ULTIMATE KCET

CRASH COURSE 2026

Mathematics | Lecture – 01

Differential Equation & Application of Derivatives

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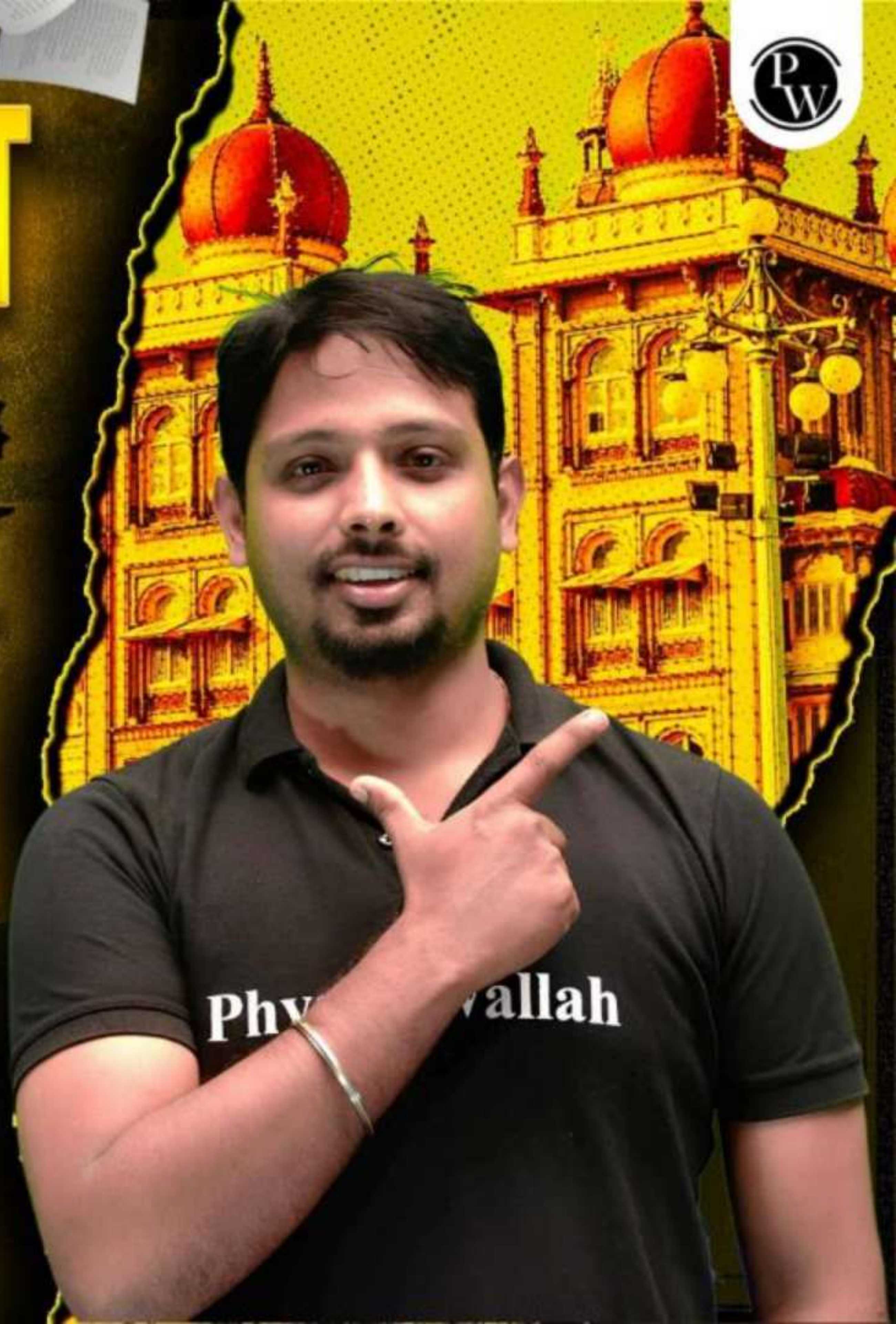
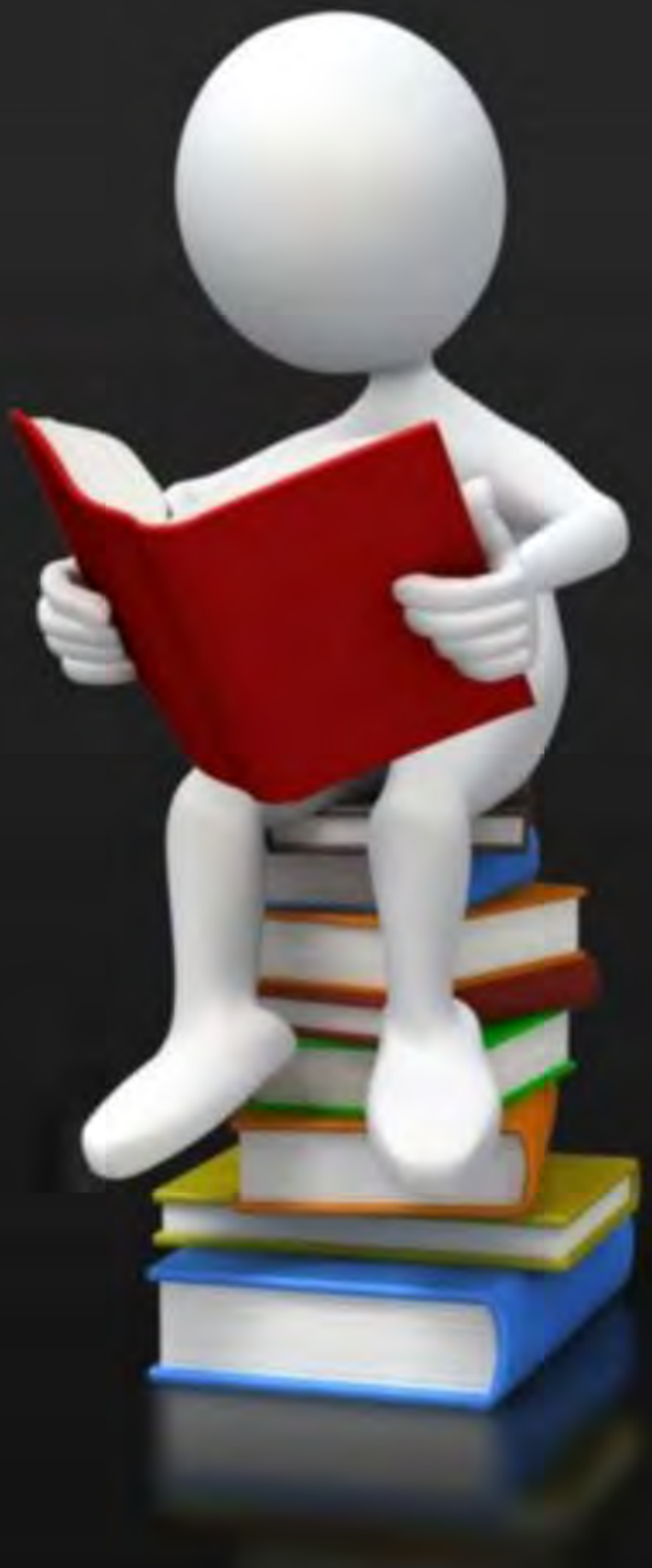


Recap of previous lecture

1. AOI
2.
3.
4.

Topics *to be covered*

1. D.E
   - Order & Degree
   - variable separable
   - Integrating Factor
   - LDE
2. AOD
3.
4.



(*) **Order:-** The ordered of the highest ordered derivation of a D.E

⇓

Here we check in a eq^n, How many times it is differentiated.

The Highest derivative will be order

Note:-

$$y' = \frac{dy}{dx} = y_1 = f'(x)$$

$$y'' = \frac{d^2y}{dx^2} = y_2 = f''(x)$$

Degree :-

The Degree of a DE is the Highest Power or exponent of highest ordered derivative of a DE, after the DE is free from fractional Powers.

Ex: $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^5 + \sin x = 0$

$\Downarrow$

order $= 2$

Degree $= 3$

(2) $\frac{d^2y}{dx^2} + \cos x = \sqrt{\frac{dy}{dx} + 2}$

$\frac{d^2y}{dx^2} + \cos x = \left(\frac{dy}{dx} + 2\right)^{1/2}$ → Fractional Power

Square on BS.

$\left(\frac{d^2y}{dx^2} + \cos x\right)^2 = \frac{dy}{dx} + 2$

$\left(\frac{d^2y}{dx^2}\right)^2 + \cos^2 x + 2\cos x\left(\frac{d^2y}{dx^2}\right) = \frac{dy}{dx} + 2$

Order = 2

Degree = 2

(3) $$\frac{d^2y}{dx^2}+2=\sqrt{\left(\frac{d^2y}{dx^2}\right)^2+\left(\frac{dy}{dx}\right)}$$

on squaring

$$\cancel{\left(\frac{d^2y}{dx^2}\right)^2}+4+4\frac{d^2y}{dx^2}=\cancel{\left(\frac{d^2y}{dx^2}\right)^2}+\frac{dy}{dx}$$

$$4+4\frac{d^2y}{dx^2}=\frac{dy}{dx}$$

order = 2

Degree = 1

* Polynomial eq^n in 'x' :-

⇓

unknown variable will be x.

⇓

we finally find value of 'x' to make LHS = RHS

Ex: $x^2 + 3x + 2 = 0$

$$\underbrace{\cos\left(\frac{dy}{dx}\right),\ e^{dy/dx},\ \log\left(\frac{d^2y}{dx^2}\right)}$$

⇓

These are not Polynomial in derivatives.

(*) **Note:-** The Degree of a DE is defined only if the given DE is a Polynomial in Derivatives

Ex:

① $\frac{d^2y}{dx} + \cos\left(\frac{dy}{dx}\right) = 2$ $\Rightarrow$ Order $= 2$

Degree $=$ ND

② $\frac{d^2y}{dx^2} + \cos y = 2$

③ $\frac{d^2y}{dx^2} + \cos x = 2$

Order $= 2$

Degree $= 1$

Ex:- (1) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + \sin x = \cos x$

$\downarrow$

order = 2

(2) $\frac{d^3y}{dx^3} + \left(\frac{d^2y}{dx^2}\right)^5 + \frac{dy}{dx} = 0$

$\downarrow$

order = 3

QUESTION

#Q. The order and degree of the differential equation $y = \frac{dp}{dx}x + \sqrt{a^2p^2 + b^2}$ where $p = \frac{dy}{dx}$ (here a and b are arbitrary constants) respectively are [2010]

(A) 1, 1

(B) 2, 2

(C) 2, 1

(D) 1, 2

$$\frac{dp}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$\frac{dp}{dx} = \frac{d^2y}{dx^2}$$

$$\left(y - \frac{dp}{dx}x\right)^2 = a^2\left(\frac{dy}{dx}\right)^2 + b^2$$

$$y^2 + \left(\frac{d^2y}{dx^2}\right)^2x^2 - 2yx\frac{d^2y}{dx^2} = a^2\left(\frac{dy}{dx}\right)^2 + b^2$$

$O = 2$

$D = 2$

QUESTION

#Q. If m and n are degree and order of $(1+{y_1}^2)^{2/3}=y_2$, then the value of $\frac{m+n}{m-n}$ is [2011]

$(1+{y_1}^2)^{2/3}=y_2$

cube on BS

$(1+{y_1}^2)^2=(y_2)^3$

$\downarrow$

$O=2=n$

$D=3=m$

$\frac{3+2}{3-2}=5$

(A) 3

(B) 4

(C) 5

(D) 2

QUESTION

#Q. If 'm' and 'n' are the order and degree of the differential equation $(y'')^5 + 4\cdot\frac{(y'')^3}{y'''} + y''' = \sin x$, then [2013]

(A) $m = 3, n = 5$

(B) $m = 3, n = 1$

(C) $m = 3, n = 3$

(D) $m = 3, n = 2$

$(y'')^5 + 4\frac{(y'')^3}{y'''} + y''' = \sin x$ (Take y''' as LCM)

$(y'')^5 y''' + 4(y'')^3 + (y''')^2 = (\sin x)y'''$

$\downarrow$

$O = 3 = m$

$D = 2 = n$

QUESTION

#Q. The order & degree of the differential equation $y = x\frac{dy}{dx} + \frac{2}{dy/dx}$ is [2014]

$$y = xy_1 + \frac{2}{y_1}$$ Take y_1 as LCM

$$yy_1 = x(y_1)^2 + 2$$

$O = 1$

$D = 2$

A 1, 2

B 1, 3

C 2, 1

D 1, 1

QUESTION

#Q. Find the order and degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2+\underline{\sin\left(\frac{dy}{dx}\right)}\right]^{3/4}=\frac{d^2y}{dx^2} \text{ is}$$ [2016]

→ Raise to the Power 4 on B.S.

↳ Degree = N.D

(A) order = 2, degree = 3

$$[1+(y_1)^2+\sin\left(\frac{dy}{dx}\right)]^3=\left(\frac{d^2y}{dx^2}\right)^4$$

(B) order = 2, degree = 4

D = N.D   O = 2

(C) order = 2, degree = 3/4

(D) order = 2, degree = not defined

QUESTION

#Q. The degree and order of the differential equation $\frac{d^2y}{dx^2} = \sqrt[3]{1 + \left(\frac{dy}{dx}\right)^2}$ respectively are [2018]

(A) 2 and 3

(B) 3 and 2

(C) 2 and 2

(D) 3 and 3

Raise to the Power 3 on BS

$(y_2)^3 = 1 + (y_1)^2$

$\downarrow$

$O = 2$

$D = 3$

QUESTION

#Q. The sum of the degree and order of the differential equation $(1 + y_1^2)^{2/3} = y_2$ is [2022]

A 4

B 5

C 6

D 7

$(1+y_1^2)^{2/3} = y_2$

cube on B S

$(1+y_1^2)^2 = (y_2)^3$

O = 2

D = 3

Sum = 5

In General soln

⇓

order of a DE

= No of Arbitrary constants

In Particular soln :

⇓

No of Arbitrary constants

= 0

Solve $\dfrac{dy}{dx} = \dfrac{x}{y} \Rightarrow \boxed{\text{order} = 1}$

$$y\,dy = x\,dx$$

$$\int y\,dy = \int x\,dx$$

$$\frac{y^2}{2} = \frac{x^2}{2} + c \rightarrow \text{general soln}$$

No of Arbitrary constants = 1

Solve $\frac{dy}{dx} = \frac{x}{y}$

given $y(1) = 4$ (conditions are given)

$x = 1 \quad y = 4$

$y(x_0) = y_0$

$\Downarrow$

we call this as initial conditions

Soln:

$y\,dy = x\,dx$

$\int y\,dy = \int x\,dx$

$\frac{y^2}{2} = \frac{x^2}{2} + c \rightarrow$ G.S

Here $x = 1$ & $y = 4$

$\frac{16}{2} = \frac{1}{2} + c$

$c = \frac{16}{2} - \frac{1}{2} = \frac{15}{2}$

$\therefore$ The particular soln is

$\frac{y^2}{2} = \frac{x^2}{2} + \frac{15}{2}$

$y^2 = x^2 + 15 \rightarrow$ Particular soln

* If G.S

$$y = c_1 e^{x} + c_2 e^{-x}$$

find order

Ans:-

Arbitrary constants are c_1 & c_2

$\therefore$ order $= 2$

* If G.S is

$$y = (c_1 + c_2) e^{x} + c_3 e^{-x}$$

find order

Soln:-

$$y = A e^{x} + c_3 e^{-x}$$

where $A = c_1 + c_2$

Arbitrary constants are A, c_3

$\therefore$ order $= 2$

QUESTION

#Q. The degree of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^2=\frac{d^2y}{dx^2} \text{ is}$$

[2017]

O = 2

D = 1

(A) 1

(B) 4

(C) 2

(D) 3

QUESTION

$x^{m+n} = x^m \cdot x^n$ | $e^{c_2+x} = e^{c_2} \cdot e^x$

#Q. The order of the differential equation $y = C_1 e^{C_2+x} + C_3 e^{C_4+x}$ is [2019]

A 1

B 3

C 2

D 4

$y = c_1 \cdot e^{c_2} e^x + c_3 e^{c_4} e^x$

$= e^x [c_1 e^{c_2} + c_3 e^{c_4}]$

$y = Ae^x$ where $A = c_1 e^{c_2} + c_3 e^{c_4}$

Arbitrary constant $= A$

$\downarrow$

order $= 1$

QUESTION

#Q. The order of the differential equation obtained by eliminating arbitrary constants in the family of curves $c_1 y = (c_2 + c_3)e^{x+c_4}$ is [2020]

(A) 2

(B) 3

(C) 4

(D) ✓ 1

$$c_1 y = (c_2+c_3)e^{c_4} \; e^x$$

$$y = \frac{(c_2+c_3)e^{c_4}}{c_1} \; e^x$$

$$y = Ae^x \quad \text{where } A = \frac{(c_2+c_3)e^{c_4}}{c_1}$$

QUESTION

Fractional Powers

#Q. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3}=4\frac{d^2y}{dx^2}$ are respectively

Cube on B S

$(1+3y_1)^2=4(y_2)^3$

$\downarrow$

$O=2$

$D=3$

A) 1, 2

B) 2, 1

C) 2, 3

D) 3, 2

QUESTION

#Q. If m and n are the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^5 + 4.\frac{\left(\frac{d^2y}{dx^2}\right)^3}{\frac{d^3y}{dx^3}} + \frac{d^3y}{dx^3} = x^2 - 1, \text{ then}$$

(A) $m = 3, n = 5$

$(y_2)^5 y_3 + 4(y_2)^3 + (y_3)^2 = (x^2-1)y_3$

$\downarrow$

$O = 3$

$D = 2$

(B) $m = 3, n = 1$

(C) $m = 3, n = 3$

(D) ✓ $m = 3, n = 2$

QUESTION

#Q. The degree of the differential equation $\frac{d^3y}{dx^3} + x^2\left(\frac{dy}{dx}\right)^3 = 4\log_e\left(\frac{d^2y}{dx^2}\right)$ is

$O = 3$

$D = ND$

(A) 3

(B) 2

(C) 1

(D) not defined

QUESTION

#Q. The order of the differential equation whose solution is
$y = c_1 \cos x + c_2 \sin x + c_3 e^{-x}$ is

(A) 1

(B) 2

(C) 3

(D) none of these

QUESTION

#Q. The degree of the differential equation $(1+y_1^2)^{\frac{3}{4}} = (y_2)^{\frac{1}{3}}$ is

A 1/3

B 4

C 9

D 3/4

LCM(4, 3) = 12

Raise to the Power 12 on BS

$(1+y_1^2)^9 = (y_2)^4$

$\downarrow$

O = 2

D = 4

QUESTION

$c_1 \cos(2x + c_2)$

$c_1[\cos 2x \cdot \cos c_2 - \sin 2x \sin c_2] = \cos 2x(B) - \sin 2x(C)$

$B = \cos c_2 (c_1)$ & $C = \sin c_2 (c_1)$

#Q. The order of the differential equation whose general solution is given by $y = c_1 \cos(2x + c_2) - (c_3 + c_4)\, a^{x + c_5} + c_6 \sin(x - c_7)$ is

(A) 3

(B) 4

(C) 5

(D) 2

$y = c_1 \cos(2x + c_2) - \underbrace{(c_3 + c_4) a^{c_5}}_{} \cdot a^x + c_6 \sin(x - c_7)$

$y = c_1 \cos(2x + c_2) - A\, a^x + c_6 \sin(x - c_7)$

$\therefore$ Arbitrary constants are

c_1, c_2, A, c_6, c_7

$\Downarrow$

order $= 5$

Variable separable form

$\Downarrow$

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

$$dy = \frac{f(x)}{g(y)}\, dx$$

$$g(y)\, dy = f(x)\, dx$$

$$\int g(y)\, dy = \int f(x)\, dx$$

→ Rules :-

1. dy & dx should always stay Nr.

2. Bring expression in y with dy & expression in x with dx

3. Integrate

QUESTION

#Q. Solve the D.E. $(x+1)\frac{dy}{dx} = 2xy$

(A) $y = (x+3) + c$

(B) $\log y = 2[x - \log|x+1|] + c$

(C) $\log y = 2[x + \log|x+1|] + c$

(D) $\log y = \sin^{-1}x + c$

$$\frac{dy}{y} = \frac{2x}{x+1}dx$$

$$\int \frac{dy}{y} = 2\int \frac{x}{x+1}dx$$

$$= 2\int \frac{x+1}{x+1} - \frac{1}{x+1}dx$$

$$\int \frac{dy}{y} = 2\int 1 - \frac{1}{x+1}dx$$

$$\log y = 2[x - \log(x+1)] + c$$

QUESTION

$\log A - \log B = \log\left(\frac{A}{B}\right)$

#Q. Solve the D.E. $(x-1)\frac{dy}{dx} = 2xy$ given that $y(2) = 1$

→ Intial condition to find Particular soln

→ $x = 2$ & $y = 1$

A) $y = (x-1)^2\, 2(x-2)$

B) $y = (x-1)^2\, e^{2(x-2)}$

C) $y = (x-1)\, e^{(x-2)}$

D) $y = x - 1$

$(x-1)\frac{dy}{dx} = +2xy$

$\int \frac{dy}{y} = +2\int \frac{x}{x-1}dx$

$\int \frac{dy}{y} = +2\int 1 + \frac{1}{x-1}dx$

$\log y = +2[x + \log|x-1|] + c$

$0 = +2[2+0] + c$

$c = -4$

$\log y = +2x + 2\log(x-1) - 4$

$\log y - \log(x-1)^2 = 2x - 4$

$\log\left[\frac{y}{(x-1)^2}\right] = 2(x-2)$

$\frac{y}{(x-1)^2} = e^{2(x-2)}$

$$\frac{y}{(x-1)^2} = e^{2(x-2)}$$

$$y = \underline{(x-1)^2} \cdot e^{2(x-2)}$$

QUESTION

#Q. Solve the D.E. $(x+y)^2\frac{dy}{dx}=1$

(A) $y-\tan^{-1}(x+y)=c$ ✓

(B) $y+\tan^{-1}(x+y)=c$

(C) $x-\tan^{-1}(x+y)=c$

(D) $x+\tan^{-1}(x+y)=c$

$\frac{dy}{dx}=\frac{1}{(x+y)^2}$

Put $x+y=t$

Diff wrt x

$1+\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{dy}{dx}=\frac{dt}{dx}-1$

$\frac{dt}{dx}-1=\frac{1}{t^2}$

$\frac{dt}{dx}=\frac{1}{t^2}+1=\frac{1+t^2}{t^2}$

$\int\frac{t^2}{1+t^2}dt=\int dx$

$\int 1-\frac{1}{1+t^2}dt=\int dx$

$t-\tan^{-1}t=x+c$

$\cancel{x}+y-\tan^{-1}(x+y)=\cancel{x}+c$

$y-\tan^{-1}(x+y)=c$

QUESTION

#Q. Solve the D.E. $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

$$\frac{dy}{dx} = \sin(x+y)$$

Put $x + y = t$

$$\frac{dy}{dx} = \frac{dt}{dx} - 1$$

$$\frac{dt}{dx} - 1 = \sin t$$

$$\frac{dt}{dx} = 1 + \sin t$$

(A) ✗ $x = \tan x + \tan y + c$

(B) ✗ $y = \tan x + \tan y + c$

(C) ✓ $x = \tan(x+y) - \sec(x+y) + c$

(D) $x = \tan(x+y) + \sec(x+y) + c$

$$\int \frac{1}{1+\sin t}\,dt = \int dx$$

$$\int \frac{1-\sin t}{\cos^2 t}\,dt = \int dx$$

$$\int \sec^2 t - \sec t \tan t \,dt = \int dx$$

$$\tan t - \sec t = x + c$$

$$x = \tan(x+y) - \sec(x+y) + c_1$$

$$c_1 = -c$$

QUESTION

#Q. Solution of differential equation $\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$ is equal to

(A) $\log\left(2 + \sec\frac{x+y}{2}\right) = x + c$

(B) $\log\{1 + \tan(x+y)\} = y + c$

(C) $\log\left(1 + \tan\frac{x+y}{2}\right) = y + c$

(D) $\log\left(1 + \tan\frac{x+y}{2}\right) = x + c$

Put $x + y = t$

$\frac{dy}{dx} = \frac{dt}{dx} - 1$

$\frac{dt}{dx} - 1 = \sin t + \cos t$

$\frac{dt}{dx} = (1 + \cos t) + \sin t$

$\int \frac{dt}{2\cos^2\frac{t}{2} + 2\sin\frac{t}{2}\cos\frac{t}{2}} = \int dx$

$\div$ by $2\cos^2 t/2$

$\int \frac{\overbrace{\frac{1}{2}\sec^2 t/2}^{f'(t)}}{\underbrace{1 + \tan t/2}_{f(t)}} dt = \int dx$

$\log\left(1 + \tan\frac{t}{2}\right) = x + c$

$\log\left[1 + \tan\left(\frac{x+y}{2}\right)\right] = x + c$

QUESTION

#Q. Solution of the differential equation $\frac{dy}{dx}=\frac{y(1+x)}{x(y-1)}$

(A) $\log|xy|+x+y=c$

(B) $\log|xy|-x+y=c$

(C) $\log|xy|+x-y=c$

(D) $\log|xy|-x-y=c$

$$\int \frac{y-1}{y}\,dy=\int \frac{1+x}{x}\,dx$$

$$\int 1-\frac{1}{y}\,dy=\int \frac{1}{x}+1\,dx$$

$$y-\log y=\log x+x+c$$

$$c_1=\log x+\log y+x-y$$

$$c_1=\underline{\log|xy|+x-y}$$

QUESTION

#Q. Solve the differential equation $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$

(A) $x + e^y \sin x = c$

(B) $\sin x(e^y + 1) = c$

(C) $y + e^x \sin x = c$

(D) $x + \sin x = c$

$\div$ by $(e^y+1)\sin x$

$\cot x\, dx + \frac{e^y}{e^y+1} dy = 0$

$\int \cot x\, dx + \int \frac{e^y}{e^y+1} dy = 0$

$\log \sin x + \log(e^y+1) = \log c$

$\log|\sin x(e^y+1)| = \log c$

$\sin x(e^y+1) = c$

QUESTION

$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

#Q. Solve the differential equation $\tan y \frac{dy}{dx} = \sin(x+y) + \sin(x-y)$

$\tan y \frac{dy}{dx} = 2\left[\sin(x)\cos(y)\right]$

(A) $\sin x + \sec y = c$

$\frac{\tan y}{\cos y} dy = 2\sin x\, dx$

(B) $2\cos x + \sec y = c$

$\int \sec y \tan y\, dy = \int 2\sin x\, dx$

(C) $\cos x + \sin x = c$

$\sec y = 2\cos x + c$

(D) $2\sin x + \sec y = e$

$\sec y + 2\cos x = c$

QUESTION

#Q. Solve the differential equation $\frac{dy}{dx} = (x + y + 1)^2$.

(A) $\tan^{-1}(1 + xy) = y + e$

(B) $\tan^{-1}(x + y + 1) = x + c$

(C) $\tan^{-1} x = x + c$

(D) $\tan^{-1}(x+y+1) = y + c$

Put $x + y + 1 = t$

$1 + \frac{dy}{dx} = \frac{dt}{dx}$

$\frac{dy}{dx} = \frac{dt}{dx} - 1$

$\frac{dt}{dx} = 1 + t^2$

$\int \frac{dt}{1+t^2} = \int dx$

$\tan^{-1} t = x + c$

$\tan^{-1}(x + y + 1) = x + c$

Integrating factors

$$\frac{dy}{dx} + Py = Q$$ (linear DE)

Rules:- ① P & Q should be func of x only
⇓
there should be no y variable in it

② $IF = e^{\int P\,dx}$

③ coefficient of $\frac{dy}{dx}$ should be one

$\frac{dy}{dx}$

Dependent variable = y

Independent variable = x

Solve a DE

$\downarrow$

means

$\Downarrow$

we are finding

Dependent variable wrt

Independent variable

$\Downarrow$

LHS $\rightarrow$ y terms

RHS $\rightarrow$ x terms

If given Linear DE

$$\frac{du}{dv} + Pu = Q$$

① Here P & Q should be func of v only

② $IF = e^{\int P\,dv}$

QUESTION

$$\frac{dy}{dx} + Py = Q$$

#Q. Integrating factor of $x\frac{dy}{dx} - y = x^4 - 3x$ is [2016]

÷ by x

$$\frac{dy}{dx} + \left(\frac{-1}{x}\right)y = x^3 - 3$$

$$P = \frac{-1}{x} \;\&\; Q = x^3 - 3$$

$$\int P\,dx = \int -\frac{1}{x}\,dx = -\log x = \log x^{-1}$$

$$IF = e^{\int P\,dx} = e^{\log x^{-1}} = x^{-1} = \frac{1}{x}$$

(A) x

(B) $\log x(c)$

(C) $1/x$

(D) $-x$

QUESTION

#Q. The integrating factor of the differential equation $x \cdot \frac{dy}{dx} + 2y = x^2$ is $(x \neq 0)$

[2017]

$P = \frac{2}{x}$

$\int P\,dx = 2\log x = \log x^2$

$IF = x^2$

(A) $e^{\log x}$

(B) $\log |x|$

(C) x

(D) x^2

QUESTION

#Q. The integrating factor of $\frac{dy}{dx}+y=\frac{1+y}{x}$ is [2018]

$\hookrightarrow Q=\frac{1+y}{x}$ (It is not a func of x only)

(A) xe^{x}

(B) $xe^{1/x}$

(C) e^{x}/x ✓

(D) x/e^{x}

$$\frac{dy}{dx}+y=\frac{1}{x}+\frac{y}{x}$$

$$\frac{dy}{dx}+y\underbrace{\left(1-\frac{1}{x}\right)}_{P}=\underbrace{\frac{1}{x}}_{Q}$$

$$\int P\,dx=\int 1-\frac{1}{x}\,dx=x-\log x$$

$$=x+\log x^{-1}$$

$$IF=e^{x+\log x^{-1}}$$

$$=e^{x}\cdot e^{\log x^{-1}}$$

$$=e^{x}(x^{-1})$$

$$=\frac{e^{x}}{x}.$$

QUESTION

#Q. The integrating factor of the differential equation $(2x+3y^2)dy = ydx (y>0)$ is [2019]

(A) $e^{\frac{1}{y}}$

(B) $\frac{1}{x}$

(C) $-\frac{1}{y^2}$

(D) $\frac{1}{y^2}$

$\frac{dy}{dx} = \frac{y}{2x+3y^2}$

$\Downarrow$

there splitting is not possible in RHS to get P & Q

$\therefore$ Reciprocal the eqn

$\frac{dx}{dy} = \frac{2x+3y^2}{y}$

$\frac{dx}{dy} = \frac{2}{y}x + 3y$

$\frac{dx}{dy} + \underbrace{\left(\frac{-2}{y}\right)}_{P} x = \underbrace{3y}_{Q}$

$\int P\,dy = -2\int \frac{1}{y}\,dy$

$= -2\log y$

$= \log y^{-2}$

$IF = e^{\log y^{-2}} = \frac{1}{y^2}$

QUESTION

#Q. Integrating factor of the differential equation $xdy - ydx + x^2e^x dx = 0$ is

$$x\,dy = dx\,(y - x^2e^x)$$

$$\frac{dy}{dx} = \frac{1}{x}y - xe^x$$

$$\frac{dy}{dx} + \underbrace{\left(\frac{-1}{x}\right)}_{P} y = \underbrace{-xe^x}_{Q}$$

$$\int P\,dx = \int -\frac{1}{x}\,dx = -\log x = \log x^{-1}$$

$$IF = e^{\log x^{-1}} = x^{-1} = \frac{1}{x}.$$

(A) $1/x$

(B) $\log\sqrt{1+x^2}$

(C) $\sqrt{1+x^2}$

(D) x

QUESTION

#Q. Integrating factor of the differential equation $(x^2+1)\frac{dy}{dx}+2xy=x^2-1$ is

[2024]

$\div$ by x^2+1

$$\frac{dy}{dx}+\underbrace{\frac{2x}{x^2+1}}_{P}y=\underbrace{\frac{x^2-1}{x^2+1}}_{Q}$$

$$\int P\,dx=\int\frac{2x}{x^2+1}dx=\log(x^2+1)$$

$f'(x)$, $f(x)$

$$IF=e^{\log(x^2+1)}=\underline{x^2+1}$$

(A) $\frac{x^2-1}{x^2+1}$

(B) $\frac{2x}{x^2+1}$

(C) x^2+1

(D) None of these

QUESTION

#Q. Integrating factor of the differential equation $(1+x^2)\frac{dy}{dx}+xy=x$ is

(A) $\frac{x}{1+x^2}$

(B) $\frac{1}{2}\log(1+x^2)$

(C) $\sqrt{1+x^2}$

(D) x

$\div$ by $1+x^2$

$$\frac{dy}{dx}+\underbrace{\frac{x}{1+x^2}}_{P}y=\underbrace{\frac{x}{1+x^2}}_{Q}$$

$$\int P\,dx=\int\frac{x}{1+x^2}dx=\frac{1}{2}\int\frac{\overset{f'(x)}{2x}}{\underset{f(x)}{1+x^2}}dx$$

$$=\frac{1}{2}\log(1+x^2)$$

$$\int P\,dx=\log(1+x^2)^{1/2}$$

$$IF=e^{\log(1+x^2)^{1/2}}$$

$$=(1+x^2)^{1/2}$$

$$=\sqrt{1+x^2}$$

QUESTION

#Q. Integrating factor of the differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$ is

$$\underbrace{\frac{dy}{dx}}_{} + \underbrace{\tan x}_{P}\, y = \underbrace{\sec x}_{Q} \quad \div \text{ by } \cos x$$

(A) $\cos x$

(B) $\tan x$

(C) $\sec x$

(D) $\sin x$

$$\int P\,dx = \int \tan x\,dx = \log \sec x$$

$$IF = \sec x$$

QUESTION

#Q. Integrating factor of the differential equation $(y\log y)dx = (\log y - x)dy$ is

(A) $1/\log y$

(B) $\log(\log y)$

(C) $\log y$

(D) $1/\log(\log y)$

$$\frac{dx}{dy} = \frac{\log y}{y\log y} - \frac{x}{y\log y}$$

$$\frac{dx}{dy} = \frac{1}{y} - \frac{1}{y\log y}x$$

$$\frac{dx}{dy} + \underbrace{\left(\frac{1}{y\log y}\right)}_{P}x = \underbrace{\frac{1}{y}}_{Q}$$

$$\int P\,dy = \int \frac{1/y \to p'(y)}{\log y \to p(y)}\,dy$$

$$= \log(\log y)$$

$$IF = e^{\log[\log y]}$$

$$= \underline{\log y}$$

QUESTION

#Q. The integrating factor of the differential equation $\frac{dy}{dx}+\frac{y}{(1-x)\sqrt{x}}=1-\sqrt{x}$ is

(A) $\frac{1-\sqrt{x}}{1+\sqrt{x}}$

(B) $\frac{1+x}{1-x}$

(C) $\frac{1+\sqrt{x}}{1-\sqrt{x}}$ ✓

(D) $\frac{\sqrt{x}}{1-\sqrt{x}}$

$P=\frac{1}{(1-x)\sqrt{x}}$

$\int P\,dx=\int\frac{1}{\sqrt{x}(1-x)}dx$

Put $\sqrt{x}=t$

$x=t^2$

$\frac{1}{2\sqrt{x}}dx=dt$

$\frac{dx}{\sqrt{x}}=2dt$

$\int P\,dx=2\int\frac{dt}{1-t^2}$

$=2\left[\frac{1}{2(1)}\log\left(\frac{1+t}{1-t}\right)\right]$

$=\log\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)$

$IF=e^{\log\left(\frac{1+\sqrt{x}}{1-\sqrt{x}}\right)}=\frac{1+\sqrt{x}}{1-\sqrt{x}}$

QUESTION

#Q. An integrating factor of the differential equation, $(1+y+x^2y)dx+(x+x^3)dy=0$ is

(A) $\log x$

(B) x

(C) e^x

(D) $1/x$

$x(1+x^2)$

$(1+y+x^2y)\,dx = -(x+x^3)\,dy$

$$\frac{1+y(1+x^2)}{x(1+x^2)} = -\frac{dy}{dx}$$

$$\frac{dy}{dx} = -\left[\frac{1}{x(1+x^2)} + \frac{y\cancel{(1+x^2)}}{x\cancel{(1+x^2)}}\right]$$

$$\frac{dy}{dx} = -\frac{1}{x(1+x^2)} - \frac{y}{x}$$

$$\frac{dy}{dx} + \underbrace{\left(\frac{1}{x}\right)}_{P} y = \underbrace{\frac{-1}{x(1+x^2)}}_{Q}$$

$$\int P\,dx = \int \frac{1}{x}\,dx = \log x$$

$$IF = e^{\log x} = x$$

Linear DE:-

$$\frac{dy}{dx} + Py = Q$$

① P & Q are func of x only

② $IF = e^{\int P\,dx}$

③ solution is

$$y(IF) = \int Q(IF)\,dx + c$$

$$\frac{du}{dv} + Pu = Q$$

① P & Q are func of v only

② $IF = e^{\int P\,dv}$

③ solution is

$$u(IF) = \int Q(IF)\,dv + c$$

QUESTION

#Q. Solve the differential equation $(x+3y^2)\frac{dy}{dx}=y(y>0)$

→ Here we cannot bring expression in y with dy & expression in x with dx

$\frac{dy}{dx}=\frac{y}{x+3y^2}$

Here we cannot split the terms in RHS

∴ Do Reciprocal

$\frac{dx}{dy}=\frac{x+3y^2}{y}$

(A) $x=3y+c$

(B) $x=y+3$

(C) $x=3y^2+cy$

(D) $x=y^2+3$

$\frac{dx}{dy}=\frac{x}{y}+3y$

$\frac{dx}{dy}+\underbrace{\left(-\frac{1}{y}\right)}_{P}x=\underbrace{3y}_{Q}$

$\int P\,dy=\int-\frac{1}{y}dy=-\log y=\log y^{-1}$

$IF=e^{\log y^{-1}}=y^{-1}=\frac{1}{y}$

Soln is

$$x(IF) = \int Q(IF)\,dy + c$$

$$\frac{x}{y} = \int 3y\left(\frac{1}{y}\right) dy + c$$

$$= \int 3\,dy + c$$

$$\frac{x}{y} = 3y + c$$

$$x = 3y^2 + cy$$

QUESTION

#Q. Solve $y\,dx - (x + 2y^2)dy = 0$.

(A) $\frac{x}{y} = 2y + c$

(B) $x = x^2 + c$

(C) $x = y + c$

(D) $\frac{x}{y} = y + c$

$\frac{dx}{dy} = \frac{x+2y^2}{y}$

$\frac{dx}{dy} + \left(\frac{-1}{y}\right)x = 2y$

$IF = \frac{1}{y}$

Soln

$\frac{x}{y} = \int 2\,dy + c$

$\frac{x}{y} = 2y + c$

QUESTION

#Q. Solve the differential equation $(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x$

$P=\frac{1}{1+x^2}$ & $Q=\frac{\tan^{-1}x}{1+x^2}$

(A) $y=1+\tan^{-1}x$

(B) $y=\tan^{-1}x-1+ce^{-\tan^{-1}x}$

$IF=e^{\tan^{-1}x}$

Soln

(C) $y=1-\tan^{-1}x$

$ye^{\tan^{-1}x}=\int e^{\tan^{-1}x}\frac{(\tan^{-1}x)}{1+x^2}dx+c$

(D) $y=-1+ce^{-\tan^{-1}x}$

Put $\tan^{-1}x=t$

$$y\,e^{\tan^{-1}x} = \int e^{t}(t)\,dt + c$$

$$y\,e^{\tan^{-1}x} = t\,e^{t} - e^{t} + c$$

$$y\,e^{\tan^{-1}x} = e^{\tan^{-1}x}\left(\tan^{-1}x - 1\right) + c$$

$$y = \underline{\left(\tan^{-1}x - 1\right)} + c\,e^{-\tan^{-1}x}$$

QUESTION

#Q. Solve the differential equation : $(x^2-1)\frac{dy}{dx}+2xy=\frac{1}{x^2-1}$

$P=\frac{2x}{x^2-1}$ | $Q=\frac{1}{(x^2-1)^2}$

$IF = x^2-1$

(A) $(x^2+1)=\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+c$

(B) $y(x^2-1)=\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+c$

(C) $y(x^2-1)=\log\left|\frac{x-1}{x+1}\right|+c$

(D) $(x^2-1)=\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+c$

Soln:

$y(x^2-1)=\int\frac{1}{x^2-1}dx+c$

$y(x^2-1)=\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+c$

QUESTION

#Q. Solve $(x + 2y^3)\frac{dy}{dx} = y$.

(A) $x = y^3 + y^2$

(B) $x = y + c$

(C) $x = y^3 + cy$

(D) $x = y^2 + xy$

$\frac{dx}{dy} = \frac{x}{y} + 2y^2$

$P = -\frac{1}{y}$ & $Q = 2y^2$

$IF = \frac{1}{y}$

Soln:

$\frac{x}{y} = \int 2y\, dy + c$

$\frac{x}{y} = y^2 + c$

$x = y^3 + cy$

QUESTION

#Q. Solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$ is

(A) $x^2y + \frac{x^4}{16} + \frac{x^4 \log x}{4} = c$

(B) $x^2y - \frac{x^4}{16} + \frac{x^4 \log x}{4} = c$

(C) $x^2y + \frac{x^4}{16} - \frac{x^4 \log x}{4} = c$

(D) $x^2y - \frac{x^4}{16} - \frac{x^4 \log x}{4} = c$

$P = \frac{2}{x}$ & $Q = x \log x$

$I.F = x^2$

Soln :-

$yx^2 = \int x^3 \log x \, dx + c$

$yx^2 = \log x\left(\frac{x^4}{4}\right) - \frac{x^4}{16} + c$

$yx^2 - \frac{x^4}{4}\log x + \frac{x^4}{16} = c$

QUESTION

#Q. Solve the differential equation $dx + xdy = e^{-y}\sec^2 y\, dy$.

$$dx = dy(e^{-y}\sec^2 y - x)$$

(A) $xe^y = \tan x + c$

(B) $xe^y = \tan y + c$

(C) $e^y = \tan y + c$

(D) $xe^x = \tan y + c$

$$\frac{dx}{dy} + (1)x = e^{-y}\sec^2 y$$

$P = 1$ & $Q = e^{-y}\sec^2 y$

$IF = e^y$

Soln is

$$xe^y = \int \sec^2 y\, dy + c$$

$$xe^y = \tan y + c$$

Increasing & Decreasing func:

① **Increasing :-**

$f'(x) \geq 0$

$\Downarrow$

open & closed Interval

$\Uparrow$

② **Decreasing :-**

$f'(x) \leq 0$

③ **Strictly Increasing :-**

$f'(x) > 0$

$\Downarrow$

only open Intervals

$\Uparrow$

④ **Strictly Decreasing :-**

$f'(x) < 0$

QUESTION

#Q. The set of real of x for which $f(x) = \frac{x}{\log x}$ is increasing, is [2010]

$$f'(x) = \frac{\log x(1) - 1}{(\log x)^2}$$

(A) empty

(B) $\{x : x \geq e\}$

(C) $\{1\}$

(D) $\{x : x < e\}$

$$f'(x) \geq 0$$

$$\log x - 1 \geq 0$$

$$\log x \geq 1$$

$$x \geq e$$

QUESTION

#Q. The function $f(x) = x^2 + 2x - 5$ is strictly increasing in the interval [2017]

$f'(x) = 2x + 2$

$f'(x) > 0$

$2x + 2 > 0$

$x > -1$

A $[-1, \infty)$

B $(-\infty, -1)$

C $(-\infty, -1]$

D $(-1, \infty)$

QUESTION

#Q. The interval in which the function $f(x) = x^3 - 6x^2 + 9x + 10$ is increasing in

[2019]

(A) $(-\infty, 1) \cup (3, \infty)$

(B) $[1, 3]$

(C) $(-\infty, 1] \cup [3, \infty)$

(D) $(-\infty, -1] \cup [3, \infty)$

$f'(x) = 3x^2 - 12x + 9$

$= 3[x^2 - 4x + 3]$

$= 3(x-3)(x-1)$

$f'(x) \geq 0$

$3(x-3)(x-1) \geq 0$

$+ \quad - \quad +$

$-\infty \quad 1 \quad 3 \quad \infty$

$x \in (-\infty, 1] \cup [3, \infty)$

QUESTION

#Q. The function $f(x) = x^2 - 2x$ is strictly decreasing in the interval [2021]

$f'(x) = 2x - 2$

$f'(x) < 0$

$x < 1$

A. $(-1, \infty)$

B. $(1, \infty)$

C. R

D. $(-\infty, 1)$

QUESTION

$(x-1)^2 \geq 0$ | $(x-1)^2 + \frac{3}{4} > 0$

$(x-1)^2 + \frac{3}{4} \geq \frac{3}{4}$

#Q. The function $f(x) = 4\sin^3 x - 6\sin^2 x + 12\sin x + 100$ strictly [2022]

$f'(x) = 12\sin^2 x\cos x - 12\sin x\cos x + 12\cos x$

$f'(x) = 12\cos x[\sin^2 x - \sin x + 1]$

$= 12\cos x[\sin^2 x - \sin x + \frac{1}{4} - \frac{1}{4} + 1]$

$f'(x) = 12\cos x[\underbrace{(\sin x - \frac{1}{2})^2 + \frac{3}{4}}_{>0}]$

(A) decreasing in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(B) increasing in $\left[\pi, \frac{3\pi}{2}\right]$

(C) decreasing in $\left[0, \frac{\pi}{2}\right]$

(D) decreasing in $\left(\frac{\pi}{2}, \pi\right)$

$f'(x)$ depends on sign of $\cos x$

$\cos x < 0$ if $x \in \left(\frac{\pi}{2}, \pi\right)$

$\therefore f'(x) < 0$ if $x \in \left(\frac{\pi}{2}, \pi\right)$

QUESTION

if $a \cdot b \geq 0$
& if $a > 0 \Rightarrow b > 0$

$(+)\frac{(+)}{} = (+)$

#Q. The function $f(x) = \log(1+x) - \frac{2x}{2+x}$ is increasing on [2022]

A (−∞, ∞)

B (−1, ∞)

C (∞, −1)

D (−∞, 0)

$$f'(x) = \frac{1}{1+x} - \frac{(2+x)2 - 2x}{(2+x)^2}$$

$$= \frac{\cancel{4} + x^2 + \cancel{4x} - [\cancel{4}(1+\cancel{x})]}{(1+x)(2+x)^2}$$

$$= \frac{x^2}{(1+x)(2+x)^2}$$

$$f'(x) = \frac{\overbrace{x^2}^{\geq 0}}{(2+x)^2}\left(\frac{1}{1+x}\right)$$

here $f'(x) \geq 0$

$$\left(\frac{x}{2+x}\right)^2\left(\frac{1}{1+x}\right) \geq 0$$

$\downarrow$
always +ve

$$\frac{1}{1+x} \geq 0$$

$\Rightarrow 1 + x > 0$

$x > -1$

if $\frac{a}{b} \geq 0$
& $a > 0$
$\Rightarrow b > 0$

QUESTION

$f(x) = a^x$

Range $= (0, \infty)$

$a^x > 0$

$\Rightarrow e^{x-x^2} > 0 \Rightarrow \forall x \in R$

$x - 2x^2 + 1$

$= -2\left[x^2 - \frac{1}{2}x - \frac{1}{2}\right]$

#Q. If $f(x) = x\,e^{x(1-x)}$ then $f(x)$ is

$f(x) = x e^{x-x^2}$

$-\frac{1}{16} - \frac{1}{2}$

$= \frac{-1-8}{16} = -\frac{9}{16}$

(A) increasing in R

(B) decreasing in R

(C) decreasing in $\left[-\frac{1}{2}, 1\right]$

(D) increasing in $\left[-\frac{1}{2}, 1\right]$

$f'(x) = x e^{x-x^2}(1-2x) + e^{x-x^2}(1)$

$= e^{x-x^2}\left[x - 2x^2 + 1\right]$

$= -2 e^{x-x^2}\left[x^2 - \frac{1}{2}x + \frac{1}{16} - \frac{1}{16} - \frac{1}{2}\right]$

$f'(x) = -2\,\underbrace{e^{x-x^2}}_{>0 \; \forall x \in R}\left[\left(x - \frac{1}{4}\right)^2 - \frac{9}{16}\right]$

$$f'(x) = -2\, e^{x-x^2}\left[\left(x-\frac{1}{4}\right)^2 - \frac{9}{16}\right]$$

Case I :- if $f'(x) \geq 0$ (condition for Increasing)

$$-2\, e^{x-x^2}\left[\left(x-\frac{1}{4}\right)^2 - \frac{9}{16}\right] \geq 0$$

$\div$ by -2 & there $e^{x-x^2} > 0 \; \forall x \in R$

$$\left(x-\frac{1}{4}\right)^2 - \frac{9}{16} \leq 0$$

$$\left(x-\frac{1}{4}\right)^2 \leq \frac{9}{16}$$

$$\left|x-\frac{1}{4}\right| \leq \frac{3}{4}$$

$$x-\frac{1}{4} \in \left[-\frac{3}{4}, \frac{3}{4}\right]$$

$$x \in \left[-\frac{3}{4}+\frac{1}{4}, \frac{3}{4}+\frac{1}{4}\right]$$

$$x \in \left[-\frac{1}{2}, 1\right]$$

$f(x) = a^x$

Domain $= R$

& Range $= (0, \infty)$

$\Rightarrow a^x > 0 \quad \forall x \in R$

$\Downarrow$

$e^{x-x^2} > 0 \quad \forall x \in R$

$f(x) = x^2 - \frac{1}{2}x + \frac{1}{2}$

here $a > 0$ | $b = -\frac{1}{2}$ & $c = \frac{1}{2}$

$b^2 - 4ac = \frac{1}{4} - 4(1)\left(\frac{1}{2}\right)$

$= \frac{1}{4} - 2 < 0$

if $f(x) = ax^2 + bx + c$

here if $a > 0$ & $b^2 - 4ac < 0$

$\Rightarrow f(x) > 0 \quad \forall x \in R$

QUESTION

if $a \cdot b > 0$
& if $a > 0 \Rightarrow b > 0$

#Q. The function x^x ; $x > 0$ is strictly increasing at

$y = x^x$

$\log y = x \log x$

$\frac{1}{y} y_1 = 1 + \log x$

$y_1 = y(1 + \log x)$

$y_1 > 0$

$\therefore \quad x^x(1 + \log x) > 0$

(A) $\forall x \in R$

(B) $x < 1/e$

(C) $x > 1/e$

(D) $x < 0$

$x^x(1 + \log x) > 0$

here $x > 0 \Rightarrow x^x > 0$

$1 + \log x > 0$

$\log x > -1$

$x > e^{-1}$

$x > \frac{1}{e}$

QUESTION

#Q. The function $f(x) = \cot^{-1}x + x$ increases in the interval

(A) $(1, \infty)$

(B) $(-1, \infty)$

(C) $(0, \infty)$

(D) ✓ $(-\infty, \infty)$

$$f'(x) = \frac{-1}{1+x^2} + 1$$

$$= \frac{-1+1+x^2}{1+x^2}$$

$$f'(x) = \frac{+x^2}{1+x^2}$$

WKT $\frac{x^2}{1+x^2} \geq 0 \quad \forall x \in R$

$f'(x) \geq 0 \quad \forall x \in R$

$\Downarrow$

$f(x)$ is increasing $\forall x \in R$

QUESTION

#Q. The function $f(x) = \cot^{-1}x + x$ decreases in the interval

$$f'(x) = \frac{-1}{1+x^2} + 1$$

(A) $(1, \infty)$

(B) $(-1, \infty)$

$$f'(x) = \frac{x^2}{1+x^2}$$

(C) ϕ ✓

since $\frac{x^2}{1+x^2} \geq 0 \ \forall x \in R$

(D) $(-\infty, \infty)$

$\Rightarrow f'(x) \geq 0 \ \forall x \in R$

$f(x)$ is never decreasing

Thank You