ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture - 01

Inequalities

By – Guru sir

Recap of previous lecture

1

2

3

4

3D — Remaing will be done on sunday morning

Topics *to be covered*

1. $f(x) = x^2$ & its Applications
2. $f(x) = |x|$
3. $f(x) = [x]$
4. $f(x) = \log_a x$ & $f(x) = \sqrt{x}$
5. Wavy curve method
6. Completing the square form

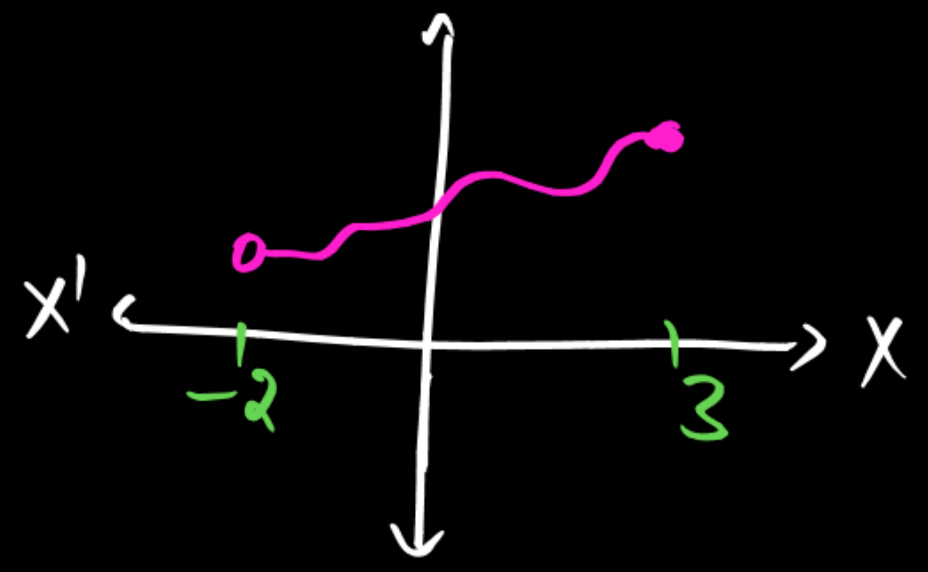
How to determine the domain & Range using graph

In graph

Domain:- The set of values of x (x-axis) for which there is a corresponding

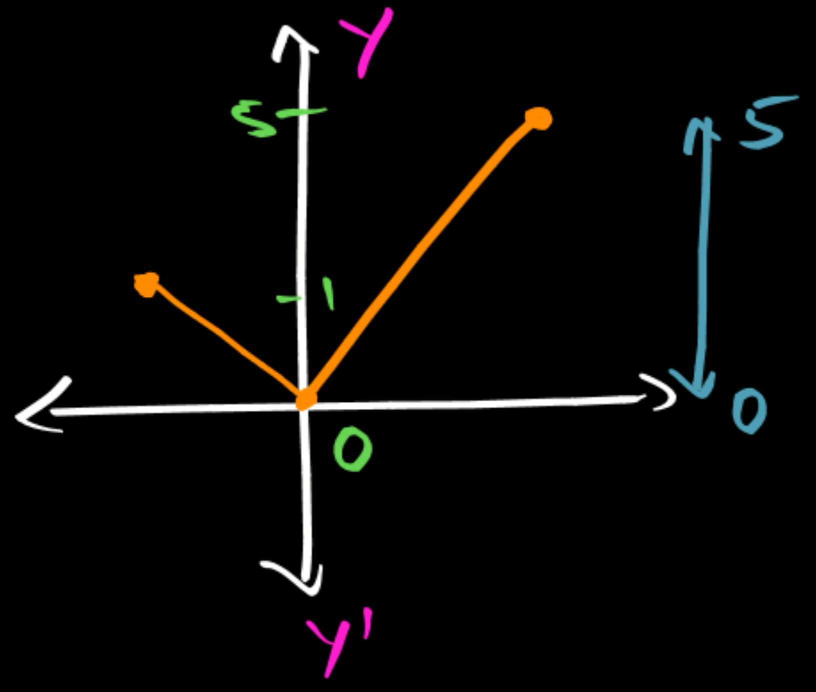
⇓

we just look into corresponding graph wrt x-axis

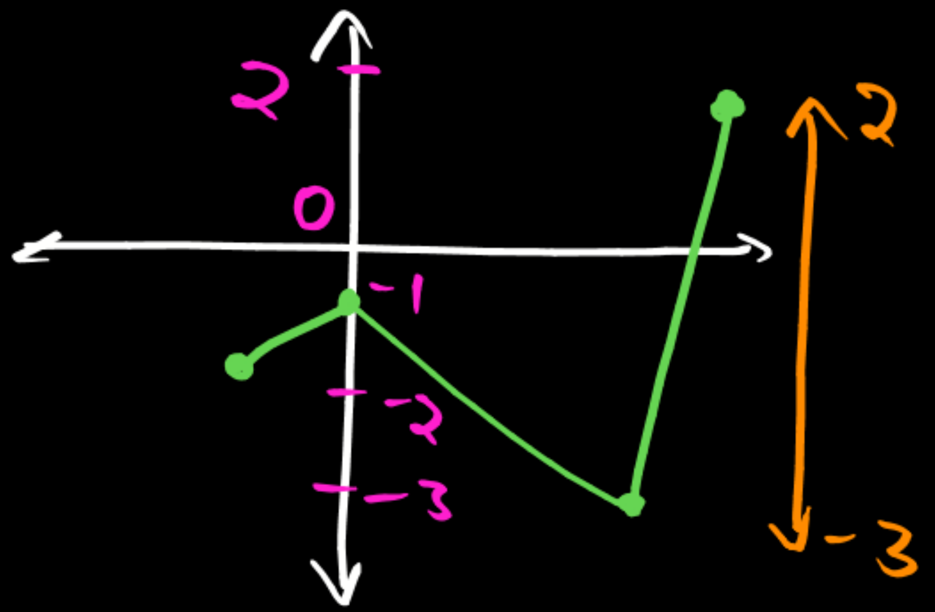


Domain = $(-2, 3]$

Range :- we look into the graph corresponding to y-axis

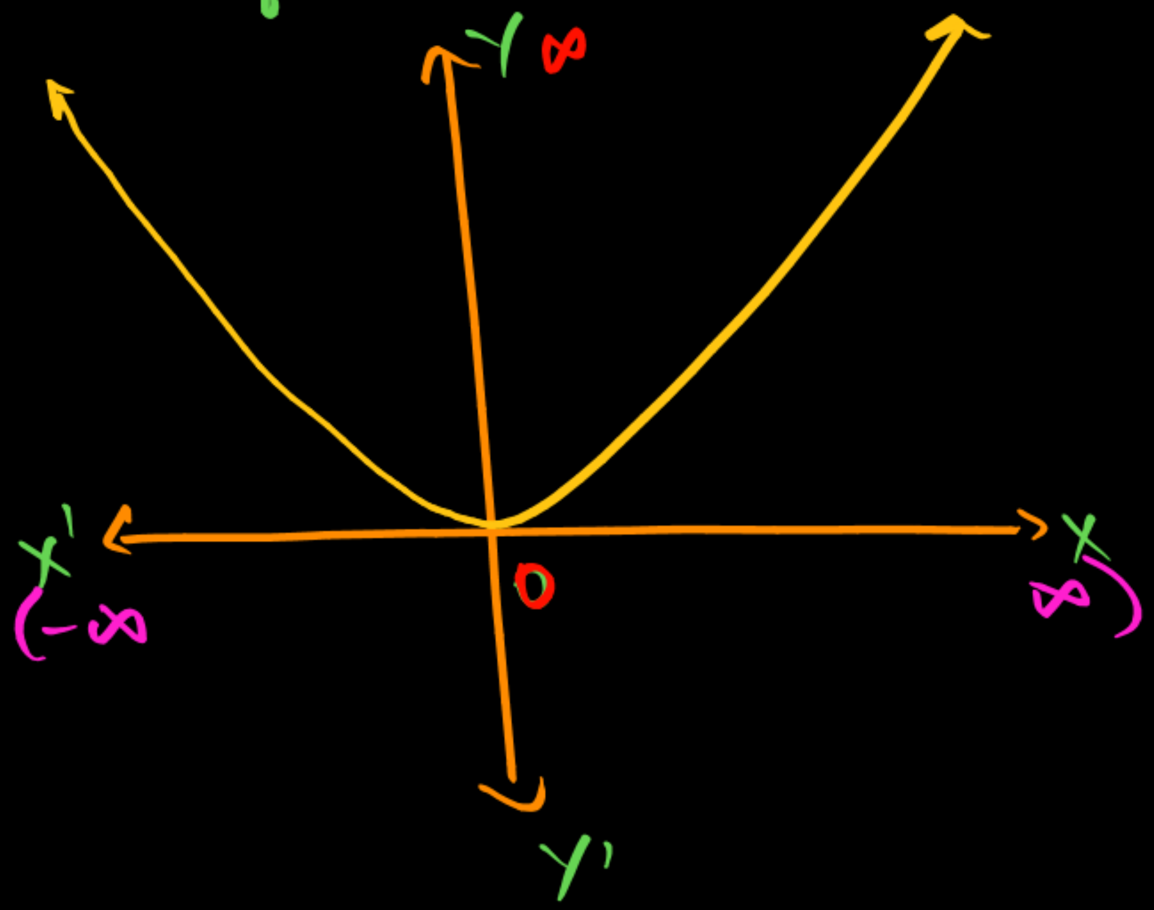


Range = $[0, 5]$



Range = $[-3, 2]$

$f(x) = x^2$



wrt (x-axis)

