ULTIMATE KCET

CRASH COURSE 2026

Mathematics | One Shot

Bayes Theorem

By – Guru sir

Topics *to be covered*

1. Bayes' Theorem
2.
3.
4.

(*) TOTAL PROBABILITY:-

If A be an event which tells us about the Job assigned and $E_1, E_2, E_3, \ldots E_n$ are the options available to complete the given Task, Then

$$P(A) = P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + \ldots + P(E_n)\,P(A|E_n)$$

(*) BAYES' THEOREM:- [Reverse Probability]

Now if any one of the options $(E_1, E_2, \ldots E_n)$ are dependent on Job A, Then

$$P(E_i/A) = \frac{P(E_i)\,P(A|E_i)}{P(A)}, \quad i = 1, 2, 3, \ldots n$$

QUESTION

#Q. A car manufacturing factory has two plants X and Y. Plant X manufactures 70% of cars and plant Y manufactures 30% of cars. 80% of cars at plant X and 90% of cars at plant Y are rated as standard quality. A car is chosen at random and is found to be standard quality. The probability that it has come from plant X is

[2021]

(A) 56/73

(B) 56/84

(C) 56/83

(D) 56/79

$P(x) = \frac{70}{100} = P(E_1)$

$P(y) = \frac{30}{100} = P(E_2)$

P(std Quality if manufactured in Plant x) $= \frac{80}{100} = P(A|E_1)$

P(— " — " — " — in plant Y) $= \frac{90}{100} = P(A|E_2)$

Let A = Car is of std. Quality

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2)$$

$$= \frac{7\cancel{0}}{10\cancel{0}}\,\frac{8\cancel{0}}{10\cancel{0}} + \frac{3\cancel{0}}{10\cancel{0}}\,\frac{9\cancel{0}}{10\cancel{0}}$$

$$= \frac{56+27}{100} = \frac{83}{100}$$

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(A)} = \frac{\frac{7\cancel{0}}{10\cancel{0}} \times \frac{8\cancel{0}}{10\cancel{0}}}{\frac{83}{100}} = \frac{56/100}{83/100} = \frac{56}{83}.$$

QUESTION

#Q. Events E_1 and E_2 form a partition of the sample space S. A is any event such that $P(E_1) = P(E_2) = \frac{1}{2}$, $P(E_2 \mid A) = \frac{1}{2}$ and $P(A \mid E_2) = \frac{2}{3}$, then $P(E_1 \mid A)$ is

[2020]

A 2/3

B 1

C 1/4

D 1/2

$$P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2)}$$

$$= \frac{\frac{1}{2}\cdot\frac{2}{3}}{\frac{1}{2}\cdot\frac{2}{3} + \frac{1}{2}\cdot\frac{2}{3}}$$

$$= \frac{\frac{1}{2}\left(\frac{2}{3}\right)}{2\left(\frac{1}{2}\left(\frac{2}{3}\right)\right)} = \frac{1}{2}$$

$$P(E_2|A) = \frac{P(E_2)\,P(A|E_2)}{P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2)}$$

$$\frac{1}{2} = \frac{\cancel{\frac{1}{2}}\cdot\frac{2}{3}}{\cancel{\frac{1}{2}}\,P(A|E_1) + \cancel{\frac{1}{2}}\cdot\frac{2}{3}}$$

$$\frac{1}{2}P(A|E_1) + \frac{1}{3} = \frac{2}{3}$$

$$P(A|E_1) = \frac{1/3}{1/2} = \frac{2}{3}$$

QUESTION

#Q. A man speaks truth 2 out of 3 times. He picks one of the natural numbers in the set $S = \{1,2,3,4,5,6,7\}$ and reports that it is even. The probability that it is actually even is

[2019]

(A) 2/5

(B) 1/10

(C) 1/5

(D) ✓ 3/5

$P(\text{Truth}) = \frac{2}{3}$

$P(\text{lie}) = \frac{1}{3}$

A = Reporting even no.

E_1 = Actual even no. $\Rightarrow P(E_1) = \frac{3}{7}$

E_2 = Actual odd no. $\Rightarrow P(E_2) = \frac{4}{7}$

$P(A|E_1) = P(\text{Truth}) = \frac{2}{3}$

$P(A|E_2) = P(\text{lie}) = \frac{1}{3}$

$$P(A) = \frac{3}{7}\frac{2}{3} + \frac{4}{7}\frac{1}{3}$$

$$P(A) = \frac{10}{21}$$

$$P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(A)} = \frac{\frac{3}{\cancel{7}}\frac{2}{\cancel{3}}}{\frac{10}{\cancel{21}}} = \frac{3\times 2}{10} = \frac{3}{5}$$

QUESTION

$S = \{1, 2, 3, 4, \underline{5, 6}\}$

#Q. A man is known to speak the truth 3 out of 5 times. He throws a dice and reports that it is a number greater than 4. Find the probability that it is actually a number greater than 4.

(A) 2/7

(B) 4/7

(C) 3/7

(D) 5/7

$P(\text{Truth}) = \frac{3}{5}$

$P(\text{lie}) = \frac{2}{5}$

$A \to$ Reporting no greater than 4

$E_1 \to$ Actual no is greater than 4 $\Rightarrow P(E_1) = \frac{2}{6}$

$E_2 \to$ Actual no is not greater than 4 $\Rightarrow P(E_2) = \frac{4}{6}$

$P(A|E_1) = P(\text{Truth}) = \frac{3}{5}$

$P(A|E_2) = P(\text{lie}) = \frac{2}{5}$

$$P(A) = \frac{2}{6}\frac{3}{5} + \frac{4}{6}\frac{2}{5}$$

$$P(A) = \frac{14}{30}$$

$$P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(A)} = \frac{\frac{2}{\cancel{6}}\frac{3}{\cancel{5}}}{\frac{14}{\cancel{30}}} = \frac{2\times 3}{14} = \frac{3}{7}$$

QUESTION

#Q. A man is known to speak the truth 3 out of 5 times. He throws a die and reports that it is '1'. Find the probability that it is actually 1 .

(A) 1/13

(B) 2/13

(C) 3/13 ✓

(D) 4/13

$P(A|E_1) = \frac{3}{5}$

$P(A|E_2) = \frac{2}{5}$

$A \rightarrow$ Reporting 1

$E_1 \rightarrow$ Actual 1 $\Rightarrow P(E_1) = \frac{1}{6}$

$E_2 \rightarrow$ Actual not 1 $\Rightarrow P(E_2) = \frac{5}{6}$

$P(A) = \frac{1}{6}\frac{3}{5} + \frac{5}{6}\frac{2}{5} = \frac{13}{30}$

$P(E_1|A) = \frac{\frac{1}{\cancel{6}}\frac{3}{\cancel{5}}}{\frac{13}{\cancel{30}}} = \frac{3}{13}$

QUESTION

1 → 2headed → $P(H) = 1$ & $P(T) = 0$
1 → Biased → $P(H) = \frac{75}{100} = \frac{3}{4}$ & $P(T) = \frac{1}{4}$
1 → Biased → $P(H) = \frac{3}{5}$ & $P(T) = \frac{40}{100} = \frac{2}{5}$

#Q. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows head. What is the probability that it was the two-headed coin?

(A) 1/47

(B) 2/47

(C) 10/47

(D) 20/47 ✓

A → coin shows head

E_1 = 2 headed coin

E_2 = 1st Biased coin

E_3 = 2nd Biased coin

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

$P(A|E_1) = 1$

$P(A|E_2) = \frac{3}{4}$

$P(A|E_3) = \frac{3}{5}$

$$P(A) = \frac{1}{3}(1) + \frac{1}{3}\left(\frac{3}{4}\right) + \frac{1}{3}\left(\frac{3}{5}\right) = \frac{1}{3}\left[1 + \frac{3}{4} + \frac{3}{5}\right] = \frac{1}{3}\left[\frac{20+15+12}{20}\right]$$

$$P(A) = \frac{47}{3(20)}$$

$$P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(A)} = \frac{\cancel{\frac{1}{3}}(1)}{\cancel{\frac{1}{3}}\left(\frac{47}{20}\right)} = \frac{20}{47}$$

QUESTION

I: 3R, 4B, 3W II: 2R, 5B, 2W

#Q. Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is:

A) 4/9

B) 5/18 ✓

C) 1/6

D) 3/10

I: 3R, 4B, 3W

Red $P(E_1) = \frac{3}{10}$ → new II: $2+1=3R$, $5B$, $2W$; $P(A|E_1) = \frac{5}{10}$

Black $P(E_2) = \frac{4}{10}$ → new II: $2R$, $5+1=6B$, $2W$; $P(A|E_2) = \frac{6}{10}$

White $P(E_3) = \frac{3}{10}$ → new II: $2R$, $5B$, $2+1=3W$; $P(A|E_3) = \frac{5}{10}$

A → Drawing Black Ball

E_1 → 1st Ball transferred is Red

E_2 → " " — Black

E_3 → " " — White

$$P(A) = \frac{3}{10}\frac{5}{10} + \frac{4}{10}\frac{6}{10} + \frac{3}{10}\frac{5}{10} = \frac{15+24+15}{100} = \frac{54}{100}$$

$$P(E_1|A) = \frac{\frac{3}{\cancel{10}}\left(\frac{5}{\cancel{10}}\right)}{\frac{54}{\cancel{100}}} = \frac{\overset{1}{\cancel{3}}\times 5}{\underset{18}{\cancel{54}}} = \frac{5}{18}$$

QUESTION

$\boxed{\begin{matrix} 3G \\ 7W \end{matrix}}$

#Q. Abagcontains 3 greenand 7 white balls. Two balls are drawn one by one at random without replacement. If the second ball drawn is green, what is the probability that the first ball drawn is also green?

(A) 5/9

(B) 4/9

(C) 2/9 ✓

(D) 8/9

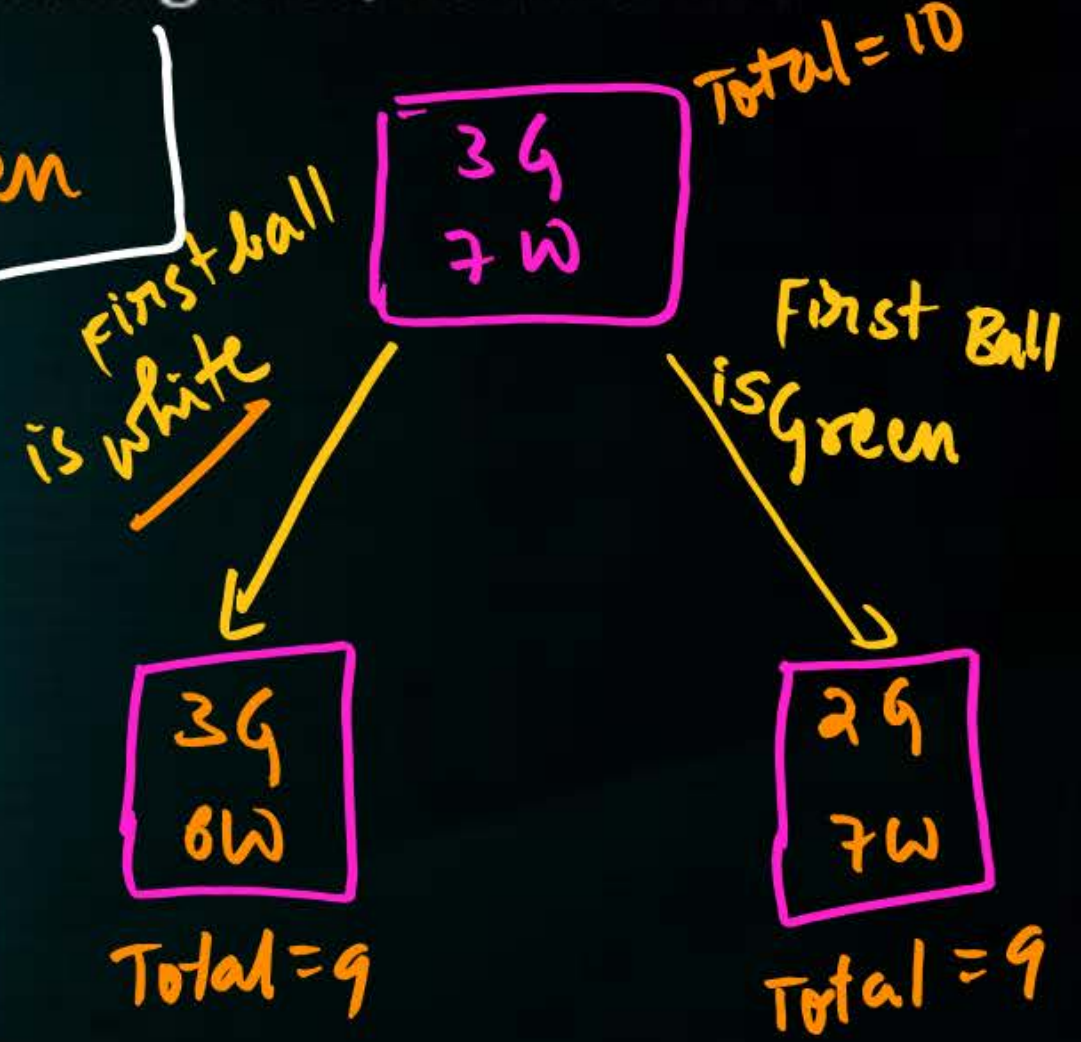
A → Second ball drawn is green

E_1 → First ball drawn is white

$P(E_1) = \frac{7}{10}$ | $P(A|E_1) = \frac{3}{9}$

E_2 → First ball drawn is Green

$P(E_2) = \frac{3}{10}$ | $P(A|E_2) = \frac{2}{9}$



$$P(A) = \frac{7}{10}\frac{3}{9} + \frac{3}{10}\frac{2}{9} = \frac{21+6}{90} = \frac{27}{90}$$

$$P(E_2|A) = \frac{P(E_2)\,P(A|E_2)}{P(A)} = \frac{\frac{3}{\cancel{10}}\left(\frac{2}{9}\right)}{\frac{27}{\cancel{90}}} = \frac{\cancel{3}\times 2}{\cancel{27}\;9} = \frac{2}{9}$$

