ULTIMATE KCET

CRASH COURSE 2026

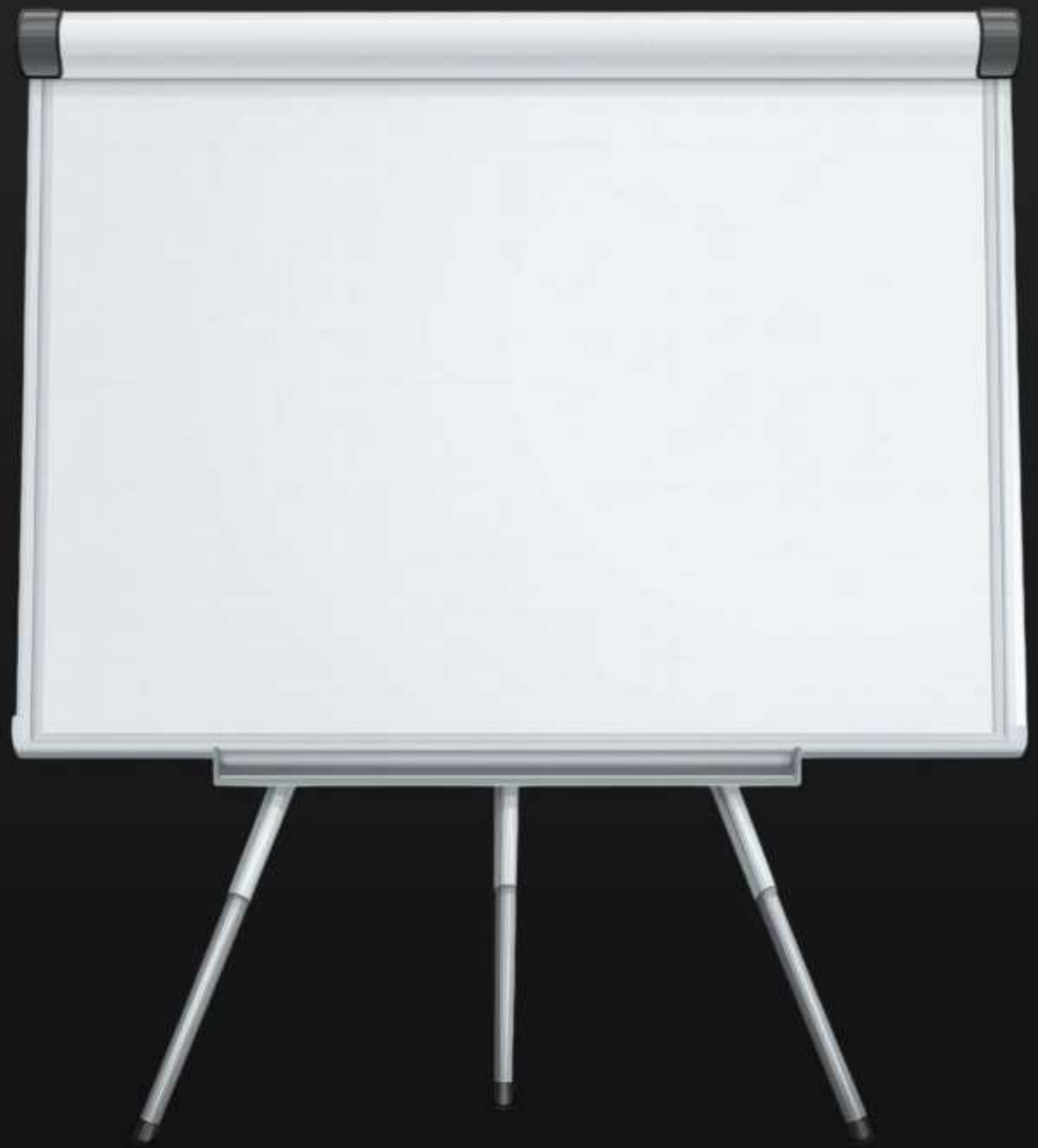
Mathematics | Lecture - 01

Inverse Trigonometric Functions

By – Guru sir

Recap of previous lecture

1. Trigonometry
2.
3.
4.



Topics *to be covered*

1. ITF
2.
3.
4.

$f(x) = \sin x$

$\Downarrow$

Here $f(x)$ is one-one & onto if

Domain = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Range = $[-1, 1]$

$\sin^{-1}(\sin x) = x$ if $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$f(x) = \sin^{-1} x$

$\Downarrow$

Here $f(x)$ is one-one & onto if

Domain = $[-1, 1]$

Range = $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin(\sin^{-1} x) = x$ if $x \in [-1, 1]$

$f(x) = \cos x$

$\Downarrow$

Here $f(x)$ is one-one & onto if

Domain = $[0, \pi]$

Range = $[-1, 1]$

$\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$

$f(x) = \cos^{-1} x$

$\Downarrow$

Here $f(x)$ is one-one & onto if

Domain = $[-1, 1]$

Range = $[0, \pi]$

$\cos(\cos^{-1} x) = x$ if $x \in [-1, 1]$

$$\sin(\sin^{-1}x) = x \; \forall x \in [-1,1]$$

$$\cos(\cos^{-1}x) = x \; \forall x \in [-1,1]$$

$$\tan(\tan^{-1}x) = x \; \forall x \in R$$

$$\cot(\cot^{-1}x) = x \; \forall x \in R$$

$$\sec(\sec^{-1}x) = x \; \forall x \in R(-1, 1)$$

$$\operatorname{cosec}(\operatorname{cosec}^{-1}x) = x \; \forall x \in R - (-1, 1)$$

$$\sin^{-1}(\sin x) = x \quad \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cos^{-1}(\cos x) = x \quad \forall x \in [0, \pi]$$

$$\tan^{-1}(\tan x) = x \quad \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\cot^{-1}(\cot x) = x \quad \forall x \in (0, \pi)$$

$$\sec^{-1}(\sec x) = x \quad \forall x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$$

$$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x \quad \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$

$$2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

$$2\tan^{-1} = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$

$$2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$2\cos^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$$

$$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$$

$$3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$$

$$3\tan^{-1}x = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$$

$$\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x + y}{1 - xy}\right)$$

$$\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x - y}{1 + xy}\right)$$

$$\sin^{-1}(-x) = -\sin^{-1}x$$

$$\tan^{-1}(-x) = -\tan^{-1}x$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$$

$$\cos^{-1}(-x) = \pi - \cos^{-1}x$$

$$\cot^{-1}(-x) = -\cot^{-1}x$$

$$\sec^{-1}(-x) = \pi - \sec^{-1}x$$

$$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$$

or

$$\sin^{-1}x = \frac{\pi}{2} - \cos^{-1}x$$

or

$$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$$

$$\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$$

or

$$\sec^{-1}x = \frac{\pi}{2} - \operatorname{cosec}^{-1}x$$

or

$$\operatorname{cosec}^{-1}x = \frac{\pi}{2} - \sec^{-1}x$$

$$\sin^{-1}\left(\frac{a}{b}\right) = \operatorname{cosec}^{-1}\left(\frac{b}{a}\right)$$

or

$$\cos^{-1}\left(\frac{a}{b}\right) = \sec^{-1}\left(\frac{b}{a}\right)$$

or

$$\tan^{-1}\left(\frac{a}{b}\right) = \cot^{-1}\left(\frac{b}{a}\right)$$

$$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$

or

$$\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x$$

or

$$\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$$

$$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$$

or

$$\tan^{-1}x = \frac{\pi}{2} - \cot^{-1}x$$

or

$$\cot^{-1}x = \frac{\pi}{2} - \tan^{-1}x$$

Trigonometric functions with following Domain and Range will be bijective

	Domain	Range
$\sin x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$[-1, 1]$
$\cos x$	$[0, \pi]$	$[-1, 1]$
$\tan x$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	R
$\cot x$	$(0, \pi)$	R
$\sec x$	$[0, \pi] - \{\pi/2\}$	$R - (-1, 1)$
$\operatorname{cosec} x$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	$R - (-1, 1)$

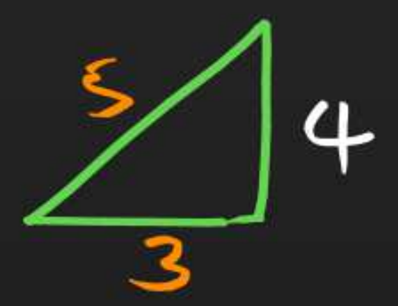
Inverse Trigonometric Functions with following Domain and Range will be bijective

Note : Here range is also called as Principal value branch.