↑

(input)

→ Domain = R

$\Downarrow$

$x \in R$

or

$x \in (-\infty, \infty)$

$(x) \rightarrow$ input

wrt (y-axis)

non negative real no.

↑

(output)

→ Range = $[0, \infty)$

$\Downarrow$

$y \in [0, \infty)$

$\Downarrow$

$f(x) \in [0, \infty)$

$\Downarrow$

$x^2 \in [0, \infty)$

$\Downarrow$

$x^2 \geq 0$

$f(x) = x^2$

$\Downarrow$

$x^2 \geq 0$

Similarly

$(x-1)^2 \geq 0$

$(x+2)^2 \geq 0$

$(3-x)^2 \geq 0$

$(2x-3)^2 \geq 0$

Find the range of

$f(x) = (x-1)^2 + 3$

Soln:

WKT $(x-1)^2 \geq 0$

Add 3

$(x-1)^2 + 3 \geq 3$

$f(x) \geq 3$

$f(x) \in [3, \infty)$

Range $= [3, \infty)$

(2) Find the range of

$$f(x) = 2 - (x+3)^2$$

Soln :-

WKT

$$(x+3)^2 \geq 0$$

$\times^{ly}$ by -1

$$-(x+3)^2 \leq 0$$

Add 2

$$2-(x+3)^2 \leq 2$$

$$f(x) \leq 2$$

Range $= (-\infty, 2]$

Remark :-

on multiplying or Dividing by a -ve value The sign of inequality get reversed

$\geq$ changes to $\leq$

$\leq$ Changes to $\geq$

$>$ Changes to $<$

$<$ changes to $>$

Similar PYQ to KCET

① If $f : R \to B$ → codomain

given by $f(x) = x^2 - 2x + 4$

is $\boxed{\text{onto}}$, find B.

$\boxed{\text{onto}}$

⇓

Codomain = Range

Solⁿ:

To find Range:-

$f(x) = x^2 - 2x + 4$

$= \underbrace{x^2 - 2x + 1^2} - 1^2 + 4$ → completing the square form

$= (x-1)^2 + 3$

Range $= [3, \infty) = B$

From previous slide

(3) Find the range of

$$f(x) = -3 - 4(x-2)^2$$

Soln:

WKT $(x-2)^2 \geq 0$

x'ly by -4

$$-4(x-2)^2 \leq 0$$

Add -3

$$-3-4(x-2)^2 \leq -3$$

$$f(x) \leq -3$$

$$\text{Range} = (-\infty, -3]$$

	Range
$\sin x$	$[-1,1]$
$\cos x$	$[-1,1]$
$\tan x$	R
$\cot x$	R
$\sec x$	$R-(-1,1)$ or $(-\infty,-1]\cup[1,\infty)$
$\operatorname{cosec} x$	$R-(-1,1)$ or $(-\infty,-1]\cup[1,\infty)$

$\Downarrow$

Trigonometric functions

$R-(-1,1)$

$\Downarrow$

$(-\infty,-1]\cup[1,\infty)$

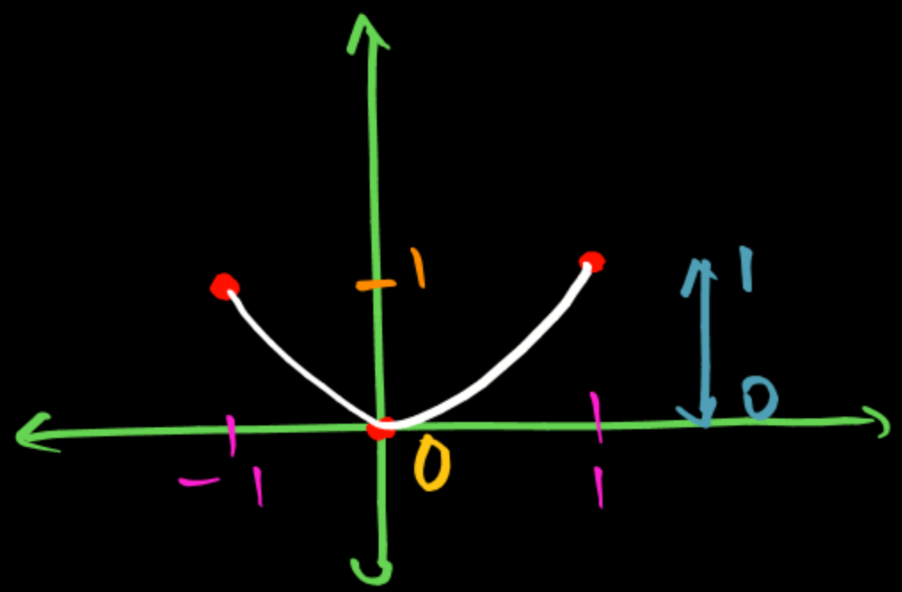
$R-[-1,1]$

$\Downarrow$

$(-\infty,-1)\cup(1,\infty)$

① if $x \in [-1,1]$

Find x^2



$x^2 \in [0,1]$

b/w $[-1,1]$

'0' is also one of the input

① if $f(x) = \sin^2 x + 3$

find Range

Soln:-

WKT

$-1 \le \sin x \le 1$

on squaring

$0 \le \sin^2 x \le 1$

Add 3

$3 \le \sin^2 x + 3 \le 4$

$3 \le f(x) \le 4$

Range = $[3, 4]$

if $t \in [2, 6]$

$t^2 \in [4, 36]$

② if $f(x) = 3 - (4 - 2\sin x)^2$

find Range

Soln:

WKT

$-1 \leq \sin x \leq 1$

x^{ly} by -2

$2 \geq -2\sin x \geq -2$

Add 4

$6 \geq 4 - 2\sin x \geq 2$

Rewrite

$2 \leq 4 - 2\sin x \leq 6$

on squaring

$4 \leq (4 - 2\sin x)^2 \leq 36$

x^{ly} by -1

$-4 \geq -(4 - 2\sin x)^2 \geq -36$

Add 3

$-1 \geq 3 - (4 - 2\sin x)^2 \geq -33$

Rewrite

$-33 \leq f(x) \leq -1$

Range

$[-33, -1]$

$[a, b]$

$\Downarrow$

$a < b$

[-1, -33]

X

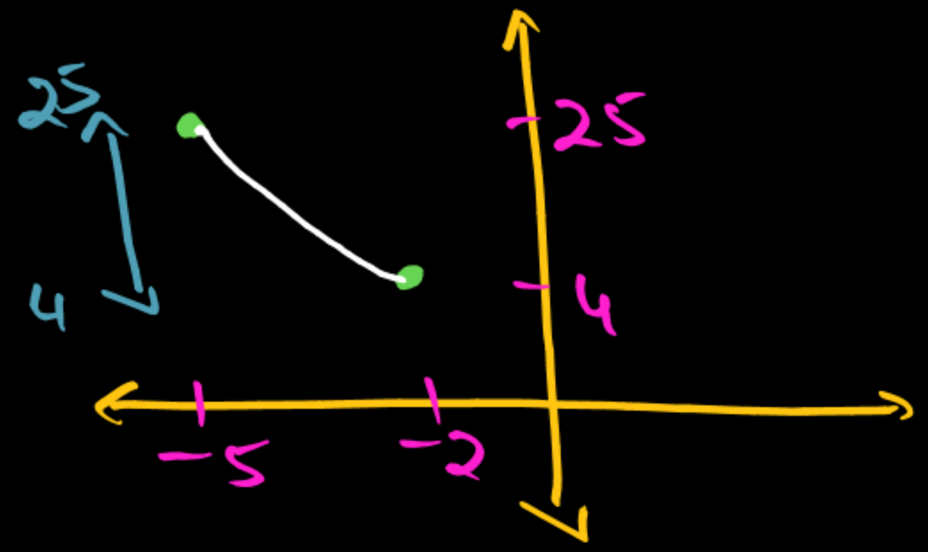
$-1 \quad k \quad -33$

(x) if $x \in [-5, -2]$

Find x^2

Soln:

$x^2 \in [4, 25]$



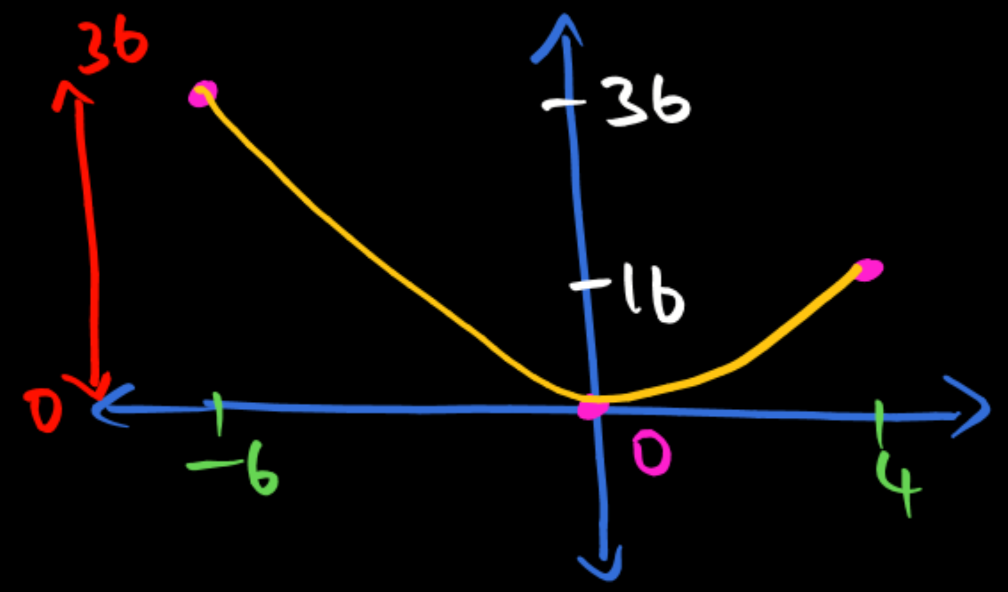
b/w -5 & -2

There is

no '0'

if $x \in [-6, 4]$

Find x^2



Ans:

$x^2 \in [0, 36]$

b/w -6 & 4

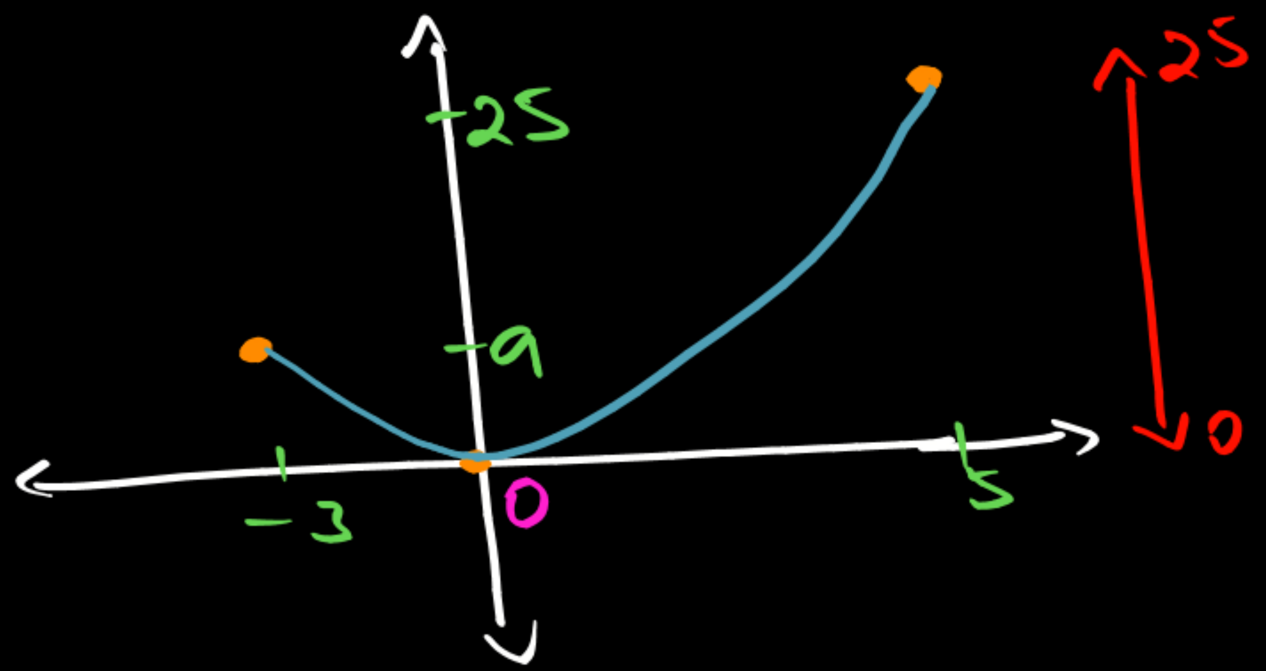
There is 0

if $x \in [-3, 5]$

Find x^2

Ans

$x^2 \in [0, 25]$



Find Range of $f(x) = (5\tan x)^2 + 4$

Soln:

WKT

$-\infty < \tan x < \infty$

$\times^{ly}$ by 5

$-\infty < 5\tan x < \infty$

on squaring

$0 \leq (5\tan x)^2 < \infty$

Add 4

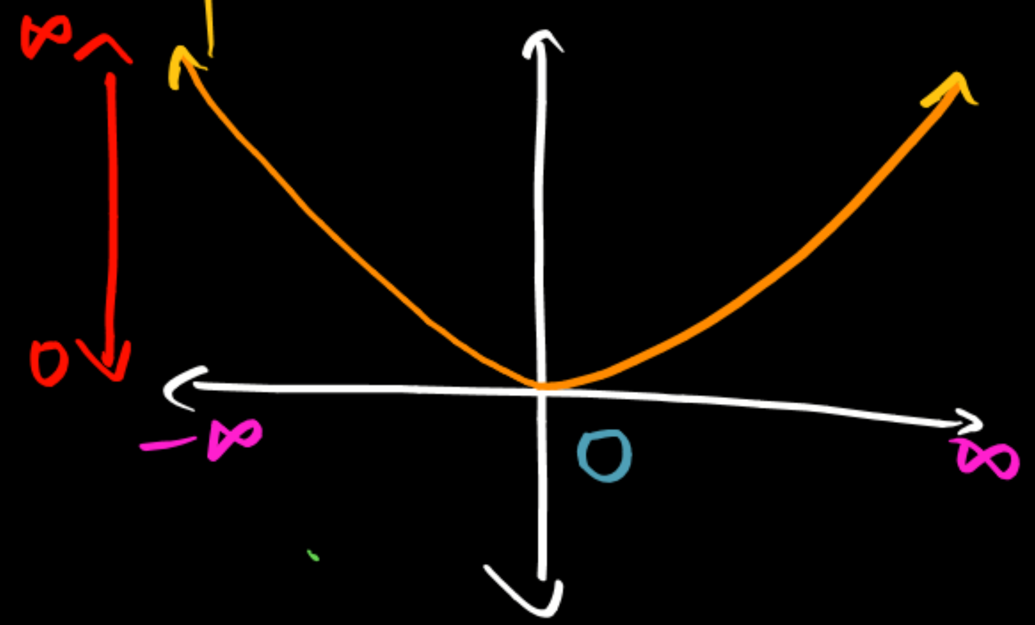
$4 \leq 4 + (5\tan x)^2 < \infty$

Range $= [4, \infty)$

if $x \in R$

$\Downarrow$

$x^2 \in [0, \infty)$

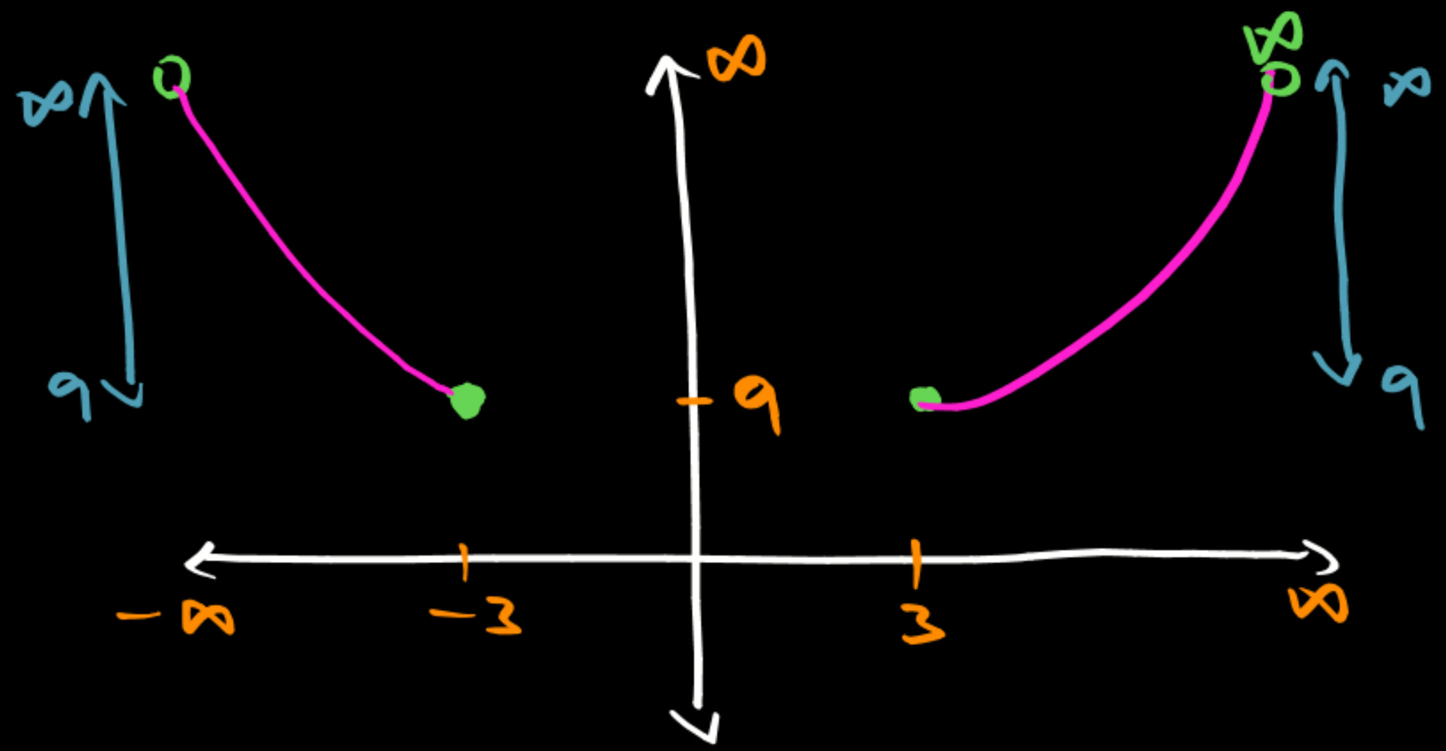


(*) if $x \in (-\infty, -3] \cup [3, \infty)$

Find x^2

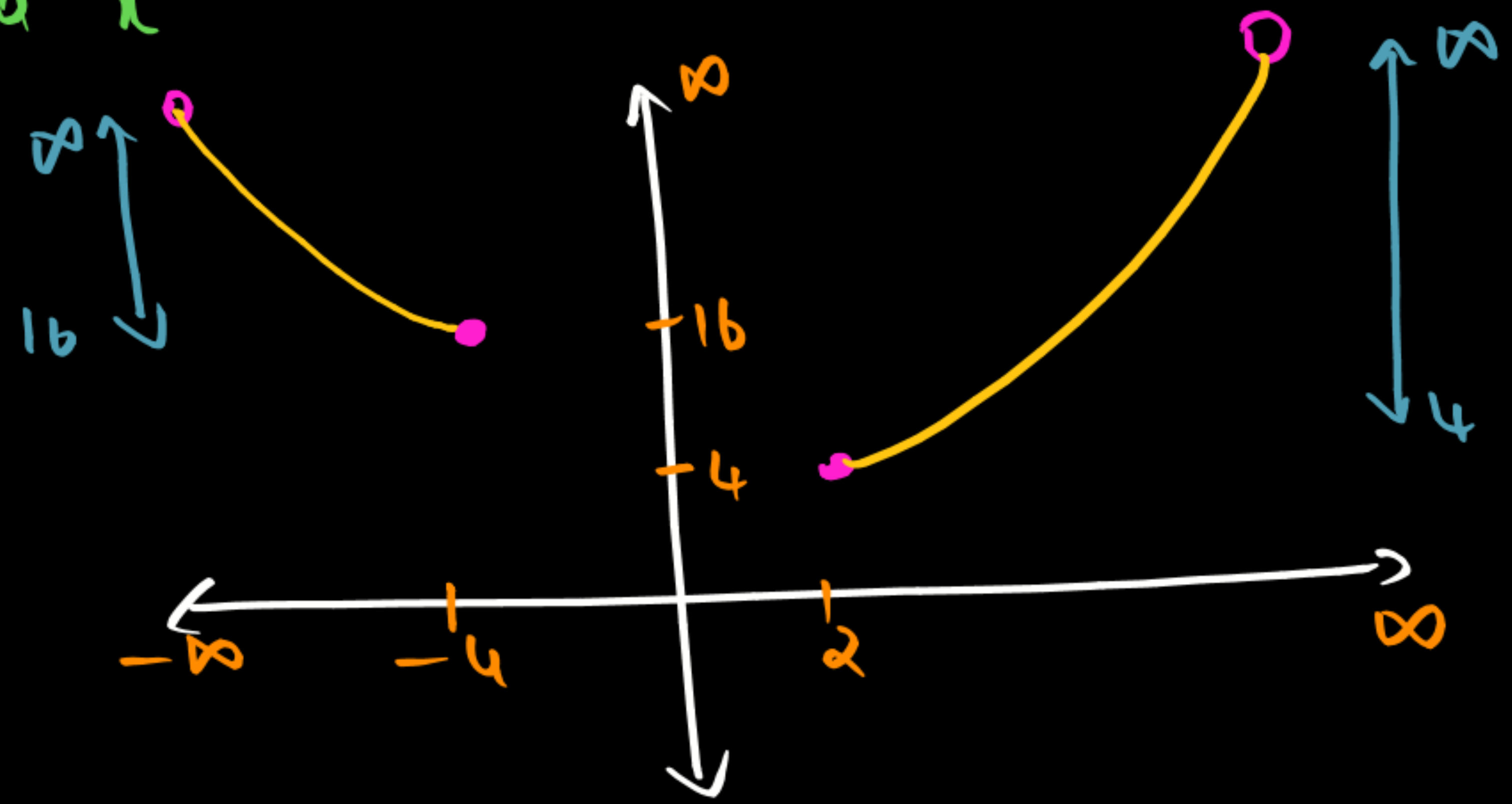
Ans

$x^2 \in [9, \infty)$



(*) if $x \in [-\infty, -4] \cup [2, \infty)$

Find x^2



$x^2 \in [16, \infty) \cup [4, \infty)$

but $[16, \infty) \subset [4, \infty)$

$\underline{x^2 \in [4, \infty)}$

if $A \subset B$

Then $A \cup B = B$

Find the range of $f(x) = \sec^2 x + 4$

Soln:-

WKT

$\sec x \in (-\infty, -1] \cup [1, \infty)$

on squaring

$\sec^2 x \in [1, \infty)$

Add 4

$\sec^2 x + 4 \in [5, \infty)$

Range $= [5, \infty)$

$\sec x \in R - (-1, 1)$

$\Downarrow$

$\sec x \in (-\infty, -1] \cup [1, \infty)$

(*) If $P = \frac{1}{5}\sin^2 x + \frac{1}{4}\cos^2 x$

Find P.

