
ULTIMATE KCET
CRASH COURSE 2026
Mathematics
Lecture – 03
Integrals
By – Guru sir

Recap *of previous lecture*

1. Indefinite Integrals
2.
3.
4.

Topics *to be covered*

1. Properties - Definite Integral
2.
3.
4.

60 marks

1. Stage 1 → 35 to 40 marks
   1. Matrices
   2. Determinants
   3. vectors
   4. 3D

   12 mark

   5. AOI → 2m
   6. Continuity & Diff → 4m

18m

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7. ITF $\rightarrow$ 2m

8. Probability $\rightarrow$ 2m

9. DE $\rightarrow$ order & Degree
   Variable separable
   IF
   LDE
   } min 2m

8m

10. R & F Class 11
    12
    } min 2 marks

(11) AOD → PYQ's
My notes → min 2 marks

(12) Integrals → (1) By Parts (PYQ's)
(2) Particular Func
(3) Definite
4m

(16)

(13) Class 11th → (1) Sets
(2) B.T
(3) C.N
10m

(4) stats
(5) St. lines
(6) Conic

(7) sequence
(8) Inequalities

gurumathswallah

Properties:-

① $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ → split func

Ex:- ① Mod

② G.I.F

② $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx \Rightarrow$ extension $\int_0^a f(x)\,dx$

$= \int_0^a f(a-x)\,dx$

$a = 0$

$b = a$

(3) $$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^{a} f(x)\,dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$

(4) $$\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^{a} f(x)\,dx & \text{if } f(-x) = f(x) \quad f(x) \to \text{even} \\ 0 & \text{if } f(-x) = -f(x) \quad f(x) \to \text{odd} \end{cases}$$

$$f(x) = \begin{cases} \sin x & 0 \le x \le \frac{\pi}{2} \\ 2x & \frac{\pi}{2} < x \le \pi \end{cases}$$

Find $\displaystyle\int_{\pi/4}^{3\pi/2} f(x)\,dx$

$\Downarrow$ split point

$$I = \int_{\frac{\pi}{4}}^{\pi/2} \sin x\,dx + \int_{\pi/2}^{3\pi/2} 2x$$

$$= \left(-\cos x\right)_{\pi/4}^{\pi/2} + \left. x^2 \right)_{\pi/2}^{3\pi/2}$$

$$= -\left[0 - \frac{1}{\sqrt{2}}\right] + \left[\frac{9\pi^2}{4} - \frac{\pi^2}{4}\right]$$

$$I = \frac{1}{\sqrt{2}} + 2\pi^2$$

$$\int_a^b f(x)\,dx$$

$\Downarrow$

Here identifying the split point is VVV Imp

$$\int_2^5 |x-4|\,dx$$

$\Downarrow$

Split point

$x=4$

$$|x-4| = \begin{cases} x-4 & \text{if } x-4 \geq 0 \\ & x \geq 4 \\ -(x-4) & \text{if } x-4<0 \\ =4-x & x<4 \end{cases}$$

$$\int_2^4 4-x\,dx + \int_4^5 x-4\,dx$$

$x<4$ $\quad$ $x>4$

$$= \left.4x-\frac{x^2}{2}\right]_2^4 + \left[\frac{x^2}{2}-4x\right]_4^5$$

$$I = 4(4-2)-\frac{1}{2}(16-4) + \left[\frac{1}{2}(25-16)-4(5-4)\right]$$

$$= 8-\frac{1}{2}(12)+\frac{9}{2}-4$$

$$= 4-\frac{3}{2} = \frac{5}{2}$$

QUESTION

#Q. The value of $\int_0^2 |1-x|dx$ is

A) 2

B) 3/2

C) 0

D) 1

$$|1-x| = \begin{cases} 1-x & 1-x \geq 0,\ x \leq 1 \\ -(1-x) = x-1 & 1-x < 0,\ x > 1 \end{cases}$$

$x = 1 \rightarrow$ split point

$$I = \int_0^1 1-x\,dx + \int_1^2 x-1\,dx$$

$x < 1$ $\quad$ $x > 1$

$$= x - \frac{x^2}{2}\Big]_0^1 + \frac{x^2}{2} - x\Big]_1^2$$

$$= 1 - \frac{1}{2} + \frac{1}{2}(4-1) - 1 = -\frac{1}{2} + \frac{3}{2} = 1$$

$$I = \int_{-3}^{4} |x+2| + |3-x| \, dx$$

$$|3-x| = \begin{cases} 3-x & x \le 3 \\ -(3-x) = x-3 & x > 3 \end{cases}$$

Split points $\Rightarrow$ $x = -2$
$x = 3$

$$I = \int_{-3}^{-2} -(x+2) + 3 - x \, dx + \int_{-2}^{3} x+2+3-x \, dx + \int_{3}^{4} x+2-3+x \, dx$$