	Domain	Range	Quadrant in which solution lies
$\sin^{-1} x$	$[-1, 1]$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$	I and IV
$\cos^{-1} x$	$[-1, 1]$	$[-1, 1]$	I and II
$\tan^{-1} x$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	I and IV
$\cot^{-1} x$	R	$(0, \pi)$	I and II
$\sec^{-1} x$	$R - (-1, 1)$	$[0, \pi] - \{\pi/2\}$	I and II
$\text{cosec}^{-1} x$	$R - (-1, 1)$	$\left[\frac{-\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	I and IV

QUESTION

Find $\sin\left[\cos^{-1} 3/5\right]$



$\sin\left[\sin^{-1}\frac{4}{5}\right]$

$= \frac{4}{5}$

$\frac{4}{5} \in [-1, 1]$

QUESTION

Find $\sin\left[\cos^{-1}\left(\frac{-3}{5}\right)\right]$

$\sin\left[\pi - \cos^{-1}\frac{3}{5}\right]$

$\pi - \theta$

$\sin(\pi - \theta) = \sin\theta$

$\sin\left[\cos^{-1}\frac{3}{5}\right]$

$\sin\left[\sin^{-1}\frac{4}{5}\right]$

$= \frac{4}{5}$

$\sin^{-1}(-x) = -\sin^{-1}x$

$\tan^{-1}(-x) = -\tan^{-1}x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\cot^{-1}(-x) = \pi - \cot^{-1}x$

$\sec^{-1}(-x) = \pi - \sec^{-1}x$

QUESTION

Find $\tan\left[\cos^{-1}\left(\frac{-3}{5}\right)\right]$

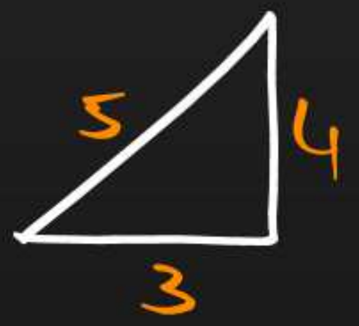
$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\tan(\pi - \theta) = -\tan\theta$

$\tan\left[\pi - \cos^{-1}\frac{3}{5}\right]$

$= -\tan\left[\cos^{-1}\frac{3}{5}\right]$

$= -\tan\left[\tan^{-1}\frac{4}{3}\right]$



$= -\frac{4}{3}$

QUESTION

Find $\sin\left[\sin^{-1}\left(\frac{-3}{5}\right)\right]$

$\sin^{-1}(-x) = -\sin^{-1}x$

$\sin\left[-\sin^{-1}\frac{3}{5}\right]$

$\sin(-\theta) = -\sin\theta$

$-\sin\left[\sin^{-1}\frac{3}{5}\right]$

$= -\frac{3}{5}$.

QUESTION

Find $\sin\left[2\sin^{-1}\left(\frac{-3}{5}\right)\right]$

$\sin^{-1}(-x) = -\sin^{-1}x$

$\sin(-\theta) = -\sin\theta$

$\sin 2A = 2\sin A\cos A$

$\sin\left[-2\sin^{-1}\frac{3}{5}\right]$

$-\sin\left[2\underbrace{\sin^{-1}\frac{3}{5}}_{A}\right]$

$= -2\sin\left[\sin^{-1}\frac{3}{5}\right]\cos\left[\sin^{-1}\frac{3}{5}\right]$

$= -2\left(\frac{3}{5}\right)\cos\left[\cos^{-1}\left(\cos\frac{4}{5}\right)\right]$

$= -\frac{6}{5}\left(\frac{4}{5}\right) = \frac{-24}{25}$

QUESTION

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$$

Find $\sin\left[\underbrace{2\tan^{-1}\frac{3}{4}}_{A}\right]$

$$= \frac{2\tan\left(\tan^{-1}\frac{3}{4}\right)}{1+\left[\tan\left(\tan^{-1}\frac{3}{4}\right)\right]^2}$$

$$= \frac{2\left(\frac{3}{4}\right)}{1+\left(\frac{3}{4}\right)^2} = \frac{\frac{6}{4}}{1+\frac{9}{16}} = \frac{6}{4}\times\frac{16}{25} = 6\times\frac{4}{25} = \underline{\frac{24}{25}}$$

$$\cos\left[\underbrace{2\tan^{-1}\frac{4}{3}}_{A}\right]$$

$$\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$$

$$= \frac{1-\left[\tan\left(\tan^{-1}\frac{4}{3}\right)\right]^2}{1+\left[\tan\left(\tan^{-1}\frac{4}{3}\right)\right]^2}$$

$$= \frac{1-\frac{16}{9}}{1+\frac{16}{9}} = \frac{-7}{25}$$

$$\cos\left(\underbrace{2\sin^{-1}\frac{1}{2}}_{A}\right)$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$= 1 - 2\left[\sin\left(\sin^{-1}\frac{1}{2}\right)\right]^2$$

$$= 1 - 2\left[\frac{1}{2}\right]^2$$

$$= 1 - 2\left(\frac{1}{4}\right)$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

$$\cos\left[\underbrace{2\cos^{-1}\frac{1}{2}}_{A}\right]$$

$$\cos 2A = 2\cos^2 A - 1$$

$$= 2\left[\cos\left(\cos^{-1}\frac{1}{2}\right)\right]^2 - 1$$

$$= 2\left(\frac{1}{2}\right)^2 - 1$$

$$= 2\left(\frac{1}{4}\right) - 1$$

$$= \frac{1}{2} - 1$$

$$= -\frac{1}{2}$$

QUESTION

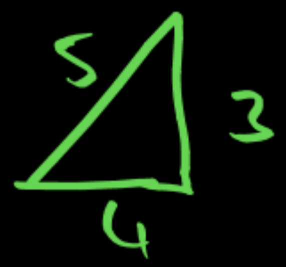
Find $\tan\left(\underbrace{2\tan^{-1}\frac{3}{4}}_{A}\right)$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$= \frac{2\tan\left(\tan^{-1}\frac{3}{4}\right)}{1-\left[\tan\left(\tan^{-1}\frac{3}{4}\right)\right]^2}$$

$$= \frac{2(3/4)}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{6}{4}}{\frac{16-9}{16}} = \frac{6}{\cancel{4}} \times \frac{\cancel{16}^{\,4}}{7} = \frac{24}{7}$$

$\cos\left(3\sin^{-1}\frac{3}{5}\right)$



$\cos 3A = 4\cos^3 A - 3\cos A$

$\Downarrow$

$\cos\left[3\cos^{-1}\frac{4}{5}\right]$

$$= 4\left[\cos\left(\cos^{-1}\frac{4}{5}\right)\right]^3 - 3\cos\left(\cos^{-1}\frac{4}{5}\right)$$

