Vernacular

Ultimate kcet crash course 2026

maths

Integrals

DPP: 4

Q1 The value of $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}} dx$ is

(A) π (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

Q2 The value of $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}} dx$ is

(A) π/4 (B) π/2

(C) π (D) 2π

Q3 Evaluate : $\int_0^{\pi/2} \frac{\cos x}{1+\cos x+\sin x} dx$

(A) $\frac{\pi}{4}-\log 2$

(B) $\frac{\pi}{2}-\log 2$

(C) $\frac{1}{2}\left[\frac{\pi}{2}-\log 2\right]$

(D) $\frac{1}{2}\left[\frac{\pi}{4}-\log 2\right]$

Q4 $\int_{0.2}^{3.5} [x] dx$ is equal to

(A) 3.5 (B) 4.5

(C) 3 (D) 4

Q5 The value of $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ is

(A) 0 (B) 1

(C) π/2 (D) π/4

Q6 Find $\int_0^1 x^3 \sqrt{1+3x^4} dx$

(A) $\frac{81}{7}$ (B) $\frac{17}{8}$

(C) $\frac{71}{8}$ (D) $\frac{7}{18}$

Q7 $\int_{-5}^{5} |x+2| dx$ is equal to

(A) 28 (B) 29

(C) 27 (D) 30

Q8 $\int_0^2 \sqrt{\frac{2+x}{2-x}} dx$ is equal to

(A) $\pi+1$ (B) $\frac{\pi}{2}+1$

(C) $\pi+\frac{3}{2}$ (D) $\pi+2$

Q9 Find $\int_1^6 \frac{dx}{\sqrt{x+3}}$

(A) $2\sqrt{2}$ (B) 2

(C) $2-\sqrt{3}$ (D) $2\sqrt{3}$

Q10 Evaluate: $\int_0^1 \frac{x \tan^{-1}}{(1+x^2)^{3/2}} dx.$

(A) $\frac{4-\pi}{2\sqrt{2}}$

(B) $\frac{4+\pi}{2\sqrt{2}}$

(C) $\frac{4-\pi}{4\sqrt{2}}$

(D) None of these

Q11 Find $\int_2^3 \frac{x}{x^2+1} dx$

(A) log (1/2) (B) log 2

(C) 2 log 2 (D) 1/2 log 2

Q12 $\int_0^1 \frac{1-x}{1+x} dx =$

(A) $1-2\log 2$ (B) $2\log 2-1$

(C) $\sqrt{2\log 2-1}$ (D) None of these

Q13 $\int_0^{\infty} e^{-x} dx$

(A) -2 (B) 2

(C) 1 (D) -1

Q14 If $f(x)=\int_{-1}^{x} |t| \, dt$ then for any $x \geq 0,\ f(x)=$

(A) $1+x^2$ (B) $\frac{1}{2}\left(1-x^2\right)$

(C) $1+x^2$ (D) $\frac{1}{2}\left(1+x^2\right)$

Q15 $\int_{-3}^{3} \cot^{-1} x\, dx =$

(A) 3π (B) 6π

(C) 0 (D) 3

Q16 The value of $\int_{-1}^{0}\left[\tan^{-1}\left(\frac{x}{x^2+1}\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)\right]dx$ is

(A) 2π (B) π

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Q17 $\int_{0}^{\pi/4} \cos 4x \cos 2x dx =$

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$

(C) $\frac{1}{6}$ (D) $\frac{1}{12}$

Q18 if $\int_{-1}^{4} f(x)dx = 4 and \int_{2}^{4}(3 - f(x))dx = 7$ then $\int_{-1}^{2} f(x)dx =$

(A) - 2 (B) 3

(C) 5 (D) 8

Q19 $\int_{0}^{3\pi/4} \sqrt{1 - \sin 2x} dx$ is

(A) $2\sqrt{2} - 1$

(B) -1

(C) 1

(D) None of these

Q20 $\int_{0}^{1} \log\left(\frac{1}{x} - 1\right)dx$ is

(A) $\log_c\left(\frac{1}{2}\right)$ (B) 1

(C) 0 (D) $\log_e 2$

Q21 $\int_{0}^{\pi} \cos 2x \ \ln \ \sin \ x \ dx$ is equal to

(A) $-\pi$ (B) $-\frac{\pi}{2}$

(C) $-\frac{\pi}{4}$ (D) π

Q22 $\int_{\pi/4}^{\pi/2} \cot x dx$ is equal to

(A) $\log 2$ (B) $2 \log 2$

(C) $\frac{1}{2}\log 2$ (D) $\frac{3}{2}\log 2$

Q23 The value of $\int_{0}^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x} dx$ is

(A) $\frac{\pi-2}{4}$

(B) $\frac{\pi-2}{8}$

(C) $\frac{\pi-1}{4}$

(D) $\frac{\pi-1}{2}$

Q24 $\int_{0}^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$

(A) log 3 (B) $\frac{1}{4}\log 3$

(C) 2 log 3 (D) $\frac{1}{2}\log 3$

Q25 The integral $\int_{1}^{2} e^x \cdot x^x \left(2 + \log_e x\right) dx$ equals

(A) e(2e – 1)

(B) e(4e – 1)

(C) $4e^2 - 1$

(D) e(4e + 1)

Q26 $\int_{0}^{\pi} [\cot x] dx$, where [.] denotes the greatest integer function, is equal to

(A) 1 (B) –1

(C) –π/2 (D) π/2

Q27 $\int_{0}^{1} x^3 \left(1 - x^2\right) dx$ is equal to

(A) $\frac{12}{5}$ (B) $\frac{1}{12}$

(C) $\frac{15}{2}$ (D) $\frac{21}{5}$

Q28 If $\int_{0}^{k} \frac{dx}{2+8x^2} = \frac{\pi}{16}$, then $k =$

(A) 1 (B) 1/2

(C) 1/4 (D) None of these

Q29 $\int_{-1}^{1} \log\left(x + \sqrt{x^2 + 1}\right) dx =$

(A) 0 (B) log 2

(C) log 1/2 (D) None of these

Q30 If $\int_{0}^{b} \frac{1}{\sqrt{1+x} - \sqrt{x}} dx = \frac{4}{3}\sqrt{2}$, then b =

(A) 8 (B) 4

(C) 2 (D) 1

Answer Key

Q1	B
Q2	A
Q3	C
Q4	B
Q5	D
Q6	D
Q7	B
Q8	D
Q9	B
Q10	C
Q11	D
Q12	B
Q13	C
Q14	D
Q15	A
Q16	C
Q17	C
Q18	C
Q19	C
Q20	C
Q21	B
Q22	C
Q23	C
Q24	B
Q25	B
Q26	C
Q27	B
Q28	B
Q29	A
Q30	D

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

Let $I=\int_0^\pi \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}} dx$ ... (1)

Also $=\int_0^\pi \frac{e^{\cos(\pi-x)}}{e^{\cos(\pi-x)}+e^{-\cos(\pi-x)}}$

$=\int_0^\pi \frac{e^{-\cos x}}{e^{-\cos x}+e^{\cos x}} dx$ ... (2)

Adding (1) and (2), we get

$2I=\int_0^\pi \frac{e^{\cos x}+e^{-\cos x}}{e^{\cos x}+e^{-\cos x}} dx=\int_0^\pi 1.dx=x\Big]_0^\pi$

$=\pi \quad \therefore I=\frac{\pi}{2}$

Video Solution:

Q2 Text Solution:

$I=\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x}+2^{\cos x}} dx$ ___ (i)

$I=\int_0^{\pi/2} \frac{2^{\sin\left(\frac{\pi}{2}-x\right)}}{2^{\sin\left(\frac{\pi}{2}-x\right)}+2^{\cos\left(\frac{\pi}{2}-x\right)}} dx=$ ___ (ii)

$\int_0^{\pi/2} \frac{2^{\cos x}}{2^{\cos x}+2^{\sin x}} dx$

Adding equations (i) and (ii), we get

$2I=\int_0^{\pi/2}\left(\frac{2^{\sin x}+2^{\cos x}}{2^{\sin x}+2^{\cos x}}\right) dx=\int_0^{\pi/2} 1 dx$

$=[x]_0^{\pi/2}=\frac{\pi}{2}$

Therefore, $I=\frac{\pi}{4}$

Video Solution:

Q3 Text Solution:

$I=\int_0^{\pi/2} \frac{\cos x}{(1+\cos x)+\sin x} dx$

$=\int_0^{\pi/2} \frac{\cos^2 x/2-\sin^2 x/2}{2\cos^2 x/2+2\sin x/2\cos x/2} dx$

$=\frac{1}{2}\int_0^{\pi/2} \frac{1-\tan^2 x/2}{1+\tan x/2} dx$

$=\frac{1}{2}\int_0^{\pi/2} 1-\tan x/2\, dx$

$=\frac{1}{2}\left[x-2\log \ \sec(x/2)\right]_0^{\pi/2}$

$=\frac{1}{2}\left[\frac{\pi}{2}-2\log\sec\left(\frac{\pi}{4}\right)\right]=\frac{1}{2}\left[\frac{\pi}{2}-\log 2\right]$

Video Solution:

Q4 Text Solution:

Here,

$\int_{0.2}^{3.5}[x]dx=\int_{0.2}^{1} 0\, dx+\int_1^2 1\, dx+\int_2^3 2\, dx+$

$\int_3^{3.5} 3\, dx$

$=0+[x]_1^2+[2x]_2^3+[3x]_3^{3.5}=1+2(1)$

$+3(0.5)=4.5$

Video Solution:

Q5 Text Solution:

$$I = \int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$$

$$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x+\sin^3 x} dx \qquad \ldots(i)$$

$$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x} \, dx$$

$$2I = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Video Solution:

Q6 Text Solution:

$$I = \int_0^1 x^3 \sqrt{1+3x^4} \, dx$$

Put $1+3x^4 = t \Rightarrow 12x^3 \, dx = dt$

When x = 0, t = 1 and when x = 1, t = 4

$$\therefore I = \int_1^4 \frac{1}{12} t^{1/2} dt = \frac{1}{12}\left[\frac{2}{3} t^{\frac{3}{2}}\right]_1^4$$

$$= \frac{1}{18}\left[4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right] = \frac{1}{18}\left[2^3 - 1\right] = \frac{7}{18}$$

Video Solution:

Q7 Text Solution:

Let $I = \int_{-5}^{5} |x+2| dx = -\int_{-5}^{-2} (x+2)dx + \int_{-2}^{5} (x+2)dx = 29$

Video Solution:

Q8 Text Solution:

$Put, x = 2\cos 2\theta \Rightarrow dx = -4\sin 2\theta d\theta \; as, x \to 0, \theta \to \frac{\pi}{4} \; as x \to 2, \theta \to 0$

$$Then\ I = \int_{\pi/4}^{0} \cot\theta(-4\sin 2\theta)d\theta$$

$$\therefore I = 8\int_0^{\pi/4} \cos^2\theta d\theta = 4$$

$$\int_0^{\pi/4}(1+\cos 2\theta)d\theta = 4\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/4}$$

$$= 4\left[\frac{\pi}{4} + \frac{1}{2}\right] = \pi + 2$$

Video Solution:

Q9 Text Solution:

$$\int_1^6 \frac{dx}{\sqrt{x+3}}$$

$$= \left[2(x+3)^{\frac{1}{2}}\right]_1^6 = 2\left[(6+3)^{1/2} - 4^{\frac{1}{2}}\right]$$

$$= 2[3-2] = 2$$

Video Solution:

Q10 **Text Solution:**

(C)

Let $\tan^{-1} x = \theta$

$\Rightarrow x = \tan\theta$

$\Rightarrow dx = \sec^2\theta\, d\theta$

Now, $x = 0 \Rightarrow \theta = 0$

and x = 1 $\Rightarrow \theta = \frac{\pi}{4}$

$$\therefore \int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}dx = \int_0^{\pi/4}\frac{\theta\tan\theta}{\sec^3\theta}\sec^2\theta\, d\theta$$

$$= \int_0^{\pi/4}\theta\sin\theta\, d\theta[-\theta\cos\theta]_0^{\pi/4} -$$

$$\int_0^{\pi/4}(-\cos\theta)d\theta$$

$$= [-\theta\cos\theta]_0^{\pi/4} + [\sin\theta]_0^{\pi/4} = \frac{4-\pi}{4\sqrt{2}}$$

Video Solution:

Q11 **Text Solution:**

$$I = \int_2^3 \frac{x}{x^2+1}dx$$

Put $x^2 + 1 = t \Rightarrow 2xdx = dt$

When x = 2, t = 5 and when x = 3, t = 10

$$\therefore I = \frac{1}{2}\int_5^{10}\frac{1}{t}dt = \frac{1}{2}[\log\ t]_5^{10}$$

$$= \frac{1}{2}[\log 10 - \log 5] = \frac{1}{2}\log\left(\frac{10}{5}\right) = \frac{1}{2}\log 2$$

Video Solution:

Q12 **Text Solution:**

$$\int_0^1 \frac{1-x-1+1}{1+x}dx = \int_0^1\left(\frac{2}{1+x} - 1\right)dx$$

$$= 2\log(1+x) - x\Big]_0^1 = 2\log 2 - 1$$

Video Solution:

Q13 **Text Solution:**

$$\int_0^\infty e^{-x}dx = \left[\frac{e^{-x}}{-1}\right]_0^\infty = -\left[e^{-\infty} - e^{-0}\right]$$

$$= -\left[\frac{1}{e^\infty} - \frac{1}{e^0}\right] = -[0-1] = 1$$

Video Solution:

Q14 **Text Solution:**

$$f(x) = \int_{-1}^{x}|t|\ \mathrm{dt}$$

$$= \int_{-1}^{0}(-t)\ \mathrm{dt} +$$

$$\int_0^x t\ dt\ \{|t| = t,\ t \geq 0 \text{ and } -t,\ t < 0\}$$

$$= \frac{1}{2}\left(x^2 + 1\right)$$

Video Solution:

Q15 **Text Solution:**

Let $I = \int_{-3}^{3}\cot^{-1} x\ dx$ ...(i)

By property of definite integral

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$I = \int_{-3}^{3} \cot^{-1}(-3+3-x)dx$$

$$I = \int_{-3}^{3} \cot^{-1}(-x)dx$$

WKT : $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$

$$I = \int_{-3}^{3} \left(\pi - \cot^{-1} x\right)dx$$

$$\Rightarrow I = \int_{-3}^{3} \pi\, dx - \int_{-3}^{3} \cot^{-1} x\, dx$$

(i) $I = \pi x|_{-3}^{3} - I$

$2I = \pi(3+3)$

$I = 3\pi$

Video Solution:

Q16 Text Solution:

$$\int_{-1}^{0}\left[\tan^{-1}\left(\frac{x}{x^2+1}\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)\right]dx$$

$$= \int_{-1}^{0}\left[cot^{-1}\left(\frac{x^2+1}{x}\right)+\tan^{-1}\left(\frac{x^2+1}{x}\right)\right]dx$$

$$= \int_{-1}^{0} \frac{\pi}{2} dx = \frac{\pi}{2}$$

Video Solution:

Q17 Text Solution:

$$\cos A \cdot \cos B = \frac{1}{2}[\cos(A+B)+\cos(A-B)]$$

$$\int_0^{\pi/4} \frac{1}{2}[\cos 6x + \cos 2x]dx = \frac{1}{2}\left\{ \left.\frac{\sin 6x}{6}\right]_0^{\pi/4} + \left.\frac{\sin 2x}{2}\right]_0^{\pi/4} \right\}$$

$$= \frac{1}{2}\left\{\frac{\sin\frac{3\pi}{2}}{6} + \frac{\sin\frac{\pi}{2}}{2}\right\} = \frac{1}{2}\left\{\frac{-1}{6} + \frac{1}{2}\right\}$$

$$= \frac{1}{2}\left[\frac{-1+3}{6}\right] = \frac{1}{2}\left[\frac{2}{6}\right] = \frac{1}{6}$$

Video Solution:

Q18 Text Solution:

$$\int_{-1}^{4} \mathrm{f(x)dx} = 4 \text{and} \int_{2}^{4} [3 - \mathrm{f(x)dx} = 7]$$

$F(4) - F(-1) = 4 \rightarrow (1)$

$\text{and} 3|x|_2^4 - [F(4) - F(2)] = 7$

$[F(4) - F(2)] = 6 - 7 = -1$

$\therefore F(4) - F(2) = -1 \rightarrow (2)$

$$\int_{-1}^{2} \mathrm{f(x)dx} = F(2) - F(-1) \rightarrow (3)$$

eqn.(1)-eqn.(2), we get$F(2) - F(-1) = 5$

$$\therefore \int_{-1}^{2} \mathrm{f(x)dx} = 5 \text{ (put value in eqn.(3)}$$

Video Solution:

Q19 Text Solution:

Given that, $\int_0^{\frac{3\pi}{4}} \sqrt{1-\sin 2x}dx$

$$= \int_0^{\frac{3\pi}{4}} \sqrt{\sin^2 x + \cos^2 x - 2\ \sin x\ \cos x dx}$$

$$= \int_0^{\frac{3\pi}{4}} \sqrt{(\sin x - \cos x)^2} dx$$

$$= \int_0^{\frac{3\pi}{4}} (\sin x - \cos x)dx$$

$$= [-\cos x - \sin x]_0^{\frac{3\pi}{4}}$$

$$= \left[-\cos\frac{3\pi}{4} - \sin\frac{3\pi}{4} + \cos 0 + \sin 0\right]$$

$$= \left[-\cos\left(\pi - \frac{\pi}{4}\right) - \sin\left(\pi - \frac{\pi}{4}\right) + 1\right]$$

$= \cos\frac{\pi}{4} - \sin\frac{\pi}{4} + 1$

$= \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} + 1$

$= 1$

Video Solution:

Q20 Text Solution:

$I = \int_0^1 \log\frac{1-x}{x}dx = \int_0^1 \log(1-x)dx - \int_0^1 \log xdx$

$= 0 \because \int_0^1 \log xdx = \int_0^1 \log(1-x)dx$, by a property

Video Solution:

Q21 Text Solution:

$I = \int_0^{\pi} \cos 2x \ \log \ \sin \ x \ dx$

$= \log\sin \ x\left[\frac{\sin 2x}{2}\right] - \int \frac{\cos x}{\sin x}\left(\frac{\sin 2x}{2}\right)dx$

$= \frac{\sin 2x}{2}\log\sin \ x - \int \cos^2 x \ dx\Big]_0^{\pi}$

$= \frac{\sin 2x}{2}\log\sin x - \int \frac{1+\cos 2x}{2}dx\Big]_0^{\pi}$

$= \frac{\sin 2x}{2}\log\sin x - \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]\Big]_0^{\pi}$

$= \left\{0 - \frac{1}{2}[\pi + 0]\right\} - 0$

$I = \frac{-\pi}{2}$

Video Solution:

Q22 Text Solution:

$\int_{\pi/4}^{\pi/2} \cot x dx = \left[\log|\sin x|\right]_{\pi/4}^{\pi/2}$

$= \log\left|\sin\frac{\pi}{2}\right| - \log\left|\sin\frac{\pi}{4}\right|$

$= \log(1) - \log\left(\frac{1}{\sqrt{2}}\right) = \log\sqrt{2} = \frac{1}{2}\log 2$

Video Solution:

Q23 Text Solution:

$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin x + \cos x}dx$

$\Rightarrow \ I = \int_0^{\pi/4} \frac{\sin^3 x + \cos^3 x}{\sin x + \cos x}dx$

$\left[\because \int_0^{2a} f(x)dx = \int_0^{a} f(x)dx + \int_0^{a} f(a-x)dx\right]$

$= \int_0^{\pi/4} \left(1 - \sin x \ \cos x\right)dx$

$= \left(x - \frac{\sin^2 x}{2}\right)_0^{\pi/4}$

$= \frac{\pi}{4} - \frac{1}{4}$

$= \frac{\pi - 1}{4}$

Video Solution:

Q24 Text Solution:

$$I = \int_{0}^{\pi/4} \frac{\sin x+\cos x}{3+\sin 2x} dx$$

$$I = \int_{0}^{\pi/4} \frac{\sin x+\cos x}{3+1-1+\sin 2x} dx = \int_{0}^{\pi/4} \frac{\sin x+\cos x}{4-(1-\sin 2x)} dx$$

$$I = \int_{0}^{\pi/4} \frac{\sin x+\cos x}{4-(\sin x-\cos x)^2} dx$$

Let $\sin x - \cos x = t$

$(cosx + \ sinx)dx = dt$ $\left| \begin{array}{l} x = 0, \quad t = -1 \\ x = \pi/4, \ t = 0 \end{array} \right.$

$$I = \int_{-1}^{0} \frac{dt}{4-t^2} = \int_{0}^{-1} \frac{dt}{t^2-4} = \frac{1}{4}\log(3)$$

Video Solution:

Q25 Text Solution:

Consider, $I = \int_{1}^{2} e^x x^x \left(2 + \log_e x\right) dx$

$$I = \int_{1}^{2} e^x x^x \left[1 + \left(1 + \log_e x\right)\right] dx$$

$$I = \int_{1}^{2} e^x \left[x^x + x^x \left(1 + \log_e x\right)\right] dx$$

$I = [e^x x^x]_1^2,$

$\left\{\int e^x \left(f(x)+f'('x)\right) dx = e^x f(x)+c\right\}$

$= e^2 \times 4 - e \times 1 = 4e^2 - e = e(4e - 1)$

Video Solution:

Q26 Text Solution:

$\mathrm{I} = \int_0^{\pi}[\cot x]\,dx$

$\mathrm{I} = \int_0^{\pi}\left[\cot\left(\pi - x\right)\right]dx = \int_0^{\pi}[-\cot x]\,dx$

Adding we have $2\mathrm{I} = \int_0^{\pi}\{[\cot x] + [-\cot x]\}\,dx$

$2\mathrm{I} = \int_0^{\pi}(-1)\,dx = -\pi \therefore \mathrm{I} = -\pi / 2$

Note that $[x] + [-x] = 0, x \in Z$ and

$= -1, x \notin Z$.

Video Solution:

Q27 Text Solution:

$I = \int_0^1 x^3 \left(1 - x^2\right) dx = \int_0^1 x^3 dx -$

$\int_0^1 x^5 dx$

$= \left[\frac{x^4}{4}\right]_0^1 - \left[\frac{x^6}{6}\right]_0^1 = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$

Video Solution:

Q28 Text Solution:

$\int_0^k \frac{1}{2+8x^2}dx = \frac{1}{2}\int_0^k \frac{dx}{1+(2x)^2} = \frac{1}{4}\int_0^{2k}\frac{dt}{1+t^2}$

[assumed t=2x dt=2dx]

$= \frac{1}{4}\left|\tan^{-1}t\right|_0^{2k} = \frac{1}{4}\tan^{-1}2k$

Comparing it with the given value,

we get $\frac{1}{4}\tan^{-1}2k = \frac{\pi}{16} \Rightarrow 2k = 1 \Rightarrow k = \frac{1}{2}$

Video Solution:

Q29 Text Solution:

Let $f(x) = \log\left(x + \sqrt{1+x^2}\right)$

Now, $f(-x) = \log\left(\sqrt{1+x^2} - x\right)$

$= \log\sqrt{(1+x^2-x)} \cdot \frac{\left(\sqrt{1+x^2}+x\right)}{\left(\sqrt{1+x^2}+x\right)}$

$= \log\frac{[(1+x^2)-x^2]}{\left(\sqrt{1+x^2}+x\right)}$

$= \log\frac{[(1+x^2)-x^2]}{\left(\sqrt{1+x^2}+x\right)} = \log 1$

$-\log\left(\sqrt{1+x^2}+x\right)$

$= -\log\left(\sqrt{1+x^2}+x\right) = -f(x)$

Hence, $\int\limits_{-1}^{1}\log\left(x+\sqrt{1+x^2}\right) = 0,$

$\left[\because \int_{-a}^{a} f(x) = 0 \text{ if } f(-x) = -f(x)\right]$

Video Solution:

Q30 Text Solution:

$I = \int\limits_a^b \frac{1}{\sqrt{1+x}-\sqrt{x}}dx = \int\limits_a^b \sqrt{1+x} + \sqrt{x}\,dx$

$= \frac{4}{3}\sqrt{2}$

$I = \frac{2}{3}\left[(x+1)^{3/2} + x^{3/2}\right]_0^b = \frac{4}{3}\sqrt{2}$

$\frac{2}{3}\left[(b+1)^{3/2} + b^{3/2} - 1\right] = \frac{4}{3}\sqrt{2}$

$(b+1)^{3/2} + b^{3/2} = 2\sqrt{2} + 1$

By option verification b = 1

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