$x < -2$ & $x < 3$ $\quad$ $x > -2$, $x < 3$ $\quad$ $x > -2$, $x > 3$

$$= \int_{-3}^{-2} -2x + 1 \, dx + \int_{-2}^{3} 5 \, dx + \int_{3}^{4} 2x - 1 \, dx$$

$$= -x^2 + x \Big]_{-3}^{-2} + 5(x)_{-2}^{3} + (x^2 - x)_{3}^{4}$$

$$I = -(4-9) + (-2+3) + 25 + (7-1)$$

$$= 5 + 1 + 25 + 6$$

$$= \underline{37}$$

$$I = \int_2^8 |x-4| - |6-x|\, dx$$

$$|6-x| = \begin{cases} 6-x & x \le 6 \\ -(6-x) = x-6 & x > 6 \end{cases}$$

$$= \int_2^4 (4-\cancel{x}) - (6-\cancel{x})\, dx + \int_4^6 (x-4)-(6-x)\, dx + \int_6^8 (\cancel{x}-4) - (\cancel{x}-6)\, dx$$

$x < 6$

$$= \int_2^4 -2\, dx + \int_4^6 2x - 10\, dx + \int_6^8 2\, dx$$

$$= -2(x)_2^4 + (x^2 - 10x)_4^6 + 2(x)_6^8$$

$$= -2(2) + (6^2 - 4^2) - 10(6-4) + 2(2)$$

$$= -4 + 20 - 20 + 4 = 0$$

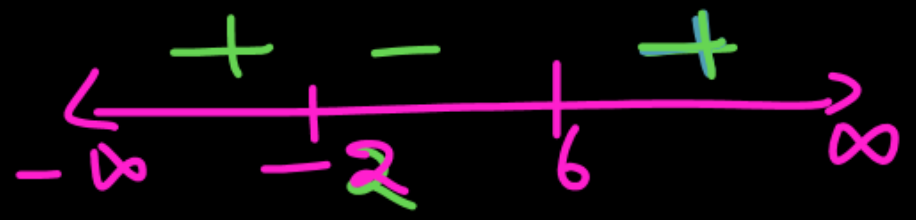
$$|x-4| = \begin{cases} x-4 & \text{if } x-4 \geq 0 \\ & \quad x \geq 4 \\ -(x-4) & \text{if } x-4 < 0 \\ = 4-x & \quad x < 4 \end{cases}$$

$$|4-x| = \begin{cases} 4-x & \text{if } 4-x \geq 0 \\ & \quad x \leq 4 \\ -(4-x) & \text{if } 4-x < 0 \\ = x-4 & \quad x > 4 \end{cases}$$

$$|x^2-4x-12| = |(x-6)(x+2)| = \begin{cases} x^2-4x-12 & x \in (-\infty,-2) \cup (6,\infty) \\ -(x^2-4x-12) \\ = 12+4x-x^2 & x \in (-2,6) \end{cases}$$

-12

$-6 \quad +2$



$$I = \int_{-6}^{4} |x^2-4x-12|\,dx = \int_{-6}^{4} |(x-6)(x+2)|\,dx$$

$$= \int_{-6}^{-2} x^2-4x-12\,dx + \int_{-2}^{4} 12+4x-x^2\,dx$$

$$= \left.\frac{x^3}{3}-2x^2-12x\right]_{-6}^{-2} + \left[12x+2x^2-\frac{x^3}{3}\right]_{-2}^{4}$$

$$= \frac{1}{3}(-8+216)-2(4-36)-12(-2+6)+12(4+2)+2(16-4)-\frac{1}{3}(64+8)$$

$$= \frac{208}{3}-\frac{72}{3}+64-48+72+24$$

$$= \frac{136}{3}+112 = \frac{136+336}{3} = \frac{472}{3}$$

QUESTION

$+ \quad - \quad +$

$-\infty \quad 0 \quad 1 \quad \infty$

#Q. The value of $\int_{-2}^{2} |x(x-1)|\,dx$ is

(A) $\frac{11}{3}$

(B) $\frac{13}{3}$

(C) $\frac{16}{3}$

(D) $\frac{17}{3}$.

$$\int_{-2}^{0} x^2 - x\,dx + \int_{0}^{1} x - x^2\,dx + \int_{1}^{2} x^2 - x\,dx$$

$$\left.\frac{x^3}{3} - \frac{x^2}{2}\right]_{-2}^{0} + \left.\frac{x^2}{2} - \frac{x^3}{3}\right]_{0}^{1} + \left[\frac{x^3}{3} - \frac{x^2}{2}\right]_{1}^{2}$$

$$= \frac{8}{3} - \frac{1}{2}(0-4) + \frac{1}{2} - \frac{1}{3} + \frac{1}{3}(7) - \frac{1}{2}(3)$$

$$= \frac{8}{3} + 2 + \frac{1}{2} - \frac{1}{3} + \frac{7}{3} - \frac{3}{2} = \frac{15-1}{3} + 1 = \frac{14}{3} + 1 = \frac{17}{3}$$

If $y = |x(x-1)|$ Find $f'(\frac{1}{2})$, $f'(\frac{1}{3})$

Soln:

At $x = \frac{1}{2}$

$$y = -[x(x-1)]$$

$$= -[x^2 - x]$$

$$y = x - x^2$$

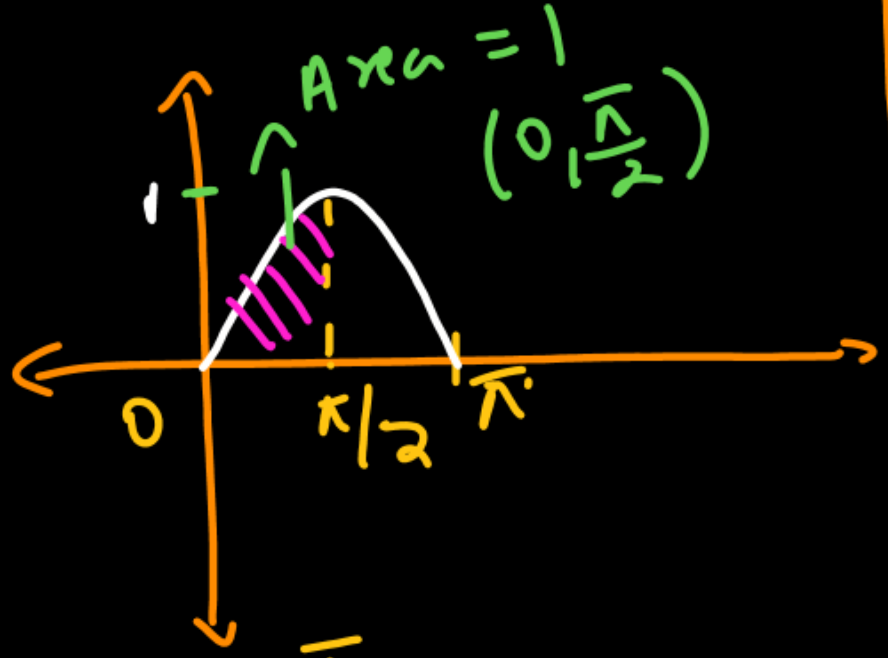
$$\frac{dy}{dx} = 1 - 2x = 1 - 2\left(\frac{1}{2}\right) = 1 - 1 = \underline{0}$$

$$\frac{dy}{dx} = 1 - 2x$$

At $x = 1/3$

$$y_1 = 1 - \frac{2}{3} = \frac{1}{3}.$$

$$|\sin x| = \begin{cases} \sin x & \text{if } x \in 1^{st} \,\&\, 2^{nd} \text{ Quad} \\ -\sin x & \text{if } x \in 3^{rd} \,\&\, 4^{th} \text{ Quad} \end{cases}$$

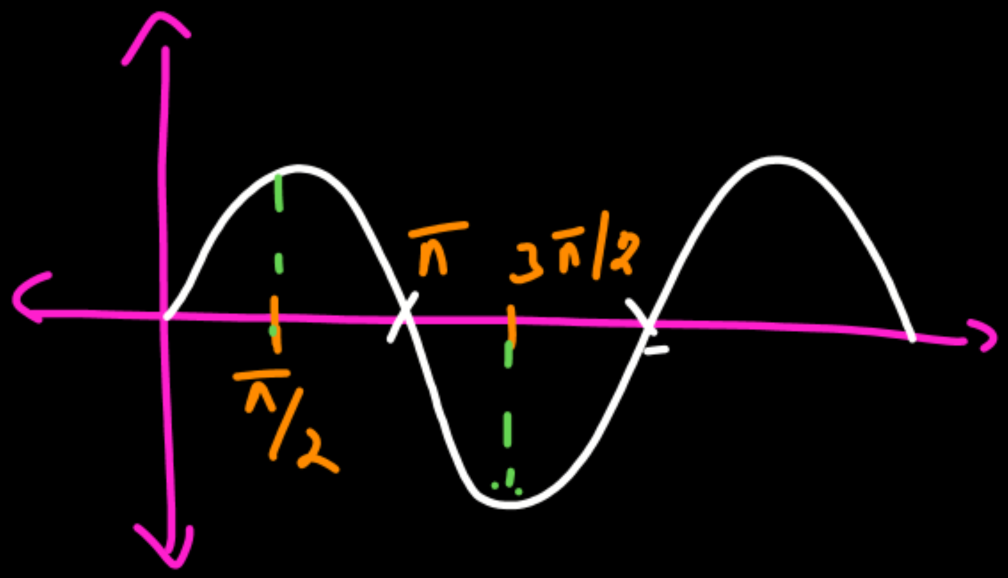


Area = 2
$x \in (0, \pi)$

$$\int_0^{\pi/2} \sin x \, dx = 1$$

$$\int_0^{\pi} \sin x \, dx = -(\cos x)_0^{\pi} = -(-1-1) = 2$$

$$\int_0^{3\pi/2}|\sin x|\,dx = 3\int_0^{\pi/2}|\sin x|\,dx = 3(-\cos x)_0^{\pi/2} = -3(0-1) = \underline{3}$$



$$\int_0^{50\pi}|\sin x|\,dx = 50\int_0^{\pi}|\sin x|\,dx$$
$$= 50(2) = 100$$

$$\int_0^{2\pi}|\sin x|\,dx = 2\underbrace{\int_0^{\pi}|\sin x|\,dx}_{} = 2(2) = 4$$

QUESTION

#Q. The value of $\int_{-\pi}^{\pi} |\sin x| dx$ is

A) 2

B) 0

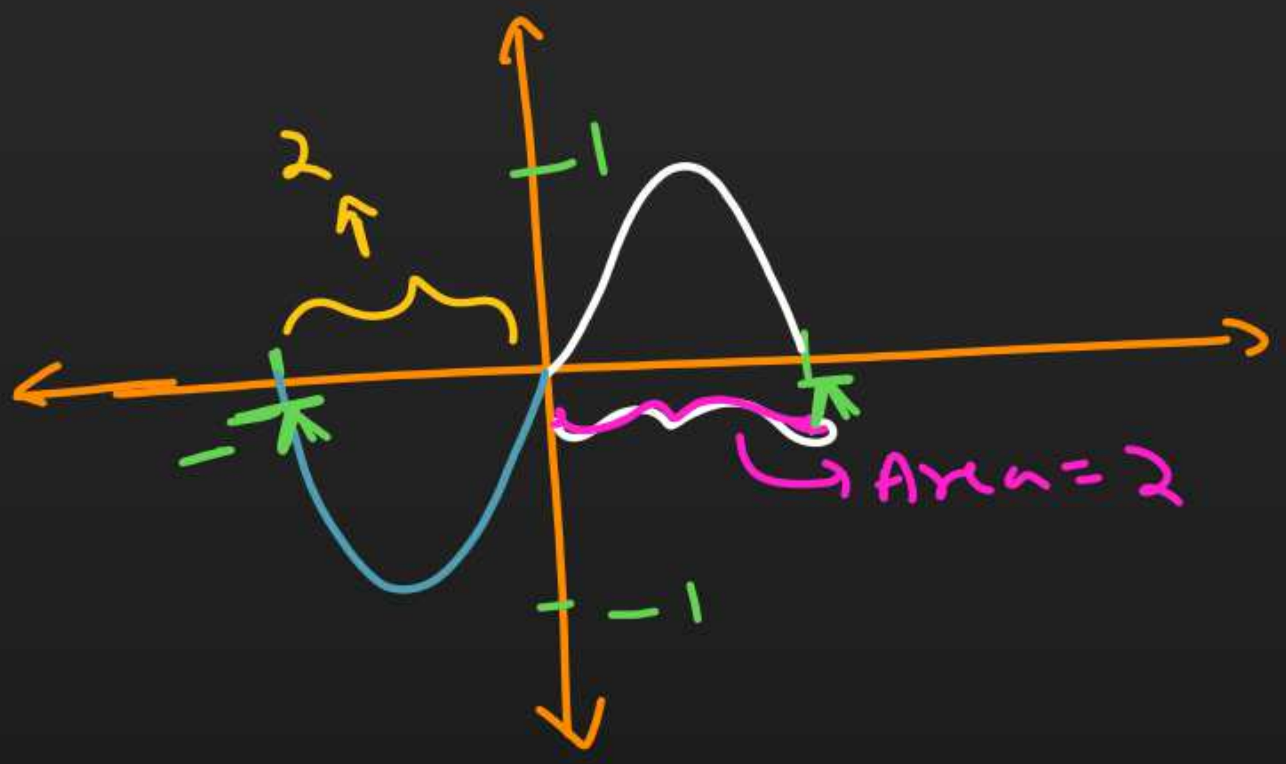
C) 4

D) 1

$2\int_0^{\pi} |\sin x| dx$

$= 2(2)$

$= 4$



$x \in (-\pi, \pi)$

Find $\int_0^{10} [x]\,dx$

$$= \int_0^1 0\,dx + \int_1^2 1\,dx + \int_2^3 2\,dx + \cdots + \int_9^{10} 9\,dx$$

$$= 0 + (x)_1^2 + 2(x)_2^3 + 3(x)_3^4 + \cdots + 9(x)_9^{10}$$

$$= 1 + 2 + 3 + \cdots + 9$$

$$= \frac{9(9+1)}{2} = 9(5) = 45$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$I = \int_{-5}^{6} [x]\,dx$$

$$I = \int_{-5}^{-4} -5\,dx + \int_{-4}^{-3} -4\,dx + \dots + \int_{0}^{1} 0\,dx + \int_{1}^{2} 1\,dx + \dots + \int_{5}^{6} 5\,dx$$

$$= -5(x)_{-5}^{-4} + (-4)(x)_{-4}^{-3} + \dots + (-1)(x)_{-1}^{0}$$

$$+ 0 + (x)_{1}^{2} + 2(x)_{2}^{3} + \dots + 5(x)_{5}^{6}$$

$$= -5(1) - 4 - 3 - 2 - 1 + 0 + 1 + 2 + 3 + 4 + 5$$

$$= \underline{0}$$

$x \in (-2, -1)$

$\Downarrow$

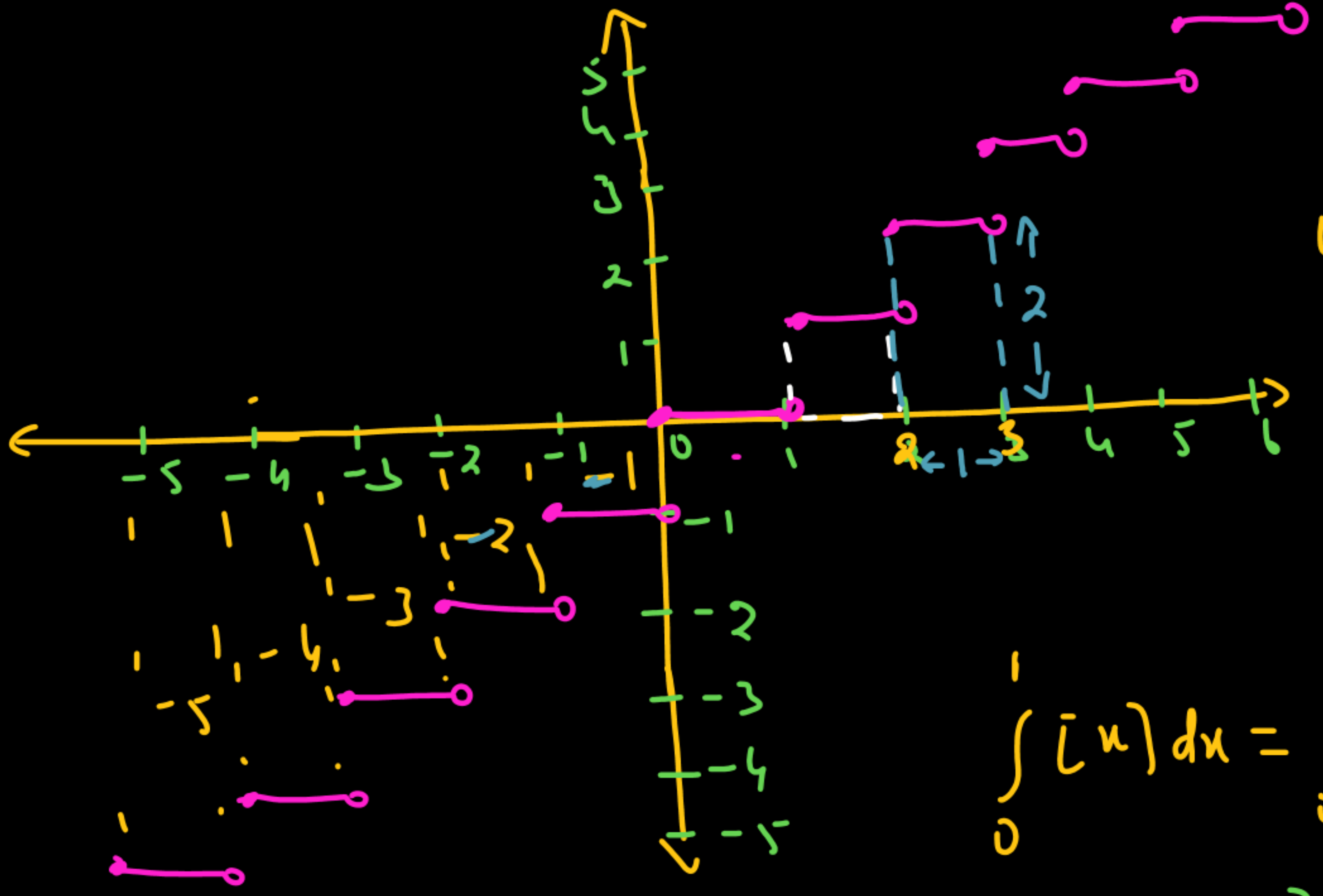
$[x] = -2$

$x \in (-5, -2)$

$x \in (-5, -4)$ $\Downarrow$ $[x] = -5$

$x \in (-4, -3)$ $\Downarrow$ $[x] = -4$

$x \in (-3, -2)$ $\Downarrow$ $[x] = -3$



$1+2+3+4+6$

$$\int_2^3 [x]\,dx = \int_2^3 2\,dx$$
$$= 2(x)_2^3$$
$$= \underline{2}$$

$$\int_0^1 [x]\,dx = \int_0^1 0\,dx = 0$$

$$\int_1^2 [x]\,dx = \int_1^2 1\,dx = x\Big)_1^2 = 1$$

QUESTION

#Q. $\int_{-1}^{1}[x]dx$, where $[x]$ is the greatest integer function not greater than x is

$\int_{-1}^{0} -1\,dx + \int_{0}^{1} 0\,dx$

$-1(x)_{-1}^{0} + 0$

$= -1(0+1)$

$x \in (-1,1)$

$x \in (-1,0) \Downarrow [x] = -1$

$x \in (0,1) \Downarrow [x] = 0$

A) 0

B) 1

C) –1

D) 2

QUESTION

#Q. The value of $\int_0^{\frac{\pi}{2}} |\text{Sin}x - \cos x| dx$ *is*

(A) 0

(B) $2(\sqrt{2}-1)$

(C) $2\sqrt{2}$

(D) $2(\sqrt{2}+1)$

$= \int_0^{\pi/4} \cos x - \sin x \, dx + \int_{\pi/4}^{\pi/2} \sin x - \cos x \, dx$

$= \sin x + \cos x \Big]_0^{\pi/4} + \Big[-\sin x - \cos x\Big]_{\pi/4}^{\pi/2}$

$= \left(\frac{1}{\sqrt{2}} - 0\right) + \left(\frac{1}{\sqrt{2}} - 1\right) - \left[\left(1 - \frac{1}{\sqrt{2}}\right) + \left(0 - \frac{1}{\sqrt{2}}\right)\right]$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}} - 2 = 2\sqrt{2} - 2$

