Vernacular

ULTIMATE KCET CRASH COURSE 2026

MATHS

DPP: 2

Vectors and 3D

Q1 If $\vec{a} = (1, -1, 2)$, $\vec{b} = (-2, 3, 5)$, $\vec{c}$ and $\hat{i} = (2, -2, 4)$ is the unit vector in the x-direction, then $\left(\vec{a} - 2\vec{b} + 3\vec{c}\right) \cdot \hat{i} =$

(A) 11 (B) 15
(C) 18 (D) 36

Q2 If $\vec{a} = i - j + 2k$, $\vec{b} = 2i + 3j$ then the $+ k$ and $\vec{c} = i - k$ magnitude of $\vec{a} + 2\vec{b} - 3\vec{c}$ -is

(A) $\sqrt{87}$ (B) $\sqrt{78}$
(C) $\sqrt{89}$ (D) $\sqrt{101}$

Q3 If $a = \hat{i} + \hat{j} - 2\hat{k}$, $b = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} - \hat{k}$ and $\vec{c} = m\vec{a} + n\vec{b}$ then $m + n =$ ______.

(A) 0 (B) 1
(C) 2 (D) –1

Q4 There non-zero non-collinear vectors *a*, *b* and *c* are such that $a + 3b$ is collinear with *c*, $3b + 2c$ is collinear with *a*. Then $a + 3b + 2c$ is equal to

(A) 0 (B) $2a$
(C) $3b$ (D) $4c$

Q5 If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$,, then sin θ/2 is

(A) $\left|\vec{a} + \vec{b}\right|$

(B) $\frac{\left|\vec{a} + \vec{b}\right|}{2}$

(C) $\frac{\left|\vec{a} - \vec{b}\right|}{2}$

(D) $\left|\vec{a} - \vec{b}\right|$

Q6 If $\left|\vec{a}\right| = 4$, $\left|\vec{b}\right| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{6}$, then $\left(\vec{a} \times \vec{b}\right)^2$ is

(A) 48 (B) 16
(C) $\vec{a}$ (D) 15

Q7 If $\vec{a}$, $\vec{b}$, $\vec{c}$ are any three mutually perpendicular vectors of equal magnitude a, $\left|\vec{a} + \vec{b} + \vec{c}\right|$ is

(A) a (B) $\sqrt{2}a$
(C) $\sqrt{3}a$ (D) 3a

Q8 The area of the triangle whose vertices are *A*(1, –1, 2), *B*(2, 1, –1) and *C*(3, –1, 2) is

(A) $4\sqrt{5}$ (B) $2\sqrt{3}$
(C) $\sqrt{13}$ (D) $\sqrt{15}$

Q9 If α, β and γ are the angles made by a vector with the coordinate axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma =$

(A) 0 (B) 1
(C) -1 (D) 2

Q10 If m_1, m_2, m_3 and m_4 are respectively, the magnitudes of the vectors

$\vec{a}_1 = 2\hat{i} - \hat{j} + \hat{k},\ \vec{a}_2 = 3\hat{i} - 4\hat{j} - 4\hat{k},$
$\vec{a}_3 = \hat{i} + \hat{j} + \hat{k}$ and $\vec{a}_4 = -\hat{i} + 3\hat{j} + \hat{k}$
then the correct order of $m_1, m_2,$
m_3 and m_4 is
(A) $m_3 < m_1 < m_4 < m_2$
(B) $m_3 < m_1 < m_2 < m_4$
(C) $m_3 < m_4 < m_2 < m_1$
(D) $m_3 < m_4 < m_2 < m_1$

Q11 If the sum of two unit vectors is a unit vector, then the magnitude of their difference is:
(A) $\sqrt{2}$ *units* (B) 2 units
(C) $\sqrt{3}$ *units* (D) $\sqrt{5}$ *units*

Q12 $\left|\vec{a} \times \vec{b}\right|^2 + \left|\vec{a} \cdot \vec{b}\right|^2 = 144$ and $\left|\vec{a}\right|$ is $= 4$ then $\left|\vec{b}\right|$ equal to
(A) 8 (B) 12
(C) 4 (D) 3

Q13 If $\vec{a}$ and $\vec{b}$ are mutually perpendicular unit vectors, then $\left(3\vec{a} + 2\vec{b}\right) \cdot \left(5\vec{a} - 6\vec{b}\right) =$
(A) 5 (B) 3
(C) 6 (D) 12