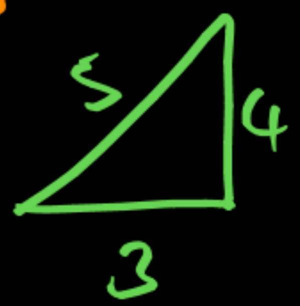
$$= 4\left(\frac{4}{5}\right)^3 - 3\left(\frac{4}{5}\right)$$

$$= \frac{4^4}{5^3} - \frac{12}{5}$$

$$= \frac{256 - 12(25)}{5^3} = \frac{256 - 300}{125} = \frac{-44}{125}$$

$$\sin\left[3\sec^{-1}\frac{5}{3}\right]$$

$$\Downarrow$$



$$\sin\left[3\underbrace{\sin^{-1}\frac{4}{5}}_{A}\right]$$

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$= 3\left(\frac{4}{5}\right) - 4\left(\frac{4}{5}\right)^3$$

$$= \frac{12}{5} - \frac{4^4}{5^3} = \frac{12(25) - 256}{125}$$

$$= \frac{300-256}{125} = \frac{44}{125}$$

$\sin^{-1}(\sin x) = x \quad \text{if} \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\frac{4\pi}{3} = \frac{3\pi+\pi}{3} = \frac{3\pi}{3}+\frac{\pi}{3} = \pi+\frac{\pi}{3}$$

$$\frac{5\pi}{3} = \frac{6\pi-\pi}{3} = 2\pi-\frac{\pi}{3}$$

$$\frac{5\pi}{6} = \frac{6\pi-\pi}{6} = \pi-\frac{\pi}{6}$$

$$\frac{17\pi}{6} = \frac{18\pi-\pi}{6} = 3\pi-\frac{\pi}{6}$$

$$\frac{43\pi}{5}$$

$$\frac{45\pi-2\pi}{5} \qquad \frac{40\pi+3\pi}{5}$$

$$= 9\pi - \frac{2\pi}{5} \qquad = 8\pi + \frac{3\pi}{5}$$

↓

acute Angle

×

$$\frac{\pi}{5} = \frac{180}{5} = 36^\circ$$

$$\frac{3\pi}{5} = 3(36) = 108^\circ$$ obtuse

$$\frac{2\pi}{5} = 2(36^\circ) = 72^\circ$$ acute

$\sin(\pi - \theta)$

$\cos(7\pi - \theta)$ $\Rightarrow$ acute

$\tan(10\pi + \theta)$

QUESTION

Find $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$

$\frac{7\pi}{6} = \frac{6\pi+\pi}{6} = \pi + \frac{\pi}{6}$

$\sin^{-1}\left[\underline{\sin\left(\pi+\frac{\pi}{6}\right)}\right]$

$\hookrightarrow \sin(\pi+\theta) = -\sin\theta$

$\sin^{-1}\left[\underline{-\sin\frac{\pi}{6}}\right]$

$\hookrightarrow \sin^{-1}(-A) = -\sin^{-1}A$

$= -\sin^{-1}\left(\sin\frac{\pi}{6}\right]$

$= -\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin^{-1}(\sin x) = x$

if $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

QUESTION

Find $\cos^{-1}\left(\cos\frac{5\pi}{6}\right)$

$\cos^{-1}(\cos x) = x$ if $x \in [0, \pi]$

$= \frac{5\pi}{6} \in [0, \pi]$

$\frac{5\pi}{6} = 5\left(\frac{\pi}{6}\right) = 5(30^\circ) = 150^\circ \in [0, 180^\circ]$

$\boxed{\theta = \pi - \alpha}$ 2nd Quad

1st Quad $\Rightarrow$ $\boxed{\theta = 2\pi + \alpha}$

π

0 (or) 2π

$\boxed{\theta = \pi + \alpha}$ 3rd Quad

4th Quad $\Rightarrow$ $\boxed{\theta = 2\pi - \alpha}$

$\alpha \rightarrow$ acute angle

QUESTION

Find $\cos^{-1}\left(\cos\frac{4\pi}{3}\right)$

$\cos^{-1}\left[\cos\left(\pi+\frac{\pi}{3}\right)\right]$

$\cos^{-1}\left[-\cos\frac{\pi}{3}\right]$

$= \pi - \cos^{-1}\left(\cos\frac{\pi}{3}\right)$

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\frac{4\pi}{3} = 240^\circ \notin [0,\pi]$

$\frac{4\pi}{3} = \frac{3\pi+\pi}{3} = \pi + \frac{\pi}{3}$

$\Downarrow$

$\pi + \alpha$

$\hookrightarrow$ 3rd Quad

$\cos(\pi+\theta) = -\cos\theta$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\cos^{-1}(\cos x) = x$ if $x \in [0,\pi]$

QUESTION

Find $\tan^{-1}\left(\tan\frac{5\pi}{4}\right)$

$\tan^{-1}(\tan x) = x$ if $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\Downarrow$

Range of $\tan^{-1}x$

$\tan^{-1}\left[\tan\left(\pi + \frac{\pi}{4}\right)\right]$

$\tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$= \frac{\pi}{4}$

$\frac{5\pi}{4} = 5(45^\circ) = 225^\circ \in 3^{rd}$ Quad

$\frac{5\pi}{4} = \frac{4\pi + \pi}{4} = \pi + \frac{\pi}{4}$

$\tan(\pi + \theta) = \tan\theta$

QUESTION

Find $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$ $= \sin^{-1}\left[\sin\left(\pi-\frac{\pi}{6}\right)\right]$

$$\frac{5\pi}{6} = \frac{6\pi-\pi}{6} = \pi - \frac{\pi}{6}$$

$$= \sin^{-1}\left[\sin\frac{\pi}{6}\right]$$

$$\sin(\pi-\theta) = \sin\theta$$

$$= \frac{\pi}{6}$$

(1) $\frac{\pi}{6}$ ✓

(2) $\frac{5\pi}{6}$

(3) $-\frac{\pi}{6}$

(4) $\frac{\pi}{3}$

$$\cos^{-1}\left[\sin\frac{5\pi}{6}\right]$$

$$=\frac{\pi}{2}-\sin^{-1}\left[\sin\frac{5\pi}{6}\right]$$

$$=\frac{\pi}{2}-\sin^{-1}\left[\sin\left(\pi-\frac{\pi}{6}\right)\right]$$

$$=\frac{\pi}{2}-\sin^{-1}\left(\sin\frac{\pi}{6}\right)$$

$$=\frac{\pi}{2}-\frac{\pi}{6}=\underline{\frac{\pi}{3}}$$

$$\cos^{-1}A+\sin^{-1}A=\frac{\pi}{2}$$

$$\cos^{-1}A=\frac{\pi}{2}-\sin^{-1}A$$

$$\sin^{-1}A=\frac{\pi}{2}-\cos^{-1}A$$

QUESTION

$\cos^{-1}\left[\cos\left(\left(-\frac{17}{15}\right)\pi\right)\right]$ is equal to

(A) $17\pi/15$

(B) $13\pi/15$

(C) $3\pi/15$

(D) $-17\pi/15$

$\cos(-\theta) = \cos\theta$

$\cos^{-1}\left[\cos\frac{17\pi}{15}\right]$

$\frac{17\pi}{15} = \frac{15\pi+2\pi}{15} = \pi + \frac{2\pi}{15}$

$\cos^{-1}\left[\cos\left(\pi+\frac{2\pi}{15}\right)\right]$

$\frac{\pi}{15} = \frac{180}{15} = 12^\circ$

$\frac{2\pi}{15} = 24^\circ \in [0,\pi]$

$\cos^{-1}\left[-\cos\frac{2\pi}{15}\right]$

$= \pi - \cos^{-1}\left(\cos\frac{2\pi}{15}\right)$

$= \pi - \frac{2\pi}{15} = \frac{13\pi}{15}$

$\cos(\pi+\theta) = -\cos\theta$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\cos^{-1}\left[\cos \frac{23\pi}{3}\right]$

$= \cos^{-1}\left[\cos\left(8\pi - \frac{\pi}{3}\right)\right]$

$= \cos^{-1}\left(\cos \frac{\pi}{3}\right)$

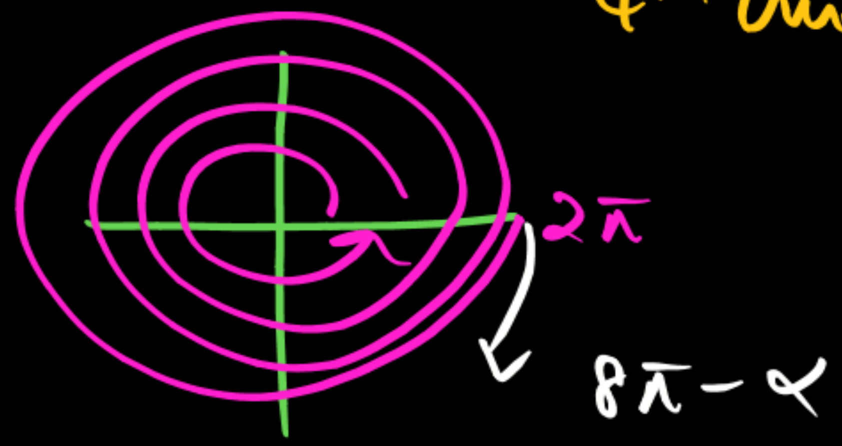
$= \frac{\pi}{3}$

$\frac{23\pi}{3}$

$\frac{21\pi + 2\pi}{3}$ $\quad$ $\frac{24\pi - \pi}{3}$

$= 7\pi + \frac{2\pi}{3}$ $\quad$ $= 8\pi - \frac{\pi}{3}$

× obtuse $\quad$ ✓ 4th Quad



QUESTION

PYQ of KCET

The value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$ is

(A) $3\pi/5$

(B) $-7\pi/5$

(C) $\pi/10$

(D) $-\pi/10$

$\frac{\pi}{2} - \cos^{-1}\left[\cos\left(7\pi - \frac{2\pi}{5}\right)\right]$

$\hookrightarrow \cos(7\pi - \theta) = -\cos\theta$

$\frac{\pi}{2} - \cos^{-1}\left[-\cos\frac{2\pi}{5}\right]$

$\frac{\pi}{2} - \left[\pi - \cos^{-1}\left(\cos\frac{2\pi}{5}\right)\right]$

$= \frac{\pi}{2} - \left[\pi - \frac{2\pi}{5}\right] = \frac{\pi}{2} - \pi + \frac{2\pi}{5}$

$= \frac{5\pi - 10\pi + 4\pi}{10} = -\frac{\pi}{10}$

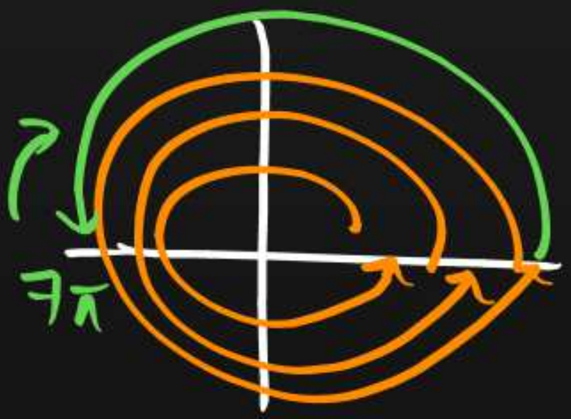
$\frac{\pi}{5} = 36^\circ$

$\frac{33\pi}{5}$

$\frac{35\pi - 2\pi}{5} = 7\pi - \frac{2\pi}{5}$ $\rightarrow$ 72° (Acute)

$\frac{30\pi + 3\pi}{5} = 6\pi + \frac{3\pi}{5}$ $\rightarrow$ 108° X

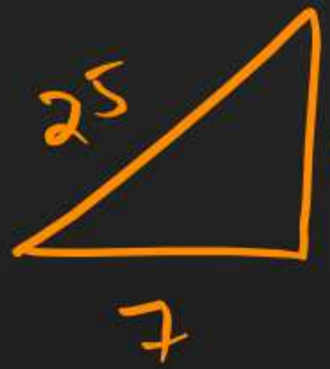
2nd Quad



$\cos^{-1}(-x) = \pi - \cos^{-1}x$

QUESTION

The value of $\cot\left[\cos^{-1}\left(\frac{7}{25}\right)\right]$ is



$\sqrt{25^2-7^2}=\sqrt{625-49}=\sqrt{576}=24$

$\cot\left[\cot^{-1}\frac{7}{24}\right]$

$=\frac{7}{24}$

A 25/24

B 25/7

C 24/25

D 7/24

QUESTION

Find the value of $\cos\left(\sin^{-1}\frac{8}{17}\right)$.

A $8/17$

B $9/17$

C $15/17$

D $17/15$

$\sqrt{289-64}$

$=\sqrt{225}=15$

$\cos\left[\cos^{-1}\left(\frac{15}{17}\right)\right]$

$=\frac{15}{17}$

PYQ

$$\cos^{-1}\left(\frac{\pi}{3}\right) + \sin^{-1}\left(\frac{\pi}{3}\right)$$

(A) 0

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) Does not exist

$$\frac{\pi}{3} = \frac{3.14}{3} > 1$$

But

$\cos^{-1}(x)$ & $\sin^{-1}(x)$

$x \in [-1, 1]$

$\Downarrow$

Domain

QUESTION

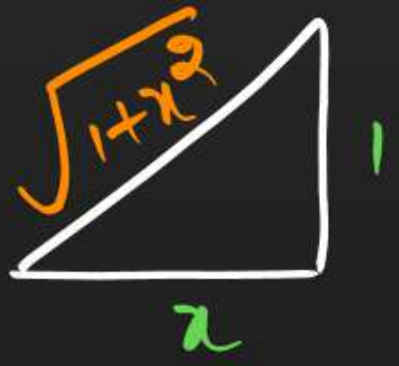
The value of $\sin\left(\cot^{-1} x\right)$ is

(A) $\dfrac{1}{\sqrt{1+x^2}}$

(B) $1+x^2$

(C) x

(D) $\dfrac{1}{1+x^2}$

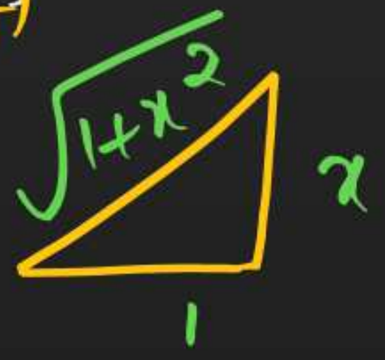


$$\sin\left[\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right]$$

$$= \frac{1}{\sqrt{1+x^2}}$$

QUESTION

The value of $\cos(\tan^{-1}x)$ is



(A) $\dfrac{x}{\sqrt{1-x^2}}$

(B) $\dfrac{x}{\sqrt{1+x^2}}$

$\cos\left[\cos^{-1}\dfrac{1}{\sqrt{1+x^2}}\right]$

(C) $\sqrt{1-x^2}$

$\dfrac{1}{\sqrt{1+x^2}}$

(D) $\dfrac{1}{\sqrt{1+x^2}}$

QUESTION

Exemplar Book Question

$\sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3)$ is equal to

$1+[\tan(\tan^{-1}2)]^2 + 1+[\cot(\cot^{-1}3)]^2$

$1+2^2+1+3^2$

$=2+4+9$

$=\underline{15}$

$\sec^2A = 1+\tan^2A$

&

$\text{cosec}^2B = 1+\cot^2B$

(A) 1

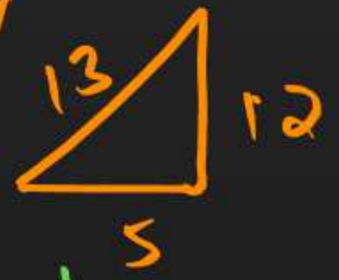
(B) 5

(C) 10

(D) 15

QUESTION

If $\cos^{-1}\left(\frac{5}{13}\right) - \sin^{-1}\left(\frac{12}{13}\right) = \cos^{-1}x$, then x is equal to



$$\sin^{-1}\frac{12}{13} - \sin^{-1}\frac{12}{13} = \cos^{-1}x$$

$$0 = \cos^{-1}x$$

$$x = \cos 0$$

$$x = 1$$

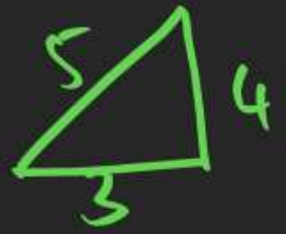
(A) 1

(B) $\frac{1}{\sqrt{2}}$

(C) 0

(D) $\frac{\sqrt{3}}{2}$

QUESTION



$\Rightarrow \theta = \cos^{-1}\frac{3}{5}$

$\cos\theta = \frac{3}{5}$

If $\theta = \sin^{-1}\left(\frac{4}{5}\right)$, then $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)+\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)$ is equal to

$\tan\left(\frac{\pi}{4}+A\right) = \frac{1+\tan A}{1-\tan A}$ | $\tan\left(\frac{\pi}{4}-A\right) = \frac{1-\tan A}{1+\tan A}$

A 5/3

B 6/5

C 10/3

D 3/5

$$\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} + \frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}} = \frac{1+1/2}{1-1/2} + \frac{1-1/2}{1+1/2}$$

$$= \frac{3/2}{1/2} + \frac{1/2}{3/2}$$

$$= 3 + \frac{1}{3}$$

$$= \frac{10}{3}$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \sqrt{\frac{1-3/5}{1+3/5}}$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{2}{8}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

QUESTION

The principal value of $\cos^{-1}\left[\cos\left\{\underline{\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)}\right\}\right]$ is

$\longrightarrow \sin^{-1}(-x) = -\sin^{-1}x$

(A) $\pi/3$

(B) $\pi/6$

(C) $2\pi/3$

(D) $-\pi/6$

$\cos^{-1}\left[\cos\left[-\sin^{-1}\frac{\sqrt{3}}{2}\right]\right]$

$\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right] \longrightarrow \cos(-\theta) = \cos\theta$

$\cos^{-1}\left(\cos\frac{\pi}{3}\right)$

$\therefore = \frac{\pi}{3}$

QUESTION

The principal value of $\cos^{-1}\left\{\sin\left(\cos^{-1}\frac{1}{2}\right)\right\}$ is

A $\pi/6$

B $\pi/4$

C $\pi/3$

D $2\pi/3$

$\cos^{-1}\left[\sin\frac{\pi}{3}\right]$

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\pi}{6}$

QUESTION

The value of $\left[\sin\left(\cot^{-1}\left(\cot\frac{17\pi}{3}\right)\right)\right]$ is

(A) $\sqrt{3}/2$

(B) $1/\sqrt{2}$

(C) $-\sqrt{3}/2$

(D) $-1/\sqrt{2}$

$\cos^{-1}(-x) = \pi - \cos^{-1}x$

$\cos(-x) = \cos x$

$\frac{17\pi}{3} = \frac{18\pi - \pi}{3} = 6\pi - \frac{\pi}{3}$

$\downarrow$ 4th Quad

$\sin\left[\cot^{-1}\left[\cot\left(\underline{6\pi - \frac{\pi}{3}}\right)\right]\right]$

$\cot(6\pi - \theta) = -\cot\theta$

$\sin\left[\underbrace{\cot^{-1}\left[-\cot\frac{\pi}{3}\right]}\right]$

$\cot^{-1}(-x) = \pi - \cot^{-1}x$

$= \sin\left[\pi - \cot^{-1}\left(\cot\frac{\pi}{3}\right)\right]$

$= \sin\left[\pi - \frac{\pi}{3}\right] = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

QUESTION

$$\frac{\pi}{20}=\frac{180}{20}=9^\circ \qquad 13(9)=117^\circ \qquad 3\left(\frac{\pi}{20}\right)=3(9^\circ)=27^\circ$$

The principal value of $\cos^{-1}\frac{1}{\sqrt{2}}\left(\cos\frac{9\pi}{10}-\sin\frac{9\pi}{10}\right)$ is equal to

A) $3\pi/20$

B) $7\pi/20$

C) $23\pi/20$

D) $17\pi/20$

$$\cos^{-1}\left[\frac{1}{\sqrt{2}}\cos\frac{9\pi}{10}-\frac{1}{\sqrt{2}}\sin\frac{9\pi}{10}\right]$$

$$\cos^{-1}\left[\sin\overset{A}{\frac{\pi}{4}}\cos\overset{B}{\frac{9\pi}{10}}-\cos\overset{A}{\frac{\pi}{4}}\sin\overset{B}{\frac{9\pi}{10}}\right]$$

$$\cos^{-1}\left[\sin\left(\frac{\pi}{4}-\frac{9\pi}{10}\right)\right]$$

$$\cos^{-1}\left[\sin\left(\frac{5\pi-18\pi}{20}\right)\right.$$

$$\cos^{-1}\left[\sin\left(\frac{-13\pi}{20}\right)\right]$$

$$\cos^{-1}\left[-\sin\frac{13\pi}{20}\right]$$

$$\pi-\cos^{-1}\left[\sin\frac{13\pi}{20}\right]$$

$$\pi-\cos^{-1}\left[\cos\left(\frac{\pi}{2}-\frac{13\pi}{20}\right)\right]$$

$$\pi-\cos^{-1}\left[\cos\left(-\frac{3\pi}{20}\right)\right] \quad \hookrightarrow \cos(-x)=\cos x$$

$$\pi-\cancel{\cos^{-1}}\left[\cancel{\cos}\frac{3\pi}{20}\right]$$

$$\pi-\frac{3\pi}{20}=\frac{17\pi}{20}$$

$$\cos^{-1}\left[\sin\left[\frac{5\pi-18\pi}{20}\right]\right]$$

$$\cos^{-1}\left[\sin\left[-\frac{13\pi}{20}\right]\right] \rightarrow \sin(-x) = -\sin x$$

$$\cos^{-1}\left[-\sin\frac{13\pi}{20}\right] \rightarrow \cos^{-1}(-x) = \pi - \cos^{-1}x$$

$$\pi - \cos^{-1}\left[\sin\frac{13\pi}{20}\right] \quad \text{WKT } \sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$$

$$\pi - \cos^{-1}\left[\cos\left(\frac{\pi}{2} - \frac{13\pi}{20}\right)\right]$$

$$\pi - \cos^{-1}\left[\cos\left[\frac{10\pi - 13\pi}{20}\right]\right]$$

$$\pi - \cos^{-1}\left[\cos\left(-\frac{3\pi}{20}\right)\right] \rightarrow \cos(-x) = \cos x$$

$$= \pi - \cos^{-1}\left(\cos\frac{3\pi}{20}\right)$$

$$= \pi - \frac{3\pi}{20}$$

$$= \frac{17\pi}{20}$$

Find $\cos^{-1}\frac{1}{2}\left[\cos\frac{7\pi}{10}-\sqrt{3}\sin\frac{7\pi}{10}\right]$

$\cos^{-1}\left[\frac{1}{2}\cos\frac{7\pi}{10}-\frac{\sqrt{3}}{2}\sin\frac{7\pi}{10}\right]$

$\cos^{-1}\left[\sin\frac{\pi}{6}\cos\frac{7\pi}{10}-\cos\frac{\pi}{6}\sin\frac{7\pi}{10}\right]$

$\cos^{-1}\left[\sin\left(\frac{\pi}{6}-\frac{7\pi}{10}\right)\right]$

$\cos^{-1}\left[\sin\left[\frac{5\pi-21\pi}{30}\right]\right]$

$\cos^{-1}\left[\sin\left[\frac{-16\pi}{30}\right]\right]$

$\cos^{-1}\left[-\sin\frac{8\pi}{15}\right]$

$\pi-\cos^{-1}\left[\sin\frac{8\pi}{15}\right]$

$\pi-\cos^{-1}\left[\cos\left(\frac{\pi}{2}-\frac{8\pi}{15}\right)\right]$

$\pi-\cos^{-1}\left[\cos\left(\frac{15\pi-16\pi}{30}\right)\right]$

$\pi-\cos^{-1}\left[\underline{\cos\left(-\frac{\pi}{30}\right)}\right] \rightarrow \cos(-x)=\cos x$

$\pi-\cos^{-1}\left(\cos\frac{\pi}{30}\right)$

$\pi-\frac{\pi}{30}=\underline{\frac{29\pi}{30}}$

2nd method

$\cos^{-1}\frac{1}{2}\left[\cos\frac{7\pi}{10} - \sqrt{3}\sin\frac{7\pi}{10}\right]$

$\cos^{-1}\left[\frac{1}{2}\cos\frac{7\pi}{10} - \frac{\sqrt{3}}{2}\sin\frac{7\pi}{10}\right]$

$\frac{\pi}{30} = \frac{180}{30} = 6^\circ$

$\cos^{-1}\left[\cos\frac{\pi}{3}\cos\frac{7\pi}{10} - \sin\frac{\pi}{3}\sin\frac{7\pi}{10}\right]$

$\therefore 31\left(\frac{\pi}{30}\right) = 31(6)$

$= 186^\circ$

$\in [0, \pi]$

$\cos^{-1}\left[\cos\left(\frac{\pi}{3} + \frac{7\pi}{10}\right)\right]$

$\cos^{-1}\left[\cos\left(\frac{10\pi + 21\pi}{30}\right)\right]$

$\cos^{-1}\left[\cos\left(\frac{31\pi}{30}\right)\right]$

$\frac{31\pi}{30} = \frac{30\pi + \pi}{30} = \pi + \frac{\pi}{30}$

$\cos^{-1}\left[\cos\left(\pi + \frac{\pi}{30}\right)\right]$

$\to \cos(\pi + \theta) = -\cos\theta$

$\cos^{-1}\left[-\cos\frac{\pi}{30}\right]$

$\to \cos^{-1}(-x) = \pi - \cos^{-1}x$

$\pi - \cos^{-1}\left(\cos\frac{\pi}{30}\right)$

$\pi - \frac{\pi}{30} = \frac{29\pi}{30}$

QUESTION

Find domain and range of $\sin^{-1}2x$

$f(x) = \sin^{-1}2x$

Domain:

$-1 \le 2x \le 1$

$-\frac{1}{2} \le x \le \frac{1}{2}$

Range:-

$\sin^{-1}2x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\downarrow$

Range.

$f(A) = \sin^{-1}A$

Domain

$\Downarrow$

$A \in [-1, 1]$

Range

$\Downarrow$

$f(A) \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Downarrow$

$\sin^{-1}A \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

QUESTION

Find domain and range of $2\sin^{-1}3x$

$f(x) = 2\sin^{-1}3x$

Domain :-

Here

$-1 \le 3x \le 1$

$\div$ by 3

$-\frac{1}{3} \le x \le \frac{1}{3}$

$x \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

Range :-

WKT

$\sin^{-1}3x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

x^{ly} by 2

$2\sin^{-1}3x \in [-\pi, \pi]$

↳ Range

QUESTION

Find domain and range of $\cos^{-1}(x^2 - 4)$

$f(x) = \cos^{-1}(x^2-4)$

$f(A) = \cos^{-1} A$

Domain $\Downarrow$ $A \in [-1, 1]$

Range $\Downarrow$ $\cos^{-1} A \in [0, \pi]$

Domain :-

$-1 \le x^2 - 4 \le 1$

Add 4

$3 \le x^2 \le 5$

$\sqrt{3} \le \sqrt{x^2} \le \sqrt{5}$

$\sqrt{3} \le |x| \le \sqrt{5}$

$x \in [-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

Range :-

$\cos^{-1}(x^2-4) \in [0, \pi]$

$\downarrow$

Range.

$a \le |x| \le b$

$\Rightarrow x \in [-b, -a] \cup [a, b]$

Domain of $f(x)=\cos^{-1}(x^2+5)$

Soln :-

Here

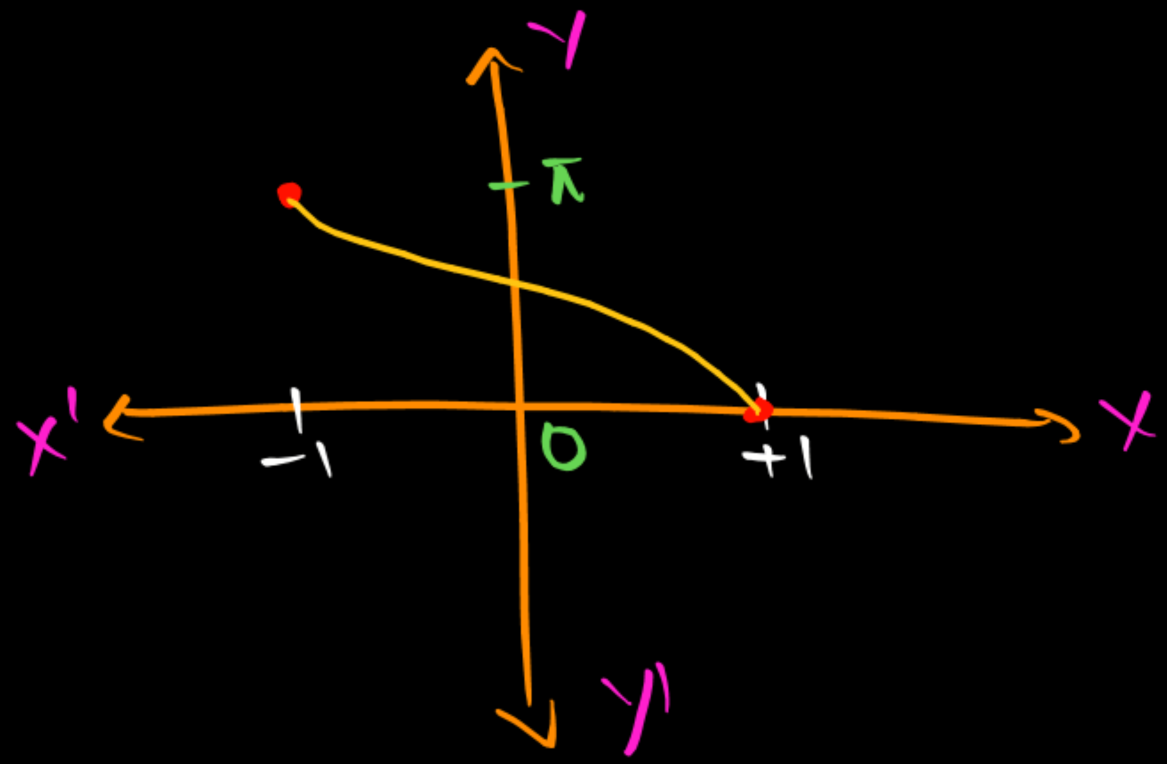
$$-1 \le x^2+5 \le 1$$

$$\Downarrow$$

$$-6 \le x^2 \le -4$$

No solution

Since x^2 cannot be -ve



$f(x) = \cos^{-1}x$

$\therefore \cos^{-1}x$ is a decreasing func^n

$\therefore$ on Applying $\cos^{-1}$

$\Downarrow$

inequality reverses

$\cos^{-1}$ is a decreasing func

QUESTION

Find domain and range of $\cos^{-1}\sqrt{x^2-9}$

$f(x) = \cos^{-1}\sqrt{x^2-9}$

Domain:-

Here

$-1 \le \sqrt{x^2-9} \le 1$

But WKT sqrt func ≥ 0

$0 \le \sqrt{x^2-9} \le 1$

on squaring

$0 \le x^2-9 \le 1$

$9 \le x^2 \le 10$

$\sqrt{9} \le \sqrt{x^2} \le \sqrt{10}$

$3 \le |x| \le \sqrt{10}$

$x \in [-\sqrt{10}, -3] \cup [3, \sqrt{10}]$

Range:- [we need to find f(x)]

Here $0 \le \sqrt{x^2-9} \le 1 \Rightarrow$ +ve input

Apply $\cos^{-1}$

$\cos^{-1} 0 \geq \cos^{-1}\sqrt{x^2-9} \geq \cos^{-1} 1$

$\frac{\pi}{2} \geq f(x) \geq 0$

$$\frac{\pi}{2} \geq f(x) \geq 0$$

$$\Downarrow$$

$$0 \leq f(x) \leq \frac{\pi}{2}$$

Range $= \left[0, \frac{\pi}{2}\right]$

function	Domain	Range	Quadrant in which solu (output) lies	
$\sin^{-1}x$	$[-1,1]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$-$ve input $x\in[-1,0]$ $\Downarrow$ $f(x)\in\left[-\frac{\pi}{2},0\right]$ 4th Quad	+ve input $x\in[0,1]$ $\Downarrow$ $f(x)\in\left[0,\frac{\pi}{2}\right]$ 1st Quad
$\cos^{-1}x$	$[-1,1]$	$[0,\pi]$	$-$ve input $x\in[-1,0]$ $\Downarrow$ $f(x)\in\left[\frac{\pi}{2},\pi\right]$ 2nd Quad	+ve input $x\in[0,1]$ $\Downarrow$ $f(x)\in\left[0,\frac{\pi}{2}\right]$ 1st Quad

For $\sin^{-1}x$

$\downarrow$

output lies in

either

1st or 4th [since range $= [-\frac{\pi}{2}, \frac{\pi}{2}]$]

Quad Quad

$\Downarrow$ $\Downarrow$

if inputs are +ve if inputs are −ve

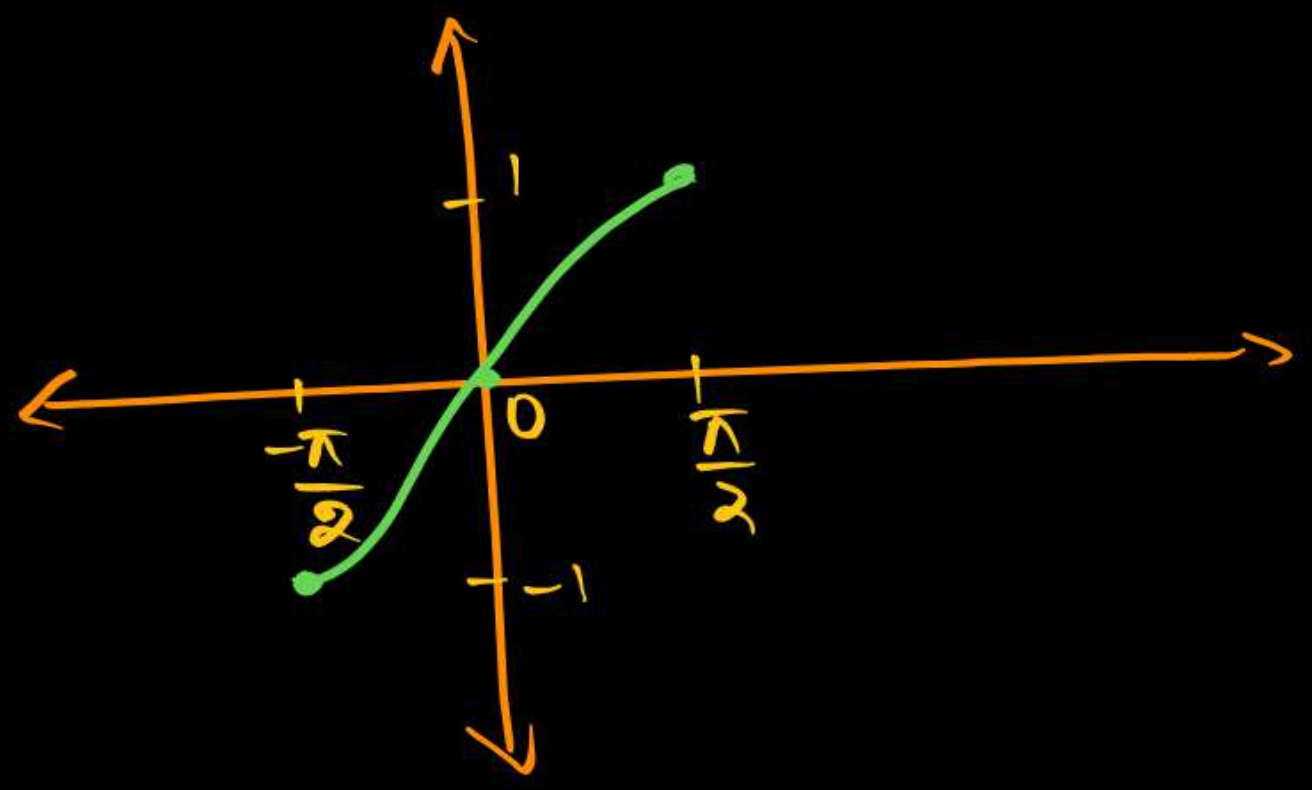
Function	Domain	Range	Quadrant in which soln lies
$\sin^{-1}x$		$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	I Quad or IV Quad
$\tan^{-1}x$		$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$\Downarrow$ +ve input ; $\Downarrow$ −ve input
$\operatorname{cosec}^{-1}x$		$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$	
$\cos^{-1}x$		$[0, \pi]$	I Quad or II Quad
$\cot^{-1}x$		$(0, \pi)$	$\Downarrow$ +ve input ; $\Downarrow$ −ve input
$\sec^{-1}x$		$[0, \pi] - \left\{\frac{\pi}{2}\right\}$	

$$\sin^{-1}\left(\underset{+ve}{\underline{\frac{1}{2}}}\right) = \frac{\pi}{6} \in 1^{st}\text{ Quad}$$

$$\sin^{-1}\left(\underset{-ve}{\underline{-\frac{1}{2}}}\right) = -\sin^{-1}\frac{1}{2} = -\frac{\pi}{6} \in 4^{th}\text{ Quad}$$

$$\cos^{-1}\left(\underset{+ve}{\underline{\frac{1}{2}}}\right) = \frac{\pi}{3} \in 1^{st}\text{ Quad}$$

$$\cos^{-1}\left(\underset{-ve}{\underline{-\frac{1}{2}}}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3} \in 2^{nd}\text{ Quad}$$



$f(x) = \sin x \rightarrow$ one - one & onto

$|x| \le a$

$x \in [-a, a]$

$y = \cos^{-1}|x|$

Domain:-

Here

$-1 \le |x| \le 1$

But mod func ≥ 0

$0 \le |x| \le 1$

$\Downarrow$

$|x| \le 1$

$x \in [-1, 1] \rightarrow$ Domain

$f(x) = \cos^{-1}|x|$

Range:-

Here

$0 \le |x| \le 1$

Apply $\cos^{-1}$

$\cos^{-1}0 \geq \cos^{-1}|x| \geq \cos^{-1}(1)$

$\frac{\pi}{2} \geq f(x) \geq 0$

$0 \le f(x) \le \frac{\pi}{2}$

Range $= \left[0, \frac{\pi}{2}\right]$

QUESTION

Find domain and range of $\sin^{-1}\sqrt{x^2-5}$

+ve func

$f(x)=\sin^{-1}x$ is a increasing func

Soln:-

Domain:-

Here

$-1 \le \sqrt{x^2-5} \le 1$

But sqrt func ≥ 0

$0 \le \sqrt{x^2-5} \le 1$

$0 \le x^2-5 \le 1$

$5 \le x^2 \le 6$

$\sqrt{5} \le |x| \le \sqrt{6}$

$x \in [-\sqrt{6}, -\sqrt{5}] \cup [\sqrt{5}, \sqrt{6}]$

Range:-

Here $0 \le \sqrt{x^2-5} \le 1$

Apply $\sin^{-1}$

$\sin^{-1}(0) \le \sin^{-1}\sqrt{x^2-5} \le \sin^{-1}(1)$

$0 \le f(x) \le \frac{\pi}{2}$

$f(x) \in [0, \frac{\pi}{2}]$

QUESTION

Find domain and range of $\sqrt{\sin^{-1}x}$

$f(x) = \sqrt{g(x)}$

$\Rightarrow g(x) \geq 0$

$f(x) = \sqrt{\sin^{-1}x}$

Soln:-

Here

$\sin^{-1}x \geq 0$

$\Downarrow$

$0 \leq \sin^{-1}x \leq \frac{\pi}{2}$ | Since $\sin^{-1}x\,[-\frac{\pi}{2}, \frac{\pi}{2}]$

$\Downarrow$ Actual range

Apply sin

$\sin 0 \leq x \leq \sin\frac{\pi}{2}$

$0 \leq x \leq 1$

Domain $= [0, 1]$

Range:-

Here

$0 \leq \sin^{-1}x \leq \frac{\pi}{2}$

Take sqrt

$0 \leq \sqrt{\sin^{-1}x} \leq \sqrt{\frac{\pi}{2}}$

$f(x) \in [0, \sqrt{\frac{\pi}{2}}]$

$\downarrow$

Range

QUESTION

Find domain and range of $\sqrt{\cos^{-1}x}$

(A) $[0,1]$ & $[0,\pi]$

(B) $[-1,1]$ & $[0,\pi]$

(C) $[-1,1]$ & $[0,\sqrt{\pi}]$

(D) $[0,1]$ & $[0,\sqrt{\pi}]$

Ans:-

Domain:-

Here

$\cos^{-1}x \geq 0$

$\Downarrow$

$0 \leq \cos^{-1}x \leq \pi$

Take cos

$\cos 0 \geq x \geq \cos\pi$

$1 \geq x \geq -1$

$\Downarrow$

$-1 \leq x \leq 1$

Domain $= [-1,1]$

Range:-

Here

$0 \leq \cos^{-1}x \leq \pi$

Take sqrt

$0 \leq \sqrt{\cos^{-1}x} \leq \sqrt{\pi}$

Range $= [0,\sqrt{\pi}]$

$$\sqrt{\cos^{-1}\left(-\frac{1}{2}\right)}$$

$$= \sqrt{\pi - \cos^{-1}\frac{1}{2}}$$

$$= \sqrt{\pi - \frac{\pi}{3}}$$

$$= \sqrt{\frac{2\pi}{3}}$$

exist

$$\sqrt{\sin^{-1}\left(-\frac{1}{2}\right)}$$

$$\sqrt{-\sin^{-1}\frac{1}{2}}$$

$$\sqrt{-\frac{\pi}{6}}$$

Does not exist

QUESTION

The domain of the function $\cos^{-1}(2x - 1)$ is

$-1 \le 2x - 1 \le 1$

Add 1

$0 \le 2x \le 2$

$\div$ by 2

$0 \le x \le 1$

A [0, 1]

B [–1, 1]

C (–1, 1)

D [0, π]

QUESTION

The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

(A) [1, 2]

(B) [–1, 1]

(C) [0, 1]

(D) None of these

$-1 \le \sqrt{x-1} \le 1$

But sqrt func ≥ 0

$0 \le \sqrt{x-1} \le 1$

$0 \le x-1 \le 1$

$1 \le x \le 2$

QUESTION

Domain of $f(x) = \sin^{-1}(x^2 - 3)$ is

(A) $\left[-\frac{5}{2}, -1\right) \cup \left(1, \frac{5}{2}\right)$

(B) $\left[-\frac{5}{2}, -1\right] \cup \left[1, \frac{5}{2}\right]$

(C) $\left(-\frac{5}{2}, -1\right) \cup \left(1, \frac{5}{2}\right)$

(D) None of these

$-1 \le x^2 - 3 \le 1$

$2 \le x^2 \le 4$

$\sqrt{2} \le \sqrt{x^2} \le \sqrt{4}$

$\sqrt{2} \le |x| \le 2$

$x \in [-2, -\sqrt{2}] \cup [\sqrt{2}, 2]$

QUESTION

The domain of the function $f(x) = \sin^{-1}\left(\frac{x+5}{2}\right)$ is

Range $= \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

A [–1, 1]

B [2, 3]

C [3, 7]

D [–7, –3]

$-1 \leq \frac{x+5}{2} \leq 1$

$-2 \leq x+5 \leq 2$

$-7 \leq x \leq -3$

QUESTION

Domain of $\sin^{-1}[x]$ (where [.] denotes G.I.F.) is

A [–1, 2]

B [–1, 2)

C (–1, 2]

D None of these

$-1 \le [x] \le 1$

$\Downarrow$

integers are $-1, 0, 1$

$[x] = -1, 0, 1$

$[-1, 0)$ $\quad$ $[1, 2)$

$[-1, 2)$

Range :-

$[x] = -1$	$[x] = 0$	$[x] = 1$
$\sin^{-1}[x] = \sin^{-1}(-1) = -\frac{\pi}{2}$	$\sin^{-1}[x] = \sin^{-1}0 = 0$	$\sin^{-1}[x] = \sin^{-1}1 = \frac{\pi}{2}$

$\text{Range} = \left\{-\frac{\pi}{2}, 0, \frac{\pi}{2}\right\}$

QUESTION

Range of $\cos^{-1}[x]$ is

here

$-1 \le [x] \le 1$

$[x] = -1$ $\quad$ $[x] = 0$ $\quad$ $[x] = 1$

$\cos^{-1}[x] = \cos^{-1}(-1)$
$= \pi - \cos^{-1} 1$
$= \pi - 0$
$= \pi$

$\cos^{-1}[x]$
$= \cos^{-1} 0$
$= \frac{\pi}{2}$

$\cos^{-1}[x]$
$= \cos^{-1} 1$
$= 0$

A $\{0, \pi\}$

B $\left\{0, \frac{\pi}{2}, \pi\right\}$

C $\left\{0, \frac{\pi}{2}\right\}$

D $\left\{\pi, \frac{\pi}{2}\right\}$

QUESTION

The domain of the function $f(x) = \frac{\overbrace{\sqrt{4-x^2}}^{g(x)}}{\underbrace{\sin^{-1}(2-x)}_{h(x)}}$ is

(A) $[0, 2]$

(B) $[0, 2)$

(C) $[1, 2)$

(D) $[1, 2]$

$g(x) = \sqrt{4-x^2}$

$4 - x^2 \geq 0$

$4 \geq x^2$

$\Downarrow$

$x^2 \leq 4$

$|x| \leq 2$

$x \in [-2, 2]$

$h(x) = \sin^{-1}(2-x)$

$\Downarrow$

$-1 \leq 2-x \leq 1$

$-3 \leq -x \leq -1$

$3 \geq x \geq 1$

$\Downarrow$

$1 \leq x \leq 3$

but $\sin^{-1}(2-x) \neq 0$

$2 - x \neq 0$

$x \neq 2$

$x \in [1, 3] - \{2\}$

Ans: $x \in [1, 2)$

QUESTION

The domain of $\sin^{-1}\left[\log_2\left(\frac{x}{12}\right)\right]$ is

(A) [2, 12]

(B) [6, 24]

(C) [1/3, 24]

(D) [2/3, 24]

$$-1 \le \log_2\left(\frac{x}{12}\right) \le 1$$

$2 > 1$

$$2^{-1} \le \frac{x}{12} \le 2$$

$$\frac{1}{2}\times 12 \le x \le 2\times 12$$

$$6 \le x \le 24$$

(1) $\log_a f(x) \ge b$

$f(x) \ge a^b$ if $a > 1$

inequality sign remains same

(2) $\log_a f(x) \ge b$

$f(x) \le a^b$ if $a \in (0,1)$

$\Downarrow$

sign of inequality reverses

Domain of

$$\sin^{-1}\left[\log_{1/3}\left(\frac{x}{6}\right)\right]$$

Soln:-

$$-1 \le \log_{1/3}\left(\frac{x}{6}\right) \le 1$$

$$\frac{1}{3} \in (0,1)$$

$\therefore$ Inequality reverses

$$\left(\frac{1}{3}\right)^{-1} \geq \frac{x}{6} \geq \left(\frac{1}{3}\right)^{1}$$

$$\frac{1}{3^{-1}} \geq \frac{x}{6} \geq \frac{1}{3}$$

$$3 \geq \frac{x}{6} \geq \frac{1}{3}$$

$\times^{ly}$ by 6

$$18 \geq x \geq 2$$

$$\Downarrow$$

$$2 \le x \le 18$$

Domain $= [2, 18]$

QUESTION

$[a, b]$

$a<b$

$[18, 2]$ ✗

$[2, 18]$ ✓

The domain of definition of the function $f(x) = \sin^{-1}(|x-1|-2)$ is

$a \le |x| \le b$

$\Downarrow$

$x \in [-b, -a] \cup [a, b]$

$-1 \le |x-1| - 2 \le 1$

$1 \le |x-1| \le 3$

$x-1 \in [-3, -1] \cup [1, 3]$

Add 1

$x \in [-2, 0] \cup [2, 4]$

(A) $[-3, 0] \cup [1, 3]$

(B) $[-2, 0] \cup [1, 4]$

(C) $[-2, 0] \cup [2, 4]$

(D) $[-2, 0] \cup [1, 2]$

Thank You