$= \underline{2(\sqrt{2}-1)}$

$x \in \left(0, \frac{\pi}{4}\right)$

$\Downarrow$

$\cos x > \sin x$

$\Downarrow$

$\cos x - \sin x > 0$

① $|\cos x - \sin x|$

$= \cos x - \sin x$

② $|\sin x - \cos x|$

$= \cos x - \sin x$

$x\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$\Downarrow$

$\cos x < \sin x$

$\Downarrow$

$\cos x - \sin x < 0$

① $|\cos x - \sin x|$

$= \sin x - \cos x$

② $|\sin x - \cos x|$

$= \sin x - \cos x$

$x \in \left(\frac{5\pi}{4}, 2\pi\right)$

$\Downarrow$

$\cos x > \sin x$

$x \in (0, 2)$

$\Downarrow$

$\sqrt{1}, \sqrt{2}, \sqrt{3} \in (0, 2)$

$1 < x < \sqrt{2}$

on squaring

$1 < x^2 < 2$

$x \in (0,1)$	$x \in (1,\sqrt{2})$	$x \in (\sqrt{2},\sqrt{3})$	$x \in (\sqrt{3},\sqrt{4})$
$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
$x^2 \in (0,1)$	$x^2 \in (1,2)$	$x^2 \in (2,3)$	$x^2 \in (3,4)$
$\Downarrow$	$\Downarrow$	$\Downarrow$	$\Downarrow$
$[x^2] = 0$	$[x^2] = 1$	$[x^2] = 2$	$[x^2] = 3$

PYQ

$$\int_0^2 [x^2]\,dx = \int_0^1 [x^2]\,dx + \int_{x=1}^{\sqrt{2}} [x^2]\,dx + \int_{x=\sqrt{2}}^{\sqrt{3}} [x^2]\,dx + \int_{\sqrt{3}}^{2} [x^2]\,dx$$

$$= \int_0^1 0\,dx + \int_1^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{\sqrt{3}} 2\,dx + \int_{\sqrt{3}}^{2} 3\,dx$$

$$= 0 + (x)_1^{\sqrt{2}} + 2(x)_{\sqrt{2}}^{\sqrt{3}} + 3(x)_{\sqrt{3}}^{2}$$

$$= \sqrt{2} - 1 + 2(\sqrt{3}-\sqrt{2}) + 3(2-\sqrt{3})$$

$$= 5 - \sqrt{2} - \sqrt{3}$$

$$I = \int_0^{3/2} [x^2]\,dx$$

b/w $(0, 3/2)$

$\Downarrow$

$\sqrt{1}, \sqrt{2}$ exists

$$= \int_0^{\sqrt{1}} [x^2]\,dx + \int_{\sqrt{1}}^{\sqrt{2}} [x^2]\,dx + \int_{\sqrt{2}}^{3/2} [x^2]\,dx$$

$$= \int_0^{\sqrt{1}} 0\,dx + \int_{\sqrt{1}}^{\sqrt{2}} 1\,dx + \int_{\sqrt{2}}^{3/2} 2\,dx$$

$$= 0 + (x)_1^{\sqrt{2}} + 2(x)_{\sqrt{2}}^{3/2}$$

$$= \sqrt{2} - 1 + 2\left(\frac{3}{2} - \sqrt{2}\right) = \sqrt{2} - 1 + 3 - 2\sqrt{2} = \underline{2 - \sqrt{2}}$$

QUESTION

#Q. The value of the integral $\int_0^{3/2}[x^2]dx$ [ ] denotes the greatest integer function, is

(A) $2+\sqrt{2}$

(B) $2-\sqrt{2}$

(C) $4+2\sqrt{2}$

(D) $4-2\sqrt{2}$

QUESTION

#Q. The value of $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} |\sin x|\,dx$ is

(A) 1

(B) 2

(C) $\sqrt{2}$

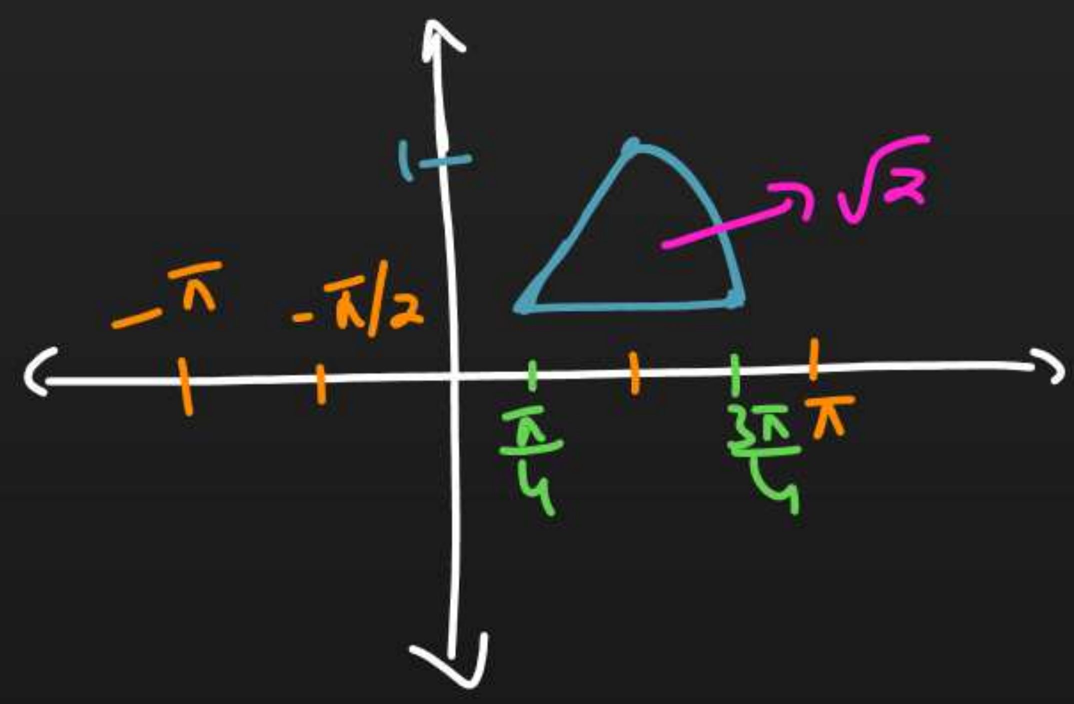
(D) $\sqrt{3}$

$\cos\frac{3\pi}{4} = \cos\left(\pi - \frac{\pi}{4}\right) = -\cos\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$

$x \in \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

$\Downarrow$

1st & 2nd Quad



$$I = \int_{\pi/4}^{3\pi/4} \sin x\, dx = -\left(\cos x\right)_{\pi/4}^{3\pi/4}$$

$$= -\left[-\cos\frac{\pi}{4} - \cos\frac{\pi}{4}\right]$$

$$= -\left[-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right] = +\frac{2}{\sqrt{2}} = \sqrt{2}$$

QUESTION

$$|1-x| = \begin{cases} 1-x & x \le 1 \\ x-1 & x > 1 \end{cases}$$

#Q. The value of $\int_0^2 |1-x|dx$ is

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

$$\int_0^1 1-x\,dx + \int_1^2 x-1\,dx$$

$$= x - \frac{x^2}{2}\Big]_0^1 + \frac{x^2}{2} - x\Big]_1^2$$

$$= \cancel{1} - \frac{1}{2} + \frac{1}{2}(3) - \cancel{1}$$

$$= \frac{-1}{2} + \frac{3}{2} = 1$$

if $x \in (1,2)$ $\quad [x] = 1$

$[1.5] = 1$

$[1.2] = 1$

$[1.3] = 1$

$x \in (1,3)$

$x \in (1,2) \Rightarrow [x] = 1$

$x \in (2,3) \Rightarrow [x] = 2$

$$\int_1^3 [x]\,dx = \int_1^2 [x]\,dx + \int_2^3 [x]\,dx$$

$$= \int_1^2 1\,dx + \int_2^3 2\,dx = x\Big)_1^2 + 2(x)_2^3$$

$$= (2-1) + 2(3-2)$$

$$= 1 + 2 = \underline{3}$$

QUESTION

#Q. The value of $\int_0^1 (|x| + |x-1|)dx$ is

A 0

B 1

C 2

D –2

$|x-1| = \begin{cases} x-1 & x-1 \geq 0 \\ & x \geq 1 \\ -(x-1) = 1-x & x-1 < 0 \\ & x < 1 \end{cases}$

$x > 0 \mid x < 1$

$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

$\int_0^1 x + [-(x-1)]\,dx = \int_0^1 x - x + 1\,dx$

$= \int_0^1 x - x + 1\,dx$

$= \int_0^1 1\,dx = (x)_0^1 = 1$

Thank

You