Q14 If $\vec{a}$, $\vec{b}$ are two mutually perpendicular unit vectors, then the value of
$\left(2\vec{a} + 3\vec{b}\right) \cdot \left(4\vec{a} - 6\vec{b}\right)$
(A) 9 (B) 10
(C) -9 (D) -10

Q15 If the projection of the vector $\vec{i} + \vec{j} + \vec{k}$ on the vector $a\vec{i} + \vec{j} + 2\vec{k}$ is $\frac{5}{3}$, then a=
(A) 7 (B) 0
(C) $\frac{1}{2}$ (D) 2

Q16 *Let* $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ *and* $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that
$\left|\vec{c} - \vec{a}\right| = 3, \left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\right| = 3$ and
the angle between $\overline{c}$ and $\vec{a} \times \vec{b}$ be 30^0.
Then $\vec{a} \cdot \vec{c}$ is equal to
(A) 2 (B) 5
(C) $\frac{1}{8}$ (D) $\frac{25}{8}$

Q17 The magnitude of the vector $6\hat{i} - 2\hat{j} + 3\hat{k}$ is
(A) 1 (B) 5
(C) 7 (D) 12

Q18 If $\vec{a} + \vec{b}$ *and* $\vec{a} - \vec{b}$ are perpendicular and $\vec{b} = 3\mathrm{i} - 4\mathrm{j} + 2\mathrm{k}$ then $\left|\vec{a}\right|$ is equal to
(A) $\sqrt{41}$ (B) $\sqrt{39}$
(C) $\sqrt{19}$ (D) $\sqrt{29}$

Q19 If the points A(3,0,p),B(-1,q,3),C(-3,3,0) are collinear, then
(A) p = 9, q = 2 (B) p = 9, q = -2
(C) p = -9, q = 2 (D) p = -9, q = -2

Q20 Find a vector $\vec{r}$ equally inclined to the three axes and whose magnitude is $3\sqrt{3}$ units.
(A) $\hat{i} + \hat{j} + \hat{k}$
(B) $\pm\sqrt{3}\left(\hat{i} + \hat{j} + \hat{k}\right)$
(C) $\pm 3\left(\hat{i} + \hat{j} + \hat{k}\right)$
(D) None of these

Q21 The angle between the lines whose direction cosines are proportional to (1,2,1) and (2,-3,6) is
(A) $\cos^{-1}\left(\frac{2}{7\sqrt{6}}\right)$ (B) $\cos^{-1}\left(\frac{1}{7\sqrt{6}}\right)$
(C) $\cos^{-1}\left(\frac{3}{7\sqrt{6}}\right)$ (D) $\cos^{-1}\left(\frac{5}{7\sqrt{6}}\right)$

Q22 If the line through the points (4, 1, 2) and (5, *k*, 0) is parallel to the line through the points (2, 1, 1) and (3, 3, –1), then *k* is equal to

(A) 3 (B) 2

(C) -2 (D) None of these

Q23 If α, β, γ be the direction angles of a vector and $\cos\alpha = \frac{14}{15}$ and $\cos\beta = \frac{1}{3}$, then $\cos\gamma =$

(A) $\pm\frac{2}{15}$ (B) $\frac{1}{5}$

(C) $\pm\frac{1}{15}$ (D) None of these

Q24 The equations of the line perpendicular from the point (-2,4,1) to the plane 7x - 2y + 3z = 1 are

(A) $\frac{x-5}{7} = \frac{y-2}{-2} = \frac{z-4}{3}$

(B) $\frac{x-2}{7} = \frac{y+4}{-2} = \frac{z+1}{-3}$

(C) $\frac{x+2}{1} = \frac{y-4}{4} = \frac{z-1}{2}$

(D) None of these

Q25 Perpendicular distance of the point (3, 4, 5) from the *y*-axis, is

(A) $\sqrt{34}$ (B) $\sqrt{41}$

(C) 4 (D) 1

Q26 The shortest distance between the skew lines

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + 3\hat{j} + 2\hat{k})$

and

$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k})$ is:

(A) 3 (B) $2\sqrt{3}$

(C) $\sqrt{3}$ (D) $\sqrt{6}$

Q27 If the lines $\frac{x+2}{4\lambda+1} = \frac{y-1}{4} = \frac{z}{-18}$ and $\frac{x}{-3} = \frac{y+1}{5\mu-3} = \frac{z-1}{6}$ are parallel to each other then the value of the pair (λ, μ) is.