QUESTION

#Q. A card is lost from a pack of 52 playing cards. From the remainder of the pack, one card is drawn and is found to be a spade. The probability, that the missing card is a spade, is

A → Drawing spade is my Job

(A) 5/17

(B) ✓ 4/17

(C) 3/17

(D) 2/17

E_1 → Lost card is spade

$P(E_1) = \frac{13}{52} = \frac{1}{4}$ | $P(A|E_1) = \frac{12}{51}$

E_2 → Lost card is not spade (Remaining)

$P(E_2) = \frac{39}{52} = \frac{3}{4}$ | $P(A|E_2) = \frac{13}{51}$

$52(\underset{\text{spade}}{13} + 39)$

Lost card is spade

Lost card is not spade

$51(\underset{\text{spade}}{12} + 39)$

$51(\underset{\text{spade}}{13} + 38)$

$$P(A) = \frac{1}{4}\left(\frac{12}{51}\right) + \frac{3}{4}\left(\frac{13}{51}\right) = \frac{12+39}{4\times 51} = \frac{51}{4\times 51}$$

$$P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(A)} = \frac{\cancel{\frac{1}{4}}\left(\frac{12}{\cancel{51}}\right)}{\frac{51}{\cancel{4\times 51}}} = \frac{12}{51} = \frac{4}{17}$$

QUESTION

#Q. A person draws two cards successively without replacement from a pack of 52 cards. He tells that both cards are aces. What is the probability that both are aces, if there are 60% chances that he speaks truth?

(A) 2/443

(B) 1/443

(C) 4/443

(D) 3/443

A → Reporting both are Ace Cards

$P(\text{Truth}) = \frac{60}{100}$

$P(\text{Lie}) = \frac{40}{100}$

E_1 → Actual both are Ace cards

$P(E_1) = \frac{\cancel{4}^{1}}{\cancel{52}_{13}} \cdot \frac{\cancel{3}^{1}}{\cancel{51}_{17}} = \frac{1}{221}$ | $P(A|E_1) = P(\text{Truth}) = \frac{60}{100}$

E_2 → Actual both are not Ace cards

$P(E_2) = 1 - P(E_1) = \frac{220}{221}$ | $P(A|E_2) = P(\text{Lie}) = \frac{40}{100}$

$$P(A) = \frac{1}{221}\left(\frac{6\cancel{0}}{10\cancel{0}}\right) + \frac{220}{221}\left(\frac{4\cancel{0}}{10\cancel{0}}\right) = \frac{6+220(4)}{221(10)}$$

$$P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(A)} = \frac{\cancel{\frac{1}{221}}\left(\frac{6}{\cancel{10}}\right)}{\frac{6+(220)4}{\cancel{221(10)}}} = \frac{6}{6+880} = \frac{6}{886} = \frac{3}{443}$$

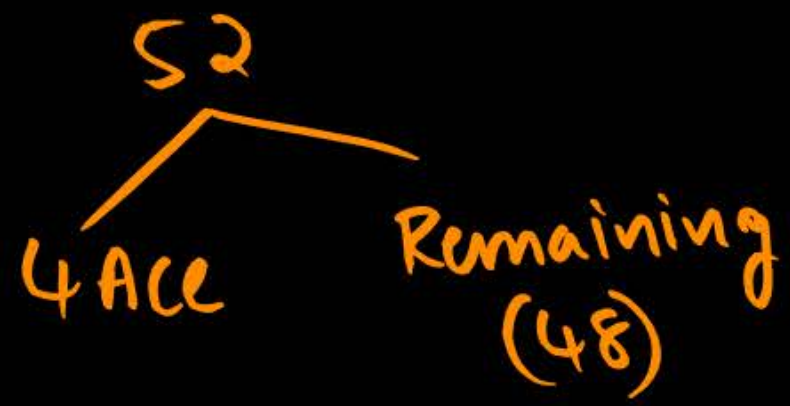
(1st one Ace, 2nd one not Ace) or (1st one not Ace, 2nd one Ace) or (Both are not Ace)

$$\frac{4}{52}\,\frac{48}{51} + \frac{48}{52}\,\frac{4}{51} + \frac{48}{52}\,\frac{47}{51}$$

$$\frac{\cancel{48}^{\cancel{12}^{4}}}{\underset{13}{\cancel{52}} \times \underset{17}{\cancel{51}}}\left[4+4+47\right]$$

$$\frac{4}{13 \times 17}\left[55\right] = \frac{220}{221}$$

$52 = 4 \times 13$



QUESTION

$$52$$
$$\text{Heap I} \swarrow \quad \searrow \text{Heap II}$$
$$26R \qquad 26B$$

#Q. Cards of an ordinary deck of playing cards are placed into two heaps. Heap-I consists of all the red cards and Heap-II consists of all the black cards. A heap is chosen at random and a card is drawn, if the card drawn is found to be a king, find the probability that the card drawn is from the Heap-II.

(A) 2/5

(B) 1/5

(C) 1/3

(D) 1/2

A → Picking King card

E_1 → Card is Picked from Heap I (Red)

$P(E_1) = \frac{26}{52} = \frac{1}{2}$ | $P(A|E_1) = \frac{2}{26}$

E_2 → Card is Picked from Heap II (Black)

$P(E_2) = \frac{26}{52} = \frac{1}{2}$ | $P(A|E_2) = \frac{2}{26}$

$$P(A) = \frac{1}{2}\frac{2}{26} + \frac{1}{2}\frac{2}{26} = \frac{4}{52}$$

$$P(E_2|A) = \frac{\frac{1}{2}\left(\frac{2}{26}\right)}{4/52} = \frac{2}{4} = \frac{1}{2}$$

QUESTION

#Q. Suppose 5 men out of 100 and 25 women out of 1000 are good orator. An orator is choosen at random. Find the probability that a male person is selected. Assume that there are equal number of men and women. → $P(\text{Men}) = \frac{1}{2}$

$P(\text{women}) = \frac{1}{2}$

Good orator

$P(\text{Men}) = \frac{5}{100} = \frac{50}{1000}$

Good orator

$P(\text{women}) = \frac{25}{1000}$

A → Good orator is selected

E_1 → Men is selected

$P(E_1) = \frac{1}{2}$ | $P(A|E_1) = \frac{50}{1000}$

E_2 = Women is selected

$P(E_2) = \frac{1}{2}$ | $P(A|E_2) = \frac{25}{1000}$

(A) 1/3

(B) 1/2

(C) 2/3 ✓

(D) None of these

$$P(A) = \frac{1}{2}\frac{50}{1000} + \frac{1}{2}\frac{25}{1000} = \frac{75}{2(1000)}$$

$$P(E_1|A) = \frac{\cancel{\frac{1}{2}}\left(\frac{50}{\cancel{1000}}\right)}{\frac{75}{\cancel{2(1000)}}} = \frac{50}{75} = \frac{2}{3}.$$

QUESTION

#Q. In a certain college, 4% of the men and 1% of the women are taller than 1.8 meters. Also 60% of the students are women. If a student selected at random is found to be taller than 1.8 meters, then the probability that the student being a woman is

A) 3/11

B) 5/11

C) 6/11

D) 8/11

QUESTION

#Q. Of the students in a school, it is known that 30% has 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he has A grade. What is the probability that the student has 100% attendance?

A) 1/2

B) 3/4

C) 2/3

D) 5/4

QUESTION

#Q. In shop A, 30 tin pure ghee and 40 tin adulterated ghee are kept for sale while in shop B, 50 tin pure ghee and 60 tin adulterated ghee are there. One tin of ghee is purchased from one of the shop randomly and it is found to be adulterated. Find the probability that it is purchased from shop B.

(A) $22/43$

(B) $21/43$

(C) $23/43$

(D) $24/43$

QUESTION

HW

#Q. Bag A contains 3 white and 2 black balls. Bag B contains 2 white and 2 black balls. One ball is drawn at random from A and transferred to B. One ball is selected at random from B and is found to be white. Find the probability that the transferred ball is white.

(A) 6/13

(B) 7/13

(C) 8/13

(D) 9/13

QUESTION

HW

#Q. There are three urns containing 2 white and 3 black balls, 3 white and 2 black balls and 4 white and 1 black ball, respectively. There is an equal probability of each urn being chosen. A ball is drawn at random from the chosen urn and it is found to be white. Find the probability that the ball drawn was from the second urn.

A. ✓ 1/3

B. 2/3

C. 1/2

D. 4/3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