Soln:-

WKT $\cos^2 x = 1 - \sin^2 x$

$$P = \frac{1}{5}\sin^2 x + \frac{1}{4}(1 - \sin^2 x)$$

$$P = \sin^2 x\left[\frac{1}{5} - \frac{1}{4}\right] + \frac{1}{4}$$

$$P = -\frac{1}{20}\sin^2 x + \frac{1}{4}$$

WKT $-1 \le \sin x \le 1$

$$\Downarrow$$

$$0 \le \sin^2 x \le 1$$

$$0 \geq -\frac{1}{20}\sin^2 x \geq -\frac{1}{20}$$

$$\frac{1}{4} \geq \frac{1}{4} - \frac{1}{20}\sin^2 x \geq \frac{1}{4} - \frac{1}{20}$$

$$\frac{1}{4} \geq P \geq \frac{5-1}{20}$$

$$\frac{1}{4} \geq P \geq \frac{4}{20}$$

$$\frac{1}{4} \geq P \geq \frac{1}{5}$$

Rewrite

$$\frac{1}{5} \le P \le \frac{1}{4}$$

$$P \in \left[\frac{1}{5}, \frac{1}{4}\right]$$

(*) Completing the square form:-

WKT

$a^2+2ab+b^2$ $\Downarrow$ $(a+b)^2$

$a^2-2ab+b^2$ $\Downarrow$ $(a-b)^2$

Remark:-

we use completing the square when factorisation is not possible

$\Downarrow$

splitting the middle term is not possible

(1) $f(x) = x^2+4x+6$

Add & subtract

$$\left(\frac{1}{2}\times\text{coefficient of } x\right)^2$$

$$=\left(\frac{1}{2}\times 4\right)^2 = 2^2$$

$$f(x) = \underbrace{x^2+4x+2^2}_{a=x \;\&\; b=2}-2^2+6 = (x+2)^2-4+6$$

$$=(x+2)^2+2$$

(*) find the range of $f(x) = x^2 - 5x + 2$

Soln:

$$f(x) = \underbrace{x^2 - 5x + \left(\frac{5}{2}\right)^2}_{} - \left(\frac{5}{2}\right)^2 + 2$$

$a = x$

$b = \frac{5}{2}$

$$a^2 - 2ab + b^2 \Downarrow (a-b)^2$$

$$f(x) = \left(x - \frac{5}{2}\right)^2 - \frac{25}{4} + 2$$

$$= \left(x - \frac{5}{2}\right)^2 - \left(\frac{25-8}{4}\right)$$

$$= \left(x - \frac{5}{2}\right)^2 - \frac{17}{4}$$

WKT $\left(x - \frac{5}{2}\right)^2 \geq 0$ Add $-\frac{17}{4}$

$$\left(x - \frac{5}{2}\right)^2 - \frac{17}{4} \geq -\frac{17}{4}$$

$$f(x) \geq -\frac{17}{4}$$

$$\text{Range} = \left[-\frac{17}{4}, \infty\right)$$

(*) Find range of $f(x) = 8 + 10x - 2x^2$

Soln:

$$f(x) = -2\left[x^2 - 5x - 4\right]$$

$$= -2\left[x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 - 4\right]$$

$$= -2\left[\left(x - \frac{5}{2}\right)^2 - \frac{25}{4} - 4\right]$$

$$= -2\left[\left(x - \frac{5}{2}\right)^2 - \frac{41}{4}\right]$$

$$f(x) = +\frac{41}{2} - 2\left(x - \frac{5}{2}\right)^2$$

(next Page)

While you are dealing with completing the square form

$\Downarrow$

Make sure coefficient of x^2 is $+1$

$$f(x) = \frac{41}{2} - 2\left(x - \frac{5}{2}\right)^2$$

WKT

$$-2\left(x - \frac{5}{2}\right)^2 \le 0$$

$$\frac{41}{2} - 2\left(x - \frac{5}{2}\right)^2 \le \frac{41}{2}$$

$$f(x) \le \frac{41}{2}$$

$$\text{Range} = \left(-\infty, \frac{41}{2}\right]$$

(*) Find range of

$$f(x) = \sin^2 x + 4\sin x + 7$$

Soln:

Put $\sin x = t$

$$f(x) = t^2 + 4t + 7$$

$$f(x) = \underbrace{t^2 + 4t + 2^2} - 2^2 + 7$$

$$f(x) = (t+2)^2 + 3$$

$$f(x) = (\sin x + 2)^2 + 3$$

WKT

$$-1 \le \sin x \le 1$$

$$1 \le \sin x + 2 \le 3$$

$$1 \le (\sin x + 2)^2 \le 9$$

$$4 \le (\sin x + 2)^2 + 3 \le 12$$

$$4 \le f(x) \le 12$$

Range $= [4, 12]$

(*) Find range of

$$f(x) = \sin^2 x + 6\cos x - 3$$

Soln:

$$f(x) = (1-\cos^2 x) + 6\cos x - 3$$

$$= -\cos^2 x + 6\cos x - 2$$

$$= -[\cos^2 x - 6\cos x + 2]$$

$$= -[t^2 - 6t + 3^2 - 3^2 + 2]$$

$$= -[(t-3)^2 - 7]$$

$$f(x) = 7 - (\cos x - 3)^2$$

WKT

$$-1 \le \cos x \le 1$$

$$-4 \le \cos x - 3 \le -2$$

on squaring

$$4 \le (\cos x - 3)^2 \le 16$$

x'ly by -1

$$-4 \ge -(\cos x - 3)^2 \ge -16$$

Add 7

$$3 \ge 7 - (\cos x - 3)^2 + 7 \ge -9$$

⇓ rewrite

$$-9 \le f(x) \le 3$$

Range $= [-9, 3]$

PYQ
of 2022

① The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ is strictly

$\cos x > 0$, $f(x)$ is $\uparrow$ — (1) ✗ decreasing in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos x < 0$, $f(x)$ is $\downarrow$ — (2) ✗ $\uparrow$ in $\left(\pi, \frac{3\pi}{2}\right)$

$\cos x > 0$, $f(x)$ is $\uparrow$ — (3) ✗ $\downarrow$ in $\left[0, \frac{\pi}{2}\right]$

$\cos x < 0$, $f(x)$ is $\downarrow$ — (4) ✓ $\downarrow$ in $\left(\frac{\pi}{2}, \pi\right)$

$$f'(x) = 12\sin^2 x\cos x - 12\sin x\cos x + 12\cos x$$

$$= 12\cos x\left[\sin^2 x - \sin x + 1\right]$$

$$= 12\cos x\left[\sin^2 x - \sin x + \left(\tfrac{1}{2}\right)^2 - \left(\tfrac{1}{2}\right)^2 + 1\right]$$

$$= 12\cos x\underbrace{\left[\left(\sin x - \tfrac{1}{2}\right)^2 + \tfrac{3}{4}\right]}_{\text{This is always +ve}}$$

Now

if $\cos x > 0 \Rightarrow f(x)$ is $\uparrow$ since $f'(x) > 0$

if $\cos x < 0 \Rightarrow f(x)$ is $\downarrow$ since $f'(x) < 0$

* Modulus function :-

$$f(x) = |x| = \begin{cases} \underbrace{x}_{} & \text{if } x \geq 0 \\ \underbrace{-x}_{\Downarrow \text{ output}} & \text{if } \underbrace{x < 0}_{\Downarrow \text{ input}} \end{cases}$$

$\Rightarrow f(x) = x$ if $\underbrace{x \geq 0}_{}$ & $f(x) = -x$ if $\underbrace{x < 0}_{}$

↳ input = +ve or 0

↳ input = −ve

output $f(x) = x$

output $f(x) = -x$

Job of Mod func

is to convert

+ve , −ve & 0

$\Downarrow$

to a +ve value or 0

$x = 5$

$\hookrightarrow +ve$

$f(x) = x$

$f(5) = 5$

$x = -3$

$f(x) = -x$

$f(-3) = -(-3)$

$= 3$

$(-ve) \times (-ve) = +ve$

$$f(x) = |x+5| = \begin{cases} x+5 & \text{if } x+5 \geq 0 \\ & x \geq -5 \\ -(x+5) & \text{if } x+5 < 0 \\ = -x-5 & x < 5 \end{cases}$$

$f(x) = x+5$ if $x \geq -5$ & $f(x) = -x-5$
if $x < 5$

$$f(x) = |x-3| = \begin{cases} x-3 & \text{if } x-3 \geq 0 \Rightarrow x \geq 3 \\ -(x-3) = 3-x & \text{if } x-3 < 0 \Rightarrow x < 3 \end{cases}$$

$\Rightarrow x=3$ is the split point

$$f(x) = |1-x| = \begin{cases} 1-x & \text{if } 1-x \geq 0,\ 1 \geq x \Rightarrow x \leq 1 \\ -(1-x) = x-1 & \text{if } 1-x < 0,\ 1 < x \Rightarrow x > 1 \end{cases}$$

$\Rightarrow x = 1$ is the split point

PYQ 2024

$$\int_1^5 |x-3|+|1-x|\,dx$$

Split points are 1 & 3

$$\int_1^3 |x-3|+|1-x|\,dx + \int_3^5 |x-3|+|1-x|\,dx$$

$x<3$ & $x>1$ $\quad$ $x>3$ & $x>1$

$$\int_1^3 -(x-3)+(x-1)\,dx + \int_3^5 x-3+x-1\,dx$$

$$\int_1^3 2\,dx + \int_3^5 2x-4\,dx$$

$$2x\Big]_1^3 + \left[x^2-4x\right]_3^5$$

$$2(2)+(25-9)-4(5-3)$$

$$4+16-8$$

$$=16-4$$

$$=\underline{12}$$

$$|\sin x| = \begin{cases} \sin x & \text{if } \sin x \geq 0 \\ & x \in [0, \pi] \\ & \hookrightarrow 1^{st} \text{ \& } 2^{nd} \\ -\sin x & \text{if } \sin x < 0 \\ & x \in [\pi, 2\pi] \\ & \hookrightarrow 3^{rd} \text{ \& } 4^{th} \text{ Quad} \end{cases}$$

$$\cos\frac{5\pi}{4} = \cos\left(\pi + \frac{\pi}{4}\right)$$

$$= -\cos\frac{\pi}{4}$$

PYQ

if $f(x) = |\sin x|$ find $f'\left(\frac{5\pi}{4}\right)$

Soln:

At $x = \frac{5\pi}{4} = 225^\circ \in 3^{rd}$ Quad

$\Downarrow$

$\sin x < 0$

$\therefore f(x) = -\sin x$

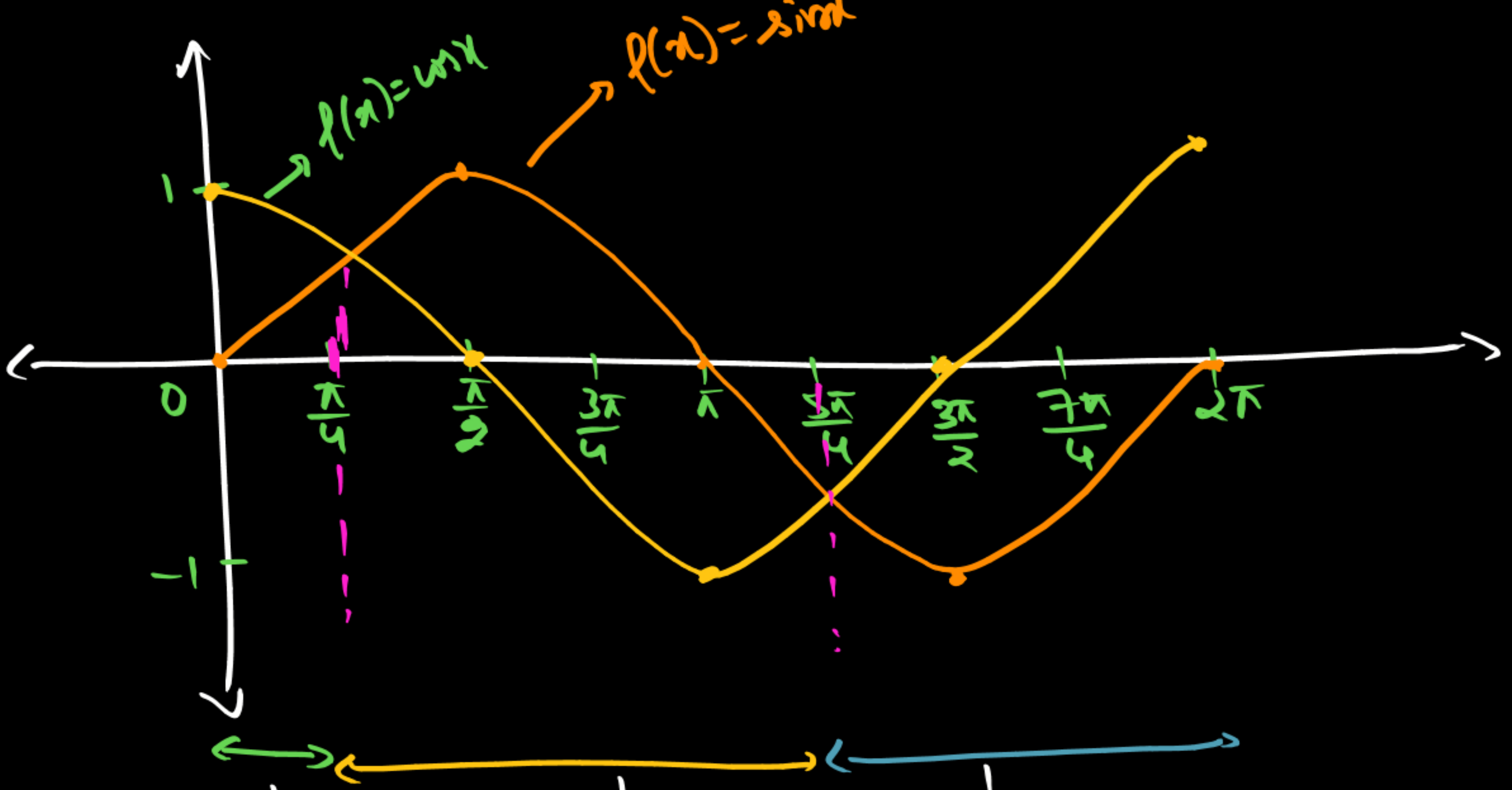
$$f'(x) = -\cos x$$

$$f'\left(\frac{5\pi}{4}\right) = -\cos\frac{5\pi}{4}$$

$$= +\cos\frac{\pi}{4} = \frac{+1}{\sqrt{2}}$$

2018 if $f(x) = |\cos x - \sin x|$ $f'\left(\frac{\pi}{6}\right)$

if $f(x) = |\sin x - \cos x|$ find $f'\left(\frac{\pi}{6}\right)$



$x \in \left(0, \frac{\pi}{4}\right)$

$\cos x > \sin x$

$x \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$\cos x < \sin x$

$x \in \left(\frac{5\pi}{4}, 2\pi\right)$

$\cos x > \sin x$

$\left(0, \frac{\pi}{4}\right)$

$\cos x > \sin x \qquad 0 > \sin x - \cos x$

$\cos x - \sin x > 0 \qquad \sin x - \cos x < 0$

$|\cos x - \sin x|$

$= \cos x - \sin x$

$|\sin x - \cos x|$

$= -(\sin x - \cos x)$

$= \cos x - \sin x$

① $f(x) = |\cos x - \sin x|$

at $x = \frac{\pi}{6} \in \left(0, \frac{\pi}{4}\right)$

$f(x) = \cos x - \sin x$

$f'(x) = -\sin x - \cos x$

$f'\left(\frac{\pi}{6}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2} = -\frac{1}{2}(1+\sqrt{3})$

② $f(x) = |\sin x - \cos x|$

at $x = \frac{\pi}{6} \in \left(0, \frac{\pi}{4}\right)$

$f(x) = \cos x - \sin x$

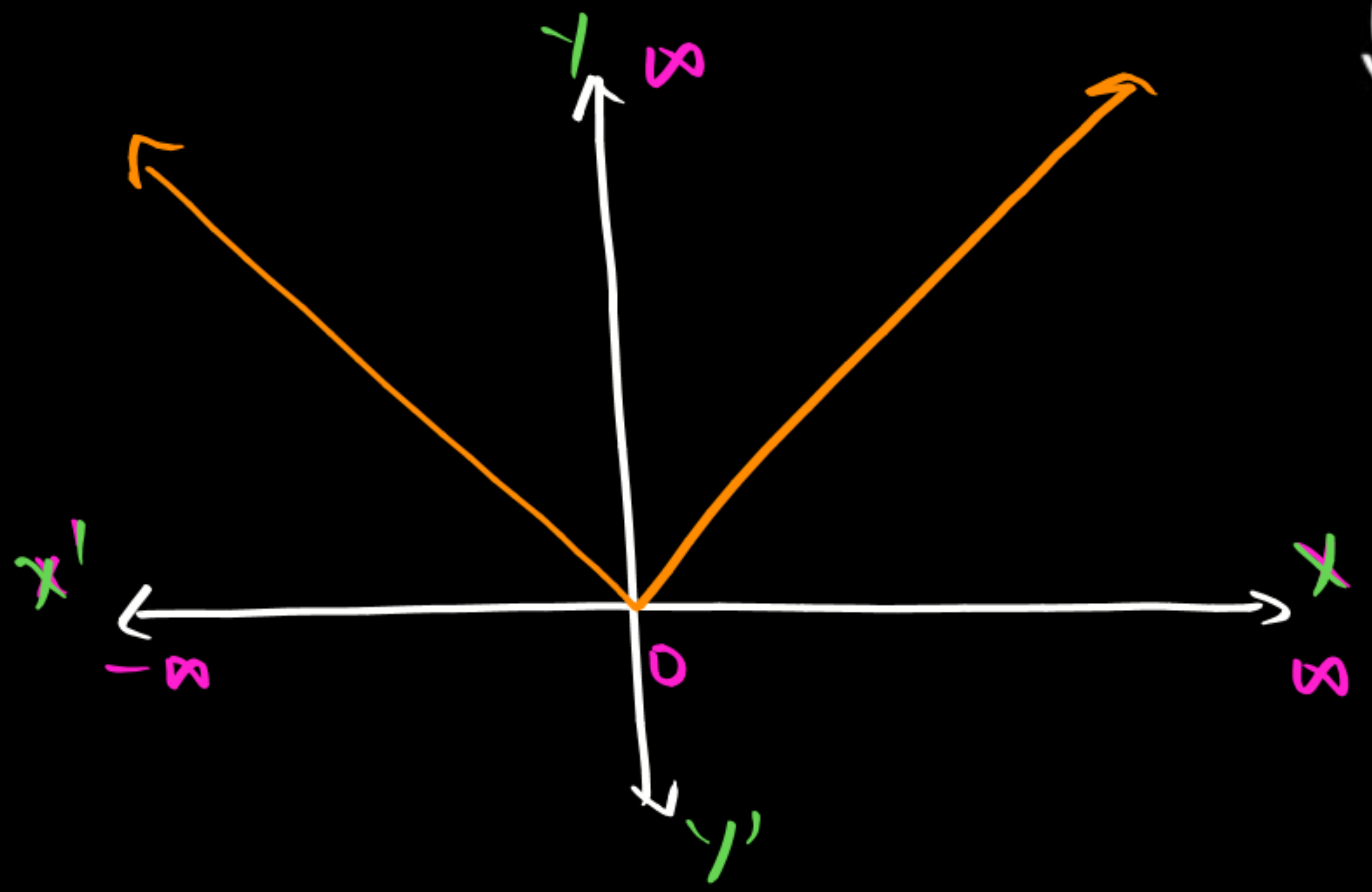
$f'(x) = -\sin x - \cos x$

$f'\left(\frac{\pi}{6}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2} = -\frac{1}{2}(1+\sqrt{3})$

* $f(x) = |x|$

domain $= R = (-\infty, \infty)$

Range $= [0, \infty)$

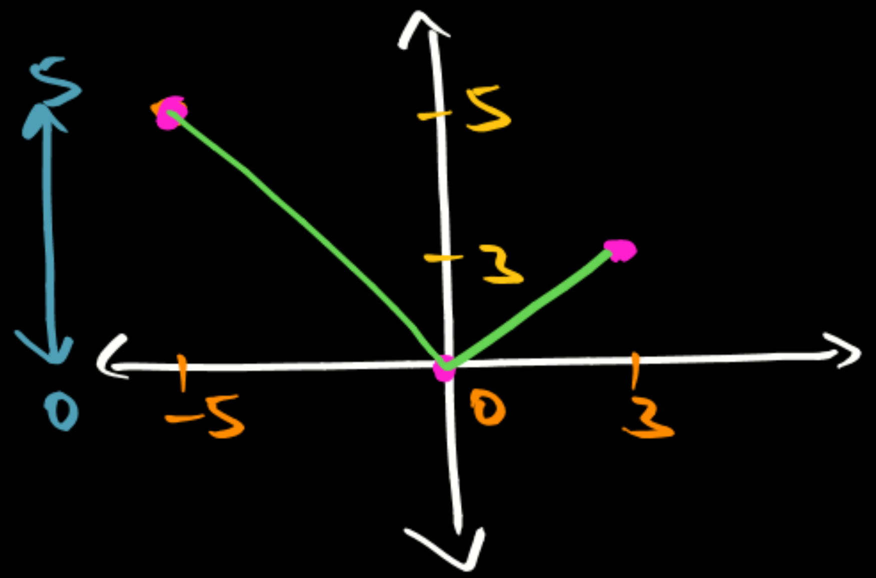


① if $x \in [-5, 3]$

Find $|x|$

Soln:

$|x| \in [0, 5]$

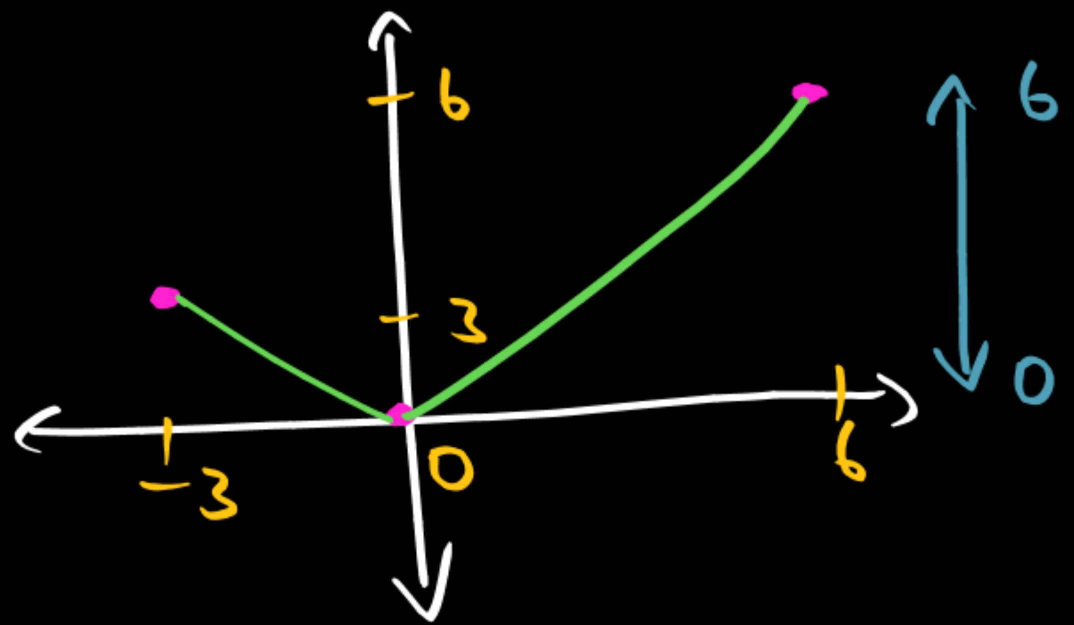


② if $x \in [-3, 6]$

Find $|x|$

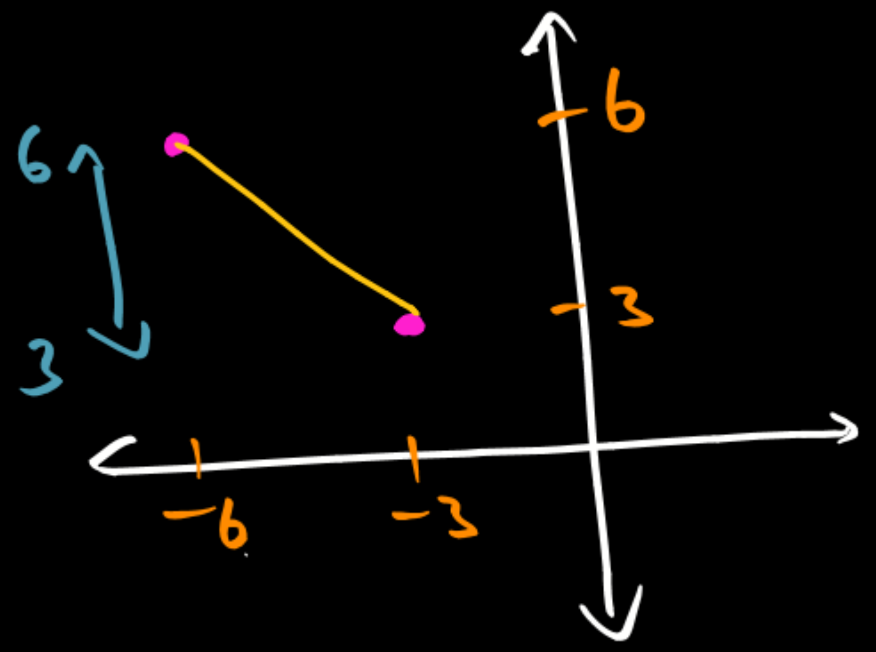
Ans:

$|x| \in [0, 6]$



③ if $x \in [-6, -3]$ $\longrightarrow$ b/w -6 & -3, '0' is NOT the input

Find $|x|$



Ans

$\therefore$

$|x| \in [3, 6]$

① Find Range of $f(x) = |-3\sin x - 4|$

<u>Soln:</u>

WKT

$$-3 \le -3\sin x \le 3$$ Add -4

$$-7 \le -3\sin x - 4 \le -1$$

Take mod

$$1 \le |-3\sin x - 4| \le 7$$

Range $= [1, 7]$

Properties:-

1. if $|x| = a$

$x = \pm a$

Ex:-

1. if $|x| = 3$

Find x

Ans:-

$x = \pm 3$

2. if $|x| = -3$

find x

$\Rightarrow$ Mod func cannot be -ve

Ans:-

Not Possible

$\Downarrow$

No soln for x

③ if $|x-3| = 5$

Find 'x'

Soln:

$$|x-3| = 5$$

$$x-3 = \pm 5$$

$x-3=5$ | $x-3=-5$

$x=8$ | $x=-2$

$$x = 8, -2$$

(4) $||x|-2| = 6$

Find x

Ans

$$||x|-2| = 6$$

$$|x|-2 = \pm 6$$

$$|x|-2 = 6$$

$$|x| = 8$$

$$x = \pm 8$$

$$|x|-2 = -6$$

$$|x| = -4$$

Not Possible

$$x = \pm 8$$

② $\sqrt{x^2} = |x|$

Ex:-

① $\sqrt{(x-1)^2} = |x-1|$

② $\sqrt{\sin^2 x} = |\sin x|$

$$I = \int_0^4 \sqrt{(x-2)^2}\,dx$$

$$= \int_0^4 |x-2|\,dx = \int_0^2 |x-2|\,dx + \int_2^4 |x-2|\,dx$$

$x<2$ $\quad$ $x>2$

$$= \int_0^2 -(x-2)\,dx + \int_2^4 (x-2)\,dx$$

$$= -\left[\frac{x^2}{2} - 2x\right]_0^2 + \left[\frac{x^2}{2} - 2x\right]_2^4$$

$$= -\left[2-4\right] + \left[\frac{1}{2}(16-4) - 2(2)\right]$$

$$= 2 + 6 - 4 = \underline{4}$$

③ if $|x| \le a$

$x \in [-a, a]$

Ex:-

$|x| \le 3$

$x \in [-3, 3]$

(or) if $|x| < a$

$x \in (-a, a)$

$|x| < 5$

$x \in (-5, 5)$

④ if $|x| \geq a$

$x \in (-\infty, -a] \cup [a, \infty)$

⑤ $|x| > a$

$x \in (-\infty, -a) \cup (a, \infty)$

Ex:-

$|x| \geq 3$

$\Downarrow$

$x \in (-\infty, -3] \cup [3, \infty)$

② $|x-3| > 2$ find x

$x-3 \in (-\infty, -2) \cup (2, \infty)$

Add 3

$x \in (-\infty, 1) \cup (5, \infty)$

(5) if $a \le |x| \le b$

$x \in [-b, -a] \cup [a, b]$

or if $a < |x| < b$

$x \in (-b, -a) \cup (a, b)$

Ex:- if

$4 \le x^2 \le 9$

Find x

Soln:-

$4 \le x^2 \le 9$

Take square root

$2 \le \sqrt{x^2} \le 3$

$2 \le |x| \le 3$

$2 \le |x| \le 3$

$\Downarrow$

$x \in [-3, -2] \cup [2, 3]$

⊛ greatest integer function:-

if $x \in [1, 2)$

$\Downarrow$

$[x] = 1$

if $x \in [1, 2]$

$\Downarrow$

$[x] =$

Ans

$x \in [1, 2)$

$\downarrow$

$[x] = 1$

$x = 2$

$\Downarrow$

$[x] = 2$

$[x] = 1, 2$

③ if $x \in [-6, -2]$

Find $[x]$.

Soln:

$x \in [-6, -5)$ $\rightarrow$ $[x] = -6$

$x \in [-5, -4)$ $\rightarrow$ $[x] = -5$

$x \in [-4, -3)$ $\rightarrow$ $[x] = -4$

$x \in [-3, -2)$ $\rightarrow$ $[x] = -3$

$x = -2$ $\rightarrow$ $[x] = -2$

$$[x] = -6, -5, -4, -3, -2$$

(*) if ① $[x] = 3$

Find x

Soln:-

$x \in [3,4)$

② $[x] = 3, 4$

Find x (You need to find all possible values of x)

Soln:-

$[x] = 3$ or $[x] = 4$

$\Downarrow$ $\Downarrow$

$x \in [3,4)$ $x \in [4,5)$

$[4.9] = 4$

$x \in [3,4) \cup [4,5)$

$\Downarrow$

$x \in [3,5)$

③ if $[x] = -5, -4$

find x

Soln:

$[x] = -5$ or $[x] = -4$

$\Downarrow$ $\Downarrow$

$x \in [-5, -4)$ $x \in [-4, -3)$

union

$x \in [-5, -3)$

PYQ 2024

④ if $[x]^2-5[x]+6=0$ find x

Soln:

Put $[x]=t$

$t^2-5t+6=0$

$$\begin{array}{c} +6 \\ -2 \quad -3 \end{array}$$

$(t-2)(t-3)=0$

$t=2$ (or) $t=3$

$[x]=2$ $[x]=3$

$x\in[2,3)$ $x\in[3,4)$

union

$x\in[2,4)$

⑤ if $[x]^2+8[x]+12=0$

find x.

Soln:

$t^2+8t+12=0$

$$\begin{array}{c} +12 \\ +6 \quad +2 \end{array}$$

$(t+6)(t+2)=0$

$t=-6$ $t=-2$

$[x]=-6$ $[x]=-2$

$x\in[-6,-5)$ $x\in[-2,-1)$

$x\in[-6,-5)\cup[-2,-1)$

2021

PYQ

$f(x)=\dfrac{1}{\sqrt{[x]^2-[x]-6}}$ find x

if $f(x)=\dfrac{1}{\sqrt{g(x)}}$

$\Rightarrow g(x)>0$

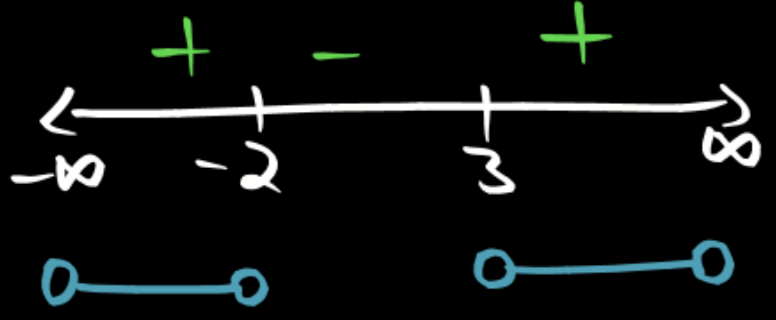
Soln: Put $[x]=t$

$t^2-t-6>0$

-6

$-3 \quad +2$

$(t-3)(t+2)>0$



$t\in(-\infty,-2)\cup(3,\infty)$

$[x]\in(-\infty,-2)\cup(3,\infty)$

next page

$[x] \in (-\infty, -2) \cup (3, \infty)$

$\Downarrow$

$[x] = -\infty, \ldots \ldots -4, -3$ or $4, 5, 6, \ldots \infty$

$(-\infty, \ldots\ldots, -2)$ $\quad$ $[4, \ldots\ldots, \infty)$

$x \in (-\infty, -2) \cup [4, \infty)$

integers in $(-\infty, -2)$

$\downarrow$

we choose only integers in the interval

$\Downarrow$

Range of $[x]$ is $\mathbb{Z}$

$[x] = -3$

$\Downarrow$

$x \in [-3, -2)$

$[x] = 4$

$x \in [4, 5)$

(*) if $[x] \in [-5, -2]$

find x

Soln:

consider integers in $[-5, -2]$

$[x] = -5, -4, -3, -2$ → consecutive integers

$[-5, \ldots\ldots, -1)$

$x \in [-5, -1)$

$[x] = -5$

$\Downarrow$

$x \in [-5, -4)$

$[x] = -2$

$x \in [-2, -1)$

④ if $[x] \in (-5,-2)$ → open interval ↳ we dont consider end point

Find x

Soln:

integers in $(-5,-2)$ are

$-4, -3$

↓

$[-4, \ldots, -2)$

$[x] = -4$

⇓

$x \in [-4, -3)$

$[x] = -3$

⇓

$x \in [-3, -2)$

$[-4, -2)$

Thank

You