Ultimate kcet crash course 2026

Maths

limits and continuity

DPP: 1

Q1 $\lim_{x\to 0} \frac{\sin(\pi\cos^2 x)}{x^2}$ is equal to
(A) 1
(B) $-\pi$
(C) π
(D) $\pi/2$

Q2 $\lim_{n\to\infty} = \frac{1^3+2^3+3^3+......n^3}{n^2(n+1)(2n+3)} =$
(A) 1/2 (B) 1/4
(C) 1/8 (D) None of these

Q3 $\lim_{x\to\infty} x\sin\frac{1}{x}$
(A) 1 (B) 0
(C) not defined (D) –1

Q4 $\lim_{x\to\frac{\pi}{2}} \frac{\cot x-\cos x}{(\pi-2x)^3}$ equals
(A) 1/16 (B) 1/8
(C) 1/4 (D) 1/24

Q5 The value of $\lim_{x\to 2} \frac{x^{10}-1024}{x^5-32}$ is
(A) 46 (B) 64
(C) 32 (D) 128

Q6 $\lim_{x\to\infty}\left(\frac{1^3+2^3+....+k^3}{k^4}\right)$ is equal to
(A) 0 (B) 2
(C) 1/4 (D) 1/3

Q7 $\lim_{x\to\tan^{-1}3} \frac{\tan^2 x-2\tan x-3}{\tan^2 x-4\tan x+3}$
(A) 1 (B) 2
(C) 0 (D) 3

Q8 $\lim_{x\to 0} \frac{\tan 2x-3x}{3x-\sin x}$ is
(A) 2 (B) 1/2
(C) -1/2 (D) 1/4

Q9 $\lim_{x\to -1}\left[1+x+x^2+....+x^{10}\right]$ is
(A) 0 (B) 1
(C) 2 (D) Does not exist

Q10 $\lim_{x\to 0^+} \frac{\sin x}{\sqrt{x}}$ is equal to
(A) 0 (B) 1/2
(C) 1 (D) Does not exist

Q11 $\lim_{x\to\frac{\pi}{4}} \frac{\sin x-\cos x}{x-\frac{\pi}{4}} =$
(A) 1 (B) $\sqrt{2}$
(C) 2 (D) $-\sqrt{2}$

Q12 $\lim_{x\to 0}\left[\frac{e^x-e^{\sin x}}{x-\sin x}\right]$ is equal to
(A) -1 (B) 0
(C) 1 (D) None of thses

Q13 $\lim_{x\to 1} \frac{1+\cos\pi x}{\tan^2\pi x}$ is equal to
(A) 0 (B) 1/2
(C) 1 (D) 2

Q14 $\lim_{x\to 0} \frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$ is
(A) 2 (B) 0
(C) 1 (D) -1

Q15 $\lim_{x\to 0} \frac{8}{x^8}\left[1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right]$
is equal to
(A) $\frac{1}{16}$ (B) $-\frac{1}{16}$
(C) $\frac{1}{32}$ (D) $-\frac{1}{32}$

Q16 $\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2} =$
(A) π/2 (B) 2/π
(C) π/3 (D) 3/π

Q17 The function f(x) = |x| is continuous at
(A) x = 0 only
(B) x = 1 only
(C) all real points
(D) for all x except at 0

Q18 The function $f(x)=\begin{cases} \frac{\sin x}{x}+\cos x, & \text{if } x \neq 0 \\ k, & \text{if } x=0 \end{cases}$ is continuous at x = 0, then the value of k is
(A) 3 (B) 2
(C) 1 (D) 1.5

Q19 If f(x) is continuous at x = 0, where $f(x)=\frac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}$, for x ≠ 0, then f(0)=
(A) 1/3 (B) 1/2
(C) 5/6 (D) 1/6

Q20 $f(x) = \sin x + \log_e x$ is continuous
(A) $\forall x$
(B) $\forall x > 0$
(C) $\forall x < 0$
(D) $\forall x \in (0,\ 1]$