(A) $\left(-2, \frac{1}{3}\right)$

(B) $\left(2, -\frac{1}{3}\right)$

(C) $\left(2, \frac{1}{3}\right)$

(D) Cannot be found

Q28 The distance between the lines $\frac{x-4}{5} = \frac{y+1}{2} = \frac{z}{1}$ and $\frac{x-1}{5} = \frac{y-2}{2} = \frac{z-3}{1}$ is

(A) $5\sqrt{129}$ (B) $\frac{\sqrt{129}}{5}$

(C) $\sqrt{\frac{129}{10}}$ (D) $\sqrt{\frac{129}{5}}$

Q29 The direction cosines of the line which is perpendicular to the lines whose direction cosines are proportional to (1, –1, 2) and (2, 1, –1) are

(A) $\frac{1}{\sqrt{35}}, \frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}$

(B) $-\frac{1}{\sqrt{35}}, \frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}$

(C) $\frac{1}{\sqrt{35}}, \frac{5}{\sqrt{35}}, -\frac{3}{\sqrt{35}}$

(D) None of these

Q30 The angle between the lines

$= (2\hat{i} - 5\hat{j} + \hat{k}) + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and

$= (7\hat{i} - 6\hat{k}) + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ is

(A) $\cos^{-1}\left(\frac{19}{21}\right)$

(B) $\cos^{-1}\left(\frac{19}{2}\right)$

(C) $\cos^{-1}\left(\frac{18}{19}\right)$

(D) $\cos^{-1}\left(\frac{9}{21}\right)$

Answer Key

Q1	(A)	Q16	(A)
Q2	(B)	Q17	(C)
Q3	(C)	Q18	(D)
Q4	(A)	Q19	(A)
Q5	(C)	Q20	(C)
Q6	(B)	Q21	(A)
Q7	(C)	Q22	(A)
Q8	(C)	Q23	(A)
Q9	(D)	Q24	(A)
Q10	(A)	Q25	(A)
Q11	(C)	Q26	(C)
Q12	(D)	Q27	(C)
Q13	(B)	Q28	(D)
Q14	(D)	Q29	(B)
Q15	(D)	Q30	(A)

Hints & Solutions

Note: scan the QR code to watch video solution

Q1 Text Solution:

$\vec{a} = (1, -1, 2),\ \vec{b} = (-2, 3, 5),\ \vec{c} = (2, -2, 4)$

So, $\vec{a} = (1, -1, 2) \equiv \hat{i} - \hat{j} + 2\hat{k};\ \vec{b} = (-2, 3, 5) \equiv -2\hat{i} + 3\hat{j} + 5\hat{k}$

$and\ \vec{c} = (2, -2, 4) \equiv 2\hat{i} - 2\hat{j} + 4\hat{k}$

$$\Rightarrow -2 + 3 = \left(\hat{i} - \hat{j} + 2\hat{k}\right) - 2\left(-2\hat{i} + 3\hat{j} + 5\hat{k}\right)$$

$$= 11\hat{i} - 13\hat{j} + 4\hat{k} \text{ and } \left(2\hat{a} - 2 + 3\hat{j} + 4\hat{c}\right) \cdot \hat{i} = 11$$

Video Solution:

Q2 Text Solution:

$\vec{a} = i - j + 2k,\ \vec{b} = 2\hat{i} + 3\hat{j} + k,\ \vec{c} = i - k$

$$\vec{a} + 2\vec{b} - 3\vec{c} = (i + 4i - 3i) + (-j + 6j) + (2k + 2k + 3k)$$

$= 2i + 5j + 7k$

$$\left|\vec{a} + 2\vec{b} - 3\vec{c}\right| = \sqrt{4 + 24 + 49} = \sqrt{78}$$

Video Solution:

Q3 Text Solution:

$\vec{a} = \hat{i} + \hat{j} - 2\hat{k},\ \vec{b} = 2\hat{i} - \hat{j} + \hat{k},\ \vec{c} = 3\hat{i} - \hat{k}$

Also, $\vec{c} = m\vec{a} + n\vec{b}$

$3\hat{i} - \hat{k} = m\left(\hat{i} + \hat{j} - 2\hat{k}\right) + n\left(2\hat{i} - \hat{j} + \hat{k}\right)$

$= (m + 2n)\hat{i} + (m - n)\hat{j} + (-2m + n)\hat{k}$

$\Rightarrow m + 2n = 3,\ m - n = 0,\ -2m + n = -1$

$\therefore m - n = 0 \Rightarrow m = n$

And $m + 2n = m + 2m = 3$

$\Rightarrow m = 1 \therefore m = n = 1 \Rightarrow m + n = 2$

Video Solution:

Q4 Text Solution:

Android App | iOS App | PW Website

condition if A collinear with B

$A = \lambda B$

Given a + 3b is collinear with c.

$a + 3b = \lambda c \quad (1)$

3b + 2c is collinear with a

$3b + 2c = \mu a \rightarrow (1).$

From (1) $a + 3b + 2c = \lambda c + 2c$

$= c(\lambda + 2) \rightarrow (3)$

From (2) $a + 3b + 2c = a + ra$

$= a(1 + \mu) \rightarrow (4)$

$c(\lambda + 2) = a(1 + \mu)$

Equate.

$1 + \mu = 0 \quad \lambda + 2 = 0.$

$\mu = -1 \quad \lambda = -2$

From (3)

$a + 3b = \lambda c$

$a + 8b = -2c$

$\Rightarrow a + 3b + 2c = 0$

Video Solution:

Q5 Text Solution:

$$\left|\vec{a} - \vec{b}\right| = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 2\vec{a} \cdot \vec{b}$$

$$= 1 + 1 - 2\cos\theta$$

$$= 2(1 - \cos\theta)$$

$$\left|\vec{a} - \vec{b}\right|^2 = 2 \cdot 2\sin^2\left(\frac{\theta}{2}\right)$$

$$\left|\vec{a} - \vec{b}\right| = 2\sin\left(\frac{\theta}{2}\right)$$

Video Solution:

Q6 Text Solution:

We have, $\left(\vec{a} \times \vec{b}\right)^2 = \left|\vec{a} \times \vec{b}\right|^2$

$$= \left|\vec{a}\right|^2 \cdot \left|\vec{b}\right|^2 \cdot \sin^2\theta$$

$$= 16 \times 4 \times \frac{1}{4} = 16$$

Video Solution:

Q7 Text Solution:

Since given $\vec{a}, \vec{b}$ & $\vec{c}$ are mutually perpendicular vector :

$$\therefore \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$$

Also, $\therefore \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

WKT

$$\left|\vec{a} + \vec{b} + \vec{c}\right|^2 = \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2 + 2\left(a \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}\right)$$

$$\therefore \left|\vec{a} + \vec{b} + \vec{c}\right|^2 = a^2 + a^2 + a^2$$

$$\therefore \left|\vec{a} + \vec{b} + \vec{c}\right|^2 = 3a^2$$

$$\therefore \left|\vec{a} + \vec{b} + \vec{c}\right| = \sqrt{3}a$$

Video Solution:

Q8 Text Solution:

Given vertices are

$A(1, -1, 2)$, $B(2, 1, -1)$ and $C(3, -1, 2)$

Area of triangle $ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

Where :

$\overrightarrow{AB} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\overrightarrow{AC} = 2\hat{i} + 0\hat{j} + 0\hat{k}$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 0 & 0 \end{vmatrix}$$

$$= \hat{i}\,(0) - \hat{j}\,(0+6) + \hat{k}\,(0-4)$$

$$= -6\hat{j} - 4\hat{k}$$

$$\therefore \left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{52} = 2\sqrt{13}$$

$$\therefore \text{ Area of triangle} = \frac{1}{2}\left(2\sqrt{13}\right) = \sqrt{13}$$

Video Solution:

Q9 Text Solution:

Let l, m, n be the d.c.'s of the vector.

Then $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$

Now, $l^2 + m^2 + n^2 = 1 \Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$$

$$\Rightarrow 3 - \left(\sin^2\alpha + \sin^2\beta + \sin^2\gamma\right) = 1$$

$$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3 - 1 = 2.$$

Video Solution:

Q10 Text Solution:

$$\text{Given, } m_1 = \left|\overrightarrow{a}_1\right| = \sqrt{2^2 + (-1)^2 + (1)^2} = \sqrt{6}$$

$$m_2 = \left|\overrightarrow{a}_2\right| = \sqrt{3^2 + (-4)^2 + (-4)^2} = \sqrt{41}$$

$$m_3 = \left|\overrightarrow{a}_3\right| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$\text{and } m_4 = \left|\overrightarrow{a}_4\right| = \sqrt{(-1)^2 + (3)^2 + (1)^2} = \sqrt{11}$$

$$\therefore m_3 < m_1 < m_4 < m_2$$

Video Solution:

Q11 Text Solution:

Let $\bar{a}, \bar{b}, \bar{c}$ bet three unit vectors such that

$\bar{c} = \bar{a} + \bar{b}$

$$\therefore \bar{c}^2 = \left(\bar{a} + \bar{b}\right)^2 \Rightarrow 1 = 1 + 1 + 2\bar{a}\cdot\bar{b}$$

$$\Rightarrow 2\,\overrightarrow{a}\,.\,\overrightarrow{b} = -1$$

$$\text{Now, } \left|\bar{a} - \bar{b}\right|^2 = 1 + 1 - 2\bar{a}\cdot\bar{b} = 3$$

$$\Rightarrow \left|\bar{a} - \bar{b}\right| = \sqrt{3} \text{ units.}$$

Video Solution:

Q12 Text Solution:

$$\left(\left|\vec{a}\right|\left|\vec{b}\right|^{2}\right) = 144$$

$$\Rightarrow \left|\vec{a}\right|\left|\vec{b}\right| = 12$$

$$\Rightarrow \left|\vec{b}\right| = \frac{12}{4} = 3$$

Video Solution:

Q13 Text Solution:

$$\left|\vec{a}\right| = \left|\vec{b}\right| = 1$$

$$15a^2 - 12b^2 = (15 \times 1) - (12 \times 1) = 15 - 12 = 3$$

Video Solution:

Q14 Text Solution:

$$\left(2\vec{a} + 3\vec{b}\right) \cdot \left(4\vec{a} - 6\vec{b}\right) = 8\left(\vec{a} \cdot \vec{a}\right)$$

$$-12\vec{a} \cdot \vec{b} + 12\vec{b} \cdot \vec{a} - 18\vec{b} \cdot \vec{b} = 8(1) - 18(1) = -10$$

Video Solution:

Q15 Text Solution:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \dot{b} = a\hat{i} + \hat{j} + 2\hat{k} \ \frac{\vec{a} \cdot \hat{b}}{|h|}$$

$$= \frac{a+1+2}{\sqrt{a^2+5}} = \frac{5}{3}$$

$$\Rightarrow 3a + 9 = 5\sqrt{a^2+5} \Rightarrow a = 2$$

Video Solution:

Q16 Text Solution:

$$\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k} \Rightarrow |a| = 3 \ and \ \vec{b} = \hat{i} + \hat{j}$$

$$Now,\ \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k} \ \therefore \left|\vec{a} \times \vec{b}\right| = 3$$

$$We\ also\ have,$$

$$\left|\left(\vec{a} \times \vec{b}\right) \times \vec{c}\ \right| = \left|\vec{a} \times \vec{b}\right|\left|\vec{c}\right| \left|\sin 30^\circ\right| \left|\hat{n}\right|$$

$$= 3\left|\vec{c}\right| \cdot \frac{1}{2}$$

$$\Rightarrow 3 = 3\left|\vec{c}\right| \cdot \frac{1}{2} \Rightarrow \left|\vec{c}\right| = 2$$

$$Since,\ \left|\vec{c} - \vec{a}\right| = 3$$

O n squaring (i), we get $\vec{c}^{\,2} + \vec{a}^{\,2} - 2\vec{c} \cdot \vec{a} = 9$

$$\Rightarrow 4 + 9 - 2\vec{a} \cdot \vec{c} = 9 \Rightarrow \vec{a} \cdot \vec{c} = 2$$

Video Solution:

Q17 Text Solution:

Given vector is $6\hat{i} - 2\hat{j} + 3\hat{k}$

$\therefore$ Its magnitude $= \sqrt{6^2 + (-2)^2 + 3^2}$

$\sqrt{36 + 4 + 9} = \sqrt{49} = 7$ units

Video Solution:

Q18 Text Solution:

Here $\left(\vec{a} + \vec{b}\right) \cdot \left(\vec{a} - \vec{b}\right) = 0$

$\left|\vec{a}\right|^2 - \left|\vec{b}\right|^2 = 0$

$\left|\vec{a}\right| = \left|\vec{b}\right|$

$\therefore \left|\vec{a}\right| = \sqrt{9 + 16 + 4}$

$\left|\vec{a}\right| = \sqrt{29}$

Video Solution:

Q19 Text Solution:

Here

$\vec{a} = 3\hat{i} + p\hat{k},\ \vec{b} = -\hat{i} + q\hat{j} + 3\hat{k},\ \vec{c} = -3\hat{i} + 3\hat{j}$

$\because$ Points A, B, C are collinear, then

$\overrightarrow{AB} = m\overrightarrow{AC} \Rightarrow \vec{b} - \vec{a} = m\left(\vec{c} - \vec{a}\right)$

$\Rightarrow -4\hat{i} + q\hat{j} + \left(3 - p\right)\hat{k}$

$= m\left(-6\hat{i} + 3\hat{j} - p\hat{k}\right)$

$\Rightarrow -6m = -4, q = 3m, -pm = 3 - p$

$\Rightarrow m = \frac{2}{3}, q = 3\left(\frac{2}{3}\right) = 2, \text{and} \frac{-2p}{3} = 3 - p$

$\Rightarrow -2p = 9 - 3p \Rightarrow p = 9$

Video Solution:

Q20 Text Solution:

We have, $\left|\vec{r}\right| = 3\sqrt{3}$

Since, $\vec{r}$ is equally inclined to three axes, so direction cosine of unit vector $\vec{r}$ will be same.

i.e. l = m = n

$\Rightarrow l^2 + l^2 +$

$\Rightarrow l$

$We\,have\ \overrightarrow{OP} =$

$\pm\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right) \left\{\because \vec{p} = \right.$

$= \left|\vec{r}\right|\overrightarrow{OP}$

$= \pm 3\sqrt{3} \times \left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right) \Rightarrow$

$\pm 3\left(\hat{i} + \hat{j} + \hat{k}\right)$

Video Solution:

Q21 Text Solution:

The angle between the lines whose direction cosines are proportional to (1,2,1) and (2,-3,6) is

$$\theta=\cos^{-1}\left\{\frac{(1)(2)+(2)(-3)+(1)(6)}{\sqrt{1^2+2^2+1^2}\cdot\sqrt{2^2+(-3)^2+6^2}}\right\}$$

$$\therefore \theta=\cos^{-1}\left(\frac{2}{7\sqrt{6}}\right)$$

Video Solution:

Q22 Text Solution:

The direction ratios of the lines are

5 – 4, k – 1, 0 – 2 and 3 – 2, 3 – 1, – 1 – 1

i.e., 1, k – 1, - 2 and 1, 2, –2

Since the lines are parallel, we have

$$\frac{1}{1}=\frac{k-1}{2}=\frac{-2}{-2}$$

i.e., k – 1 = 2 Þ k = 3

Video Solution:

Q23 Text Solution:

We know,

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$$

$$\Rightarrow \cos\gamma=\sqrt{1-\left(\frac{14}{15}\right)^2-\left(\frac{1}{3}\right)^2}$$

$$=\sqrt{\frac{8}{9}-\left(\frac{196}{225}\right)}=\pm\frac{2}{15}$$

Video Solution:

Q24 Text Solution:

Normal to the plane had drs (7, -2, 3) by verifying the options we get option (a) is the required equation.

Video Solution:

Q25 Text Solution:

Distance of (a, b, g) from y-axis is given by

$$d=\sqrt{\alpha^2+\gamma^2}$$

$\therefore$ Distance of (3, 4, 5) from y-axis is

$$d=\sqrt{3^2+5^2}=\sqrt{9+25}=\sqrt{34}$$

Video Solution:

Q26 Text Solution:

Shortest distance = $\left|\frac{\left(\vec{a}_2-\vec{a}_1\right)\cdot(b_1\times b_2)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&3&2\\2&3&1\end{vmatrix}=\hat{i}\left(-3\right)-\hat{j}\left(-3\right)+\hat{k}\left(-3\right)$$

$= -3\hat{i}+3\hat{j}-3\hat{k}$

$\therefore\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{9+9+9}=\sqrt{27}=3\sqrt{3}$

$$\left(\vec{a}_2-\vec{a}_1\right)\cdot\left(\vec{b}_1\times\vec{b}_2\right)$$

$=\left(3\hat{i}+3\hat{j}+3\hat{k}\right)\cdot\left(-3\hat{i}+3\hat{j}-3\hat{k}\right)$

$= -9+9-9=-9$

$\therefore$ Shortest distance $=\frac{9}{3\sqrt{3}}=\frac{3}{\sqrt{3}}=\sqrt{3}$

Video Solution:

Q27 Text Solution:

Consider, $L_1\ :\ \frac{x+2}{4\lambda+1}=\frac{y-1}{4}=\frac{z}{-18}$

And $L_2:\frac{x}{-3}=\frac{y+1}{4\mu-3}=\frac{z-1}{6}$

If two lines are parallel, then their direction ratios are proportional.

$\therefore\frac{4\lambda+1}{-3}=\frac{4}{5\mu-3}=\frac{-18}{6}\Rightarrow\frac{4\lambda+1}{-3}=-3$ and $\frac{4}{5\mu-3}=-3$

$\Rightarrow 4l+1=9$ and $4=-15\mu+9$

$\Rightarrow 4l=8$ and $15\mu=5$

$\Rightarrow\ \lambda=2$ and $\mu=\frac{1}{3}$

So, the value of the pair (λ,μ) is $\left(2,\ \frac{1}{3}\right)$

Video Solution:

Q28 Text Solution:

Given lines are

$\frac{x-4}{5}=\frac{y+1}{2}=\frac{z}{1}$ and $\frac{x-1}{5}=\frac{y-2}{2}=\frac{z-3}{1}$

Here, $\vec{a}_1=4\hat{i}-\hat{j},\ \vec{a}_2=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=5\hat{i}+2\hat{j}+\hat{k}$

$\therefore\ \vec{a}_2-\vec{a}_1=\left(\hat{i}+2\hat{j}+3\hat{k}\right)-\left(4\hat{i}-\hat{j}\right)$

$=-3\hat{i}+3\hat{j}+3\hat{k}$

$\vec{b}\times\left(\vec{a}_2-\vec{a}_1\right)=\hat{i}\left(6-3\right)-\hat{j}\left(15+3\right)+\hat{k}\left(15+6\right)$

$=3\hat{i}-18\hat{j}+21\hat{k}$

and $\left|\vec{b}\right|=\sqrt{5^2+2^2+1^2}=\sqrt{25+4+1}$

$=\sqrt{30}$

Now, shortest Distance $=\left|\frac{\vec{b}\times\left(\vec{a}_2-\vec{a}_1\right)}{\left|\vec{b}\right|}\right|$

$=\frac{1}{\sqrt{30}}\left|3\hat{i}-18\hat{j}+21\hat{k}\right|$

$=\frac{1}{\sqrt{30}}\sqrt{3^2+(-18)^2+21^2}$

$=\sqrt{\frac{774}{30}}=\sqrt{\frac{129}{5}}$

Video Solution:

Q29 Text Solution:

If l, m, n are the direction cosines of the line perpendicular to the given lines, we have

$l + m + 2n = 0$ and $2l + m - n = 0$

The give $\frac{l}{-1} = \frac{m}{5} = \frac{n}{3} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{1+25+9}}$

$= \frac{1}{\sqrt{35}}$

Hence, the direction cosines are

$\left(\frac{-1}{\sqrt{35}}, \frac{5}{\sqrt{35}}, \frac{3}{\sqrt{35}}\right)$

Video Solution:

Q30 Text Solution:

Let $\vec{a}$ and $\vec{b}$ be the vectors parallel to the lines

$\vec{r} = \left(2\hat{i} - 5\hat{j} + \hat{k}\right) + \lambda\left(3\hat{i} + 2\hat{j} + 6\hat{k}\right)$ and

$\vec{r} = \left(7\hat{i} - 6\hat{k}\right) + \mu\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ respectively.

$\therefore\ \vec{a} = 3\hat{i} + 2\hat{j} + 6\hat{k}, \vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore\ \vec{a} \cdot \vec{b} = (3)(1) + (2)(2) + (6)(2) = 19$

And $|\vec{a}| = \sqrt{9 + 4 + 36} = 7, |\vec{b}| = \sqrt{1 + 4 + 4} = 3$

Let θ be the acute angle between the two given lines.

$\therefore \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{19}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right)$

Video Solution: