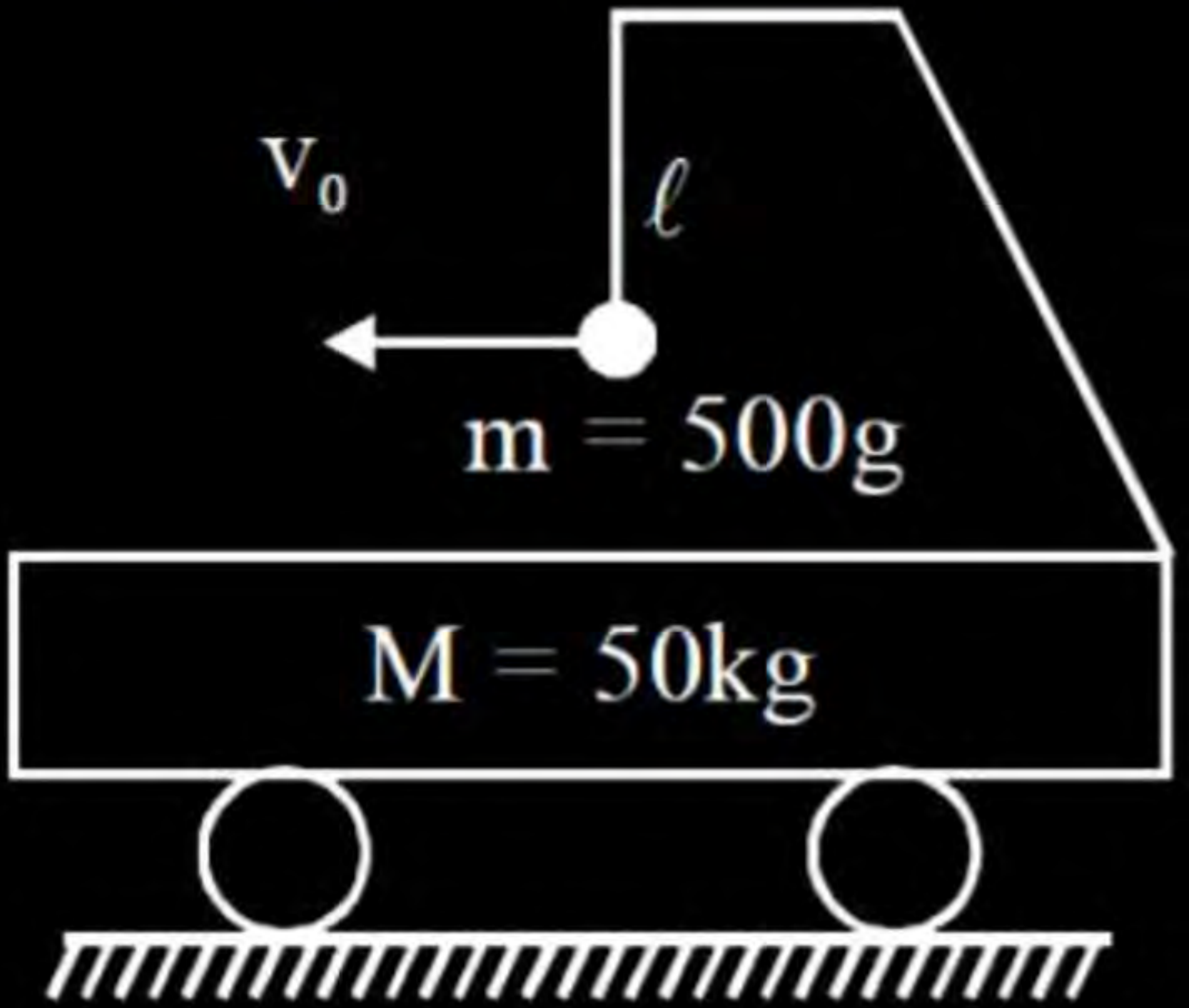
The frame of mass 6 m is initially at rest. A particle of mass m is attached to the end of the light rod, which pivots freely at A. If the rod is released from rest in the horizontal position shown, determine the velocity v_{rel} (in m/s) of the particle with respect to the frame when the rod is vertical. (Length of the rod is 2.1 m)



Ans. 7

Question

A pendulum bob of length 1 and mass $m(500\text{ g})$ is suspended on a smoothly running trolley of mass $M(50\text{ kg})$. The pendulum bob is given an impulse so as to impart it a horizontal velocity v_0. Maximum velocity attained by trolley in subsequent motion is $\frac{K}{101}v_0$. Assume that pendulum bob oscillates subsequently find the value of K^2.



Ans. 4

Question

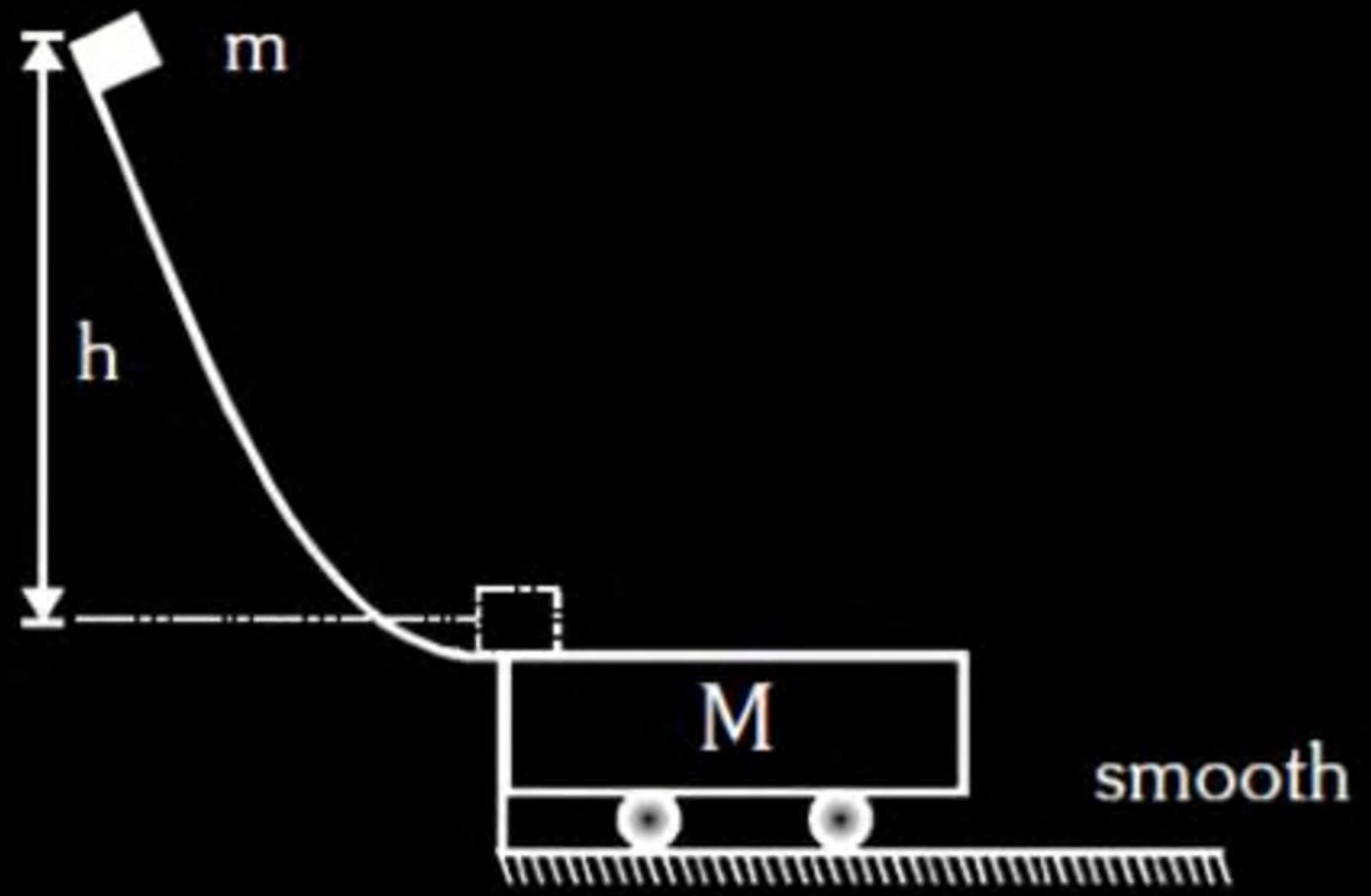
A carriage of mass M and length ℓ is joined to the end of a slope as shown in the figure. A block of mass m is released from the slope from height h . It slides till end of the carriage. The friction between the mass and the slope and also friction between carriage and horizontal floor is negligible. Coefficient of friction between block and carriage is μ. Find h in the given terms.

(A) $\mu\left(1+\frac{M}{m}\right)\ell$

(B) $2\mu\left(1+\frac{M}{m}\right)\ell$

(C) $\mu\left(2+\frac{m}{M}\right)\ell$

(D) $\mu\left(1+\frac{m}{M}\right)\ell$



Ans. D

Question

A small ball of mass m is placed in a circular tube of negligible width, mass M and radius R which is kept on a horizontal plane in gravity free space. Friction is absent between tube and ball. Ball is given a velocity v_0 as shown. Then,

(A) Path of ball from center of mass of system (ball + tube) will be circular.

(B) Path of ball from center of mass of system (ball + tube) will be elliptical

(C) Radius of curvature of ball at the time of projection of ball is $\frac{MR}{m+M}$

(D) Normal force between tube and ball if $M = 2m$, at the time of projection of ball is $\frac{2mV_0^2}{3R}$

Ans. AD

Question

Paragraph for Next 3 Questions

A small object of mass $m = 1$ kg is released from rest at the top of an inclined plane that connects to a horizontal plane without an edge. it slides onto a cart of mass M that has a semi-cylindrical surface of radius $R = 0.36$ m fixed to the middle of it, as shown in the figure.

The small object reaches the topmost point of the semi-cylinder and stops there. It continues to move vertically with free fall and hits the cart exactly at the edge. All friction can be neglected.

Ans. 4

Question

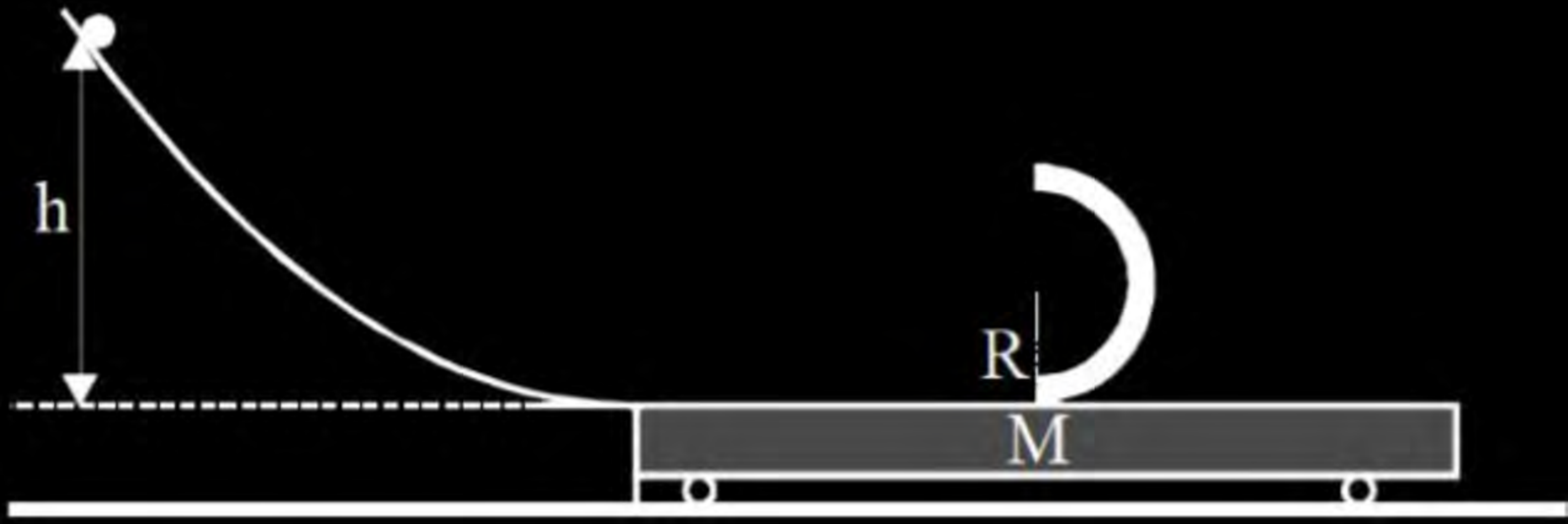
What is the minimum possible length of the cart?

(A) R

(B) 2 R

(C) $3R$

(D) 4 R



Ans. D

Question

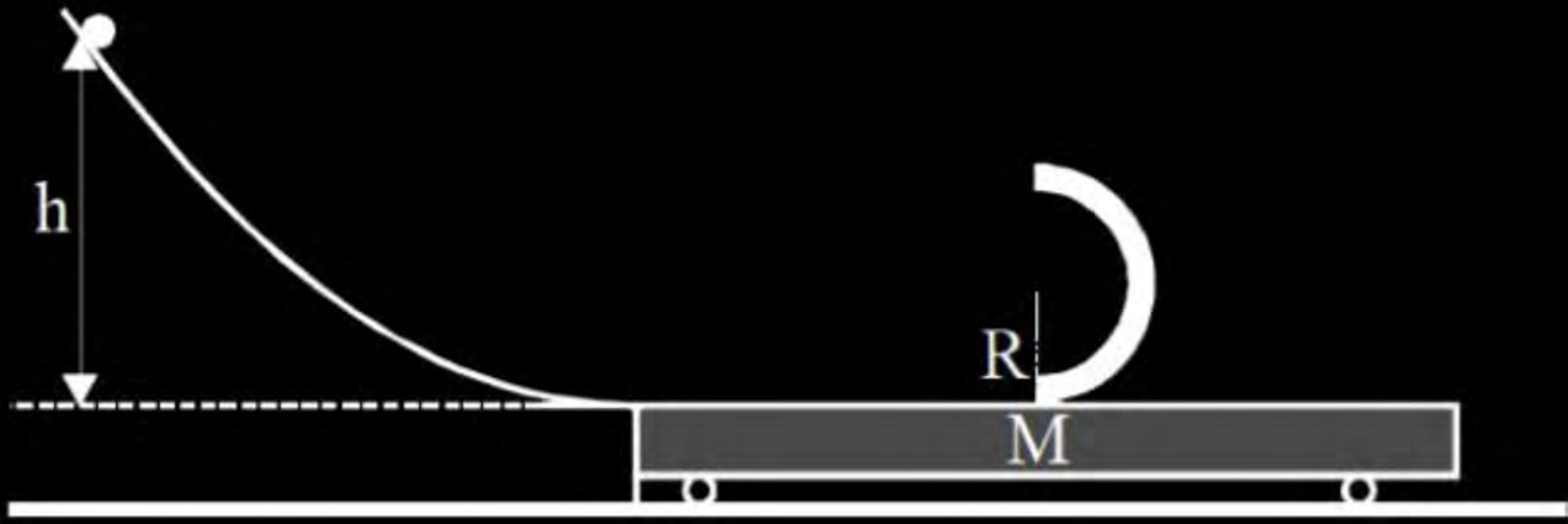
At what height h was the small object released ?

(A) $h = \frac{1}{2}\left(\frac{m}{M}\right)^2 R$

(B) $h = \frac{1}{2}\left(\frac{M}{m}\right)^2 R$

(C) $h = \frac{1}{2}\left(\frac{m}{M}\right) R$

(D) $h = \left(\frac{M}{m}\right)^2 R$



Ans. B

Question

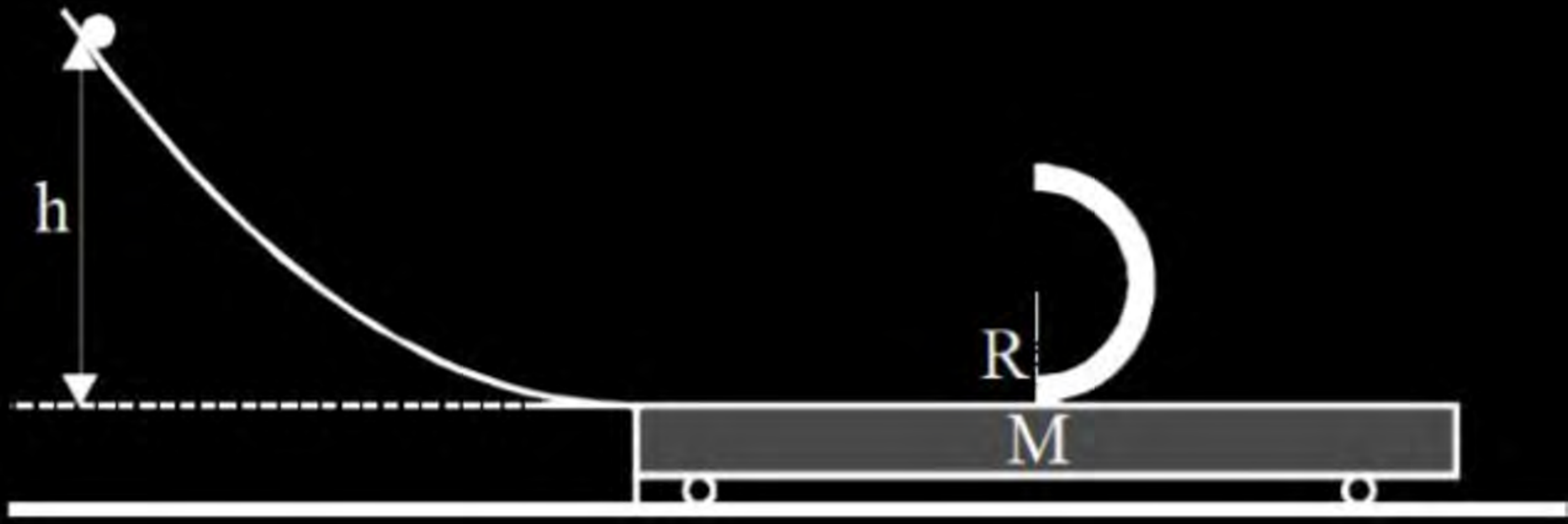
What is the ratio of mass of cart and mass of the particle ?

(A) $k = \frac{1+\sqrt{17}}{2}$

(B) $k = \frac{1+\sqrt{14}}{2}$

(C) $k = \frac{1+\sqrt{16}}{2}$

(D) $k = \frac{1+\sqrt{18}}{2}$



Ans. A

Question

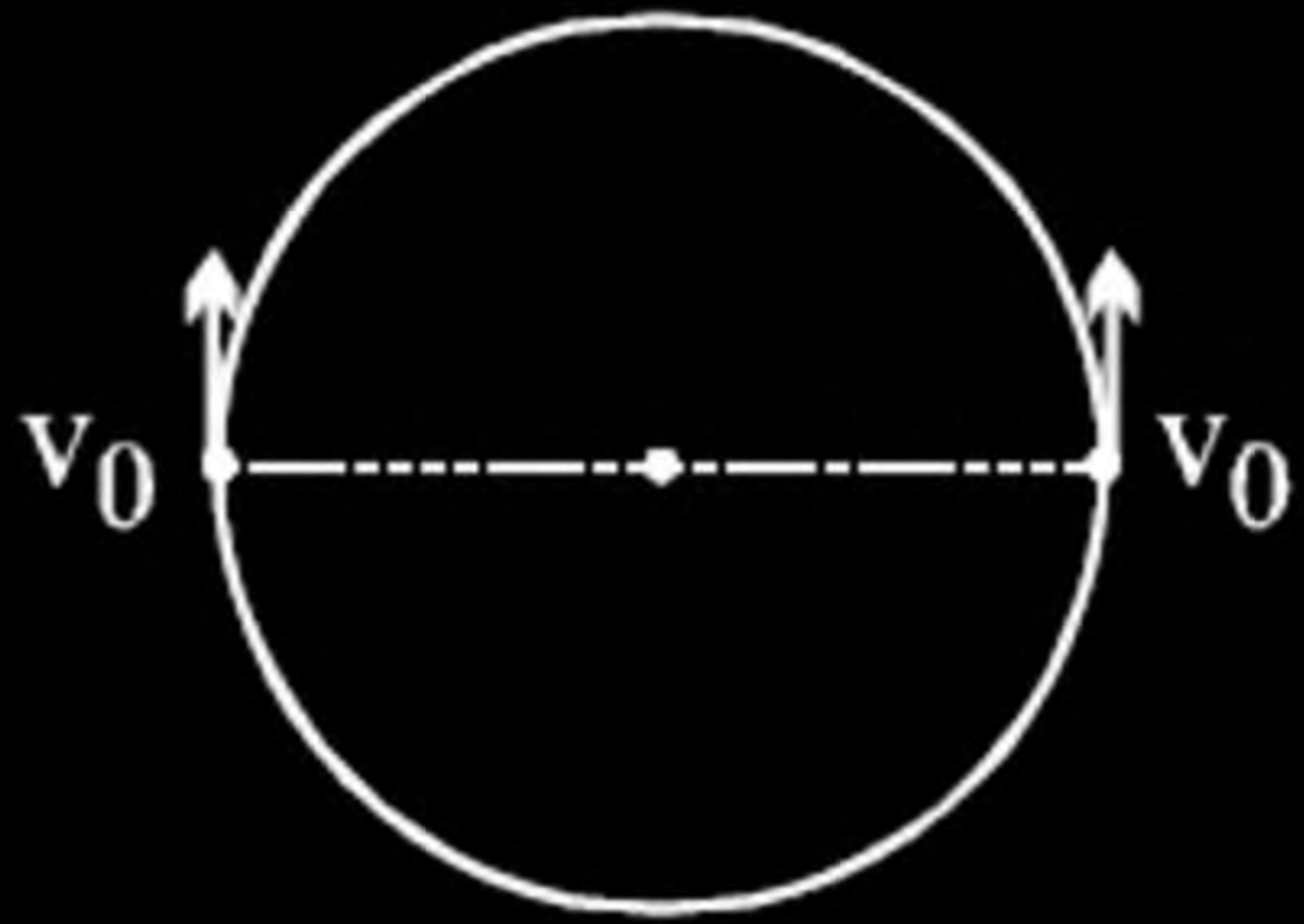
Two paticles of mass m , constrained to move along the circumference of a smooth circular hoop of equal mass m , are initially located at opposite ends of a diameter and given equal velocities v_0 shown in the figure. The entire arrangement is located in gravity free space. Their relative velocity just before collision is:

(A) $\frac{1}{\sqrt{3}}v_0$

(B) $\frac{\sqrt{3}}{2}v_0$

(C) $\frac{2}{\sqrt{3}}v_0$

(D) none of these

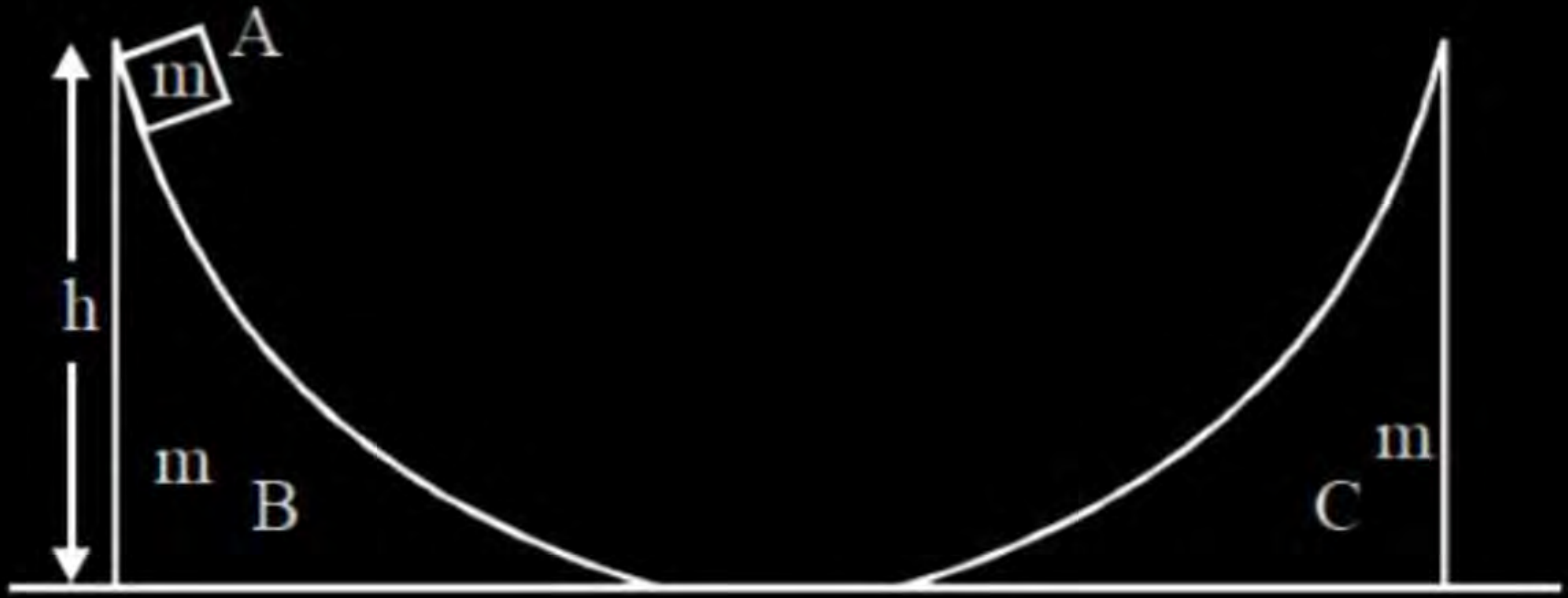


Ans. C

Question

Paragraph for next 3 Questions

Block A is placed on wedge B at a height h above ground. Block and the two wedges are all of same mass m . Neglect friction every where :



Question

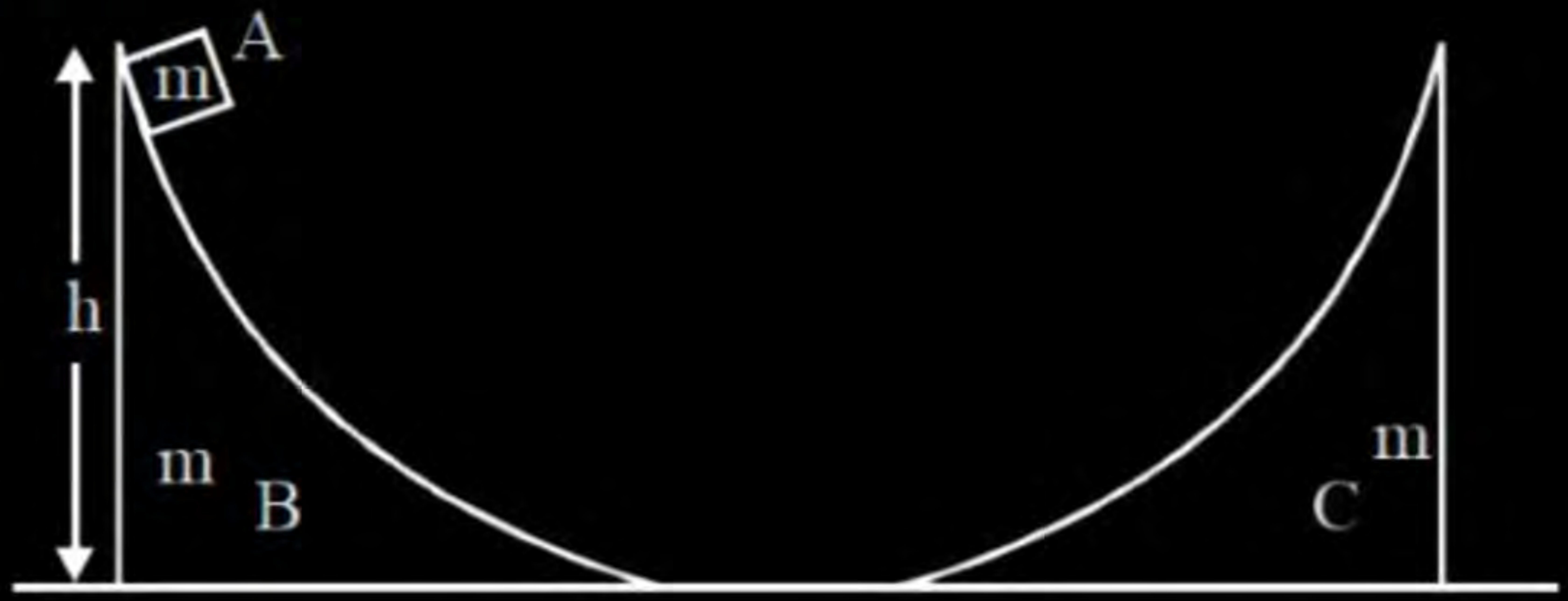
Find velocity of B when A has slid down from it :

(A) $\sqrt{gh}$

(B) $\sqrt{\frac{gh}{2}}$

(C) $\frac{\sqrt{gh}}{2}$

(D) None of these



Ans. A

Question

Find maximum height upto which block A rises on wedge C :

(A) h

(B) $\frac{h}{2}$

(C) $\frac{h}{4}$

(D) None of these

Ans. C

Question

Find velocity of A when it has slid down to ground from wedge C :

(A) 0

(B) $\sqrt{\frac{gh}{2}}$

(C) $\sqrt{\frac{gh}{4}}$

(D) None of these

Ans. A

THANK YOU
BAWAL
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