Q21 If $f(x)=\begin{cases} \frac{x^2-9}{x-3}, & \text{if } x \neq 3 \\ 2x+K, & \text{otherwise} \end{cases}$,is continuous at x = 3 , then k =
(A) 3 (B) 0
(C) -6 (D) 1/6

Q22 Find k, so that the function f(x) is continuous at x = 1, where $f(x)=\begin{cases} kx^2, & \text{for } x \geq 1 \\ 4, & \text{for } x < 1 \end{cases}$
(A) 0 (B) 4
(C) 2 (D) 6

Q23 If $f(x) = x^3$ is continuous at x = 2, then find $f(2)$
(A) 2 (B) 3
(C) 4 (D) 8

Q24 If f(x) is continuous at x=0, where $f(x)=(1+2x)^{\frac{1}{x}}$, for $x \neq 0$, then find f(0)
(A) 1/e (B) $1/e^2$
(C) e^2 (D) 2e

Q25 If $f(x)=\begin{cases} \frac{\sin x}{x} & \text{if } x<0 \\ x+1 & \text{if } x \geq 0 \end{cases}$ is
(A) Discontinuous at x = 0
(B) Continuous at x = 0
(C) f(0) = 0
(D) LHL ≠ 1

Q26 If $f(x)=\begin{cases} \frac{1-\sin x}{\pi-2x}, & x \neq \frac{\pi}{2} \\ \lambda, & x=\frac{\pi}{2} \end{cases}$, be continuous at x = $\pi/2$, then value of λ is
(A) -1 (B) 1
(C) 0 (D) 2

Q27 If $f(x)=\begin{cases} cx+1 & x \leq 3 \\ cx^2-1 & x > 3 \end{cases}$ is continuous at x = 3 then C is equal to
(A) 1/3 (B) 2/3
(C) 3/2 (D) 3

Q28 $f(x)=\begin{cases} 3x-8 \text{ if } x \leq 5 \\ 2k \quad \text{if } x > 5 \end{cases}$ is continous, find k
(A) 7/2 (B) 4/7
(C) 3/7 (D) 2/7

Q29 If $f(x)=\frac{1}{x},\ x \neq 0$ is
(A) Continuous in the domain of f
(B) Continuous in [-1, 1]
(C) Discontinuous
(D) None of these

Q30 If $f(x)=\begin{cases} x+1 & \text{if } x \leq 0 \\ \frac{\tan x}{x} & \text{if } x > 0 \end{cases}$ is
(A) f(0) ≠ LHL
(B) RHL ≠ LHL
(C) Continuous at x = 0
(D) None of these

Answer Key

Q1	C
Q2	C
Q3	A
Q4	A
Q5	B
Q6	C
Q7	B
Q8	C
Q9	B
Q10	A
Q11	B
Q12	C
Q13	B
Q14	C
Q15	C
Q16	B
Q17	C
Q18	B
Q19	D
Q20	B
Q21	B
Q22	B
Q23	D
Q24	C
Q25	B
Q26	C
Q27	A
Q28	A
Q29	A
Q30	C

Vernacular

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

$$\lim_{x\to 0}\frac{\sin\left(\pi\cos^2 x\right)}{x^2}=\lim_{x\to 0}\frac{\sin\left(\pi-\pi\cos^2 x\right)}{x^2}$$

$$=\lim_{x\to 0}\frac{\sin\left(\pi\sin^2 x\right)}{x^2}=\lim_{x\to 0}\frac{\sin\left(\pi\sin^2 x\right)}{(\pi\sin x)}\cdot\pi\frac{\sin^2 x}{x^2}$$

$$=1\cdot\pi\cdot 1=\pi$$

$$\because \lim_{x\to 0}\frac{\sin x}{x}=1.$$

Video Solution:

Q2 Text Solution:

$$\lim_{n\to\infty}\frac{1^3+2^3+3^3+\ldots\ldots\ldots+n^3}{n^2(n+1)(2n+3)}=\lim_{n\to\infty}\frac{n^2(n+1)^2/4}{n^2(n+1)(2n+3)}$$

$$=\lim_{n\to\infty}\frac{(n+1)}{4(2n+3)}=\lim_{n\to\infty}\frac{n(1+1/n)}{4n(2+3/n)}$$

$$=\frac{1}{4.2}=\frac{1}{8}$$

Video Solution:

Q3 Text Solution:

$$\lim_{x\to\infty} x\sin\frac{1}{x}$$

$$\text{put } \frac{1}{x}=t \quad \text{then } x=\frac{1}{t}$$

$$\text{If } x\to\infty \text{ then } t\to 0$$

$$\lim_{t\to 0}\frac{\sin t}{t}=1$$

Video Solution:

Q4 Text Solution:

We have, $\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}$

$$=\lim_{h\to 0}\frac{(-\tan h)-(-\sin h)}{(-2h)^3}$$

$$=-\frac{1}{8}\lim_{h\to 0}\frac{\sin h-\tan h}{h^3}$$

$$=\frac{1}{8}\lim_{h\to 0}\frac{\tan h(1-\cos h)}{h^3}$$

$$=\frac{1}{8}\lim_{h\to 0}\left(\frac{\tan h}{h}\right)\left(\frac{2\sin^2\frac{h}{2}}{h^2}\right)=\frac{1}{8}\cdot 1\cdot 2\frac{1}{4}=\frac{1}{16}$$

Video Solution:

Q5 Text Solution:

$$\lim_{x\to 2}\frac{x^{10}-1024}{x^5-32}=\lim_{x\to 2}\frac{x^{10}-2^{10}}{x^5-2^5}=\lim_{x\to 2}\frac{\frac{x^{10}-2^{10}}{x-2}}{\frac{x^5-2^5}{x-2}}$$

$$=\frac{\lim_{x\to 2}\frac{x^{10}-2^{10}}{x-2}}{\lim_{x\to 2}\frac{x^5-2^5}{x-2}}=\frac{10\times 2^{10-1}}{5\times 2^{5-1}}=64$$

Video Solution:

Q6 Text Solution:

$\lim_{x\to\infty}\left(\frac{1^3+2^3+\cdots+k^3}{k^4}\right)$ is equal to

$$1^3+2^3+\cdots+k^3=\left[\frac{k(k+1)}{2}\right]^2=\frac{k^2(k+1)^2}{4}$$

$$\lim_{x\to\infty}\frac{k^2(k+1)^2}{4k^4}$$

$$\lim_{x\to\infty}\frac{k^4\left(1+\frac{1}{k}\right)^2}{4k^4}=\frac{(1+0)^2}{4}=\frac{1}{4}$$

Video Solution:

Q7 Text Solution:

$$\lim_{x\to\tan^{-1}3}\frac{(\tan x-3)(\tan x+1)}{(\tan x-3)(\tan x-1)}$$

$$=\lim_{x\to\tan^{-1}}\frac{\tan x+1}{\tan x-1}=\frac{3+1}{3-1}=2$$

Video Solution:

Q8 Text Solution:

$$\lim_{x\to 0}\frac{\frac{\tan 2x}{2x}\times 2x-3x}{3x-\frac{\sin x}{x}\times x}=\lim_{x\to 0}\frac{2x-3x}{3x-x}=\lim_{x\to 0}\frac{-x}{2x}=-\frac{1}{2}$$

Video Solution:

Q9 Text Solution:

$$\lim_{x\to -1}\left[1+x+x^2+....+x^{10}\right]$$

$$=1+(-1)+(-1)^2+.....+(-1)^{10}=1-1+1-....+1=1$$

Video Solution:

Q10 Text Solution:

$$\lim_{x\to 0^+}\frac{\sin x}{\sqrt{x}}=\lim_{x\to 0^+}\frac{\sin x}{\sqrt{x}\cdot\sqrt{x}}\sqrt{x}=\lim_{x\to 0^+}\frac{\sin x}{x}\cdot\sqrt{x}=1(0)=0$$

Video Solution:

Q11 Text Solution:

Using L'Hospitals rule

$$\lim_{x \to \frac{\pi}{4}} \frac{\cos x + \sin x}{1} = \cos\frac{\pi}{4} + \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= \sqrt{2}$$

Video Solution:

Q12 Text Solution:

$$\lim_{x \to 0}\left[\frac{e^{x} - e^{\sin x}}{x - \sin x}\right]\left(\frac{0}{0}\text{form}\right)$$

Using L − Hospital's rule three times, then

$$\lim_{x \to 0} \frac{e^{x} - e^{\sin x}\cdot\cos x}{1 - \cos x} = \lim_{x \to 0} \frac{e^{x} - e^{\sin x}\cos^2 x + \sin x \cdot e^{\sin x}}{\sin x}$$

$$= \lim_{x \to 0} \frac{e^{x} - e^{\sin x}\cdot\cos^2 x + e^{x} 2\cos x \sin x + e^{\sin x}\cdot\cos x \sin x + e^{\sin x}\cdot\cos x}{\cos x}$$

$$= 1$$

Video Solution:

Q13 Text Solution:

$$\lim_{x \to 1} \frac{1 + \cos \pi x}{\tan^2 \pi x} = \lim_{x \to 2} \frac{-\pi \sin \pi x}{-2\pi \tan \pi x \sec^2 \pi x}$$ [using L - Hospital's rule]

$$\lim_{x \to 1} \frac{-1}{2}\cos^3 \pi x = \frac{1}{2}$$

Video Solution:

Q14 Text Solution:

$$\lim_{x \to 0} \frac{\sin x}{\sqrt{x+1} - \sqrt{1-x}}$$

$$= \lim_{x \to 0} \frac{\sin x\left[\sqrt{1+x} + \sqrt{1-x}\right]}{2x}$$

$$= \frac{1}{2}\lim_{x \to 0}\left(\frac{\sin x}{x}\right)\left[\sqrt{1+x} + \sqrt{1-x}\right]$$

$$= \frac{1}{2} \times 1 \times 2 = 1$$

Video Solution:

Q15 Text Solution:

$$\lim_{x \to 0} \frac{8}{x^8}\left[1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right]$$

$$= \lim_{x \to 0} \frac{8}{x^8}\left(1 - \cos\frac{x^2}{2}\right)\left(1 - \cos\frac{x^2}{4}\right)$$

$$= \lim_{x \to 0} \frac{32}{x^8}\left(\frac{\sin\frac{x^2}{4}}{\frac{x^2}{4}}\right)^2 \cdot \left(\frac{x^2}{4}\right)^2$$

$$\cdot\left(\frac{\sin\frac{x^2}{8}}{\frac{x^2}{8}}\right)^2\left(\frac{x^2}{8}\right)^2 = \frac{1}{32}$$

Video Solution:

Q16 Text Solution:

$$\lim_{x \to 1} \frac{(1-x)}{\cot\frac{\pi x}{2}}\left(\frac{0}{0}\text{ Form }\right)$$

$$= \lim_{x \to 1} \frac{-1}{-\operatorname{cosec}^2\left(\frac{\pi x}{2}\right)\cdot\left(\frac{\pi}{2}\right)} = \frac{2}{\pi}$$

Video Solution:

Q17 Text Solution:

The modulus function is continuous for all the real values.

Video Solution:

Q18 Text Solution:

Since $f(x)$ is continuous at $x = 0$

$$\lim_{x\to 0} f(x) = f(0) \text{ (by definition)}$$

$$\lim_{x\to 0}\left(\frac{\sin x}{x} + \cos x\right) = k$$

$$\lim_{x\to 0}\frac{\sin x}{x} + \lim_{x\to 0}\cos x = k$$

$$1 + 1 = k$$

$$\Rightarrow k = 2$$

Video Solution:

Q19 Text Solution:

apply L HOSPITAL rule

$$f(0) = \lim_{x\to 0}\frac{\frac{1}{2\sqrt{1+x}} - \frac{1}{3\sqrt{1+x}}}{1}$$

$$= \frac{1}{6}$$

Video Solution:

Q20 Text Solution:

Since the given function is defined $\forall x > 0$, hence it is continuous $\forall x > 0$

Video Solution:

Q21 Text Solution:

$$\lim_{x\to 3} f(x) = \lim_{x\to 3}\frac{x^2-9}{x-3} = \lim_{x\to 3}(x+3) = 6$$

$$and\ f(3) = 2(3)+k = 6+k$$

$$\because f\ continuous\ x = 3;\quad \therefore 6+k = 6 \Rightarrow k = 0$$

Video Solution:

Q22 Text Solution:

For $x \geq 1,\ f(x) = kx^2$

$\therefore f(1) = k(1)^2 = k$

For x < 1, f(x) = 4

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1}(4) = 4$

Since f is continuous at x = 1.

$\therefore \lim_{x\to 1^-} f(x) = f(1)\ \therefore 4 = k$

Video Solution:

Q23 Text Solution:

f(x) is continuous at x = 2, then f(2) = $\lim\limits_{x \to 2} f(x)$

$\therefore \lim\limits_{x \to 2} x^3 = 2^3 = 8$

Video Solution:

Q24 Text Solution:

Since, f(x) is continuous at x = 0

$$f(0) = \lim_{x \to 0} f(x) = \lim_{x \to 0} (1 + 2x)^{\frac{1}{x}}$$

$$= \lim_{x \to 0} \left[(1 + 2x)^{\frac{1}{2x}}\right]^2 = \left[\lim_{x \to 0} (1 + 2x)^{\frac{1}{2x}}\right]^2$$

$$= e^2$$

Video Solution:

Q25 Text Solution:

$$LHL = \lim_{x \to 0^-} f\left(x\right) = \lim_{x \to 0^-} \frac{\sin x}{x} = 1$$

$$RHL = \lim_{x \to 0^+} f\left(x\right) = \lim_{x \to 0^+} x + 1 = 0 + 1$$

$$= 1$$

$f(0) = 0 + 1 = 1$

$LHL = \mathrm{RHL} = f(0)$

$f(x)$is continuous at$x = 0.$

Video Solution:

Q26 Text Solution:

f(x) is continuous at $x = \frac{\pi}{2}$, *then* $\lim\limits_{x \to \pi/2} f\left(x\right) = f\left(0\right)$ *or* $\lambda = \lim\limits_{x \to \pi/2} \frac{1 - \sin x}{\pi - 2x}, \left(\frac{0}{0}, form\right)$ *Applying L − Hospital's rule,* $\lambda = \lim\limits_{x \to \pi/2} \frac{-\cos x}{-2} \Rightarrow \lambda = \lim\limits_{x \to \pi/2} \frac{\cos x}{2} = 0$

Video Solution:

Q27 Text Solution:

$\mathrm{f}\left(\mathrm{x}\right) = \begin{cases} \mathrm{cx} + 1 & \mathrm{x} \leq 3 \\ \mathrm{cx}^2 - 1 & \mathrm{x} > 3 \end{cases}$ is continuous at x = 3.

$\mathrm{LHL} = \lim\limits_{\mathrm{x} \to 3^-} \mathrm{cx} + 1 = 3\mathrm{c} + 1$

$\mathrm{RHL} = \lim\limits_{\mathrm{x} \to 3^+} \mathrm{Cx}^2 - 1 = \mathrm{C}9 - 1$

$3\mathrm{c} + 1 = 9\mathrm{c} - 1$

6c = 2

c = 1/3

Video Solution:

Q28 Text Solution:

LHL=RHL

15-8=2k

K=$\frac{7}{2}$

Video Solution:

Q29 Text Solution:

$given\ the\ domain\ x \neq 0$

$f(x) = \frac{1}{x}$

$\Rightarrow f(c) = \frac{1}{c} \quad where\ c \neq 0$

$\lim_{x \to c} f(x) = f(c)$

$f\ is\ contonuous\ in\ the\ domain\ of\ f$

Video Solution:

Q30 Text Solution:

$$LHL = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} x + 1 = 0 + 1$$

$$= 1$$

$$RHL = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\tan x}{x} = 1$$

$$f(0) = 0 + 1$$

$$LHL = RHL = f(0)$$

$\therefore$ $f(x)$ is continuous at $x = 0$

Video Solution: